

Structural change patterns and development in open economies*

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Abstract

The share of manufacturing in output follows an inverted U shape over the course of development. However, both the timing and the magnitude of structural change differ substantially across countries. I show a simple open economy model of structural change can explain why countries with lower aggregate productivity industrialize more slowly and start de-industrialization at a lower share of manufacturing in output.

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Keywords: structural change; international trade; productivity differences

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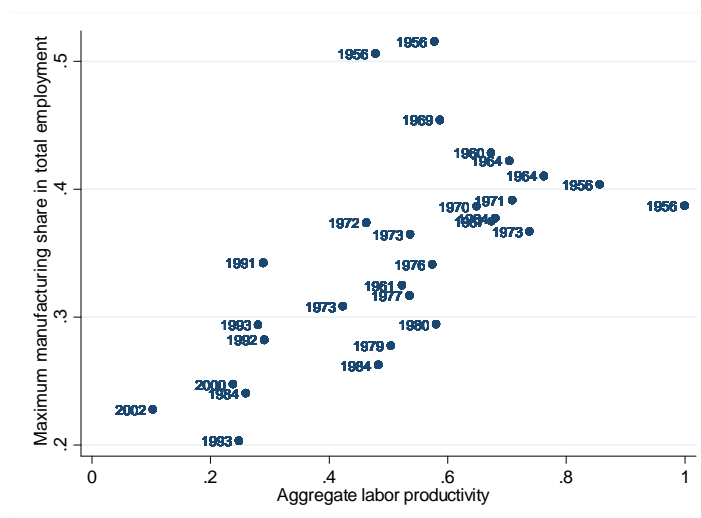


Figure 1: Maximum manufacturing share in employment (%) as a function of aggregate labor productivity (relative to the US) in the year the peak was attained. Source: author’s calculations, Duarte and Restuccia (2010).

1 Introduction

As economies develop, the sectoral composition of their employment and output follows a well documented pattern: the agriculture share declines, the share of the service sector increases and the manufacturing share follows an inverted U (Kuznets (1966)). However, despite structural change being qualitatively similar across countries, its timing and amplitude vary systematically with the income level.

Figure 1 plots the maximum share of manufacturing employment and the year this was attained versus an index of aggregate labor productivity, measured relative to the United States. A striking pattern emerges: low aggregate productivity countries industrialize relatively late and the peak manufacturing share in output is lower than in high productivity countries. The correlation is large (.65) and strongly significant. While developed countries have reached a peak manufacturing share of up to 50%, in countries that have completed this stage of structural change later, the peak manufacturing shares are as low as 20%. Recently, Rodrik (2015) has also documented a positive and significant correlation between the peak manufacturing employment share and the income at which this peak is reached. He also finds that low income countries industrialize relatively late and the peak manufacturing share in output is lower than in high productivity countries.

In the following, I study whether trade openness can explain this pattern of (de)industrialization across income levels. In the simple framework introduced below, sectoral reallocations (first away from agriculture, then away from manufacturing and

into services) occur even in autarky, due to non-homothetic preferences and sectoral differences in productivity. However, international trade can slow down or precipitate these shifts depending on the country's productivity level, which also defines its comparative advantage. When cross-country productivity gaps differ across sectors, I show that under trade less advanced countries industrialize more slowly and start de-industrialization at a lower share of manufacturing in output.

Recent research has shed some light on the role of openness in shaping structural change as well as the patterns of trade. Matsuyama (2009) shows that an open economy model of manufacturing decline yields different time-series and cross-country predictions when manufacturing and services display either income elasticity or productivity growth differentials. In this paper I study the peak manufacturing share in output across income levels and show that international trade interacts with both sectoral productivity and income elasticity differences to generate structural change patterns similar to those found in the data. Rodrik (2015) focuses on two particular cases, a large economy with an exogenous trade balance and a small open economy, meant to capture the experiences of developed and developing countries, respectively. In contrast, here I endogenize international trade to study the country-specific (de)industrialization paths arising in general equilibrium. Moreover, the empirical analysis shows comparative advantage to be a direct and significant correlate of structural change in a sample of developed and developing economies over the period 1950 – 2004.

2 The model

This section proposes a tractable model of Ricardian trade and structural change, following Matsuyama (2009). The world consists of two economies $i = 1, 2$. Time is discrete and infinite $t = 1, 2, \dots$. There are four goods, non-tradable agriculture (a), manufacturing (m), services (s) as well as a numeraire good (o), which can be thought of as tradable agricultural products or natural resources. Each economy is endowed with one unit of labor that can be allocated freely across the first three sectors. There is no production of the numeraire - each economy is endowed with y_{it} units. The a and s goods are not tradable, while manufacturing and the numeraire are costlessly tradable. Production sectors display linear technologies with country (i), sector (k) and time (t) specific labor productivities:

$$Y_{it}^k = A_{it}^k l_{it}^k, \quad k \in \{a, m, s\}. \quad (1)$$

Henceforth, country and time subscripts are used only as needed. Let the international price of manufactured goods be p^m and the country specific prices of agriculture and services be p_i^a and p_i^s respectively. Wage equalization across sectors implies:

$$w_i = p^m A_i^m = p_i^a A_i^a = p_i^s A_i^s, \quad i = 1, 2.$$

The representative consumer in each country has preferences over the four goods:

$$U(a_i, m_i, s_i, o_i) = \log(a_i - \bar{a}) + \log m_i + \log(s_i + \bar{s}) + \log o_i, \quad (2)$$

where \bar{a} and \bar{s} are common parameters across countries. The parameter \bar{a} , interpreted as a minimum consumption threshold is standard in the structural change literature (see Matsuyama (2009), Buera and Kaboski (2009)) while \bar{s} can be interpreted as household production available even when no services are purchased in the market (see Duarte and Restuccia (2010), Rogerson (2008)). Non-homotheticity in a and s together with sectoral productivity differences offer a transparent way to generate structural change even in a closed economy. International trade in manufacturing generates structural change processes that are interdependent across countries. The consumer maximizes utility subject to the budget constraint:

$$a_i p_i^a + m_i p^m + s_i p_i^s + o_i = w_i + y_i. \quad (3)$$

Each period, product and labor market clearing conditions are respectively:

$$\sum_{i=1}^2 m_i = \sum_{i=1}^2 Y_i^m, \quad \sum_{i=1}^2 o_i = \sum_{i=1}^2 y_i, \quad a_i = Y_i^a, \quad s_i = Y_i^s \quad \text{and} \quad \sum_{k \in \{a, m, s\}} l_i^k = 1, \quad i = 1, 2. \quad (4)$$

Solving the consumer's problem yields allocations:

$$\begin{aligned} a_i &= \frac{w_i + y_i + p_i^s \bar{s} + 3p_i^a \bar{a}}{4p_i^a}; & m_i &= \frac{w_i + y_i + p_i^s \bar{s} - p_i^a \bar{a}}{4p^m}; \\ s_i &= \frac{w_i + y_i - 3p_i^s \bar{s} - p_i^a \bar{a}}{4p_i^s}; & o_i &= \frac{w_i + y_i + p_i^s \bar{s} - p_i^a \bar{a}}{4}. \end{aligned} \quad (5)$$

Combining wage equalization conditions yields relative prices:

$$p_{it}^a = p_t^m A_{it}^m / A_{it}^a \quad \text{and} \quad p_{it}^s = p_t^m A_{it}^m / A_{it}^s. \quad (6)$$

Substituting (5) and (6) in the market clearing conditions (4) yields the world price of manufactured goods:

$$p_{it}^m = 3(y_{1t} + y_{2t}) / (A_{1t}^m B_{1t} + A_{2t}^m B_{2t}), \quad (7)$$

where $B_{it} = 1 + \bar{s}/A_{it}^s - \bar{a}/A_{it}^a$.

Assumption 1. $\bar{a} < (A_t^a/A_t^s) \bar{s}, \forall t > 0$.

This is a sufficient condition to ensure positive manufacturing prices.¹

¹In a multi-country economy, it is sufficient that Assumption 1 holds for the poorest country

The aggregate output in country i is:

$$Y_{it} = p_{it}^a Y_{it}^a + p_t^m Y_{it}^m + p_{it}^s Y_{it}^s + y_{it} = p_t^m A_{it}^m + y_{it}. \quad (8)$$

where the last equality obtains after substituting the technologies (1), relative prices (6) and using the labor market clearing condition. Before turning to the role of international trade, I first explain how structural change arises in a closed economy.

3 Structural change in a closed economy

Dropping country subscripts, let sectoral technologies grow at constant, sector specific growth rates: $A_t^a = A_0^a e^{g_a t}$, $A_t^m = A_0^m e^{g_m t}$ and $A_t^s = A_0^s e^{g_s t}$ where $g_k > 0$ and $A_{i0}^k > 0$ are some initial values for $k \in \{a, m, s\}$. Also let the numeraire endowment increase over time: $y_t = y_0 e^{g_y t}$.

The price of the manufactured good (7) becomes $p_t^m = 3y_t / (A_t^m B_t)$ and the manufacturing share in output is:

$$\nu_t^m = \frac{p_t^m m_t}{p_t^m A_t^m + y_t} = \frac{1}{1 + 3B_t^{-1}} = \frac{1}{1 + 3(1 + (1/A_t^s)(\bar{s} - \bar{a}A_t^s/A_t^a))^{-1}}. \quad (9)$$

Examining (9) reveals the mechanism at work in this model of structural change. The substitution of labor away from agriculture increases with the subsistence threshold \bar{a} and decreases with the productivity of agriculture. This effect pushes up the manufacturing labor and output share. Second, the home production buffer in services \bar{s} induces a larger income elasticity for services than other goods. This lowers ν_t^m , ceteris paribus. The overall effect depends on the relative productivities. Empirical studies document that productivity growth in services lags that in manufacturing which lags that in agriculture.² Thus, the ratio A_t^s/A_t^a goes to zero asymptotically and ν_t^m goes up with A_t^s . However, given \bar{a} is large enough, at least in a first transition phase, the effect from agriculture dominates and ν_t^m increases. As income increases, the agriculture transition tapers off and the second transition - from manufacturing to services - takes off. Thus, different sectoral productivity and income elasticities generate the inverted U shape of the manufacturing output share.

Assumption 2. $g_a \geq g_m > g_s$.

As previously discussed, this ranking of productivity growth rates across sectors is consistent with the observed patterns of structural change. The following proposition

(lowest A_{at}).

²For example, average sectoral productivity growth rates in the sample used in Duarte and Restuccia (2010), covering 29 countries during the period from 1956 to 2004 are 3.97% in agriculture, 3.03% in manufacturing and 1.27% in services.

summarizes the industrial dynamics in a closed economy.

Proposition 1. *Structural change under autarky implies: i) a steady decline in the output share of agriculture: $\partial \nu_t^a / \partial t < 0, \forall t > 0$; ii) a rise in the service share: $\partial \nu_t^s / \partial t > 0, \forall t > 0$; iii) an inverted U shape manufacturing share, peaking at $t^* = \log(\bar{a}g_a A_0^s / \bar{s}g_s A_0^a) / (g_a - g_s)$.*

Proof. See Appendix. □

Next, I analyze how international trade affects the process of structural change when countries with different aggregate and sectoral productivity levels engage in international trade.

4 Structural change and international trade

Let $i = 1$ denote the advanced economy and $i = 2$ the developing economy. Sectoral relative productivities in country 2 are $\lambda_k = A_2^k / A_1^k, 0 < \lambda_k \leq 1$, for $k \in \{a, m, s\}$. Initial levels are normalized to unity in country 1 without loss of generality. Duarte and Restuccia (2010) document that across sectors, agriculture displays the largest gaps between developed and developing countries. To focus on this pattern in relative productivities, I assume $\lambda_a < \lambda_m \leq \lambda_s \leq 1$. Under international trade, the manufacturing price level becomes:

$$p_t^{mo} = 3(y_{1t} + y_{2t}) / (A_{1t}^m B_{1t} + A_{2t}^m B_{2t}), \quad (10)$$

where $B_{it} = 1 + \bar{s} / A_{it}^s - \bar{a} / A_{it}^a$. The price now reflects the world purchasing power $y_{1t} + y_{2t}$ as well as a linear combination of manufacturing technologies in the two countries, weighted by their respective (autarky) structural change biases, B_{it} . With homothetic preferences ($B_{1t} = B_{2t} = 1$) the international price level depends only on endowments and manufacturing productivity levels. Otherwise, p_t^{mo} depends on international differences in productivity *across all sectors*, and thus on the industrial structure of each economy. In equilibrium, this produces additional effects on sectoral employment beyond the direct effects stemming from different income elasticities and productivity growth rates. Focusing on the manufacturing output share $\nu_{it}^{mo} = p_t^{mo} m_{it} / (p_t^{mo} A_{it}^m + y_{it})$, the main result of the paper can be stated.

Proposition 2. *The peak manufacturing share in output under international trade is lower (higher) in developing (developed) economies relative to autarky if: $\lambda_y > \theta \lambda_m$ for $\theta > 1$.*

Proof. See Appendix. □

Intuitively, the condition in Proposition 2 requires the developing country to have a strong comparative advantage in the good y . Conversely, the advanced economy is a net exporter of manufacturing. The proposition states that international trade, interacting with the domestic mechanisms of structural change, distorts the structure of production in different directions, depending on the level of aggregate productivity.

Given the pattern of trade described above, the relative price of manufacturing is lower in the developing country compared to autarky. This implies lower relative wages in the manufacturing sector and places the economy on a transition path characterized by a faster relocation into services at the same time as it slows down the exit from agriculture. Opposite trends arise in the advanced economy.

Note that using the expression for aggregate output (8), the condition above can be reinterpreted, given y_{it} and a world manufacturing price as relating to an aggregate productivity threshold, below which openness slows down the transition into manufacturing.

Next, I look at how international trade affects the pace of structural change.

Proposition 3. *Under international trade, the manufacturing share in output peaks later (earlier) in developing (developed) economies relative to autarky.*

Proof. See Appendix. □

The result is driven by the larger cross-country gap in agriculture productivity relative to the gap in services. Note that here international trade acts a source of divergence both for the industrial structure of nations and for their aggregate productivity and income levels. Relative to autarky, both the price of manufacturing and, by equation (8), the output are higher (lower) in the developed (developing) economy. On the one hand, the lower income in the developing country is the direct result of trade, which leads to lower manufacturing wages. On the other hand, in general equilibrium, higher income elasticity in services together with lower income imply manufacturing workers move mainly into agriculture, which is the relatively backward sector. This decreases income further, slowing down the structural change process.

5 Empirical analysis

So far I have shown theoretically that international trade introduces a wedge between the paths of the manufacturing shares in output and employment across countries depending on their comparative advantage in manufacturing vs agriculture or natural resources.

In the following I assess the effect of comparative advantage on observed structural change patterns in a panel of 29 developed and developing economies over the period

1950 – 2004. The data, assembled in Duarte and Restuccia (2010) contains the employment (hours) shares of agriculture, manufacturing and services as well as indices of sectoral and aggregate productivity, allowing for the computation of the value added share of each sector. Population and per capita output data are obtained from the WDI database.

I use FAO data on aggregate agriculture exports to construct a revealed comparative advantage index in agriculture (RCA_a) defined for country i at time t as:

$$RCA_{ait} = \frac{X_{it}^a / X_{it}}{\sum_i X_{it}^a / \sum_i X_{it}}$$

where the numerator is the share of country i 's agriculture exports in i 's total exports and the denominator is the same share computed for the world economy. Higher values (above one) imply stronger comparative advantage in agriculture. As expected, RCA_a and output per capita are negatively correlated (-0.41 and significant at 1%): as countries develop they tend to acquire comparative advantage in other sectors.

Focusing on the behavior of the manufacturing shares in output and employment, the theory predicts hump shaped evolutions as per capita income (and productivity) increases. Thus, I estimate:

$$shIND_{it} = \beta_1 INC_{it} + \beta_2 INC_{it}^2 + \beta_3 RCA_{it} + \beta_4 POP_{it} + \mu_i + T_t + \varepsilon_{it}$$

where $shIND_{it}$ is the (output) employment share of manufacturing, INC_{it} is real income per capita, POP_{it} is the population and μ_i and T_t are country and time fixed effects.

The first four columns of Table 1 show least squares (LS) and instrumental variable (IV) estimates for the manufacturing output share. I use first lags to instrument for output per capita and comparative advantage. Table 2 reports similar estimates for the employment share. Standard errors robust to heteroskedasticity and autocorrelation are included within parentheses. The Wald F statistic is above 170 in all specifications, suggests the model does not suffer from weak identification.³ Since the IV and LS estimates are quite similar, in the following I focus on the least squares ones.

Columns (1) and (2) in Table 1, controlling only for population size, time and country fixed effects, show indeed that the manufacturing share in output peaks at an income level of around 32,000 constant U.S. dollars. Table 2 implies the employment share peaks at around 26,000 constant dollars.

Columns (3) and (4) show that revealed comparative advantage in agriculture has a negative and significant effect on industrialization after controlling for income per capita. The effect is also quantitatively important. Everything else equal, moving

³See Kleibergen and Paap (2006).

Table 1: Manufacturing output share

	(1)	(2)	(3)	(4)	(5)	(6)
	<i>LS</i>	<i>IV</i>	<i>LS</i>	<i>IV</i>	<i>LS</i>	<i>IV</i>
<i>INC</i>	1.859*** (0.14)	1.845*** (0.14)	1.711*** (0.14)	1.690*** (0.14)	1.107*** (0.13)	1.064*** (0.13)
<i>INC</i> ²	-0.029*** (0.00)	-0.029*** (0.00)	-0.027*** (0.00)	-0.026*** (0.00)	-0.021*** (0.00)	-0.020*** (0.00)
<i>POP</i>	10.470*** (1.45)	10.589*** (1.46)	9.594*** (1.46)	9.641*** (1.48)	8.461*** (1.42)	8.646*** (1.44)
<i>RCAa</i>			-0.772*** (0.14)	-0.775*** (0.15)	-0.222* (0.13)	-0.140 (0.14)
<i>Y/L_m</i>					0.363*** (0.02)	0.355*** (0.02)
<i>Y/L_a</i>					-0.529*** (0.05)	-0.521*** (0.05)
<i>Trade Open</i>					8.138*** (0.79)	9.150*** (0.91)
Observations	1251	1222	1227	1198	1218	1189
Country fixed effects	yes	yes	yes	yes	yes	yes
Time fixed effects	yes	yes	yes	yes	yes	yes
R-squared	0.23	0.24	0.26	0.27	0.50	0.51

Notes:

The dependent variable is the share of manufacturing in value added. *INC* is real GDP per capita in thousand constant 2005 U.S. dollars. *POP* is the logarithm of population. *RCAa* is an index of revealed comparative advantage in agriculture. *Y/L* is hourly productivity in manufacturing (*m*) and agriculture (*a*). *Trade open* is total trade flows as a share of output. Sectoral value added and productivity data are from Duarte and Restuccia (2009). *RCAa* is computed from FAO data. The other variables are from WDI. First lags used as instruments. Heteroskedasticity and autocorrelation robust standard errors within parantheses. The Wald F statistic is above 200 in all specifications. * indicates significance at the 10 percent level, ** indicates significance at the 5 percent level, *** indicates significance at the 1 percent level.

from an *RCAa* in the 95th percentile (5.59) to a 5th percentile index (.08) increases the value added (employment) share of manufacturing by 4.3 (4.6) percentage points. These are quantitatively important effects given the median shares of manufacturing value added and employment in the data are 0.301 and 0.28 respectively.

While output per capita determines the size of the manufacturing sector via the demand channel, sectoral productivity levels could have additional effects on labor reallocation via relative prices. Also, for a given comparative advantage, openness

Table 2: Manufacturing employment share

	(1)	(2)	(3)	(4)	(5)	(6)
	<i>LS</i>	<i>IV</i>	<i>LS</i>	<i>IV</i>	<i>LS</i>	<i>IV</i>
<i>INC</i>	1.334*** (0.13)	1.303*** (0.13)	1.098*** (0.14)	1.061*** (0.14)	1.198*** (0.15)	1.167*** (0.16)
<i>INC</i> ²	-0.025*** (0.00)	-0.024*** (0.00)	-0.021*** (0.00)	-0.020*** (0.00)	-0.018*** (0.00)	-0.017*** (0.00)
<i>POP</i>	26.153*** (1.41)	26.486*** (1.40)	24.772*** (1.58)	24.945*** (1.60)	13.076*** (1.64)	13.439*** (1.67)
<i>RCAa</i>			-0.832*** (0.19)	-0.819*** (0.20)	-0.716*** (0.15)	-0.692*** (0.17)
<i>Y/L_m</i>					-0.185*** (0.03)	-0.176*** (0.03)
<i>Y/L_a</i>					-0.500*** (0.06)	-0.480*** (0.05)
<i>Trade Open</i>					0.179 (1.00)	0.171 (1.11)
Observations	1251	1222	1227	1198	1218	1189
Country fixed effects	yes	yes	yes	yes	yes	yes
Time fixed effects	yes	yes	yes	yes	yes	yes
R-squared	0.39	0.40	0.42	0.43	0.54	0.54

Notes:

The dependent variable is the share of manufacturing in employment, measured as worked hours. *INC* is real GDP per capita thousand constant 2005 U.S. dollars. *POP* is the logarithm of population. *RCAa* is an index of revealed comparative advantage in agriculture. *Y/L* is hourly productivity in manufacturing (*m*) and agriculture (*a*). *Trade open* is total trade flows as a share of output. Sectoral employment and productivity data are from Duarte and Restuccia (2009). *RCAa* is computed from FAO data. The other variables are from WDI. First lags are used as instruments. Heteroskedasticity and autocorrelation robust standard errors within parantheses. The Wald F statistic is above 200 in all specifications. * indicates significance at the 10 percent level, ** indicates significance at the 5 percent level, *** indicates significance at the 1 percent level.

to international trade can also affect the industrial structure. Thus, in columns (5) and (6) I control for hourly productivity measures in manufacturing and agriculture as well as for trade openness, defined as the trade flows share in GDP. Given income per capita reflects average sectoral productivity, productivity in services is left out to avoid multicollinearity issues. As expected, the higher the productivity in manufacturing, the lower the employment share and the higher the value added. Higher productivity in agriculture lowers the size of manufacturing in terms of valued added while it lowers the manufacturing employment share. Trade openness increases the

value added share of manufacturing without boosting the corresponding employment share. Importantly, the coefficient on $RCAa$ stays negative and significant throughout.

6 Structural change paths and development (to be added)

7 Conclusion

The paper shows a simple model of structural change and international trade can provide an explanation for the systematic differences in the (de)industrialization process across countries and over time. In line with these findings, the empirical analysis documents that comparative advantage in agriculture lowers the size of the manufacturing sector controlling for sectoral productivity, income per capita and overall trade openness. Further work is needed to understand, both theoretically and empirically, the interactions between structural change and openness at different stages of development.

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8 Appendix

Proof of Proposition 1: i) $\partial \nu_t^a / \partial t = \frac{-3A_0^a A_0^s e^{(g_a+g_s)t} (A_0^a e^{g_a t} \bar{s} g_s + \bar{a} (3A_0^s e^{g_s t} g_a + (g_a - g_s) \bar{s}))}{(\bar{a} A_0^s e^{g_s t} - A_0^a e^{g_a t} (4A_0^s e^{g_s t} + \bar{s}))^2} < 0$, ii) $\partial \nu_t^s / \partial t = \frac{3A_0^a A_0^s e^{(g_a+g_s)t} (3A_0^a e^{g_a t} \bar{s} g_s + \bar{a} (A_0^s e^{g_s t} g_a + (g_a - g_s) \bar{s}))}{(\bar{a} A_0^s e^{g_s t} - A_0^a e^{g_a t} (4A_0^s e^{g_s t} + \bar{s}))^2} > 0$, iii) $\partial B_t / \partial t \geq 0 \Leftrightarrow g_a \bar{a} A_0^s e^{g_s t} - \bar{s} g_s A_0^a e^{g_a t} \geq 0 \Leftrightarrow g_a \bar{a} A_0^s / \bar{s} g_s A_0^a \geq e^{(g_a - g_s)t} \Leftrightarrow t \leq \log(\bar{a} g_a A_0^s / \bar{s} g_s A_0^a) / (g_a - g_s)$.

Proof of Proposition 2: Note Assumption 1 implies $\bar{s} / (\lambda_s e^{g_s t}) - \bar{a} / (\lambda_a e^{g_a t}) > 0$. This is sufficient for $\bar{s} / e^{g_s t} - \bar{a} / e^{g_a t} > 0$ since $\lambda_s > \lambda_a$.

i) $\nu_{1t}^{mo} > \nu_{1t}^m, \forall t \geq 0$ implies:

$$\frac{3(e^{g_s t} \bar{a} - e^{g_a t} \bar{s}) (e^{g_a t} \bar{s} \lambda_a (\lambda_m - \lambda_s \lambda_y) + e^{g_s t} \lambda_s (e^{g_a t} \lambda_a (\lambda_m - \lambda_y) + \bar{a} (-\lambda_m + \lambda_a \lambda_y)))}{4P_0 (e^{g_a t} \bar{s} \lambda_a (\lambda_m + \lambda_s) + e^{g_s t} \lambda_s (-\bar{a} (\lambda_a + \lambda_m) + e^{g_a t} \lambda_a (4 + \lambda_m + 3\lambda_y)))} > 0,$$

where $P_0 = -e^{g_s t} \bar{a} + e^{g_a t} (4e^{g_s t} + \bar{s})$. The denominator is positive since $P_0 > \bar{s} e^{g_a t} - \bar{a} e^{g_s t} > 0$ where the latter inequality follows from Assumption 1 and

$\lambda_a \lambda_s e^{(g_a+g_s)t} \left((4 + \lambda_m + 3\lambda_y) + (\lambda_a + \lambda_m) \left(\frac{\bar{s}}{\lambda_s e^{g_s t} (\lambda_a + \lambda_m)} - \frac{\bar{a}}{\lambda_a e^{g_a t}} \right) \right) > 0$ since $\lambda_s > \lambda_a \Leftrightarrow (\lambda_m + \lambda_s) / (\lambda_a + \lambda_m) > 1$ and $\bar{s} / (\lambda_s e^{g_s t}) - \bar{a} / (\lambda_a e^{g_a t}) > 0$. Since $e^{g_s t} \bar{a} - e^{g_a t} \bar{s} < 0$ by Assumption 1, the numerator is positive when:

$$\bar{s} \lambda_a (\lambda_m - \lambda_s \lambda_y) e^{g_a t} + e^{(g_a+g_s)t} \lambda_s \lambda_a (\lambda_m - \lambda_y) + \bar{a} e^{g_s t} \lambda_s (-\lambda_m + \lambda_a \lambda_y) < 0. \quad (11)$$

A sufficient condition for the latter inequality is:

$$\lambda_y > \lambda_m \theta. \quad (12)$$

where $\theta > 1$ is obtained solving for λ_y in (11): $\lambda_y > \lambda_m n / d$ where $n(t) = 1 + \bar{s} / (\lambda_s e^{g_s t}) - \bar{a} / (\lambda_a e^{g_a t})$ and $d(t) = 1 + \bar{s} / e^{g_s t} - \bar{a} / e^{g_a t}$. Define $f(t) = n(t) / d(t)$ and $f' = \partial f / \partial t$. Then $\lim_{t \rightarrow \infty} f(t) = 1$ and $\lim_{t \rightarrow \infty} f' = \lim_{t \rightarrow \infty} (n' - n d' / d) / d < 0$, since $d(t) > 0, \forall t \geq 0$, and $\lim_{t \rightarrow \infty} (n' - n d' / d) = \bar{s} (1 - 1 / \lambda_s) / e^{g_s t} \leq 0$ as $\lambda_s \leq 1$. Therefore $\max f(t) = \theta \geq 1$.

ii) $\nu_{2t}^{mo} < \nu_{2t}^m, \forall t \geq 0$ implies:

$$\frac{3P_1 (e^{g_a t} \bar{s} \lambda_a (\lambda_m - \lambda_s \lambda_y) + e^{g_s t} \lambda_s (e^{g_a t} \lambda_a (\lambda_m - \lambda_y) + \bar{a} (-\lambda_m + \lambda_a \lambda_y)))}{4P_2 (e^{g_a t} \bar{s} \lambda_a - e^{g_s t} (\bar{a} - 4e^{g_a t} \lambda_a) \lambda_s)} < 0,$$

where $P_1 = e^{g_a t} \bar{s} \lambda_a - e^{g_s t} \bar{a} \lambda_s$ and $P_2 = (-e^{g_s t} \bar{a} (\lambda_a + \lambda_m) \lambda_s + e^{g_a t} \bar{s} \lambda_a (\lambda_m + \lambda_s)) \lambda_y + e^{(g_a + g_s) t} \lambda_a \lambda_s (\lambda_y + \lambda_m (3 + 4\lambda_y))$. Following i), the inequality holds for (12). Since 12 implies $\nu_{1t}^{mo} > \nu_{1t}^m$ and $\nu_{2t}^{mo} < \nu_{2t}^m, \forall t \geq 0$, it is sufficient to also rank the peak manufacturing shares.

Proof of Proposition 3: $\partial \nu_{2t}^m / \partial t = 0$ implies the date of the peak manufacturing share in autarky t_2^* solves $-e^{g_a t} \bar{s} g_s \lambda_a + e^{g_s t} \bar{a} g_a \lambda_s = 0$. Similarly, t_1^* solves $-e^{g_a t} \bar{s} g_s + e^{g_s t} \bar{a} g_a = 0$. Then, for $\lambda_a < \lambda_s$:

$$\begin{aligned} \left. \frac{\partial \nu_{1t}^{mo}}{\partial t} \right|_{t=t_1^*} &= \frac{3e^{t(g_a + g_s)} \lambda_a \lambda_s (1 + \lambda_y) \bar{a} \bar{s} (g_a - g_s) \lambda_m (\lambda_a - \lambda_s)}{4(\lambda_a E_1 - \lambda_s E_2 + \lambda_s \lambda_a e^{t(g_a + g_s)} e^{g_a t} (4 + \lambda_m + 3\lambda_y))^2} < 0, \\ \left. \frac{\partial \nu_{2t}^{mo}}{\partial t} \right|_{t=t_2^*} &= -\frac{3e^{t(g_a + g_s)} \lambda_a \lambda_m \lambda_s (1 + \lambda_y) \bar{a} \bar{s} (g_a - g_s) (\lambda_a - \lambda_s) \lambda_y}{4((\lambda_a E_1 - \lambda_s E_2) \lambda_y + e^{t(g_a + g_s)} \lambda_a \lambda_s (\lambda_y + \lambda_m (3 + 4\lambda_y)))^2} > 0. \end{aligned}$$

where $E_1 = e^{g_a t} \bar{s} (\lambda_m + \lambda_s)$ and $E_2 = e^{g_s t} \bar{a} (\lambda_a + \lambda_m)$.