Diversity, Volatility and Innovation

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Abstract

In this paper I build a bridge between the level of diversity of an economy and its level of innovation. I do so, building a multisector endogenous growth model. I consider two key aspects of the creative process of innovation that have not been considered in the literature. First, investment in R&D is irreversible, and second, inventors can partially transport previous irreversible investment to a different technology class. The fact that R&D investment is irreversible implies that when bad shocks hit an entrepreneur he cannot undo his previous capital accumulation in the same way he builds up more capital when good shocks hit her. Anticipation of this situation leads her to refrain to accumulate too much capital. Diversification of the economy provides a way out. While previous investment cannot be undone, it can be transported to a sector that can recycle past innovation effort. The negative link between volatility and growth arises naturally in this setup. I complement the predictions of the model with empirical evidence using data at patent-inventor level, which confirms the positive effect of diversity and negative effect of volatility on innovation rates.

JEL Codes: O3, O4, E2

Keywords: Innovation, Irreversibility, Volatility, Complexity, Economic Growth

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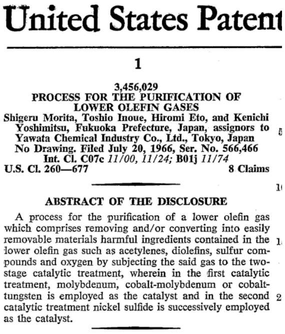
1 Introduction

Innovators are at the center of the creation of new ideas, improving processes, productivity and ultimately economic growth. These people invest real resources, their own human capital, their time and their focus, to accumulate a specific set of skills and knowledge (which I will refer to as "research capital"), which allows them to, eventually, have a breakthrough. I will argue that, in highly diversified economies it will be less risky, for these inventors, to engage in very specific R&D activities. Therefore, they will invest and innovate more. In more diversified economies, if a big shock hits the inventor, she may have the option to recycle partially her previous efforts in a related sector. In a less diversified economy, hit by the same negative shock, this inventor would simply see her previous effort go to waste. Consider two examples.

In 2014 the Airbus company started the design of the Aerion AS2 ultrasound business jet, using proprietary supersonic laminar flow technology of the firm Aerion. This jet promises to fly from London to New York in three hours or less, and its development cost is projected to be around $100 million. While its first test flight is programmed for 2019, meeting this deadline hinges on the the success of the research agenda that aims to create a light resistant material, called forged carbon fiber composite. The Airbus team of engineers currently works on it, in the Aerion base in Reno, Nevada. What would happen if the world demand for supersonic jets suddenly disappeared in 2017?. Not all would be lost. All the investment and learning of this engineering team could be recycled. For instance, the "Automobili Lamborghini Advanced Composite Structures Laboratory" at the University of Washington, Seattle works in a similar research agenda and could benefit from the advances and errors from the Airbus team if they were to fail. Luckily Seattle and Nevada are less than two hours away, share the same language and institutional framework. What would happen instead, if Airbus' project failed, and it was located in Chile or Australia?

Less hypothetical is the following example, taken from the sample used in this paper (see figure 1). There exists a very specific skill: *manipulation and degradation of poisonous gases*. In 1969, Shigeru Morita, a Japanese inventor in metallurgy, applied to patent a "process for the purification of lower olefin gases which comprises removing and/or converting into easily removable materials harmful ingredients". He patented several related methods in the following years until the oil crisis hit Japan in 1973. The economy transited to more knowledge intensive industries and abandoned energy-intensive technologies. For year 2000 Mr. Morita had moved

(a) Abstract in patent US3456029 in year 1969


United States Patent

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3,456,029
PROCESS FOR THE PURIFICATION OF LOWER OLEFIN GASES
Shigeru Morita, Toshio Inoue, Hiromi Eto, and Kenichi Yoshimitsu, Fukuoka Prefecture, Japan, assignors to Yawata Chemical Industry Co., Ltd., Tokyo, Japan
No Drawing. Filed July 20, 1966, Ser. No. 566,466
Int. Cl. C07c *11/00, 11/24;* B01j *11/74*
U.S. Cl. 260—677 8 Claims

ABSTRACT OF THE DISCLOSURE

A process for the purification of a lower olefin gas which comprises removing and/or converting into easily removable materials harmful ingredients contained in the lower olefin gas such as acetylenes, diolefins, sulfur compounds and oxygen by subjecting the said gas to the two-stage catalytic treatment, wherein in the first catalytic treatment, molybdenum, cobalt-molybdenum or cobalt-tungsten is employed as the catalyst and in the second catalytic treatment nickel sulfide is successively employed as the catalyst.

(b) Main scheme in patent US6844179 in year 2000

Fig.2

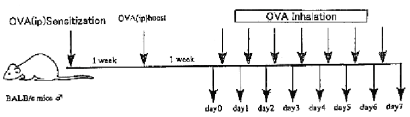

Figure 1: Mr. Morita's mobility: This Japanese inventor transited from pure metallurgic applications to sediments, to biology (proteins and DNA encoding) sector because he possessed very specific skills in dangerous gas manipulation which he could recycle in a different sector. Panel *(a)* shows his initial inventions in metallurgy and panel *(b)* show the plan of medical trial where inhalation of a gas was the chosen technique of medication

to a promising sector which was unrelated to his initial one, but where his expertise and skill would prove valuable. He collaborated and applied for a patent which claims the invention of a DNA encoding protein to treat infectious airborne diseases such as asthma and chronic obstructive pulmonary disease. The medical trials required infecting mice with these airborne diseases: a task that Mr. Morita could handle, recycling his skill and expertise in poisonous gas degradation. Going to pharmaceutical sector was not a lucky deviation from his metallurgy initial sector. It was a planned and informed decision when circumstances changed, and Morita was able to do this, because he could transport his expertise to an existing sector. What if Morita had worked and lived in Venezuela in year 2015 instead of Japan in year 2000? He would not have been able to recycle his cumulative research capital as easily. More importantly, the theory in this paper predicts that he would not have invested as much as he did in the first place, anticipating this possibility. Many of his inventions would not have been created.

This paper builds a bridge between the degree of *diversity* in production activities in a given economy, and its level of *innovation* and productivity *growth.* I claim that more diverse economies, are more likely to be more innovative. I also claim that the (negative) effect of sectoral volatility on productivity growth is lower in more diversified economies[1].

What are the main ingredients in this story? Investment in research-capital can be different from physical capital investment. In particular, because it takes time to acquire, and more importantly because "*... a substantial part of R&D outlays might be extremely irreversible due to being project - specific, and not merely to firm - specific or industry - specific.*" (Dixit and Pindyck (1994), p. 424). Inventors in a non-diverse economy, will refrain from accumulating research capital if they expect a positive probability of a bad event in the future, because if that shock were to realize, they would find themselves with a more-than-optimal amount of research capital, which they cannot undo. Anticipating this scenario they curtail research-capital accumulation today. On the other hand, while inventors in diverse economies share the same incentives, and may ultimately face the same shocks, the fact that they operate in more complex economies, gives them the possibility to more easily recycle their research capital. While they cannot undo the investment, they can transport it to a sector which demands similar skills and knowledge. This provides an option value which enters into the accumulation decision today, and results in higher levels of investment in R&D today. Put differently, the interaction of irreversibility and volatility hinders research effort, however, diversity partially offsets this effect.

While the simplest version of the model in this paper, presents the number of sectors in a given economy as fixed, there exists evidence that research effort has a positive and non-negligible probability of producing innovations in different sectors, and also creating new sectors. That is, innovation can make an economy more complex (see Akcigit et al., (2014) for a model in which firms may have incentives to be in many sectors simultaneously). This feedback channel would only make the argument of this paper even more relevant. The two channels combined would result in a virtuous circle of diversification, or a poverty trap of non-diversification. In the basic version of the model I will initially abstract from this feedback effect and leave it as an extension. There are two reasons to do so. First, pedagogically, abstracting from the feedback channel highlights the

[1] The literature agrees on the empirical fact that volatility and growth are negatively related. The empirical observation, initially raised by Ramey and Ramey (1995) states that this negative correlation is strong in emerging economies but vanishes when we look at OECD economies. Aghion, et al. (2005) argue that this difference emerges because financial development is low in emerging economies. They build a model financial frictions interact with volatility to hinder growth. In my model, the link between volatility and growth emerges naturally in non diversified economies, without assuming financial frictions. While this paper does not rule out the mechanism in Aghion et al. (2005), it provides a complementary explanation to theirs.

main economic mechanism of the effect of diversity on innovation. Second, we can think of this model as one that explains innovation dynamics taking diversification of countries as initial condition, and which allows us to think about comparative statics, or, we can think that while the feedback channel is present, diversification of economies evolves slowly.

I start by documenting that inventors do not stay in one sector only, but that it is not unusual for them to move sectors. They move once and for all, in one direction and back, or simply transit a few sectors before retiring. Second, I document that the intellectual distances (for which I will later define an empirical measure) that inventors transit, are smaller in highly diversified economies than in non-diversified ones. I use this observation to build a multi-sector endogenous growth model with irreversible research capital, in which the intellectual distance from one sector to another is key in determining how difficult it is for an inventor to recycle her previous investment, and partially escape irreversibility. In section 4, I explore the validity of the main predictions of the model using patent - inventor data, as well as cross country aggregate data. Section 5 concludes.

1.1 Empirical Foundations

The examples presented above are far from outliers in the data. The model in section 2 builds on three empirical observations about the behavior of the investment process in R&D.

1. Investment in R&D is highly irreversible.

2. Conditional on moving sectors, inventors in more diversified economies move shorter intellectual distances than those in low diversified economies.

3. The fraction of inventors who move sectors is non-negligible.

1.- Investment in R&D is highly irreversible. It is reasonable to argue that among all types of investment outlays, R&D is among the most irreversible. This is partly because of its intangible nature, and also because it usually is very project-specific (Dixit and Pindyck, 1994). Testing for irreversibility however, is hard. The main effect of irreversibility is that even one the investor wishes to adjust downwards its capital holdings, she cannot. When we observe the data we cannot see that unfulfilled intention. There are ways to go around this limitation. Goel and Ram (2001) use the methodology proposed in Pindyck and Solimano (1993) which infers indirectly the irreversibility of different kinds of investment in the presence of uncertainty. They use data from 9 OECD economies in R&D expenditure from the National Science Board and conclude that, R&D investment is irreversible while there is no ground to say the same about physical capital investment. The notion of research-capital used in this paper includes the definition in Goel and Ram (2001) and includes intellectual and time investment on the side of the inventor, which is by definition, also irreversible.

2.- Conditional on moving sectors, inventors in more diversified economies move shorter intellectual distances than those in low diversified economies. Think about a patent as lying within some technological class and call this technology class E. Empirically, this technological class can be represented by the first two digits of its International Patent Classification (IPC) code (IPC2). I use data from the NBER-USPTO Patent Grant Database which contains information about application of patents, the technology class, and their inventors. I combine this information with data on cross-citations from Lai et al. (2013).

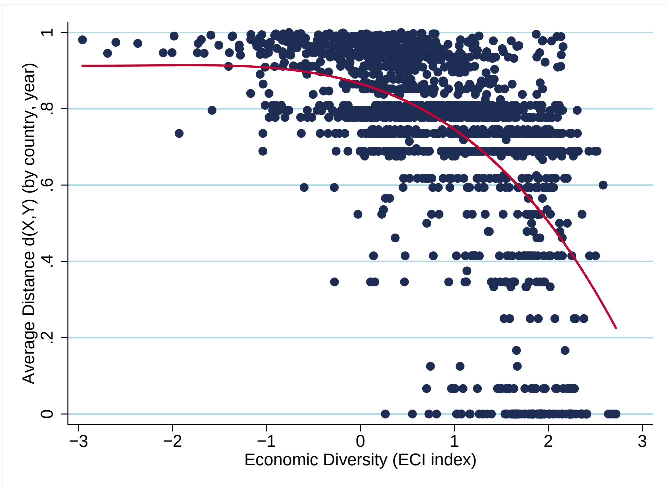

Figure 2: Intellectual distance of innovators who migrate sectors and economic diversification. Source: NBER-USPTO Patent Grant Database and Lai et al. (2012).

Consider an inventor who wants to migrate from sector E to sector F, and define $\#cit(E, F)$ as the number of patents in sector F that cite any patent in sector E. Also, let $\#cit(F)$ be the number of patents in sector F. Then, I will define the intellectual distance to go from E to F as[2],

$$d(E,F) = \left[1 - \frac{\#cit(F,E)}{\#cit(F)}\right] \tag{1}$$

with $0 < d(E,F) < 1$. Note this distance measure is not symmetric which captures the fact that it could be easier to move in one direction than the in the opposite. This measure is also intuitive. If an inventor considers moving to sector F from sector E, and every patent in sector F cites at least one patent in E, then it should be very easy for that inventor to transit to F. His knowledge in E is valuable in F. Distance in this case is equal to zero. On the other hand if an inventor wants to transit from sector E to F, but not one patent in F cites one in E, intellectual sector F does not feed on knowledge from E. The intellectual distance in such case is one. All other cases fall in between. In the vertical axis of figure (2) I compute the the country-year average of the intellectual distance transited by those inventors who decided to move to a different sector. In the horizontal axis I use the "Economic Complexity Index", a measure of economic diversity proposed by Haussman et al. (2012) using UN Comtrade data[3]. The ECI index goes from less to more diversification. Figure (2) shows inventors who move to another sector, in highly diversified economies usually do not move long distances. Alternatively we can also say that short distance movements are only possible in highly diversified economies.

3.- Inventors do move sectors during their lifetime. Empirical fact #2 only considers those inventors who change sectors from one patent to another. But how frequent are these events? For this we need to consider events in which inventors switch and events in which they do not switch sectors in their next patent application. Consider the unconditional probability of innovating in a different sector than the original sector X for every two consecutive innovations $\{a_n, a_{n+1}\}$ by the same inventor $\mathcal{A}$.

$$P[a_{n+1} \notin X | a_n \in X] = \frac{1}{\#\mathcal{A}} \sum_{i \in \mathcal{A}} \mathbf{1}\{a_{n+1} \in X, a_n \in X\} \tag{2}$$

[2]This measure of distance is similar to equation (1) in Akcigit et al. (2015) who consider instead a symmetric distance (distance from A to B is equal to the distance from B to A). I consider the asymmetric distance, as it seems more reasonable to think about recycling research-capital. For instance, consider an applied mathematician (M) who wants to move to finance (F). The distance from M to F is shorter as much of previous investment can be recycled in this direction but not in the opposite way.

[3]I provide a complete explanation of the Haussman et al. (2012) index in Appendix D

Note that equation (2) considers every two consecutive inventions by any inventor. If an inventor switches a few times only and $n \to \infty$, then this probability tends to zero. If this inventor switches sectors every time she applies to a new patent, (2) tends to one as the number of inventions grows large[4].

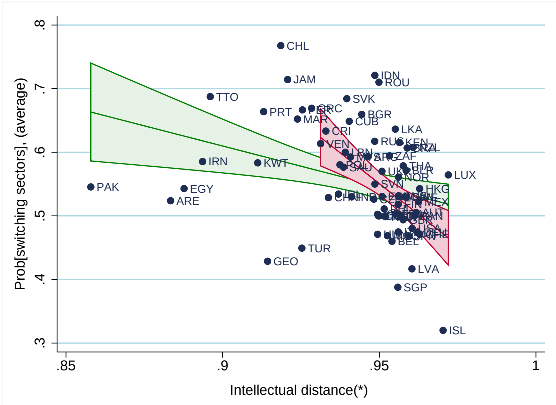

Figure 3: Source: NBER-USPTO Patent Grand Database and Lai et al. (2013). Note: Censoring implies we cannot observe the distance when an inventor wants to move but decides it is too costly. Mean imputation method is used

In figure (3) I plot the country average of this probability and plot it against the average intellectual distance transited by the same inventors. The main take-away is that moving sectors is, by no means an exception. We can also see that this probability is negatively correlated with the distance inventors have to move. This last observation, however hast to be taken with a grain of salt. Inventors who choose not to move because the distance to another sector is too large, are unobservable. This negative correlation is taken conditional on actually moving sectors.

1.2 Position in the Literature

This paper inserts in the endogenous growth literature which considers explicitly firm dynamics. Its main contribution is to build a bridge between the level of diversification and innovation, which is new in the literature. In order to do this, I consider explicitly the possibility of migration across sectors of the economy on the face of stochastic realizations of expected profit. It also considers the role

[4]Note that this way of computing the probability of switching sectors avoids the ad hoc decision of labeling an inventor to belong to a sector by any of their inventions.

of sectoral volatility on the sector migration decision and its ultimate impact on innovation, building a novel competing explanation for the link between volatility and growth.

This paper contributes on the theory side to the literature that considers explicitly a role for industry heterogeneity on the process of innovation. Previous research on growth theory that considers the existence of more than one industry in the economy is abundant, starting with the seminal work of Grossman and Helpman (1991). However, work that considers an explicit role for the number of heterogeneous characteristics of these industries is thin. Akcigit et al. (2013) consider a model to contrast basic and applied research by firms who can pursue research in several industries. Their model teaches us that basic research, which more likely produces radical innovations, can be used in several industries while applied research is more specific to a given industry. They give an explicit role to the number of industries in which a firm develops research agendas. A few other papers giving a role to industry heterogeneity in the context of innovation are Aghion and Howitt (1996), Ngai and Samaniego (2011) and Kongsamut et al. (2001), but none gives a role to the abundance – or lack thereof – of sectors in the innovation process. This paper feeds on the literature of firm dynamics within an industry, exposed in detail in Hopenhayn (1992), but limits its contribution to the implications on the innovation process.

This paper also contributes, on the theory side, to the discussion of the link of growth and volatility, which has been mostly empirical. Ramey and Ramey (1995) present empirical evidence, that there is negative and significant relation between volatility and growth in non OECD economies. For OECD economies this relationship seems to vanish, which motivates Aghion et al. (2005) to make the case that financial development would be a determinant on the causality of volatility on lower growth. Under the assumption that the key difference between OECD economies and the rest of the world is that the former are more financially developed than the latter, Aghion et al. (2005) develop a model in which volatility is related to growth via the financing channel of R&D. If entrepreneurs need to finance research and they are subject to credit constraints proportional to their output, recessions make these constraints harsher and curtail research, generating a causal link of volatility to innovation. My model provides the same result for free, without assumptions on frictions on the financing of research endeavors. If AABM's mechanism exists, and is important, it would only reinforce the mechanism in this paper. All in all, AABM's paper, is mute about the positive and robust relation between economic diversification and innovation.

On the empirical side, I contribute to the literature providing evidence on inventor mobility across sectors using micro data, showing how inventor productivity and mobility depends on their environment. This paper is not alone in this extension. Recent data availability on inventor identities has resulted in a fruitful research agenda . Perhaps one of the most interesting examples of it is recent work by Akcigit et al. (2015) which considers geographical migration of inventors and their tax burden.

2 A Simple Model

The simplest possible model which can account for the stylized facts in Section 1.1 consists of a consumer which supplies labor inelastically and every period consumes a good c_t; a composite different sectoral final goods. In each industry, the final good requires labor and a continuum of intermediate capital goods. These latter intermediate goods are subject to improvement via innovation. In particular, each of them is produced only by the entrepreneur who holds the latest blueprint (vintage) and therefore, can do it most efficiently. The monopolist in each intermediate capital good remains so, until her blueprint is no longer the most efficient and another entrepreneur replaces her. This is the engine of economic growth in this model, just as in the basic Schumpeterian model in Aghion and Howitt (1992).

To make the main argument in this paper – that diversity is positively related to different rates of innovation–, I require a richer structure for the innovation process. In the standard endogenous growth literature, we usually equate the static marginal cost of engaging into innovation with the marginal value of it, as though we could decide on the level of research inputs with full flexibility. Research capital accumulation, is a process that takes time. But more importantly, R&D investment is highly irreversible; a strong feature from which the literature has abstracted so far. This characteristic will complicate the investment decision but will prove to be useful in understanding the connection between economic diversity, volatility and innovation moments.

2.1 Preferences

The representative household supplies inelastic labor which I normalize to unity, and consumes final goods c_t for which he derives instant utility $u(c_t)$, with $u(\cdot)$

being non-decreasing, strictly concave, and that satisfies $u(0) = 0$, and Inada conditions $\lim_{c\to 0} u_c(\cdot) = \infty$ and $\lim_{c\to\infty} u_c(\cdot) = 0$. The sources of income for this household are her wage, interest payments on past savings in uncontingent assets (a_t), and net profits from firms, $\bar{\Pi}_t$. In particular, the problem of the household is as follows. Taking as given wages, interest rates and profits, $\{w_t, r_t, \bar{\Pi}_t\}_{t=0}^{\infty}$, the representative household chooses $\{c_t, a_{t+1}\}_{t=0}^{\infty}$, to solve the following program,

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\} \tag{3}$$

subject to

$$c_t + a_{t+1} \leq (1 + r_t)a_t + w_t L_t + \bar{\Pi}_t \tag{4}$$

where β is the intertemporal discount factor and the instant utility function is constant relative risk aversion with parameter γ, $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$. The first order condition of this problem is the usual Euler Equation which relates consumption in t and $t+1$,

$$\left(\frac{c_{t+1}}{c_t}\right)^{\gamma} = \beta(1 + r_{t+1}) \tag{5}$$

2.2 Technology

2.2.1 Final Good Producer

The production technology in this model follows standard models of endogenous growth. Production of the final good that is ready for consumption, is an aggregation of the final goods in every industry m, $m \in \mathcal{M}_t = \{1, 2, \ldots, M_t\}$, with M_t countable and finite. Let us use Y_t for the final good – which can be readily consumed–, and $y_{m,t}$ for the final good in sector m, such that,

$$Y_t = \left(\sum_{m \in \mathcal{M}_t} y_{m,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{6}$$

therefore, the final goods produced in each industry can either be perfect complements (if ϵ approaches zero), perfect substitutes (when ϵ approaches infinity) or be somewhere in between.

In each sector m, sectoral output $y_{m,t}$ is produced using a continuum of varieties of intermediate capital goods $x_{m,i,t}$, $i \in [0,1]$ using the same technology in Grossman and Helpman (1991),

$$\log y_{m,t} = \int_0^1 \log x_{m,i,t} di \tag{7}$$

The problem of the (sectoral) final good producer is static as intermediate goods $x_{m,i,t}$ are fully depleted in the production of the final good. In particular, given prices of intermediate goods $\{p_{m,i,t}\}_{m\in\mathcal{M}_t, i\in[0,1], t=0}^{\infty}$, the (sectoral) final good producer chooses intermediate goods $\{x_{m,i,t}\}_{t=0}^{\infty} \geq 0$ to solve the following program,

$$\max_{\{x_{m,i,t}\geq 0\}} \exp\left(\int_0^1 \log x_{m,i,t} \mathrm{d}i\right) - \int_0^1 p_{m,i,t} x_{m,i,t} \mathrm{d}i \tag{8}$$

which readily yields the following first order condition (demand for intermediate goods),

$$x_{m,i,t}^D = \left(\frac{p_{m,t}}{p_{m,i,t}}\right) y_{m,t} \tag{9}$$

2.2.2 Intermediate Good Producer

For each sector m, there is a continuum of intermediate goods whose production efficiency can be improved using R&D. Each variety of intermediate good $i \in [0,1]$ is produced with a linear technology on labor,

$$x_{m,i,t} = (\ell_{m,i,t} + \xi_{m,t}) q_{m,i,t} \tag{10}$$

where $\ell_{m,i,t}$ is the amount of labor used in the production of this specific variety, $\xi_{m,t}$ is an additive cost shock which is sector-specific. The production of the i variety grows proportionally to its quality $q_{m,i,t}$, which in turn grows with innovation according to a quality ladder of step size σ.

$$q_{m,i,t+1} = \begin{cases} (1+\sigma) q_{m,i,t} & \text{if} \quad \text{there exists innovation} \\ q_{m,i,t} & \qquad \text{otherwise} \end{cases} \tag{11}$$

with $\sigma > 0$. The labor augmenting shock $\xi_{m,t}$ evolves exogenously according to

$$\xi_{m,t+1} = \rho_\xi \xi_{m,t} + \sigma_\xi \epsilon_{m,t+1} \tag{12}$$

where $\epsilon_{m,t} \sim N(0,1)$, and is independent from $\epsilon_{n,t}$ for $n \neq m$. Also note that the unconditional mean of this process is zero which means that if $\sigma_\xi = 0$ we are back to the usual deterministic production function. Total cost of production of each variety will then be,

$$w_t \ell_{m,i,t} = w_t \left(\frac{x_{m,i,t}}{q_{m,i,t}} - \xi_{m,t} \right)$$

The incumbent producer will be use optimal pricing consistent with Bertrand competition. Price will be set equal to the marginal cost of the closest follower in order to deter her from producing. Denote the efficiency of the closest follower by $\tilde{q}_{m,i,t}$, then:

$$p_{m,i,t} = \frac{w_t}{\tilde{q}_{m,i,t}} \tag{13}$$

In every variety i, the owner of the latest blueprint takes home the monopolistic profit $\pi_{m,i,t}$ every period, which we can calculate in the following way:

$$\pi_{m,i,t} = \left[\frac{w_t}{q_{m,i,t}} (1+\sigma) x_{m,i,t} - w_t \frac{x_{m,i,t}}{q_{m,i,t}} + w_t \xi_{m,t} \right]$$

which can be expressed in terms of state variables and total output in the economy,

$$\pi_{m,i,t} = \left[\left(\frac{\sigma}{1+\sigma} \right) \left(\frac{Q_t}{Q_{m,t}} \right)^{1-\varepsilon} \left(1 + \sum_m \xi_{m,t} \right) + \frac{\xi_{m,t}}{1+\sigma} \right] Q_t \tag{14}$$

We can readily see that profits are sector-specific and not firm-specific. This is a result of assuming sector-specific aggregate shocks and not firm-specific idiosyncratic shocks. Appendix A2 elaborates on the algebra that leads to expression (14).

2.3 Research and Development

Up until this point, the ingredients of the model are standard and the results, derivative. The problem of the researcher, however, is novel and deviates substantially from the previous literature in two main aspects. First, I acknowledge that research capital is accumulated through time. Ideas are not produced with a one shot investment, but require accumulation of experience, errors, and skill development. Second, and more important, investment in research capital is irreversible.

Each country is a finite collection of industries (sectors) $m \in \mathcal{M}_t$. The cardinality of this set will play a role on innovation even if it remains fixed. In the basic model I will abstract from M_t growing to highlight that the main mechanism in this paper is not related with expanding variety models. Instead, M_t will be a measure of economic diversification and will affect the decision of investment in R&D when combined with irreversibility. Thus, even with a fixed number of sectors, more complex economies will feature higher innovation rates.

Accumulation of project-specific knowledge is a time consuming activity which cannot be bought at one moment in time, but rather takes time to accumulate. This idea is captured in the model by distinguishing research effort $R_{m,t}$ in sector m, from research capital $w_{m,t}$. In particular, for any given entrepreneur in sector m, research capital law of motion is,

$$\omega_{m,t+1} = \omega_{m,t}(1-\delta) + f\left(\frac{R_{m,e,t}}{Q_{m,t+1}}\right) \tag{15}$$

where $\log Q_{m,t} = \exp(\int_0^1 \log q_{m,i,t})di$, is the average quality in sector m. The reason to use average and not any particular quality $q_{m,i,t}$ is that once an innovator is successful in sector m, she will become the incumbent of a random variety in the sector. This simplifies the problem drastically at no cost. Research capital (ω_t) can never be less than the un-depreciated part of previous holdings, because of the irreversibility. It can, however, be augmented with industry specific research effort in period t, $R_{m,t}$. This effort has to be proportional to the quality of the sector $Q_{m,t}$, which captures the fact that more complex goods are usually harder to innovate upon. Function $f(\cdot)$ transforms this normalized innovation effort into stock of knowledge, and is assumed to be increasing, concave and to satisfy Inada conditions; $\lim_{x\to 0} f'(x) = \infty$ and $\lim_{x\to\infty} f'(x) = 0$. These assumptions on the functional form of $f(\cdot)$ capture the fact that while investment in one period can increase cumulative know-how, large amounts of know-how require several periods.

Assumption 1. *A given researcher can only do research in one industry $m \in \mathcal{M}_t$ at the time and will produce in only one variety in such a sector, at the same time.*

Assumption 1 is clearly an abstraction. Several firms can innovate in different sectors of the economy as documented in Akcigit et al. (2014), who defend the idea that, for a given firm, performing research in several industries raises the value of basic research, which generates the most radical innovations. Here I am abstracting from this mechanism and emphasizing a different one. Then, when researcher e has $\omega_{m,t}$ know-how in sector m, she can produce ideas or innovations in sector $m \in \mathcal{M}_t$ only, and does so with probability:

$$\mu_{m,t} = \mu(\omega_{m,t}) \tag{16}$$

where $\mu(\cdot)$ satisfies $\mu_x(x) > 0$, $\mu_{xx}(x) < 0$ and is bounded to $\mu \in [0, 1]$, $\lim_{x\to\infty} \mu(x) = 1$, $\lim_{x\to 0} \mu(x) = 0$[5]. . In particular, I will use the following functional form for the probability of innovation given in equation (16),

$$\mu(\omega_m) = \{1 - \exp(-\nu\omega_m)\} \quad , \quad \omega_m \geq 0 \tag{17}$$

Once an entrepreneur decides the amount of research capital for the following period, she cannot alter it again for the same period. The timing of events (figure 4) is as follows. Any entrepreneur enters the period with ω_t research capital. Nature decides if she is going to succeed according to equation (17) and, if she is, assigns her randomly to any variety in the sector and will improve the quality of that variety by a factor $(1 + \sigma)$. Nature also decides on the cost shock $\xi_{m,t}$. Lucky incumbents will receive a profit flow $\pi_{m,t}$ and start planning for the next period. First, she chooses if she stays in the same sector or migrates to a different one. If she moves, part of her research capital will be eroded (more on this later). Then, she also decides how much research capital to bring to the next period, ω_{t+1}

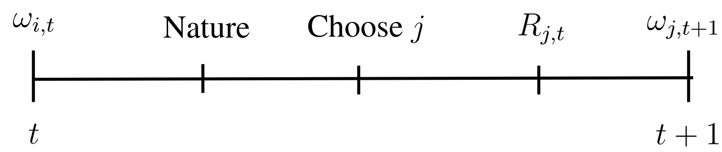

Figure 4: Timeline of Events in Innovation

Conditional on already choosing a sector for next period, the second decision (ω_{t+1}) is constrained by non negativity of research capital investment. When the entrepreneur moves to a different sector, she must consider that her cumulative research capital was specific to her original industry and it may not be fully portable. More research capital will be destroyed the farther away are the origin/destination technology classes. We can think of technology classes in the

[5]Note that the functional form chosen is actually the cumulative distribution function of a truncated (> 0) exponential random variable. Only one parameter, ν, governs this distribution, which is why choose it. While we could use more flexible functional forms such as the Weibull or Shifted Gompertz distributions, that use two parameters, these are not always weakly concave and are usually convex for low levels of the argument.

economy as in the location model originally developed by Salop (1979), and also used in Akcigit et al. (2015). Each sector is a node distributed – not necessarily evenly – on the circle of radius $1/\pi$ as depicted in figure (5). Moving between two sectors that are next to each other, imply very little destruction of research capital. Moving to a sector on the other side of the circle implies that cumulative research capital is not useful in the destination sector. More generally, the size of knowledge destruction is a fraction $0 < \phi_{jm} < 1$, which is the theoretical counterpart of $d(m, j)$ in equation (1). Note that the more nodes there are in this circle, the more diverse the economy is, and the smaller average distance between nodes. This is one of the fundamental milestones of this model. In spite of being only a partial one, diversification provides an escape from irreversibility for a researcher who wants to abandon her industry.

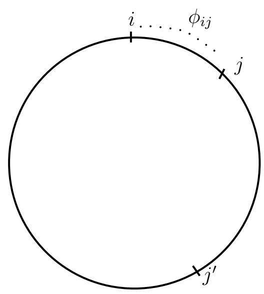

Figure 5: Distribution of sectors in each economy $\mathcal{M}_t$

The modified law of motion of research capital when an entrepreneur moves from sector m to j.

$$\omega_{j,t+1} = \omega_{m,t}(1-\delta)(1-\phi_{mj}) + f\left(\frac{R_{j,t}}{Q_{j,t+1}}\right) \tag{18}$$

2.3.1 Entrepreneurs across sectors

The collection of all entrepreneurs will be referred to as $\mathcal{E}$, and has mass E, which we will later normalize to M. An entrepreneur e is said to be active in industry m, when $\omega_{m,t}(e) > 0$. In particular, the collection of all entrepreneurs in such industry m is denoted by $\mathcal{E}_m = \{e \in \mathcal{E} : \omega_{m,t} > 0\}$, and has mass $E_m E$, where by construction E_m is the proportion of entrepreneurs in sector m. Hence, for all t, it must always be

$$\sum_{m \in \mathcal{M}_t} E_m = E \tag{19}$$

The fact that research capital is irreversible means that even when an entrepreneur is not successful and is replaced by a better entrepreneur, she may still hold positive amounts of research capital. She will be an outsider. The problem of the researcher, thus, is going to be different if she is an incumbent or she is an outsider. Denote $\mathbf{J}(\cdot)$ values for incumbents and $\mathbf{H}(\cdot)$, values for outsiders.

We can think of the decisions of every entrepreneur as happening in two stages. First, they decide to stay in their sector or migrate. Second, they decide how much research capital to bring to the next period. Once we solve the second problem it is easy to solve the first one. In particular, the first problem is reduced to comparing two alternatives: moving or staying, conditional on doing what is best in each of these cases. The following problems, determine how much research capital to bring to the next period in every possible case, and after nature has decided on the cost shocks and on the success/failure of innovation in the current period.

Incumbent's Problem. Consider first the problem of the current producer of good $x_{m,i,t}$, who holds the latest patent for its production. The aggregate state variables will be the vector of qualities in every sector, $Q = \{Q_m\}_{m\in\mathcal{M}}$ and shocks, $Z = \{\xi_m\}_{m\in\mathcal{M}}$. The individual state variable will be the current level of research capital ω. Let $J^S(Q, Z, \omega|m)$ be the value of being an incumbent in period t in sector m, and deciding to stay in sector m after learning the realization of nature in the current period. For this decision the only control variable is the stock of research capital next period in m, ω'. In particular, the problem for this entrepreneur is: Given, current research capital, qualities and shocks for the current period $\{Q, Z, \omega\}$, and aggregate variables, $\{r', \lambda_m\}$, the incumbent entrepreneur who decides to stay in the same sector m, chooses ω' to solve the program,

$$J^S(Q, Z, \omega|m) = \max_{\{\omega'\}} \left\{ \begin{array}{c} \pi(Q_m, \xi_m) - Q'_m f^{-1}(\omega'_m - (1-\delta)\omega_m) \\ +\frac{1}{1+r'} \int_{Z'|Z} [(1 - \lambda_m + \mu(\omega')\lambda_m) J^0(Q', Z', \omega'|m) \\ +(1 - \mu(\omega'))\lambda_m H^0(Q', Z', \omega'|m)] \, dF(Z') \end{array} \right\} \quad (20)$$

subject to

$$\omega' \geq (1-\delta)\omega$$

Equation (20) is the value of staying in sector m for the incumbent. After nature has revealed the (un)successful innovations and the cost shock in period t, the incumbent observes $\{Q, Z\}$, the incumbent is entitled to produce and obtain the flow profit. Since she decides to stay in sector m – and therefore her cumulative know-how does not erode – her research efforts are given by the term in parenthesis in the first line of (20). The rest of the lines stand for the continuation value of being an incumbent in sector m and deciding to stay in it. With probability $\mu(\omega)$ the entrepreneur is successful and gets a random variety i to extract profits from, and with probability $1 - \lambda_m$ nobody innovates and therefore the current incumbent remains so. Finally, with probability $(1 - \mu(\omega))\lambda_m$ the entrepreneur loses her monopoly to an entrant and becomes herself an outsider. The perceived laws of motion for the state variables are straightforward for ω' as it is the control variable. The perceived law of motion of the probability of being replaced is given by $\mathcal{L}^E(Q, Z)$ The cost shocks have an exogenous structure. Finally, while quality is stochastic, the assumption that we have a continuum of firms in each sector results in non-stochastic sector-aggregate quality evolution.

$$\begin{array}{lcll} \omega' & = & \mathcal{G}^S(Q, Z, \omega|m) & \\ \lambda_m & = & \mathcal{L}^E(Q, Z, \omega) & \\ \xi'_m & = & \rho_\xi \xi_m + \sigma_\xi \epsilon & \forall m \in \mathcal{M} \\ \log Q'_m & = & \lambda_m \log(1+\sigma) + \log Q_m & \forall m \in \mathcal{M} \end{array}$$

The incumbent entrepreneur, however, may also choose to abandon sector m for sector n which is at distance ϕ_{mn} (see figure 5). Then, given, current research capital, qualities and shocks for the current period $\{Q, Z, \omega\}$, and aggregate variables, $\{r', \lambda_m\}$, the incumbent entrepreneur who decides to move to sector $n \in \mathcal{M}$, chooses ω' to solve the program

$$J_n^M(Q, Z, \omega|m) = \max_{\{\omega'\}} \left\{ \begin{array}{c} \pi(Q_m, \xi_m) - Q'_n f^{-1}(\omega' - (1-\delta)(1-\phi_{mn})\omega) \\ +\frac{1}{1+r} \int_{Z'|Z} [\mu(\omega') J^0(Q', Z', \omega'|n) \\ +(1-\mu(\omega'))H^0(Q', Z', \omega'|n)]\, dF(Z') \end{array} \right\} \quad (21)$$

subject to

$$\omega' \geq (1-\delta)(1-\phi_{mn})\omega$$

and using analog perceived laws of motion for aggregate state variables as in (20). Conditional on moving from sector m to sector n, original know-how in sector m is destroyed not only because of depreciation, by a factor δ, but because a fraction ϕ_{mn} of the cumulative knowledge is not portable. When moving to n the entrepreneur is automatically an outsider in the new sector. Next period, with probability $\mu(\omega')$ she will become an incumbent to the sector and will acquire the value of being an incumbent with the latest innovation of quality in some variety. If she cannot innovate then she remains an outsider with positive know-how in sector n, whose value is given by the third line in equation (21).

Given equations (20) and (21), the researcher $e \in \mathcal{E}_m$ must decide if she wants to stay in her industry, or she wants to move. Furthermore, if she decided to move, she must decide where to. This option is summarized in the following program. Given (20) and (21), the value of having the choice is

$$J^0(Q, Z, \omega|m) = \max\{J^S(Q, Z, \omega|m), \{J^M_n(Q, Z, \omega|m)\}_{n \neq m, n \in \mathcal{M}}\} \tag{22}$$

It is because of the fact that we need to compare all industries at the time in equation (22), that we need to keep track of the whole vector of aggregate qualities and cost shocks, $\{Q, Z\}$ for each value function. Computationally, M_t being finite and small, makes this problem possible to solve. Else, a distribution of industries – which is in essence an infinite dimensional object–, would make the problem too hard to solve in practice.

Outsider's Problem. The problem of the outsider entrepreneur $e \in \mathcal{E}_m$ in industry m ($\omega_m > 0$), who decides to stay in industry m is different from that of the incumbent in that she does not enjoy the flow of profits. Let $H^S(Q, Z, \omega|m)$ denote this value. Given, current research capital, qualities and shocks for the current period $\{\omega, Q, z\}$, and aggregate variables, $\{r', \lambda_m\}$, the outsider entrepreneur who decides to stay in the same sector m, chooses ω' to solve the program,

$$H^S(Q, Z, \omega|m) = \max_{\{\omega'\}} \left\{ \begin{array}{c} -Q_m(1+\sigma)^{\tilde{\lambda}_m} f^{-1}(\omega' - (1-\delta)\omega) \\ +\frac{1}{1+r'} \int_{Z'|Z} [\mu(\omega')J^0(Q', Z', \omega'|m) \\ +(1-\mu(\omega'))H^0(Q', Z', \omega'|m)]\, dF(Z') \end{array} \right\} \tag{23}$$

subject to

$$\omega \geq (1-\delta)\omega$$

There are no flow of profits, as the researcher is not an incumbent. The first line is the cost of engaging into research to expand the initial research capital ω_m. The second line is the value of being successful and the third is the value of not being able to innovate and thus, remaining an outsider in the sector. Again, the perceived laws of motion will be analogs to those in (20).

In a similar way, an entrepreneur $e \in \mathcal{E}_i$ who is an outsider to sector m and holds research capital ω_m may choose to move to another sector that promises better prospects. If he decides to move, it will be effective from next period. Given, current research capital, qualities and shocks for the current period $\{\omega, Q, Z\}$, and aggregate variable, r', the outsider entrepreneur who decides to move to n, chooses ω' to solve the program,

$$H_n^M(Q, Z, \omega|m) = \max_{\{W_n'\}} \left\{ \begin{array}{c} -Q_n' f^{-1}(\omega' - (1-\delta)(1-\phi_{mn})\omega_m) \\ +\frac{1}{1+r'} \int_{Z'|Z} [\mu(\omega')J^0(Q', Z', \omega'|n) \\ +(1-\mu(\omega'))H^0(Q', Z', \omega'|n)]\, dF(Z') \end{array} \right\} \tag{24}$$

subject to

$$\omega' \geq (1-\delta)(1-\phi_{mn})\omega$$

Finally, once the outsider has the value of moving to each of the available sectors in the country, she will choose the one that gets her the largest value,

$$H^0(Q, Z, \omega|m) = \max\{H^S(Q, Z, \omega|m), \{H_n^M(Q, Z, \omega|m)\}_{n \neq m, n \in \mathcal{M}}\} \tag{25}$$

Given equations (23), (24) and (25), the outsider researcher decides whether to persist in industry m or to move to a different sector n.

2.3.2 Entry, Exit and Innovation Aggregation

We are now in position to examine the dynamics of λ_m, the probability of being replaced by a successful innovator in the same sector. While each entrepreneur uses a perceived law of motion $\mathcal{L}^E(\cdot)$, in the definition of the recursive competitive equilibrium this is actually an equilibrium object $\mathcal{L}(Q, Z)$. Recall there is a mass of E_m entrepreneurs in sector m. Also, let $\#I_m^S$, and $\#O_m^S$ denote the mass of incumbents and outsiders, respectively, who decide to stay in sector m. Therefore, the outflow of researchers who leave sector m is given by construction by $E_m - \#I_m^S - \#O_m^S$, which will be in turn the inflow of some other sector.

$$\lambda_m(Q,Z) = (\#I_m^S)\mu(\omega'|J^S) + (\#O_m^S)\mu(\omega'|H^S) + \sum_{j\neq m} 1(E_j - \#I_j^S - \#O_j^S > 0)\mu(\omega'|H_m^M(\cdot|j)) \quad (26)$$

which is simply the average probability of innovation of those who stayed in the sector, and those who decided to become new outsiders. Note that the populations – given initial conditions – should satisfy several restrictions. For instance, the sum of outflows should be equal to the sum of inflows. We can pinpoint exactly these flows if we know the vector of future populations E'_m, to which we turn next.

Free entry condition for every industry should hold. This means that no entrepreneur should have incentive to migrate in equilibrium, because everyone who wanted to move already has. In order to solve for the population of entrepreneurs in each sector, it will be key to use a result of the equilibrium, that outsiders move before incumbents because they have more to gain of it. Actually, even when incumbents can move, all adjustment will be given by outsiders. Outsiders migration will change the value $\lambda_m(Q,Z)$ and therefor the value of moving for incumbents, until they do not wish to anymore. We require that in every sector outsider entrepreneurs do not have the incentive to move. Thus the vector $\{E'_i\}$ will be the solution (in equilibrium) to the system of equations

$$H^S(Q,Z,\omega) = H^0(Q,Z,\omega) \quad (27)$$

along with $\sum_j E_j = E$.

Proposition 1. *If an incumbent with research capital W has the incentive to move from sector m to n, then outsiders with the same level of research capital, in the same sector have a greater incentive, and move first.*

Proof. In Appendix B. □

Note that in every case (incumbent/outsider, staying/moving) I have assumed that all entrepreneurs in the same group behave identically. This is not an assumption. The absence of idiosyncratic shocks implies that a transition function for the distribution of ω' is degenerate. Every entrepreneur in the same group is identical, therefore their policy function and decisions will be identical.

3 Equilibrium

3.1 Mechanism and Intuition

Before computing the fully fledged equilibrium dynamics of the model, it is useful to distill the main mechanism. This is easier to do in a simplified version of the model in section (2). Let there only be one sector (so that we can drop the m index), let also $\xi_t = \xi$ be deterministic, the quality ladder be flat $\sigma = 0$, and the function $f(\cdot)$ be the identity function. This means that the states $\{q, \xi\}$ collapse to being parameters we do not need to keep track of in the recursive formulation of this simple case. In particular, the only state to keep track of is the stock of research capital, ω. The value of being an incumbent/outsider in $t-1$, who starts period t with capital ω are given by,

$$
\begin{aligned}
J(\omega) \quad = \quad & \max_{\omega' \geq (1-\delta)\omega} \left\{ -q[\omega' - (1-\delta)\omega] + \mu(\omega)\left[\pi + \frac{J(\omega')}{1+r}\right] \right. \\
& \left. +(1-\mu(\omega))\left[\lambda \frac{H(\omega')}{1+r} + (1-\lambda)(\pi + \frac{J(\omega')}{1+r})\right]\right\} \qquad (28)
\end{aligned}
$$

$$
\begin{aligned}
H(\omega) \quad = \quad & \max_{\omega' \geq (1-\delta)\omega} \left\{ -q[\omega' - (1-\delta)\omega] + \mu(\omega)\left[\pi + \frac{J(\omega')}{1+r}\right] \right. \\
& \left. +(1-\mu(\omega))\frac{H(\omega')}{1+r} \right\} \qquad (29)
\end{aligned}
$$

It will be helpful for further proofs to note that being an incumbent is more valuable than being an outsider.

Lemma 1. *The value of being an incumbent will be higher than the value of being an outsider, i.e.* $J(\omega) > H(\omega)$.

Proof. In Appendix B □

One of the main predictions of this model is that irreversibility of research capital will hinder previous accumulation of it. Later, I will make the argument that diversification relaxes this constraint, thereby offsetting the following result, at least partially. Note that I will be using, to get a closed form result, the simplified model of equations (28) and (29).

Proposition 2. *Irreversibility of research capital investment results in lower previous capital accumulation of research capital, ω'.*

Proof. Let the Lagrange multipliers for equation (28) and (29) be χ and $\bar{\chi}$. Momentarily focus on the problem of the incumbent in equation (28). Note the first order condition with respect to ω is given by,

$$q + \beta(1 - \lambda + \lambda\mu(\omega))J'(\omega') + \beta\lambda(1 - \mu(\omega))H'(\omega') + \chi = 0 \tag{30}$$

with $\beta = \frac{1}{1+r}$, and complementary slackness conditions, $\chi\omega' = 0$, $\chi > 0$ and $\omega' > 0$. Note that if the constraint binds and χ is positive, there is a wedge between the marginal benefit of ω' and the marginal cost q. In particular, the benefit is lower than the cost, and while the researcher would like to lower the value of ω' for next period, she is constrained from below by previous investment $(1 - \delta)\omega$. This is at the core of this paper: the very anticipation of hitting this constraint that will reduce future investment. In particular, the envelope conditions are

$$\begin{aligned}
\frac{\partial J'(\omega')}{\partial\omega'} &= (1 - \delta)q + \mu'(\omega')[\pi\lambda + \beta\lambda J(\omega'') - \lambda\beta H(\omega'')] - (1 - \delta)\chi \\
\frac{\partial H'(\omega)}{\partial\omega'} &= (1 - \delta)q + \mu'(\omega')(\pi + \beta J(\omega'')\beta H(\omega'')) - (1 - \delta)\bar{\chi}
\end{aligned}$$

which if we plug in in the first order condition, results in

$$\begin{aligned}
\frac{q}{\beta} &= (1 - \lambda + \lambda\mu(\omega))[(1 - \delta)q + \mu'(\omega')(\pi\lambda + \beta\lambda J(\omega'') - \lambda\beta H(\omega'')) - (1 - \delta)\chi'] \\
&\quad \lambda(1 - \mu(\omega))[(1 - \delta)q - \mu'(\omega')(\pi\beta J(\omega'') - \beta H(\omega'') - (1 - \delta)\bar{\chi'}]
\end{aligned}$$

By the implicit function theorem, we have that

$$\frac{\partial\mu'(\omega')}{\partial\chi'} = \frac{1 - \delta}{A + B} > 0 \tag{31}$$

with $A = \lambda(1 - \lambda + \lambda\mu(\omega))(\pi + \beta J(\omega'') - \beta H(\omega'')) > 0$ and $B = \lambda(1 - \mu(\omega))(\pi + \beta J(\omega'') - \beta H(\omega'')) > 0$ by Lemma 1 . Then, when $\chi' > 0$ the constraints starts binding. This means that the optimal level of research capital ω' decision is lower than in the case the constraint does not exist.: the anticipation of being constrained in the future, reduces equilibrium research in the present. □

Irreversibility of investment decisions has been studied in previous work, keeping in mind mostly physical productive capital, K and thus, keeping in mind production functions which exhibit constant returns to scale. Notably, Pindyck (1988), focuses on capacity choice under irreversibility highlighting the sunk cost of forfeiting the value of the option to expand at a later date (a call option). This *option value* notion is carefully expanded in Abel et al. (1996), to correct the simple net present value discount rate, not only by the call option, but the put option of reselling at a lower price the invested marginal unit. On the macroeconomic implications of irreversibility, while Olson (1979) introduces irreversibility in the representative agent model, investment dynamics are not variable enough for irrevesibility to have a bite. Bertola and Caballero (1994), on the other side argue that it is idiosyncratic (micro) irreversibilities which can help explain aggregate dynamics in the presence of important sources of idiosyncratic irreversibility. Put differently, efforts have been directed towards understanding the effects of irreversibility on volatility.

This paper takes a different direction, assessing the effect of irreversibility on the level, rather than volatility, of innovation, income and ultimately consumption. It takes this previous intuition to model a type of investment which have not yet thought about in this way, i.e. research capital, and Proposition 2 presents the main effect of this constraint on the level of research effort. But this paradigm, allows us also to address a second stylized fact in the literature: the link of volatility and the level of growth. This property is a natural result of the proposed framework and is the second major contribution of this paper, as it links second moments with the level of innovation, in a natural way that does not require assuming borrowing constraints, or any other source of financial frictions.

Proposition 3. *Higher volatility on the industry idiosyncratic shock will lower the value of holding research capital ω', and hence, slower innovation rates.*

Proof. The proof is by construction, and is straightforward. Consider equation (28) and without loss of generality, consider π^* such that $\omega' = (1-\delta)\omega + \frac{\epsilon}{2}$ and $\chi = 0$. In the assumptions of this simplification of the model we know that π is deterministic, and thus degenerate. Assume that next period, there is a mean preserving spread of the distribution of π such that it can take two values $\pi \in \{\pi^H, \pi^L\}$ with equal probability. Further, assume that this change happens in $t+1$ and never again, and also, the researcher anticipates it. If $\epsilon = |\pi^H - \pi| = |\pi^L - \pi|$, then the variance is ϵ^2. Increasing variance from zero to $\epsilon^2 > 0$ will have detrimental effects on the value of being an incumbent and investing in innovation.

The first case is such that $\pi = \pi^H$. From the first order conditions we can infer that equilibrium ω' is higher for both the incumbent and the outsider, and by equation (26), the overall level of competition is also higher as everyone is doing more research. This latter effect tames the effect of higher profits. Consider the other possibility in which $\pi = \pi^L$ and entrepreneurs know about it. Then, symmetrically they see the value of being a researcher with capital ω' decreases, and would like to disinvest. However, the constraint $\omega' \geq (1-\delta)\omega$ binds and prevents research capital to plummet. Researchers want to cut investment but they cannot. Put differently, good shocks are vanished by free entry, and bad shocks have to be endured. Raising the volatility – even if it is mean preserving – only raises the left tail risk of the innovation process. It affects asymmetrically the behavior of the researcher. Volatility lowers the value of research without assuming credit constraints as in Aghion et al. (2005). This is the second important message in this paper; volatility will affect growth but will affect it less the less severe the irreversibility is. This will happen when there is a way to exit the industry by moving to a sister industry, namely when the economy grows in diversity. □

3.2 Competitive Equilibrium

Having introduced the main components of the model we can proceed to examine the definition of equilibrium and examine the existence and characterization of an expected balanced growth path (EBGP) for this economy.

Definition 1 (Equilibrium). *A competitive equilibrium for this economy with innovation and inventor mobility, is aggregate allocations* $\{c_t, a_t, Y_t\}_{t=0}^{\infty}$, *sectoral allocations* $\{y_{m,t}\}_{m\in\mathcal{M}_t,t=0}^{\infty}$, $\{\ell_{m,i,t}, x_{m,i,t}\}_{i\in[0,1],m\in\mathcal{M}_t,t=0}^{\infty}$, *profits* $\{\pi_{m,t}\}_{m\in\mathcal{M}_t,t=0}^{\infty}$, *prices* $\{r_t, w_t, p_{m,t}, p_{m,i,t}^x\}_{i\in[0,1],m\in\mathcal{M}_t,t=0}^{\infty}$, *distribution of investors population* $\{E_{m,t}\}_{t=0,m\in\mathcal{M}_t}^{\infty}$ *value functions* $\{J^S(\cdot|m), H^S(\cdot|m), J_n^M(\cdot|m), H^M{}_n(\cdot|m)\}_{n\in\mathcal{M}_t}$, *and qualities and probabilities* $\{Q_m, \Lambda(\cdot|m)\}_{t=0,m\in\mathcal{M}_t}^{\infty}$, *such that*

1. Given $\{w_t, r_t, \Lambda_t, \bar{\Pi}_t\}$, and initial assets a_0, the consumer chooses $\{c_t, a_{t+1}\}_{t+0}^{\infty}$ to solve (3) subject to (4)

2. Given $\{p_{m,i,t}^x\}_{i\in[0,1],m\in\mathcal{M}_t,t=0}^{\infty}$, the final good producer of sectoral output, $y_{m,t}$ demands each variety of $\{x_{m,i,t}^D\}_{i\in[0,1]}$ to solve program (8) in every t

3. Given $\{w_t\}$ and $\{\xi_{m,t}, q_{m,i,t}\}$ the producer of $x_{m,i,t}^S$ sets price $p_{m,i,t}^x$, $\forall m, i, t$ according to Bertrand Monopolistic Competition, according to (13), and earning profits in (14)

4. Given $\{\lambda_m = \mathcal{L}^E(Q, Z|m)\}_{m\in\mathcal{M}_t}$ and perceived laws of motion for $\{Q, Z\}$, then values $\{J^S, H^S, J_n^M, H_n^M\}$, $\forall n \in \mathcal{M}_t$ and policy functions $\{\mathcal{G}_m\}, \forall m \in \mathcal{M}$, solve problems (20), (21), (22), (23), (24) and (25), subject to the pertinent irreversibility constraints in each case.

5. Markets clear

$$x^D_{m,i,t} = x^S_{m,i,t}, \qquad \forall m, i, t \tag{32}$$

$$\sum_{m\in\mathcal{M}_t} \int_0^1 \ell_{m,i,t}\mathrm{d}i = L \tag{33}$$

$$a_{t+1} = 0 \qquad \forall t \tag{34}$$

6. Consistency is required. Therefore, the perceived laws of motion for $\{Q_m, \xi_m\}$, $\forall m \in \mathcal{M}_t$ are in fact

$$\xi'_m = \rho_\xi \xi_m + \sigma_\xi \epsilon, \qquad \forall m \in \mathcal{M} \tag{35}$$

$$\log Q'_m = \lambda_m \log(1+\sigma) + \log Q_m, \qquad \forall m \in \mathcal{M} \tag{36}$$

$$\mathcal{L}^E(Q, Z) = \mathcal{L}(Q, Z) \tag{37}$$

where $\mathcal{L}(Q, Z)$ is pinned down by the free entry condition (26)

7. Share of entrepreneurs are given by (27)

Definition 2 (Expected Balanced Growth Path). *The economy is in balanced growth path (EBGP) in period τ, if it is in a trajectory such that, $\forall t > \tau$, aggregate variables $\{c_t, w_t, Y_t, \{y_{m,t}\}_{m\in\mathcal{M}_t}\}_{t>\tau}$ grow at a constant average common rate, prices $\{\{P_{m,t}\}_{m\in\mathcal{M}_t}, r_t\}_{t>\tau}$, probabilities $\{\tilde{\lambda}_{m,t}\}_{m\in\mathcal{M}_t,t>\tau}$ and size of populations $\{E_{m,t}\}_{m\in\mathcal{M}_t,t>\tau}$ are stationary around a constant.*

This economy lacks a balance growth path. The fundamental reason for it is that it lacks a continuum of sectors which average out, and on which dimension we can invoke a law of large numbers. Instead, sectoral stationary shocks will hit this economy. For any variable which would be a constant in the BGP, $x_t = x$, the expected balanced growth path definition revolves around $\bar{x} = \lim_{b\to\infty} \frac{1}{b}\sum_{\tau=t}^{t+b} x_\tau = x$. It can be shown that most of the markovian stability notions are extended to this case (see Stokey and Lucas (1989), chapter 11 & 12).

The system of equations which represents the economy in EBGP is presented in detail in Appendix C for the interested reader. However, some insights are worth being mentioned. First, note that this economy resembles a collection of multisector Aghion-Howitt economies, in which the innovation rate is determined in each sector by the distribution of research capital $\{\lambda_m\}_{m\in\mathcal{M}_t}$, in particular by equation (26). While this innovation rate is not constant, the average of a finite sequence $\{\lambda_m^j\}_{j=1}^N$ of size N is, i.e. $\bar{\lambda}_m = \frac{1}{N}\sum_{\tau=t}^{t+N}\lambda_{m,\tau}$. Second, in the long run we require that all sectors m grow at the same rate. This means $\bar{\lambda}_m = \bar{\lambda}_n$, for $n \neq m$. If not, the relative size of one sector with respect to the whole economy will diverge and become negligible.

The system of equations characterizing equilibrium in Appendix C are all expressed in stationary notation. The growth rate of the economy is equal to the growth rate of Q, where we know $Q = \left(\sum_m Q_M^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}$. At the same time, the growth rate of Q_m can be calculated by noting that $\log Q_m' = \int_0^1 \log q_{m,i,t}' di$, which implies that the growth rate $g_{m,t}$ of every sector in each period of time is equal to

$$(1 + g_{m,t}) = \lambda_{m,t}\log(1+\sigma), \qquad \forall m$$

Since the average innovation rate is $\bar{\lambda}_m$, then the average of a sequence of sectoral growth also converges to a constant $\bar{g}_m$, such that $\bar{g} = \bar{g}_m$. This completes the description of the equilibrium of this economy and its main predictions. Next section investigates further how valid these predictions are in the data.

4 Empirical Analysis

Two main predictions distill from this model. First, diverse economies provide more incentives to innovation and therefore are more innovative. This must be true at the inventor level. Second, volatility hinders growth and innovation but it does so more in less diverse economies. In this section I will use data from patent and inventor applications to test these predictions. I will also consider macro data to investigate the traction of this mechanism at the aggregate level.

4.1 Data Sources

The main sources of information are the NBER-USPTO Utility Patents Grand Data. The United States Patent and Trademark Office grants a property right on those inventors applying for patent protection. It gives the holder a temporal right to sell or use of her idea. It is up to the patent holder to enforce her own rights if she is granted a patent. The main types of patents are (a) Utility patents, (b) Design patents, and (c) Plant patents. Utility patents are those we think of when we think about economic growth. These are issued for the invention of a "new and useful process, machine, manufacture, or composition of matter...", and cover around 90% of the patent documents issued by the USPTO. Each of these applications also contains the IPC (International Patent Classification) code which pins each invention down to a technology class.

The second source of information is the Disambiguated Inventor Data (DID) by Lai et al. (2012) which contains data on around 3.1 million inventors for the period 1975-2010. It also contains data con cross citations of patents. If patent A cites patent B, then we observe a forward citation from A to B, and backward citation from B to A.

For the macro level data, I use data from PATSTAT, also known as the EPO Worldwide Patent Statistical Database, which has the widest availability of country offices registering patent filing. While there are many more countries with many patents in the PATSTAT dataset which are not present in the USPTO, the latter has more complete information on the countries it contains. Thus, not any dataset dominates the other.

Finally I consider data on diversification from Hausmann et al. (2013). These authors, consider UN Comtrade data to construct an index that captures both, how many products an economy produces, and how difficult these are to produce; diversity and ubiquity. They label their index the "economic complexity index", which I will use as a proxy of diversification in my empirical analysis. These data is available since 1975 to 2013.

4.2 Evidence from micro data: inventors' data

In this section I will try to show that the predictions of the model are present in the data. For these I will use inventor level data and track down each invention they did. Data for inventors comes from the Lai et al. (2013) who build on data from " NBER Patent Data Project". This data contains the history of more than two million inventors, and almost seven million patents that can be traced back to authors (see Table 1). Inventors in more complex economies behave differently than their counterparts in simpler economies. In particular, looking at micro evidence we can see that,

Table 1: Productivity & Diversity (I). Inventors are more productive in complex economies. High(Low) complexity group is defined as countries in percentiles 75 to 95 (05 to 25), in the 1980 measure of the ECI. Serial inventor sample is the truncation of inventors to those with more than one patent

	Number	*Proportion (%)*	*High diversification*	*Low diversification*
Total Inventors	2107420	100		
Once in a lifetime inventors	1139231	54.06	54.64	64.41
Inventions per inventor	3.3	100	3.21	2.11
Inventions per serial-inventor	6.0		5.89	4.13

Empirical fact 1: *Inventors in more diverse economies are more innovative. (Propositions 2)*

Inventors are, on average, more productive in a diverse economy. In table (1) some summary statistics are presented, which show that inventors in the top 75-95 percentile range of cross-country diversity, are more innovative than those in the 5-25 percentile. While an average inventor produces 2.1 patents in the latter, she produces 3.21 in the former group. If we truncate the sample to those inventors who have filed at least two applications, the number of patents goes from 4.1 to 5.9 patents on average in low and high diversified economies, respectively.

For a more detailed analysis of productivity let us consider table (2), which regresses the (log) number of inventions of an inventor in her lifetime with the diversification and volatility of the economy where she resided at the time of invention, and other controls for scale and initial wealth. Volatility is calculated in two different ways. First, using the standard deviation of the PPP GDP growth rate of the 1975-2011 sample for each country. Second, following Neumeyer and Perri (2005) I use a time variant measure of deviation from trend. The sample size is about 2 million inventors. The effect of higher diversity (ECI index) ranges from inventors being from 13 to 22 percent more productive in terms of the number of patents they produce in a lifetime when the ECI index goes up by one unit. This amounts to going from being as diverse as Chile to being as diverse as Canada. Volatility, on the other hand is detrimental. However, the effect is ameliorated by the interaction of diversification.

Empirical fact 2: *Volatility reduces innovation, but does so more in less diverse economies. (Propositions 3)*

Table 2: Productivity & Complexity II. Observations are at inventor level, dependent variable is the number of innovations per inventor (log). Volatility measures are the standard deviation of yearly PPP GDP and the Neumeyer & Perri (2005) measure of country volatility. Standard errors in parentheses, $**p < 0.01, *p < 0.05, +p < 0.1$

	(1)	(2)	(3)	(4)	(5)
Diversification (*)	0.217**	-0.012*	0.011*	0.033**	0.051**
	(0.001)	(0.005)	(0.005)	(0.004)	(0.004)
Volatility measure 1		-2.906**	-2.800**		
		(0.160)	(0.160)		
Volatility measure 2				-1.570**	-1.545**
				(0.083)	(0.083)
*Diversification * Volatility 1*		5.038**	4.572**		
		(0.113)	(0.113)		
*Diversification * Volatility 2*				2.166**	1.980**
				(0.054)	(0.054)
Employment (log)		0.063**	0.058**	0.058**	0.053**
		(0.001)	(0.001)	(0.001)	(0.001)
Per capita GDP		0.065**	0.061**	0.065**	0.061**
		(0.003)	(0.003)	(0.003)	(0.003)
Constant	0.216**	-0.494**	-0.589**	-0.480**	-0.571**
	(0.003)	(0.030)	(0.030)	(0.030)	(0.030)
Year dummies	No	No	Yes	No	Yes
Observations	2,024,284	2,010,802	2,010,802	2,010,802	2,010,802
R-squared	0.011	0.015	0.018	0.014	0.017

Empirical fact 3: *There is a higher probability that an inventor switches sectors (classified by IPC2 industries) in a more diverse economies.*

Empirical fact 3, is not actually testing for a prediction of the model but for the assumption that inventors will switch more often in more diverse economies because they observer smaller intellectual distances to transit. That is, diversification of an economy, largely determines average intellectual distances, and the difficulty to move sectors. The decision to move is binary, and so a discrete response model is estimated to assess marginal probabilities. Fact 3 can be seen in Table (3) which models the probability of an inventor changing sectors at the IPC2 level. The observations are at inventor-patent level. That is, there is one observation for every patent of every inventor. In this exercise I control for overall inventor productivity, scale effect, initial wealth, education and trade openness of the economies these inventors reside in. I also exclude super-star inventors, defined as those who are on the top 5% of number of patents produced, in the last two columns.

Table 3: Productivity & Complexity III. Logit regression. $\mathbb{P}$(Inventor changed IPC2 sector from last invention$= 1|X = x$). Observations are at inventor/patent level. Volatility measures are the standard deviation of yearly PPP GDP, and the Neumeyer & Perri (2005) measure of country volatility. Standard errors in parentheses,$**p < 0.01, *p < 0.05, +p < 0.1$ Italics are marginal effects evaluated at the mean of regressors.

	(1) *All*	(2) *All*	(3) *All*	(4) *Exclude Superstars*	(5) *Exclude Superstars*
Diversification	0.020**	0.021**	0.034**	0.012*	0.023**
	(0.003)	(0.006)	(0.006)	(0.006)	(0.007)
	0.0047	*0.0049*	*0.0078*	*0.0029*	*0.0053*
Volatility measure 1		1.710**		1.911**	
		(0.564)		(0.574)	
		0.4031		*0.4495*	
*Diversification * Volatility 1*		0.014		-0.167	
		(0.319)		(0.326)	
		0.0032		*-0.0392*	
Volatility measure 2			3.064**		3.070**
			(0.578)		(0.589)
			0.7182		*0.7173*
*Diversification * Volatility 2*			-0.822*		-0.851*
			(0.341)		(0.349)
			-0.1913		*-0.1975*
Number of innovations (log)	0.014**	0.014**	0.014**	-0.005**	-0.005**
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Employment (log)	-0.018**	-0.014**	-0.015**	-0.008**	-0.008**
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
GDP per capita 1990	0.001	0.029**	0.032**	0.053**	0.056**
	(0.007)	(0.007)	(0.007)	(0.007)	(0.007)
Human Capital (Barro Lee)	0.032**	0.012+	0.010	0.008	0.005
	(0.006)	(0.007)	(0.007)	(0.007)	(0.007)
Trade Openness	-0.001**	-0.001**	-0.001**	-0.001**	-0.001**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Constant	0.327	0.032	-0.016	-0.201	-0.249
	(0.295)	(0.298)	(0.298)	(0.298)	(0.298)
Yearly Dummies	Yes	Yes	Yes	Yes	Yes
Observations	4,601,873	4,599,487	4,601,297	4,281,097	4,282,850
Pearson p-value	0.00	0.00	0.00	0.00	0.00
Hosmer-Lemeshow p-value	0.00	0.00	0.00	0.00	0.00

In Table (3), the third line under the estimation of the logit model is the marginal effect of such variable evaluated at the mean of other regressors. There are two things to learn from this table. First, volatility affects positively the probability of changing sectors. In particular, a 1% increase in output growth volatility implies roughly that one out of 4 inventors will switch sectors. Second, diversification also implies higher mobility. Marginal probabilities imply that a 1 out of 10 Turkish inventors who would not switch sector, will do so if they were in Canada. The interaction of these two variables generates a negative coefficient. We learn that if volatility increases, the effect on higher mobility of inventors is lower if diversification is already high. Presumably, because high diversification already implies high mobility. A different interpretation is that higher diversification raises mobility less in highly volatile economies, presumably because volatile economies already experience high sector mobility of inventors to begin with.

4.3 Evidence from macro data

How important are the predictions of this model at the macro level? Even if diversity has traction at the inventor level, how do these numbers add up at the aggregate level. Let us consider aggregate innovation rates at country level. Figure 6 shows in the vertical axis the logarithm of the number of patents per worker in eight different industries, defined by the first disaggregation of the International Patent Classification (IPC-1), administered by the World Intellectual Property Organization (WIPO). This data is from the European Patent Office (PATSTAT) and is the most comprehensive data of innovation for cross-country comparisons. Innovation per worker is contrasted with the standard deviation of output growth in the 1975-2011 period (from PWT) and with the ECI. Diversity is (strongly) positively related to innovation, and this latter one is in turn, negatively related to volatility. While figure 6 uses year 1993 for the ECI, any base year will preserve the main fact: the more complex an economy, the higher the innovation rate. At the same time, the less volatile the economy, the higher the innovation rate. To consider how diversification evolves in this empirical fact, let us consider Table 4 .

We can formalize this graphical analysis. In Table 4 the same message is delivered. The first four regressions regress the log of patents per worker, for country i in year t to diversification and output volatility, specific to each country. The final column uses country level averages to elicit the cross sectional variation component. In these regressions I control also for initial income and domestic credit as a ratio of GDP, to make this evidence comparable to Aghion et al. (2005) who claim this latter variable is a key driver of the joint dynamics of output growth and volatility relation. Notably, volatility hurts innovation and is robust to all specifications, but so is diversification, with a different sign. Furthermore

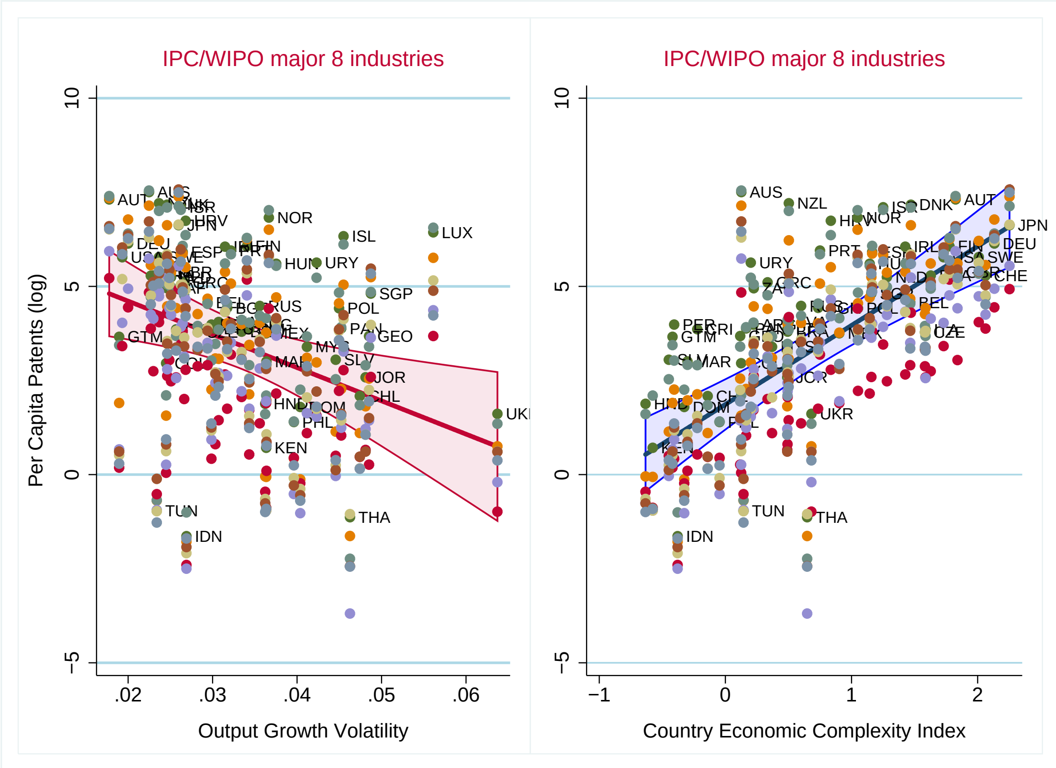

Figure 6: Patent creation per employee vs. **(a)** Output Growth Volatility and **(b)** Economic Complexity. Each country has eight observations in the scatter plot, for the eight IPC1 categories which are: Human necessities, Transportation, Chemistry/ Metallurgy, Textile/Paper, Fixed Construction, Mechanical/Engineering, Physics, Electricity. Data sources are Patstat Online, The Atlas of Economic Complexity and Penn World Tables

it appears that complexity's role is more important when volatility is higher as the coefficients of the interaction of both variables, is too, positive and significant. The analysis survives the introduction of a proxy to financial development, namely domestic credit as a ratio to GDP. That is, the story in this paper is at least, complementary to the story in Aghion et al. (2005), and both channels may well be operative simultaneously.

5 Conclusions

In this paper I have developed the idea that diversification of an economy has a dominant role in determining its prosperity. In particular, I argue that diversifi-

Table 4: Innovation per worker: Per worker granted patents, 1975-2011 from European Patent Office (PATSTAT data). Countries which are not included are those who did not submit patenting information. First three specifications are based on country-year observations, while specification 4 is based on cross country observations. Restricted to Utility Models. Standard errors in parenthesis, $***p < 0.01, **p < 0.05, *p < 0.1$.

	(1)	(2)	(3)	(4)	(5)
Diversification	1.840***	1.784***	0.827***	0.575***	2.167**
	(0.074)	(0.080)	(0.161)	(0.158)	(1.074)
Volatility	-0.237***	-0.251***	-0.482***	-0.484***	-0.774**
	(0.039)	(0.041)	(0.052)	(0.051)	(0.316)
*Diversification * Volatility*			0.339***	0.365***	0.052
			(0.050)	(0.048)	(0.328)
Domestic Credit /GDP		0.002	0.003**	0.009***	-0.006
		(0.001)	(0.001)	(0.001)	(0.009)
Initial Income	-0.156***	-0.176***	-0.169***	-0.213***	-0.354*
	(0.046)	(0.047)	(0.047)	(0.045)	(0.195)
Constant	4.993***	5.187***	5.777***	6.184***	8.663***
	(0.542)	(0.555)	(0.552)	(0.704)	(2.644)
Year dummies	No	No	No	Yes	NA
Observations	1,316	1,272	1,272	1,272	54
R-squared	0.400	0.401	0.422	0.478	0.551

cation raises the value of innovation and therefore implies higher rates of research effort. In order to do so, I construct a multi-sector model of endogenous growth with inventor mobility across sectors which also captures the fact that (a) investment in R&D is the most irreversible form of investment, (b) innovations are the result of cumulative research rather than an instantaneous process. Inventors build up research capital along time. The main mechanism in the model is as follows. Irreversibility of research capital, coupled with volatility of profits generates a pervasive effect in non-diversified economies. When a bad shock hits the entrepreneur she has no way to undo previous capital investment. In a diversified economy, she cannot either, but she can transport partially her cumulative research capital to a new more promising sector. Anticipation of this situation in low-diversification economies, makes entrepreneurs behave rationally and curtail R&D investment today, in the face of a possibility of having ended up over-invested. I provide empirical support both, at the micro level and the macro level data. On the micro side I use information on patent-inventor data set from the NBER-USTPO Utility

Patent Grand Data, and the Atlas of Economic Complexity (to proxy diversification). I find support of the main predictions of my model: diversification enhances innovation and volatility hinders it.

Appendix

A: Derivations

A1: Preferences

The problem of the consumer is given by program (3) subject to budget constraint (4). Given prices r_t, w_t, transfers Λ_t, and profits $\bar{\Pi}_t$ the constrained optimization problem is

$$\mathfrak{L} = \sum_{t=0}^{\infty} \beta^t u(c_t) + \vartheta_t\{(1 + r_t)a_t + w_t L_t + \bar{\Pi}_t\}$$

from which we obtain

$$\beta(1 + r_{t+1}) = \left(\frac{c_{t+1}}{c_t}\right)^\gamma$$

In BGP, we know that c_t grows at the constant economy rate $1 + g$ and therefore we can pin down the interest rate along the balance trajectory: $r_{t+1} = \frac{(1+g)^\gamma}{\beta} - 1$.

A2: Technology

From the main text we know that there are three levels of production; final good, (M_t) sectoral final goods, and a continuum of intermediate goods for each sector. The final good firm produces Y_t which is ready to be consumed and, is therefore in the same units as c_t. Its price is normalized to one, $p_t = 1$. I will first characterize the demand for each of these markets, then interact them with the production technologies and finally impose market clearing in each market, and the labor market.

The final good firm operates in perfect competition, aggregates the sectoral final goods in a basket, and sells it competitively to the consumer. In particular, given the price of final goods in each sector m, $\{p_{m,t}\}_{m\in\mathcal{M}_t}$ the final good producer chooses $\{y_{m,t}\}$ to solve the following program,

$$\max_{\{y_m\}_{m\in\mathcal{M}}\geq 0} \left(\sum_m y_{m,t}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} - \sum_m p_{m,t} y_{m,t}$$

which yields the first order condition that determines the demand for each sectoral final good $y^D_{m,t}$

$$y^D_{m,t} = Y_t \left(\frac{1}{p_{m,t}} \right)^{\varepsilon}$$

We can learn more about if we calculate total cost of assembling the final good.

$$\begin{aligned}\sum_m y_{m,t} p_{m,t} &= \sum_m Y_t^{\frac{1}{\varepsilon}} y_{m,t}^{1-\frac{1}{\varepsilon}} \\ &= Y_t^{\frac{1}{\varepsilon}} \left(\sum_m y_{m,t}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1} \frac{\varepsilon-1}{\varepsilon}} \\ &= Y_t\end{aligned}$$

which will be handy when we calculate equilibrium wages, w_t.

Next, let us characterize the demand for intermediate goods $x^D_{m,i,t}$. Given sequences of prices $\{p_{m,t}\}$, $\{p_{m,i,t}\}$ the sectoral final good producer chooses $\{x_{m,i,t}\}_{i \in [0,1]}$ to solve the program,

$$\max_{\{x_{m,i,t}\}_{i\in[0,1]} \geq 0} p_{m,t} \exp \left(\int_0^1 \log x_{m,i,t} \mathrm{d}i \right) - \int_0^1 p_{m,i,t} x_{m,i,t} \mathrm{d}i$$

from which we can solve the first order condition to find an expression for demand of intermediate goods, $x^D_{m,i,t}$,

$$x^D_{m,i,t} = \left(\frac{p_{m,t}}{p_{m,i,t}} \right) y_{m,t}$$

finally, the demand for labor will be pinned down by the equilibrium production of variety $x_{m,i,t}$,

$$\ell^D_{m,i,t} = \frac{x_{m,i,t}}{q_{m,i,t}} - \xi_{m,t}$$

Let us start to introduce supply functions and market clearings. First, in the sectoral final good market we know that pricing is not competitive, but instead Bertrand pricing will pin down the level of production. The incumbent will price in a way such that the owner of an older vintage does not want to compete. The closest follower owner will not compete if the marginal cost of producing is lower than $w_t/\tilde{q}_{m,i,t}$ where $\tilde{q}_{m,i,t}$ is the quality of the best follower. Then pricing is given by equation (13). Plugging this result in $x^D_{m,i,t}$ yields equilibrium production of intermediate good $x_{m,i,t} = \frac{p_{m,t} y_{m,t}}{(1+\sigma) w_t} q_{m,i,t}$.

Note that we can plug in the previous result on the labor demand function

$\ell^D_{m,i,t}$, which results in

$$\ell^D_{m,i,t} = \frac{p_{m,t}y_{m,t}}{(1+\sigma)w_t} - \xi_{m,t}$$

which is the same across varieties. This way we can aggregate across varieties and across sectors to get,

$$\sum_m \int_0^1 \ell_{m,i,t}\mathrm{d}i = \sum_m \frac{p_{m,t}y_{m,t}}{(1+\sigma)w_t} - \sum_m \xi_{m,t}$$

Using market clearing for the labor market, we can then obtain an expression for wages

$$\begin{aligned} w_t &= \frac{\sum_m p_{m,t}y_{m,t}}{(1+\sigma)(1+\sum_m \xi_{m,t})} \\ &= \frac{Y_t}{(1+\sigma)(1+\sum_m \xi_{m,t})} \end{aligned}$$

We have by now equilibrium expressions for $x_{m,i,t}$ and w_t. We can calculate the equilibrium quantity of sectoral final output, $y_{m,t}$. For this, define average sectoral quality by $\log Q_{m,t} = \int_0^1 \log q_{m,i,t}\mathrm{d}i$. Use equation (7) and the solution for $x_{m,i,t}$ to obtain

$$p_{m,t} = \frac{(1+\sigma)w_t}{Q_{m,t}}$$

which can be further plugged in the demand for $y_{m,t}$ to obtain

$$y_{m,t} = Y_t \left[\frac{Q_{m,t}(1+\sum_m \xi_{m,t}}{Y})\right]^{\varepsilon}$$

and finally, we can obtain two expressions for sectoral final output and final output that are functions only of state variables,

$$Y_t = \left(1 + \sum_m \xi_{m,t}\right) Q_t \tag{38}$$

$$y_{m,t} = \left(1 + \sum_m \xi_{m,t}\right) Q_t^{1-\varepsilon} Q_{m,t}^{\varepsilon} \tag{39}$$

where the economy-wide average quality is defined by $Q_t = \left(\sum_m Q_{m,t}^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}$

Finally, we can inform our profit function with all these equilibrium results, and obtain equation (14).

B: Proofs

Proposition 1. *If an incumbent with research capital ω has the incentive to move from sector m to n, then outsiders with the same level of research capital, in the same sector have a greater incentive, and move first.*

Proof. Define A to be the set of all research capital values, $\{\omega_m\}$ for which it is convenient to move from m to sector v, given that the entrepreneur is an incumbent. That is

$$A = \{\omega_m \in \mathcal{W} : J_v^M(Q, Z, \omega|m) > J^S(Q, Z, \omega|m)\}$$

Define, as well, the set of all values of research capital for which it is convenient to move to sector v from sector m, if the entrepreneur is an outsider rather than an incumbent,

$$\tilde{A} = \{\omega_m \in \mathcal{W} : H_v^M(Q, Z, \omega|m) > H^S(Q, Z, \omega|m)\}$$

It is sufficient to prove that $A \subseteq \tilde{A}$. Let notation be,

$$\omega_1^* = \arg\max J_v^M(\omega'; \omega) \tag{40}$$
$$\omega_2^* = \arg\max J_m^S(\omega'; \omega) \tag{41}$$
$$\omega_3^* = \arg\max H_v^M(\omega'; \omega) \tag{42}$$
$$\omega_4^* = \arg\max H_m^M(\omega'; \omega) \tag{43}$$

Note that because of the independence of profits of research capital, $\omega_1^* = \omega_3^*$. By definition, we have that,

$$\mu(\omega_1^*)EJ_v^0(\omega_1^*) + (1-\mu(\omega_1^*))EH_v^0(\omega_1^*) - Q'_m f^{-1}(\omega_1^* - (1-\delta)(1-\phi)\omega) \quad >$$
$$(1-\lambda+\mu(\omega_2^*)\lambda)EJ_m^0(\omega_2^*) + (1-\mu(\omega_2^*))\lambda EH_m^0(\omega_2^*) - Q'_m f^{-1}(\omega_2^* - (1-\delta)\omega)$$

and we need to show that

$$\mu(\omega_3^*)EJ_v^0(\omega_3^*) + (1-\mu(\omega_3^*))EH_v^0(\omega_3^*) - Q'_m f^{-1}(\omega_3^* - (1-\delta)(1-\phi)\omega) \quad >$$
$$\mu(\omega_4^*)EJ_v^0(\omega_4^*) + (1-\mu(\omega_4^*))EH_v^0(\omega_4^*) - Q'_m f^{-1}(\omega_4^* - (1-\delta)(1-\phi)\omega)$$

Noting the properties of a maximum, and $\lambda > 0$ it is straightforward to see that

$$\begin{aligned}
(1-\lambda+\mu(\omega_2^*)\lambda)EJ_m^0(\omega_2^*) + (1-\mu(\omega_2^*))\lambda EH_m^0(\omega_2^*) - Q'_m f^{-1}(\omega_2^* - (1-\delta)\omega) &\geq \\
(1-\lambda+\mu(\omega_4^*)\lambda)EJ_m^0(\omega_4^*) + (1-\mu(\omega_4^*))\lambda EH_m^0(\omega_4^*) - Q'_m f^{-1}(\omega_4^* - (1-\delta)\omega) &> \\
\mu(\omega_4^*)EJ_v^0(\omega_4^*) + (1-\mu(\omega_4^*))EH_v^0(\omega_4^*) - Q(1+\sigma)f^{-1}(\omega_4^* - (1-\delta)(1-\phi)\omega) &= \\
H_m^S(Q,Z,\omega|m) &
\end{aligned}$$

Then, $\omega \in A$ implies $\omega \in \tilde{A}$. That is $A \subseteq \tilde{A}$. Outsiders have a greater incentive to move first. □

Lemma 1. *The value of being an incumbent will be higher than the value of being an outsider, i.e.* $J(\omega) > H(\omega)$.

Proof. Without loss of generality consider $\omega = \bar{\omega}$, for an arbitrary $\bar{\omega}$. Also, for equations (28) and (29) consider $\omega^* = \text{argmax}\, J(Q,Z,\omega;\omega')$ and $\hat{\omega} = \text{argmax}\, H(Q,Z,\omega;\omega')$. Then,

$$\begin{aligned}
J(\omega) &= -q[\omega^* - (1-\delta_\omega)\omega] + \mu(\omega)\left[\pi + \frac{1}{1+r}J(\omega^*)\right] \\
&\quad +(1-\mu(\omega))\left[\lambda\frac{1}{1+r}H(\omega^*) + (1-\lambda)(\pi + \frac{1}{1+r}J(\omega^*)\right] \\
&\geq -q[\hat{\omega} - (1-\delta_\omega)\omega] + \mu(\omega)\left[\pi + \frac{1}{1+r}J(\hat{\omega})\right] \\
&\quad +(1-\mu(\omega))\left[\lambda\frac{1}{1+r}H(\hat{\omega}) + (1-\lambda)(\pi + \frac{1}{1+r}J(\hat{\omega})\right] \\
&= H(\omega) + (1-\mu(\omega))(1-\lambda)\left[\pi + \frac{1}{1+r}J(\hat{\omega}) + \frac{1}{1+r}H(\hat{\omega})\right] \\
&> H(\omega) + \frac{(1-\mu(\omega))(1-\lambda)}{1+r}\left[\frac{1}{1+r}J(\hat{\omega}) + \frac{1}{1+r}H(\hat{\omega})\right] \qquad (44)
\end{aligned}$$

Then, we know that it must be the case that $J(\omega) - H(\omega) > \frac{(1-\mu(\omega))(1-\lambda)}{1+r}[J(\hat{\omega}) - H(\hat{\omega})]$. Consider the first case in which $J(\hat{\omega}) - H(\hat{\omega}) > 0$, which then results in $J(\omega) - H(\omega) > 0$ as required. In the alternative case we could assume that $J(\hat{\omega}) - H(\hat{\omega}) < 0$. Then, it is also useful to consider the sequence conformed by the optimal policy function result for $H(\cdot)$: $\{\omega_i\}_{i=1}^N = \{\omega_1 = \omega, \omega_2 = \hat{\omega}, \omega_3 = \omega'(\omega_2), \omega_4 = \omega'(\omega_3), \ldots, \omega_N\}$. Then we know from (44) that

$$J(\omega_1) - H(\omega_1) > \frac{(1-\mu(W))(1-\lambda)}{1+r}\left[J(\omega_2) - H(\omega_2)\right]$$
$$J(\hat{W}) - H(\hat{W}) > \frac{(1-\mu(W))(1-\lambda)}{1+r}\left[J(\omega_3) - H(\omega_3)\right]$$

In particular, for $A_i < \infty$ (which we know holds),

$$\begin{aligned}
J(\omega_2) - H(\omega_2) &= \frac{(1-\mu(W))(1-\lambda)}{1+r}\left[J(\omega_3) - H(\omega_3)\right] + A_2 \\
&= \frac{(1-\mu(W))(1-\lambda)}{1+r}\left[\frac{(1-\mu(W))(1-\lambda)}{1+r}[J(\omega_4) - H(\omega_4)] + A_3\right] + A_2 \\
&= \left(\frac{(1-\mu(W))(1-\lambda)}{1+r}\right)^{N-2}\left[J(\omega_N) - H(\omega_N)\right] \\
&\quad + \sum_{i=2}^{N-1}\left(\frac{(1-\mu(W))(1-\lambda)}{1+r}\right)^{i-2} A_i \\
&> \left(\frac{(1-\mu(W))(1-\lambda)}{1+r}\right)^{N-2}\left[J(\omega_N) - H(\omega_N)\right]
\end{aligned}$$

The right hand side goes to zero as N goes to infinity, as $\frac{(1-\mu(W))(1-\lambda)}{1+r} \in (0,1)$. Then we get that $J(\omega_2) > H(\omega_2)$, which contradicts the premise of the second case, therefore we can conclude that $J(\omega_1) = J(\omega) > H(\omega) = H(\omega_1)$ □

C: System in BGP

The stationary system is[6],

$$
\begin{aligned}
(r_{t+1})_{bgp} &= \frac{(1+g)^{\gamma}}{\beta} - 1 \\
\left(\frac{w_t}{Y_t}\right)_{bgp} &= \frac{1}{1+\sigma}\left[\frac{\sum\left(\frac{Q_m}{Y_t}\right)^{\varepsilon-1}}{1+\sum \xi_m}\right]^{1/\varepsilon} \\
\left(\frac{\pi_{m,t}}{Y_t}\right)_{bgp} &= \frac{\sigma}{(1+\sigma)^{1+\varepsilon}}\left[\frac{w_t}{Q_{m,t}}\right]^{-\varepsilon} + \left(\frac{w_t}{Y_t}\right)\xi_{m,t} \\
\left(\frac{c_t}{Y_t}\right)_{bgp} &= 1 \\
\left(\frac{\mathbf{J^S}(W_m)}{Y_t}\right)_{bgp} &= \max_{W'_m}\left\{\frac{\pi}{Y_t} - \frac{Q_m}{Y_t}((1+\sigma)^{\lambda} f^{-1}(\cdot)\right. \\
&+ \left.\frac{1+g}{1+r}\mathbb{E}\left[(1-\tilde{\lambda}_m+\mu_m\tilde{\lambda}_m)\frac{\mathbf{J^0}(W'_m)}{Y_{t+1}} + (1-\mu_m)\tilde{\lambda}\frac{\mathbf{H^0}(W'_m)}{Y_{t+1}}\right]\right\} \\
\left(\frac{\mathbf{J^M_m}(W_m)}{Y_t}\right)_{bgp} &= \max_{W'_n}\left\{\frac{\pi}{Y_t} - \frac{Q_n}{Y_t}((1+\sigma)^{\lambda} f^{-1}(\cdot)\right. \\
&+ \left.\frac{1+g}{1+r}\mathbb{E}\left[\mu_n\frac{\mathbf{J^0}(W'_m)}{Y_{t+1}} + (1-\mu_n)\frac{\mathbf{H^0}(W'_m)}{Y_{t+1}}\right]\right\} \\
\left(\frac{\mathbf{H^S}(W_m)}{Y_t}\right)_{bgp} &= \max_{W'_m}\left\{-\frac{Q_m}{Y_t}((1+\sigma)^{\lambda} f^{-1}(\cdot)\right. \\
&+ \left.\frac{1+g}{1+r}\mathbb{E}\left[\mu_m\frac{\mathbf{J^0}(W'_m)}{Y_{t+1}} + (1-\mu_m)\frac{\mathbf{H^0}(W'_m)}{Y_{t+1}}\right]\right\} \\
\left(\frac{\mathbf{H^M_n}(W_m)}{Y_t}\right)_{bgp} &= \max_{W'_n}\left\{-\frac{Q_n}{Y_t}((1+\sigma)^{\lambda} f^{-1}(\cdot)\right. \\
&+ \left.\frac{1+g}{1+r}\mathbb{E}\left[\mu_n\frac{\mathbf{J^0}(W'_n)}{Y_{t+1}} + (1-\mu_n)\frac{\mathbf{H^0}(W'_n)}{Y_{t+1}}\right]\right\} \\
\left(\frac{\mathbf{J^0}(W_m)}{Y_t}\right)_{bgp} &= \max\left\{\left(\frac{\mathbf{J^S}(W_m)}{Y_t}\right)_{bgp}, \left(\frac{\mathbf{J^M}(W_m)}{Y_t}\right)_{bgp}\right\} \qquad (45) \\
\left(\frac{\mathbf{H^0}(W_m)}{Y_t}\right)_{bgp} &= \max\left\{\left(\frac{\mathbf{H^S}(W_m)}{Y_t}\right)_{bgp}, \left(\frac{\mathbf{H^M}(W_m)}{Y_t}\right)_{bgp}\right\} \qquad (46)
\end{aligned}
$$

[6]For presentation simplicity note I am omiting state variables in the value functions and arguments of profits/probabilities which are functions rather than numbers. This appendix is meant only to show how to twist the competitive equilibrium definition into the balanced trajectory.

D: Supporting Figures and Tables

Table 5: Statistics for Economic Complexity Index. Source: Atlas of Economic Complexity, by Hausmann et al. (2013)

Year	Mean	Percentile 10	Median	Percentile 90
1980	2.023	-1.856	2.044	2.3066
1993	2.038	-2.12	1.961	2.364
2000	1.990	-1.404	1.942	2.352

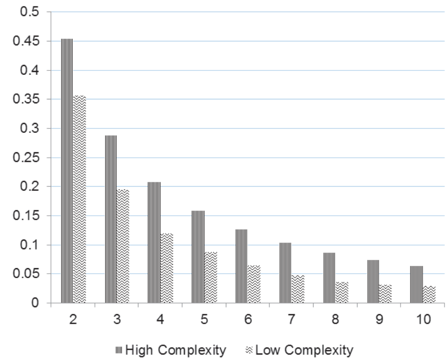

Figure 7: Inventor Productivity. The horizontal axis (n) is the number of patents. The vertical representes the proportion of inventors that have n or more patents. "High Complexity" group comprises innovators in countries between the 75 and 95 percentile, and the "Low Complexity" innovators in the 5-25 percentile.

E: Data definitions and sources.

Diversification is understood in the model of this paper as the number of sector which cohabit an economy. Thinking about it in the data is harder. We require comprehensive and comparable data on what every economy produces. There is not such a metric. There exists, however, a way to circumvent this data limitation by looking at trade data from UN Comtrade. In particular, I take the measure of economic diversification using the definitions on Hausmann et al. (2013) which I briefly explain here, for completeness. According to the authors, an economy should be, by a good metric, considered more complex than another one, if it produces a larger number of different kinds of goods, and/or each of these is harder

Table 6: Output Growth in the AABM Model, sample restricted 1960-2000. First two regressions mimic the AABM model. While volatility is still significant, financial development looses economic significance. Standard errors in parenthesis, $p < 0.01, **p < 0.05, *p < 0.1$

	per capita (1)	*aggregate* (2)	*per capita* (3)	*aggregate* (4)
Complexity			0.010**	-0.005
			(0.004)	(0.004)
Volatility	-0.677***	-0.751***	-0.337*	-0.564***
	(0.205)	(0.141)	(0.197)	(0.166)
Dom Cred / GDP	-0.000	-0.000**	-0.000**	-0.000*
	(0.000)	(0.000)	(0.000)	(0.000)
*Credit*Vol*	0.009**	0.012***	0.011***	0.012***
	(0.004)	(0.003)	(0.004)	(0.003)
*Comp*Vol*			0.072	0.179**
			(0.101)	(0.085)
Initial Income	0.000	-0.000	0.001	0.000
	(0.001)	(0.001)	(0.001)	(0.001)
Constant	0.023*	0.060***	0.019	0.047***
	(0.013)	(0.009)	(0.015)	(0.013)
Observations	101	101	80	80
R-squared	0.324	0.359	0.495	0.400

to elaborate. The first intrinsic characteristic of a complex economy is then, "diversity". The second, is the difficulty involved in producing each of these goods, proxied by its 'ubiquity".

Economic Complexity Index is calculated using the two concepts: diversity and ubiquity. Consider, for country c and product p, an indicator variable which takes the value one (1) if country c produces p and zero (0) otherwise. Then stack all these indicator variables in a matrix to be labeled $M_{c,p}$. If we sum horizontally, then we get a measure of diversity of any country c: the number of products it produces. If we sum vertically for a given product p, then we obtain a measure of how many countries know how to produce product p: its ubiquity. The less countries producing p, the more likely p is a hard-to-produce good[7]. We can correct

[7]For instance, the hardest to produce items using this methodology would be (at SITC4 product level): Machines and appliances for specialized industries, appliances based on the use of X-ray and radiation, appliances for physical or medical analysis, and those on the easies to produce side would be: crude oil, cotton, cocoa beans, sesame seeds and tin ores.

Table 7: Productivity & Complexity IV. Logit regression. Dependent binary variable (Serial inventor=1, Once in lifetime inventor=0). Observations are at inventor level. Volatility measures are the standard deviation of yearly PPP GDP and the Neumeyer & Perri (2005) measure of country volatility. Standard errors in parentheses,$**p < 0.01, *p < 0.05, ***p < 0.1$ Italics are marginal effects evaluated at the mean of regressors.

	(1)	(2)	(3)	(4)	(5)
Complexity (base year 93)	0.450**	-0.041**	0.011	0.085**	0.126**
	(0.004)	(0.013)	(0.013)	(0.010)	(0.010)
	0.1118	*-0.0102*	*0.0027*	*0.0211*	*0.0314*
Volatility measure 1		-7.690**	-7.394**		
		(0.439)	(0.439)		
		-1.9127	*-1.8482*		
Volatility measure 2	-0.547**			-3.422**	-3.373**
	(0.061)			(0.216)	(0.217)
	-.1360716			*-0.85*	*-0.8430*
*Complexity * Volatility1*		11.074**	10.060**		
		(0.307)	(0.308)		
		2.7544	*2.5146*		
*Complexity * Volatility 2*				4.387**	4.012**
				(0.140)	(0.141)
				1.091	*0.989*
Employment (log)		0.115**	0.104**	0.104**	0.094**
		(0.002)	(0.002)	(0.002)	(0.002)
		0.0287	*0.0260*	*0.1040*	*0.0234*
Per capita GDP		0.158**	0.153**	0.157**	0.152**
		(0.007)	(0.007)	(0.007)	(0.007)
		0.0394	*0.0384*	*0.1575*	*0.0380*
Constant	-1.014**	-2.655**	-2.897**	-2.679**	-2.917**
	(0.008)	(0.074)	(0.075)	(0.074)	(0.076)
Year dummies	No	No	Yes	No	Yes
Pearson p-value	0.00	0.00	0.00	0.00	0.00
Hosmer-Lemeshow p-value	0.00	0.00	0.00	0.00	0.00
Observations	2,023,962	2,010,802	2,010,802	2,010,802	2,010,802

(weight) the diversity measure by the ubiquity measure and the ubiquity measure with the diversity measure. Finally, we end up with an index for the diversity in production of an economy which already embeds the difficulty of producing each item. This index is called the ECI (Economic Complexity Index)[8]. The ECI variable is defined on the complete real line, as it is the eigenvector of a matrix. In practice however, and in this sample in particular, it is pretty much bounded, with mean equal to 1.89 and standard deviation of 0.40. For reference, a country like Chile has an average ECI of 0.011, Honduras of -0.98, while Canada has 1.05 and Germany 2.21. Thus all comparative statics from the exercises below, when we think a unit change in complexity can be thought of as moving an inventor from Chile to Canada[9].

[8]The interested reader on the specifics of this index is referred to page 24, box 2.1 of Hausmann et al. (2013)

[9]Interested readers can refer to table 5 for more detailed statistics on the ECI sample.

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