International Trade Models with Endogenous Terms of Trade, World Equilibrium Allocations, and Trade Patterns.

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1 Introduction

The Heckscher-Ohlin-Samueslson (HOS) model of two goods, two factors and two countries (2 x 2 x 2) is used in any standard textbook exposition of international trade theory, and it is the workhorse for deriving the patterns of trade in goods between countries differing in their factor endowments. But in most expositions, this (2 x 2 x 2) HOS model is only analyzed and solved graphically, which means that just the qualitative properties of the model are examined in various comparative static analyses (illustrations). The quantitative size the endogenous terms of trade (free trade equilibrium price ratio) is not calculated. Neither doe the classical "Offer curve diagram" techniques allow any possibilty of a quantitative determination of the general equilibrium terms of trade of two large trading countries. This is a serious "state of art" for international trade theory, as it is the world market equilibrium price ratio (terms of trade) alone that determines the inner workings (factor allocations, factor prices, consumer demands, export/import, potential specialization) of the domestic economies. With free trade (law of one commodity price in both countries), the two countries are inseparable and their general (Walrasian) equlibrium cannot methodologically be solved as in two-sector autarky models.

The many applied trade models - multisector computable general equilibrium models - of international trade organisation or academic institutions are quite distinct from the HOS model with free trade above. Usually applied modelers have dropped the law of one price (same good, perfect substitutes at home and abroad) and instead adopted the Armington (1969) assumption of allowing domestically produced and foreign produced goods to be imperfect substitutes in use, and so the countries never specializes in a few goods. Many theoretical trade models, e.g., Mundell (1960) also adopts an apriori (exogenous) division between the export and import goods of each country.

Our main object will be to show how solve quantitaveley and explicitly for the endogenous terms of trade and present analytically the international general (Walrasian) equilibrium solutions in both the (2 x 2 x 2) HOS model and its analytic parametric extensions to assuming different international technologies and consumer preferences.

2 The structure of two domestic economies

There are two countries in the world, A and B. Each country produces two goods, $i = 1, 2$, which are fully homogeneous throughout the world. In both countries, there are two production factors, labor and capital, which are supplied in fixed quantities. National labor supply in country $J = A, B$ is L_J, while capital endowment is K_J. International migration of factors is in any case excluded, while reallocation among sectors is always possible and fully frictionless.

It is assumed that both factors are fully employed in each country :

$$K_{1J} + K_{2J} = K_J \ ; \quad L_{1J} + L_{2J} = L_J \ , \quad J = A, B \tag{1}$$

$$k_J = K_J/L_J = \lambda_{L_{1J}} k_{1J} + \lambda_{L_{2J}} k_{2J} \ , \quad J = A, B \tag{2}$$

$$\lambda_{L_{1J}} \ = \ \frac{L_{1J}}{L_J} \ = \ \frac{k_J - k_{2J}}{k_{1J} - k_{2J}} \ ; \ \lambda_{L_{2J}} \ = \ 1 - \lambda_{L_{1J}} \ , \quad J = A, B \tag{3}$$

$$\lambda_{K_{1J}} \ = \ \frac{K_{1J}}{L_J} \ = \ \frac{k_{1J}}{k_J} \lambda_{L_{1J}} \ ; \ \lambda_{K_{2J}} \ = \ 1 - \lambda_{K_{1J}} \ , \quad J = A, B \tag{4}$$

where $\lambda_{L_{iJ}} = L_{iJ}/L_J$, $\lambda_{K_{iJ}} = K_{iJ}/K_J$, are, respectively, the fraction of workers (capital stock) of country J that are employed in sector i, and $k_{iJ} \equiv K_{iJ}/L_{iJ}$ is the capital intensity in the same sector. Since the model is static and there is no public sector, final consumptions are the only types of demand.

Technology exhibits constant returns to scale (CRTS) in both countries. Since factor markets are perfectly competitive, the Euler theorem ensures that the total monetary (sales) value (revenue) from production in each sector equates the remuneration of primary factors, which is also total production cost (C_{iJ}),

$$P_{iJ}\, Y_{iJ} \ = \ w_J\, L_{iJ} + r_J\, K_{iJ} \ = \ C_{iJ} \ , \quad i = 1, 2 \ , \quad J = A, B \tag{5}$$

with the sectorial cost shares :

$$\epsilon_{L_{iJ}} = \frac{w_J\, L_{iJ}}{C_{iJ}} \ , \ \ \epsilon_{K_{iJ}} = \frac{r_J\, K_{iJ}}{C_{iJ}} \ \ ; \ \epsilon_{L_{iJ}} + \epsilon_{K_{iJ}} = 1 \ , \quad i = 1, 2 \ , \quad J = A, B \tag{6}$$

Total national income (gross domestic product, GDP), (Y_J), is obtained as

$$Y_J = P_{1J}\, Y_{1J} + P_{2J}\, Y_{2J} \ = \ w_J\, L_J + r_J\, K_J \ , \quad J = A, B \tag{7}$$

Macro factor income shares are identically linked to the endowment ratio as,

$$\delta_{LJ} = \frac{w_J\, L_J}{Y_J} \ , \ \ \delta_{KJ} = \frac{r_J\, K_J}{Y_J} \ ; \ k_J \equiv \left[\frac{w_J}{r_J}\right] \frac{\delta_{KJ}}{\delta_{LJ}} \ ; \ \ \delta_{LJ} + \delta_{KJ} = 1 \ , \ \ J = A, B \tag{8}$$

Let Q_{iJ}, $i = 1, 2$, denote the quantitative size of the domestic demands (absorption level) for good 1 and good 2, and they are respectively equal to domestic production, Y_{iJ}, minus net exports, X_{iJ}, i.e.,

$$Q_{1J} = Y_{1J} - X_{1J}, \quad Q_{2J} = Y_{2J} - X_{2J} \ , \quad J = A, B \tag{9}$$

The trade balance is assumed to satisfy the constraint,

$$P_{1J}X_{1J} + P_{2J}X_{2J} = 0 \ ; \quad \text{i.e.} \quad Y_J = P_{1J}Q_{1J} + P_{2J}Q_{2J} \ , \quad J = A, B \tag{10}$$

i.e., balanced trade prevails with no foreign borrowing/lending allowed.
The composition of GDP, (10), into final demand (expenditure) shares, s_{iJ}, is

$$s_{iJ} \ = \ P_{iJ}Q_{iJ}/Y_J \ ; \quad \sum_{i=1}^{2} s_{iJ} \ \equiv \ \sum_{i=1}^{2} P_{iJ}Q_{iJ}/Y_J = 1 \ , \ J = A, B \tag{11}$$

The macro factor income shares δ_{LJ}, δ_{KJ}, (8), are GDP expenditure-weighted, (50), combinations of sectorial factor (cost) shares, $\epsilon_{L_{iJ}}$, $\epsilon_{K_{iJ}}$,

$$\delta_{LJ} \ = \ \sum_{i=1}^{2} s_{iJ}\epsilon_{L_{iJ}}, \quad \delta_{KJ} \ = \ \sum_{i=1}^{2} s_{iJ}\epsilon_{L_{iJ}}, \quad \delta_{LJ} + \delta_{KJ} = 1 \, , \quad J = A, B \tag{12}$$

The factor allocation fractions, (3), (4), can then be restated as,

$$\lambda_{L_{iJ}} = \ s_{iJ}\epsilon_{L_{iJ}}/\delta_{LJ} \ , \qquad \lambda_{K_{iJ}} \ = \ s_{iJ}\epsilon_{K_{iJ}}/\delta_{KJ} \ , \quad i = 1, 2 \ , \quad J = A, B \tag{13}$$

The total factor endowment ratio, K/L, satisfies the identity, cf. (8), (12):

$$K_J/L_J \ = \ k_J \ = \ \frac{\omega_J \, \delta_{KJ}}{\delta_{LJ}} \ = \ \omega_J \sum_{i=1}^{2} s_{iJ}\epsilon_{K_{iJ}} \Big/ \sum_{i=1}^{2} s_{iJ}\epsilon_{L_{iJ}} \ , \quad J = A, B \tag{14}$$

which is a convenient representation of Walras's law (identity) and (2).

We study the international interactions in the case of a Cobb-Douglas (CD) production (cost) function and in the case of a technology with constant elasticity of substitution (CES).

2.1 Sector technologies, cost functions and domestic prices

Cobb-Douglas production-, cost functions and the relative prices

For sector $i = 1, 2$ in country $J = A, B$, we assume standard CD technologies:

$$Y_{iJ} = \gamma_{iJ}(L_{iJ})^{1-a_{iJ}}(K_{iJ})^{a_{iJ}} \ , \ y_{iJ} = \gamma_{iJ}(k_{iJ})^{a_{iJ}} \ , \quad i = 1, 2 \ , \quad J = A, B \quad (15)$$

where Y_{iJ} is the domestic output of sector i in country J - with sectoral labour productivity, $y_{iJ} \equiv Y_{iJ}/L_{iJ}$, and capital intensity, $k_{iJ} \equiv K_{iJ}/L_{iJ}$. The marginal rates of substitution, ω_J (here equal to relative factor prices, w_J/r_J) are :

$$\omega_J \equiv \frac{w_J}{r_J} = \frac{1-a_{iJ}}{a_{iJ}} k_{iJ} \ ; \quad k_{iJ} = \frac{a_{iJ}}{1-a_{iJ}} \omega_J \quad (16)$$

The standard dual CD cost function of (15-16) is,

$$C_{iJ}(w_J, r_J, Y_{iJ}) = \frac{1}{\gamma_{iJ}} \left[\frac{w_J}{1-a_{iJ}}\right]^{1-a_{iJ}} \left[\frac{r_J}{a_{iJ}}\right]^{a_{iJ}} Y_{iJ} \ , \quad i = 1, 2 \ , \ J = A, B \quad (17)$$

and the sectorial cost shares (6) are :

$$\epsilon_{L_{iJ}} = 1 - a_{iJ} \ , \ \epsilon_{K_{iJ}} = a_{iJ} \ ; \ \epsilon_{L_{iJ}} + \epsilon_{K_{iJ}} = 1 \ , \quad i = 1, 2 \ , \quad J = A, B \quad (18)$$

The domestic relative commodity prices (unit costs) is derived from (17) as

$$p_J = \frac{P_{1J}}{P_{2J}} = \frac{C_{1J}/Y_{1J}}{C_{2J}/Y_{2J}} = \frac{c_{1J}(\omega_J)}{c_{2J}(\omega_J)} = \frac{\gamma_{2J}}{\gamma_{1J}} \frac{(\omega_J)^{a_{2J}-a_{1J}}}{\bar{a}_J} \ , \quad J = A, B \quad (19)$$

where

$$\bar{a}_J \ = \ \frac{(a_{1J})^{a_{1J}} (1-a_{1J})^{1-a_{1J}}}{(a_{2J})^{a_{2J}} (1-a_{2J})^{1-a_{2J}}} \ , \quad J = A, B \quad (20)$$

Next, we can use the inverse of (19), i.e.,

$$\omega_J \ = \ \left[\frac{\gamma_{1J}}{\gamma_{2J}} \bar{a}_J \, p_J\right]^{\frac{1}{a_{2J}-a_{1J}}} \ , \quad J = A, B \quad (21)$$

and insert (21) into (16) to get the sectoral capital intensities and sectoral labour productivities with the relative good price (p_J) as the 'independent' variable,

$$k_{iJ}(p_J) = \frac{a_{iJ}}{1-a_{iJ}} \left[\frac{\gamma_{1J}}{\gamma_{2J}} \bar{a}_J \, p_J\right]^{\frac{1}{a_{2J}-a_{1J}}} \ , \quad \frac{k_{1J}(p_J)}{k_{2J}(p_J)} = \frac{a_{1J}(1-a_{2J})}{a_{2J}(1-a_{1J})} \quad (22)$$

and hence by (22) and (15),

$$y_{iJ}(p_J) = \gamma_{iJ} \left[\frac{a_{iJ}}{1-a_{iJ}}\right]^{a_{iJ}} \left[\frac{\gamma_{1J}}{\gamma_{2J}} \bar{a}_J \, p_J\right]^{\frac{a_{iJ}}{a_{2J}-a_{1J}}} \ , \ i = 1, 2 \ , \quad J = A, B \quad (23)$$

Next rewrite (3) as

$$\lambda_{L_{1J}}(p_J) \quad = \quad \frac{\frac{k_J}{k_{2J}(p_J)} - 1}{\frac{k_{1J}(p_J)}{k_{2J}(p_J)} - 1} \quad , \qquad J = A, B \tag{24}$$

and use (22) to get the allocation fractions of labour (24) and capital (4) in p_J:

$$\lambda_{L_{1J}}(p_J) = \frac{a_{2J}\,(1-a_{1J})}{a_{1J}\,(1-a_{2J}) - a_{2J}\,(1-a_{1J})} \left[\frac{1-a_{2J}}{a_{2J}} \left[\frac{\gamma_{1J}}{\gamma_{2J}}\, \bar{a}_J\, p_J \right]^{\frac{1}{a_{1J}-a_{2J}}} k_J - 1 \right] \tag{25}$$

$$\lambda_{K_{1J}}(p_J) \;=\; \frac{k_{1J}(p_J)}{k_J}\, \lambda_{L_{1J}}(p_J) \;\; , \qquad J = A, B \tag{26}$$

The relative prices (p_J) with CD (19) can range from zero to infinity, cf. Fig. 1, case 1-2. A diversified economy requires that its factor endowment ratio (k_J) belongs to the 'diversification cone' - limits set by the capital intensities (22):

$$a_{1J} > a_{2J} : \frac{a_{2J}}{1-a_{2J}} \left[\frac{\gamma_{1J}}{\gamma_{2J}}\, \bar{a}_J\, p_J \right]^{\frac{1}{a_{2J}-a_{1J}}} < k_J < \frac{a_{1J}}{1-a_{1J}} \left[\frac{\gamma_{1J}}{\gamma_{2J}}\, \bar{a}_J\, p_J \right]^{\frac{1}{a_{2J}-a_{1J}}} \tag{27}$$

$$a_{2J} > a_{1J} : \frac{a_{1J}}{1-a_{1J}} \left[\frac{\gamma_{1J}}{\gamma_{2J}}\, \bar{a}_J\, p_J \right]^{\frac{1}{a_{2J}-a_{1J}}} < k_J < \frac{a_{2J}}{1-a_{2J}} \left[\frac{\gamma_{1J}}{\gamma_{2J}}\, \bar{a}_J\, p_J \right]^{\frac{1}{a_{2J}-a_{1J}}} \tag{28}$$

CES production-, cost functions and relative prices

The CES version of the model adopts the technology specification:

$$Y_{iJ} = \gamma_{iJ} \left[(1-a_{iJ})(L_{iJ})^{\frac{\sigma_{iJ}-1}{\sigma_{iJ}}} + a_{iJ}(K_{iJ})^{\frac{\sigma_{iJ}-1}{\sigma_{iJ}}} \right]^{\frac{\sigma_{iJ}}{\sigma_{iJ}-1}} , \; i = 1,2, \;\; J = A, B \tag{29}$$

with the sectoral labour productivities on the intensive form as:

$$y_{iJ} = \gamma_{iJ} \Big[1 - a_{iJ} + a_{iJ}(k_{iJ})^{\frac{\sigma_{iJ}-1}{\sigma_{iJ}}} \Big]^{\frac{\sigma_{iJ}}{\sigma_{iJ}-1}} , \;\; i = 1,2, \;\; J = A, B \tag{30}$$

Analogously to the CD case, (16), we have the CES expressions:

$$\omega_J \equiv \frac{w_J}{r_J} = \frac{1-a_{iJ}}{a_{iJ}}\, k_{iJ}^{1/\sigma_{iJ}} \;;\;\; k_{iJ} = \left[\frac{a_{iJ}}{1-a_{iJ}}\, \omega_J \right]^{\sigma_{iJ}} , \; i = 1,2, \quad J = A, B \tag{31}$$

Cost minimiztion with (29), (31), yields the standard CES cost function:

$$C_{iJ}(w_J, r_J, Y_{iJ}) = (1/\gamma_{iJ}) \left[(1-a_{iJ})^{\sigma_{iJ}} w_J^{1-\sigma_{iJ}} + a_{iJ}^{\sigma_{iJ}} r_J^{1-\sigma_{iJ}} \right]^{\frac{1}{1-\sigma_{iJ}}} \; Y_{iJ} \tag{32}$$

The CES sectoral cost shares (6) are wellknown and given by:

$$\epsilon_{K_{iJ}} = \frac{1}{1 + [(1-a_{iJ})/a_{iJ}]^{\sigma_{iJ}}\, \omega_J^{1-\sigma_{iJ}}} \;\; ; \;\; \epsilon_{L_{iJ}} + \epsilon_{K_{iJ}} = 1 \; , \quad i = 1,2 \tag{33}$$

The domestic relative commodity prices (unit costs) follows from (32) as,

$$p_J = \frac{P_{1J}}{P_{2J}} = \frac{c_{1J}(\omega_J)}{c_{2J}(\omega_J)} = \frac{\gamma_{2J}}{\gamma_{1J}} \frac{\left[a_{1J}^{\sigma_{1J}} + (1-a_{1J})^{\sigma_{1J}}\, \omega_J^{1-\sigma_{1J}}\right]^{\frac{1}{1-\sigma_{1J}}}}{\left[a_{2J}^{\sigma_{2J}} + (1-a_{2J})^{\sigma_{2J}}\ \omega_J^{1-\sigma_{2J}}\right]^{\frac{1}{1-\sigma_{2J}}}} \tag{34}$$

If the sectoral elasticities of substitution are the same in both sectors, i.e. $\sigma_{1J} = \sigma_{2J} = \sigma_J$, then the relative commodity prices (34) simplify to :

$$p_J = \frac{P_{1J}}{P_{2J}} = \frac{c_{1J}(\omega_J)}{c_{2J}(\omega_J)} = \frac{\gamma_{2J}}{\gamma_{1J}} \left[\frac{a_{1J}^{\sigma_J} + (1-a_{1J})^{\sigma_J}\, \omega_J^{1-\sigma_J}}{a_{2J}^{\sigma_J} + (1-a_{2J})^{\sigma_J}\ \omega_J^{1-\sigma_J}}\right]^{\frac{1}{1-\sigma_J}}, \quad J = A, B \tag{35}$$

The inverse of (35) gives the relative factor prices from the relative prices as,

$$\omega_J = \left[\left[\frac{a_{1J}}{1-a_{1J}}\right]^{\sigma_J} \frac{(p_J)^{\sigma_J-1} - \left[\frac{\gamma_{2J}}{\gamma_{1J}}\right]^{\sigma_J-1}\left[\frac{a_{2J}}{a_{1J}}\right]^{\sigma_J}}{\left[\frac{\gamma_{2J}}{\gamma_{1J}}\right]^{\sigma_J-1}\left[\frac{1-a_{2J}}{1-a_{1J}}\right]^{\sigma_J} - (p_J)^{\sigma_J-1}}\right]^{\frac{1}{1-\sigma_J}}, \quad J = A, B \tag{36}$$

Using (36) in (31) yields the capital intensity in sector (i) of country (J) :

$$k_{iJ}(p_J) = \left[\frac{a_{iJ}}{1-a_{iJ}}\right]^{\sigma_J}\left[\left[\frac{a_{1J}}{1-a_{1J}}\right]^{\sigma_J} \frac{(p_J)^{\sigma_J-1} - \left[\frac{\gamma_{2J}}{\gamma_{1J}}\right]^{\sigma_J-1}\left[\frac{a_{2J}}{a_{1J}}\right]^{\sigma_J}}{\left[\frac{\gamma_{2J}}{\gamma_{1J}}\right]^{\sigma_J-1}\left[\frac{1-a_{2J}}{1-a_{1J}}\right]^{\sigma_J} - (p_J)^{\sigma_J-1}}\right]^{\frac{\sigma_J}{1-\sigma_J}} \tag{37}$$

In case of sector $i = 1$, it simplifies to,

$$k_{1J}(p_J) = \left[\frac{a_{1J}}{1-a_{1J}} \frac{(p_J)^{\sigma_J-1} - \left[\frac{\gamma_{2J}}{\gamma_{1J}}\right]^{\sigma_J-1}\left[\frac{a_{2J}}{a_{1J}}\right]^{\sigma_J}}{\left[\frac{\gamma_{2J}}{\gamma_{1J}}\right]^{\sigma_J-1}\left[\frac{1-a_{2J}}{1-a_{1J}}\right]^{\sigma_J} - (p_J)^{\sigma_J-1}}\right]^{\frac{\sigma_J}{1-\sigma_J}}, \quad J = A, B \tag{38}$$

and for sector $i = 2$, it becomes,

$$k_{2J}(p_J) = \left[\frac{a_{2J}}{1-a_{2J}}\right]^{\sigma_J}\left[\left[\frac{a_{1J}}{1-a_{1J}}\right]^{\sigma_J} \frac{(p_J)^{\sigma_J-1} - \left[\frac{\gamma_{2J}}{\gamma_{1J}}\right]^{\sigma_J-1}\left[\frac{a_{2J}}{a_{1J}}\right]^{\sigma_J}}{\left[\frac{\gamma_{2J}}{\gamma_{1J}}\right]^{\sigma_J-1}\left[\frac{1-a_{2J}}{1-a_{1J}}\right]^{\sigma_J} - (p_J)^{\sigma_J-1}}\right]^{\frac{\sigma_J}{1-\sigma_J}} \tag{39}$$

Note in this CES case (37) that the relative capital intensities of a country only depends on its own technology parameters:

$$\frac{k_{1J}(p_J)}{k_{2J}(p_J)} = \left[\frac{1-a_{2J}}{1-a_{1J}}\frac{a_{1J}}{a_{2J}}\right]^{\sigma_J}, \quad J = A, B \tag{40}$$

The output of sector $i = 1$ as a function of p_J is given by, cf. (38), (30),

$$y_{1J}(p_J) = \gamma_{1J} \left[\frac{\gamma_{2J}}{\gamma_{1J}}\right]^{\sigma_J} \left[\frac{(1-a_{1J})^{\sigma_J - 1}(a_{1J})^{\sigma_J}}{[(1-a_{2J})\,a_{1J}]^{\sigma_J} - [(1-a_{1J})\,a_{2J}]^{\sigma_J}}\right]^{\frac{\sigma_J}{1-\sigma_J}}$$
$$\left[(p_J)^{\sigma_J - 1} - \left[\frac{\gamma_{2J}}{\gamma_{1J}}\right]^{\sigma_J - 1}\left[\frac{a_{2J}}{a_{1J}}\right]^{\sigma_J}\right]^{\frac{\sigma_J}{1-\sigma_J}} \tag{41}$$

and output of sector $i = 2$ as, cf. (39), (30),

$$y_{2J}(p_J) = \gamma_{2J}(1-a_{2J})^{\frac{\sigma_J}{\sigma_J - 1}}$$
$$\left[1 - \left[\frac{1-a_{1J}}{1-a_{2J}}\,\frac{a_{2J}}{a_{1J}}\right]^{\sigma_J} \frac{(p_J)^{\sigma_J-1} - \left[\frac{\gamma_{2J}}{\gamma_{1J}}\right]^{\sigma_J-1}\left[\frac{1-a_{2J}}{1-a_{1J}}\right]^{\sigma_J}}{(p_J)^{\sigma_J-1} - \left[\frac{\gamma_{2J}}{\gamma_{1J}}\right]^{\sigma_J-1}\left[\frac{a_{2J}}{a_{1J}}\right]^{\sigma_J}}\right]^{\frac{\sigma_J}{\sigma_J-1}} \tag{42}$$

The fractions of workers in sectors, $i = 1, 2$, become by (38-40), (24),

$$\lambda_{L_{1J}}(p_J) = \frac{[(1-a_{1J})a_{2J}]^{\sigma_J}}{[(1-a_{2J})a_{1J}]^{\sigma_J} - [(1-a_{1J})a_{2J}]^{\sigma_J}} \cdot$$
$$\left[\left[\frac{1-a_{2J}}{a_{2J}}\right]^{\sigma_J}\left[\left[\frac{1-a_{1J}}{a_{1J}}\right]^{\sigma_J} \frac{\left[\frac{\gamma_{2J}}{\gamma_{1J}}\right]^{\sigma_J-1}\left[\frac{1-a_{2J}}{1-a_{1J}}\right]^{\sigma_J} - (p_J)^{\sigma_J-1}}{(p_J)^{\sigma_J-1} - \left[\frac{\gamma_{2J}}{\gamma_{1J}}\right]^{\sigma_J-1}\left[\frac{a_{2J}}{a_{1J}}\right]^{\sigma_J}}\right]^{\frac{\sigma_J}{1-\sigma_J}} k_J - 1\right] \tag{43}$$

$$\lambda_{L_{2J}}(p_J) = \frac{[(1-a_{2J})a_{1J}]^{\sigma_J}}{[(1-a_{1J})a_{2J}]^{\sigma_J} - [(1-a_{2J})a_{1J}]^{\sigma_J}} \cdot$$
$$\left[\left[\frac{1-a_{1J}}{a_{1J}}\, \frac{\left[\frac{\gamma_{2J}}{\gamma_{1J}}\right]^{\sigma_J-1}\left[\frac{1-a_{2J}}{1-a_{1J}}\right]^{\sigma_J} - (p_J)^{\sigma_J-1}}{(p_J)^{\sigma_J-1} - \left[\frac{\gamma_{2J}}{\gamma_{1J}}\right]^{\sigma_J-1}\left[\frac{a_{2J}}{a_{1J}}\right]^{\sigma_J}}\right]^{\frac{\sigma_J}{1-\sigma_J}} k_J - 1\right] \tag{44}$$

Finally, fractions of capital allocated to sectors, $i = 1, 2$, are, cf. (43), (38), (4),

$$\lambda_{K_{1J}}(p_J) \;=\; \frac{k_{1J}(p_J)}{k_J}\,\lambda_{L_{1J}}(p_J) \;, \qquad J = A, B \tag{45}$$

The relative prices (p_J) can with CES condition (35) only display bounded variation, cf. Fig. 1, case 3-6. The factor endowment ratios (k_J) belong to the 'diversification cone' - limits set by the capital intensities (38), (39) - only when:

$$a_{1J} > a_{2J} : k_{2J}(p_J) < k_J < k_{1J}(p_J) \;;\;\; a_{2J} > a_{1J} : k_{1J}(p_J) < k_J < k_{2J}(p_J) \tag{46}$$

2.2 CD consumer preferences and demand functions

In each country $J = A, B$, we have a represenative consumer with Cobb-Douglas preferences u_J and country-specific consumption parameters:

$$u_J = U_J(Q_{1J}, Q_{2J}) = (Q_{1J})^{\alpha_J}(Q_{2J})^{1-\alpha_J}, \quad J = A, B \tag{47}$$

where Q_{iJ} is the domestic demand for good i in country J.

Maximization of utility (47) under the budget constraint, cf. (7):

$$P_{1J} \cdot Q_{1J} + P_{2J} \cdot Q_{2J} = Y_J, \quad J = A, B \tag{48}$$

yields the optimal demanded quantities and final demand shares,

$$Q_{1J} = \alpha_J \cdot (Y_J/P_{1J})\ ; \quad Q_{2J} = (1-\alpha_J) \cdot (Y_J/P_{2J}), \quad J = A, B \tag{49}$$

$$s_{1J} = P_{1J}Q_{1J}/Y_J = \alpha_J\ ;\ s_{2J} = P_{2J}Q_{2J}/Y_J = 1-\alpha_J\ ; \quad \textstyle\sum_{i=1}^{2} s_{iJ} = 1 \tag{50}$$

2.3 General equlibrium of two autarky economies

When each country, $J = A, B$, are autarky economies, then the domestic demand for good i in country J, must also in general equilibrium be equal to production (output) of good i in country J, i.e.,

$$Q_{iJ} = Y_{iJ}, \quad i = 1, 2, \quad J = A, B \tag{51}$$

$$\delta_{LJ} =, \ \delta_{KJ} =, \ k_J \equiv \frac{\omega_J \delta_{KJ}}{\delta_{LJ}} = \frac{\omega_J\, \delta_{KJ}(\omega_J)}{\delta_{LJ}(\omega_J)} = \Psi_J(\omega_J) \quad , \ J = A, B \tag{52}$$

With CD technology

$$k_J = \Psi_J(\omega_J) = [\frac{\alpha_J a_{1J} - (1-\alpha_J)a_{2J}}{\alpha_J(1-a_{1J}) + (1-\alpha_J)(1-a_{2J})}]\, \omega_J \quad , \ J = A, B \tag{53}$$

With CES technology

$$\begin{aligned} k_J &= \Psi_J(\omega_J) = \frac{\omega_J \delta_{KJ}(\omega_J)}{\delta_{LJ}(\omega_J)} \quad , \ J = A, B \\ &= \frac{\omega_J^{2\sigma_J} + \alpha_J[\frac{1-a_{2J}}{a_{2J}}\,\omega_J]^{\sigma_J}\,\omega_J + (1-\alpha_J)[\frac{1-a_{1J}}{a_{1J}}\,\omega_J]^{\sigma_J}\,\omega_J}{\alpha_J[\frac{1-a_{1J}}{a_{1J}}\,\omega_J]^{\sigma_J} + (1-\alpha_J)[\frac{1-a_{2J}}{a_{2J}}\,\omega_J]^{\sigma_J} + [\frac{1-a_{1J}}{a_{1J}}\frac{1-a_{2J}}{a_{2J}}]^{\sigma_J}\,\omega_J} \end{aligned} \tag{54}$$

3 The trade balance and world market prices

We now assume free trade between the two countries with perfect integration of the national commodity markets. Due to the absence of frictions in international trade the *law of one price* applies:

$$P_{iA} = P_{iB} = P_i\ , \quad i = 1, 2\ : \quad p_A = P_1/P_2 = p_B = p \tag{55}$$

International trade patterns follow the conventional law of comparative advantage, which in this model specification derives from differences in preferences, technology and in factor endowments. Trade is always balanced.
As commodity markets are fully integrated, world market equilibrium implies:

$$X_{iA} = Y_{iA} - Q_{iA} = -X_{iB} = -\left(Y_{iB} - Q_{iB}\right) \ ; \quad i = 1,2 \tag{56}$$

where X_{iJ} are net exports of good i by country J.

In order to derive the equation of the world trade balance and its terms of trade, re-write the optimal consumption demand for good 1 (49), using the definition of net exports in equation (56) and (7):

$$P_1 \cdot (Y_{1J} - X_{1J}) = \alpha_J \cdot (P_1 Y_{1J} + P_2 Y_{2J}) \tag{57}$$

Solving for X_{1J} yields

$$P_1 X_{1J} = (1 - \alpha_J)\, P_1 Y_{1J} - \alpha_J \cdot P_2 Y_{2J}. \tag{58}$$

In real terms, (58) reads as

$$X_{1J} = L_J \left[(1 - \alpha_J)\, \lambda_{L_{1J}} \cdot y_{1J} - \frac{\alpha_J}{p} \lambda_{L_{2J}} \cdot y_{2J} \right], \quad J = A, B \tag{59}$$

Let v_A, v_B represent the country *shares* of world labor force (population), i.e.,

$$v_A = L_A/(L_A + L_B), \qquad v_A + v_B = 1. \tag{60}$$

Lemma 1. *For two competitive trading economies, with CD utility functions and regular sector technologies, y_{iJ}, i=1,2 , J=A,B, (production functions), the international equilibrium terms of trade, $p = P_1/P_2$, satisfies the condition :*

$$p = \frac{v_A\, \alpha_A \cdot \lambda_{L_{2A}} \cdot y_{2A} + v_B\, \alpha_B \cdot \lambda_{L_{2B}} \cdot y_{2B}}{v_A\, (1 - \alpha_A)\, \lambda_{L_{1A}} \cdot y_{1A} + v_B\, (1 - \alpha_B)\, \lambda_{L_{1B}} \cdot y_{1B}} \tag{61}$$

$$= \frac{y_{2A}}{y_{1A}} \frac{v_A\, \alpha_A \cdot \lambda_{L_{2A}} + v_B\, \alpha_B \cdot \lambda_{L_{2B}} \cdot (y_{2B}/y_{2A})}{v_A\, (1 - \alpha_A) \cdot \lambda_{L_{1A}} + v_B\, (1 - \alpha_B)\, \lambda_{L_{1B}} \cdot (y_{1B}/y_{1A})} \tag{62}$$

and with the same sector technologies, $y_{iA} = y_{iB} = y_i$, $(i = 1, 2)$, in A and B :

$$p = \frac{y_2}{y_1} \frac{v_A\, \alpha_A \cdot \lambda_{L_{2A}} + v_B\, \alpha_B \cdot \lambda_{L_{2B}}}{v_A\, (1 - \alpha_A) \cdot \lambda_{L_{1A}} + v_B\, (1 - \alpha_B)\, \lambda_{L_{1B}}} \tag{63}$$

Proof. Inserting (59) in (56) gives the world market equilibrium condition (61).

With Leontief sector technologies, we get

$$p = \Phi(k_A, k_B) = \frac{y_2}{y_1} \frac{(v_A\alpha_A + v_B\alpha_B)k_1 - v_A\alpha_A k_A - v_B\alpha_B k_B}{(v_A\alpha_A + v_B\alpha_B)k_2 + v_A(1-\alpha_A)\, k_A + v_B(1-\alpha_B)\, k_B} \tag{64}$$

The *terms* of *trade surfaces* (TTS), (64), have the functional form,

$$p = \frac{A_0 + A_1\, k_A + A_2\, k_B}{B_0 + B_1\, k_A + B_2\, k_B}, \tag{65}$$

which belongs to the family of quadratics in three variables (conic surfaces). The shape of (65) is a *hyperbolic paraboloid*, upon which hyperbolas appear for fixed k_A or fixed k_B.

Traditionally, the terms of trade are determined by the intersection of reciprocal demand (offer) curves, Oniki and Uzawa (1965). In a growth context, the shifting offer curve technique is rather cumbersome. The same applies to the long-run offer curve methodology, Atsumi (1971).

4 Computation of endogenous terms of trade on world markets

4.1 CD sector technologies

The general solution with different CD sector technologies

The ratios of sectoral labour productivities within countries are given by (23):

$$\frac{y_{2J}}{y_{1J}} = \frac{1-a_{1J}}{1-a_{2J}}\, p\,, \quad J = A, B \tag{66}$$

The ratios of sectoral labour productivities between countries become by (23):

$$\frac{y_{iA}}{y_{iB}} = D_i\, p^{\left[\frac{a_{1A}\, a_{2B} - a_{1B}\, a_{2A}}{(a_{2A}-a_{1A})(a_{2B}-a_{1B})}\right]}\,, \quad i = 1,2 \tag{67}$$

where,

$$D_i = \frac{\gamma_{iA}\left[\frac{\gamma_{1A}}{\gamma_{2A}}\right]^{\frac{a_{iA}}{a_{2A}-a_{1A}}}}{\gamma_{iB}\left[\frac{\gamma_{1B}}{\gamma_{2B}}\right]^{\frac{a_{iB}}{a_{2B}-a_{1B}}}}\;\frac{\left[\frac{a_{iA}}{1-a_{iA}}\right]^{a_{iA}}}{\left[\frac{a_{iB}}{1-a_{iB}}\right]^{a_{iB}}}\;\frac{[\bar{a}_A]^{\frac{a_{iA}}{a_{2A}-a_{1A}}}}{[\bar{a}_B]^{\frac{a_{iB}}{a_{2B}-a_{1B}}}}\,, \quad i = 1,2 \tag{68}$$

Inserting (66-67) into (62), and using $\lambda_{L_{2J}} = 1 - \lambda_{L_{1J}}$, we get :

$$p^{\left[\frac{a_{1B}\cdot a_{2A} - a_{1A}\cdot a_{2B}}{(a_{2A}-a_{1A})(a_{2B}-a_{1B})}\right]} =$$
$$\frac{\upsilon_A\left[(1-\alpha_A)(1-a_{2A}) + \alpha_A(1-a_{1A})\right]\lambda_{L_{1A}} - \upsilon_A\alpha_A(1-a_{1A})}{\upsilon_B\left[\frac{\alpha_B(1-a_{1A})}{D_2}(1-\lambda_{L_{1B}}) - \frac{(1-\alpha_B)(1-a_{2A})}{D_1}\lambda_{L_{1B}}\right]} \tag{69}$$

which is an implicit equation of the terms of trade (p) in the factor endowments.

Theorem 1. *For two competitive trading economies, with CD utility functions, (47), and CD sector technologies, (15), the international equilibrium terms of trade, $P_1/P_2 = p = \Phi(k_A, k_B)$, is an implicit given function of the factor endowments, k_A and k_B, and determined by: $\Omega(p, k_A, k_B) = 0$, where $\Omega(p, k_A, k_B)$ is given by:*

$$\begin{aligned} & \upsilon_A \frac{a_{1B} - a_{2B}}{a_{2B}\,(1 - a_{1B})} \left[\frac{1}{\bar{a}_A} \frac{\gamma_{2A}}{\gamma_{1A}}\right]^{\frac{1}{a_{2A} - a_{1A}}} \left[(1 - \alpha_A)\,(1 - a_{2A}) + \alpha_A\,(1 - a_{1A})\right] k_A \cdot p^{\frac{1}{a_{1A} - a_{2A}}} \\ - \;& \upsilon_B \frac{a_{1A} - a_{2A}}{(1 - a_{1A})\,(1 - a_{2A})} \left[\frac{a_{1B}\,(1 - a_{2B})}{a_{2B}\,(1 - a_{1B})}\,(1 - a_{1A})\,\frac{\alpha_B}{D_2} + (1 - a_{2A})\,\frac{1 - \alpha_B}{D_1}\right] p^{\frac{a_{2A} \cdot a_{1B} - a_{1A} \cdot a_{2B}}{(a_{1A} - a_{2A})(a_{1B} - a_{2B})}} \\ + \;& \upsilon_B \frac{a_{1A} - a_{2A}}{1 - a_{1A}} \frac{1 - a_{2B}}{a_{2B}} \left[\frac{1}{\bar{a}_B} \frac{\gamma_{2B}}{\gamma_{1B}}\right]^{\frac{1}{a_{2B} - a_{1B}}} \left[\frac{1 - \alpha_B}{D_1} + \frac{1 - a_{1A}}{1 - a_{2A}} \frac{\alpha_B}{D_2}\right] k_B \cdot p^{\frac{a_{1A}(1 - a_{2B}) - a_{2A}(1 - a_{1B})}{(a_{1A} - a_{2A})(a_{1B} - a_{2B})}} \\ - \;& \upsilon_A \frac{a_{1B} - a_{2B}}{a_{2B}\,(1 - a_{1B})} \left[a_{1A}\alpha_A + (1 - \alpha_A)\,a_{2A}\right] \end{aligned} \tag{70}$$

With the same CD production elasticities in both countries, i.e. the parameters:

$$a_{iA} = a_{iB} = a_i\,, \;\; i = 1, 2 \;:\; \bar{a}_A = \bar{a}_B = \bar{a} \tag{71}$$

the equation $\Omega(p, k_A, k_B) = 0$ with $\Omega(p, k_A, k_B)$ given by (70) leads to an explicit analytic terms of trade formula:

$$p = \Phi(k_A, k_B) = \frac{1}{\bar{a}} \left[\frac{\bar{\gamma}\,\upsilon_A \left[\frac{\gamma_{2A}}{\gamma_{1A}}\right]^{\frac{1}{a_2 - a_1}} [1 - \bar{\alpha}_A]\,k_A + \upsilon_B \left[\frac{\gamma_{2B}}{\gamma_{1B}}\right]^{\frac{1}{a_2 - a_1}} [1 - \bar{\alpha}_B]\,k_B}{\bar{\gamma}\,\upsilon_A \bar{\alpha}_A + \upsilon_B \bar{\alpha}_B}\right]^{a_2 - a_1} \tag{72}$$

$$\text{where} \qquad \bar{a} = \frac{a_1^{a_1}\,(1 - a_1)^{1 - a_1}}{a_2^{a_2}\,(1 - a_2)^{1 - a_2}}; \quad \bar{\gamma} = \left[\left[\frac{\gamma_{2B}}{\gamma_{2A}}\right]^{a_1} \left[\frac{\gamma_{1A}}{\gamma_{1B}}\right]^{a_2}\right]^{\frac{1}{a_2 - a_1}}; \tag{73}$$

$$\text{and} \qquad \bar{\alpha}_J \;\equiv\; \alpha_J\,a_1 + (1 - \alpha_J)\,a_2\,, \quad J = A, B \tag{74}$$

If also the CD total factor productivities are the same in the two countries, i.e

$$\gamma_{iA} \;=\; \gamma_{iB} \;=\; \gamma_i\,,\;\; i=1,2\;:\;\bar{\gamma}\;=\;1 \tag{75}$$

then (72) becomes,

$$p=\Phi(k_A,k_B)=\frac{1}{\bar{a}}\frac{\gamma_2}{\gamma_1}\left[\frac{\upsilon_A\,(1-\bar{\alpha}_A)\,k_A+\upsilon_B\,(1-\bar{\alpha}_B)\,k_B}{\upsilon_A\cdot\bar{\alpha}_A+\upsilon_B\cdot\bar{\alpha}_B}\right]^{a_2-a_1} \tag{76}$$

the terms of trade (p) with the same CD sector technologies in both countries.

With the same tastes [CD preferences, utility functions], (47), and by (74):

$$\bar{\alpha}_A \;=\; \bar{\alpha}_B \;=\; \bar{\alpha} \;\equiv\; \alpha\, a_1+(1-\alpha)\,a_2 \tag{77}$$

the terms of trade (76) becomes,

$$p=\frac{1}{\bar{a}}\frac{\gamma_2}{\gamma_1}\left[\frac{1-\bar{\alpha}}{\bar{\alpha}}\;(\upsilon_A k_A+\upsilon_B k_B)\right]^{a_2-a_1} \tag{78}$$

With also the same size of the two countries : $\upsilon_A=\upsilon_B=\frac{1}{2}$*, then (78) gives,*

$$p=\frac{1}{\bar{a}}\frac{\gamma_2}{\gamma_1}\left[\frac{1-\bar{\alpha}}{2\,\bar{\alpha}}\;(k_A+k_B)\right]^{a_2-a_1} \tag{79}$$

Finally, with the same relative factor endowments: $k_J=k_A=k_B$*, and by (79),*

$$p=\frac{1}{\bar{a}}\frac{\gamma_2}{\gamma_1}\left[\frac{1-\bar{\alpha}}{\bar{\alpha}}\;k_J\right]^{a_2-a_1} \tag{80}$$

The terms of trade (80) is the same as the relative Walrasian general equilibrium prices in autarky, cf.

Proof. Consider finally that

$$\lambda_{L_{1J}}=\frac{k_J-k_{2J}}{k_{1J}-k_{2J}} \tag{81}$$

and the following equation for the terms of trade

4.2 CES technology

The general solution

Due to the analytical complexity of the model, we assume identical elasticities of substitution throughout the world in both sectors:

$$\sigma_A = \sigma_B = \sigma \tag{82}$$

Under the assumption, (82), the relative labor productivity within countries is

$$\frac{y_{2J}}{y_{1J}} = \left[\frac{\gamma_{2J}}{\gamma_{1J}}\right]^{1-\sigma} \left[\frac{1-a_{1J}}{1-a_{2J}}\right]^{\sigma} p^{\sigma} \tag{83}$$

The relative labor productivity between countries takes the general form:

$$\frac{y_{iB}}{y_{iA}} = \frac{\gamma_{iB}}{\gamma_{iA}} \left[\frac{1-a_{iB}}{1-a_{iA}} \frac{\frac{p^{\sigma-1}\left[1-\left[\frac{a_{iB}}{a_{1B}}\frac{1-a_{1B}}{1-a_{iB}}\right]^{\sigma}\right]-\left[\frac{\gamma_{2B}}{\gamma_{1B}}\right]^{\sigma-1}\left[\left[\frac{a_{2B}}{a_{1B}}\right]^{\sigma}-\left[\frac{a_{iB}}{a_{1B}}\frac{1-a_{2B}}{1-a_{iB}}\right]^{\sigma}\right]}{p^{\sigma-1}-\left[\frac{\gamma_{2B}}{\gamma_{1B}}\right]^{\sigma-1}\left[\frac{a_{2B}}{a_{1B}}\right]^{\sigma}}}{\frac{p^{\sigma-1}\left[1-\left[\frac{a_{iA}}{a_{1A}}\frac{1-a_{1A}}{1-a_{iA}}\right]^{\sigma}\right]-\left[\frac{\gamma_{2A}}{\gamma_{1A}}\right]^{\sigma-1}\left[\left[\frac{a_{2A}}{a_{1A}}\right]^{\sigma}-\left[\frac{a_{iA}}{a_{1A}}\frac{1-a_{2A}}{1-a_{iA}}\right]^{\sigma}\right]}{p^{\sigma-1}-\left[\frac{\gamma_{2A}}{\gamma_{1A}}\right]^{\sigma-1}\left[\frac{a_{2A}}{a_{1A}}\right]^{\sigma}}} \right]^{\frac{\sigma}{\sigma-1}} \tag{84}$$

In case of sector $i = 1$, the expression (84) becomes,

$$\frac{y_{1B}}{y_{1A}} = \left[\left[\frac{a_{1B}}{a_{1A}}\right]^{\sigma} \frac{[(1-a_{1A})\,a_{2A}]^{\sigma}-[(1-a_{2A})\,a_{1A}]^{\sigma}}{[(1-a_{1B})\,a_{2B}]^{\sigma}-[(1-a_{2B})\,a_{1B}]^{\sigma}}\right]^{\frac{\sigma}{1-\sigma}} \left[\frac{\gamma_{1B}}{\gamma_{1A}}\right]^{1-\sigma} \cdot$$
$$\left[\frac{\gamma_{2B}}{\gamma_{2A}}\frac{1-a_{1A}}{1-a_{1B}}\right]^{\sigma} \left[\frac{p^{\sigma-1}-\left[\frac{\gamma_{2B}}{\gamma_{1B}}\right]^{\sigma-1}\left[\frac{a_{2B}}{a_{1B}}\right]^{\sigma}}{p^{\sigma-1}-\left[\frac{\gamma_{2A}}{\gamma_{1A}}\right]^{\sigma-1}\left[\frac{a_{2A}}{a_{1A}}\right]^{\sigma}}\right]^{\frac{\sigma}{1-\sigma}} \tag{85}$$

and for sector $i = 2$:

$$\frac{y_{2B}}{y_{2A}} = \left[\left[\frac{a_{1B}}{a_{1A}}\right]^{\sigma} \frac{[(1-a_{1A})\,a_{2A}]^{\sigma}-[(1-a_{2A})\,a_{1A}]^{\sigma}}{[(1-a_{1B})\,a_{2B}]^{\sigma}-[(1-a_{2B})\,a_{1B}]^{\sigma}}\right]^{\frac{\sigma}{1-\sigma}} \frac{\gamma_{2B}}{\gamma_{2A}} \cdot$$
$$\left[\frac{1-a_{2A}}{1-a_{2B}}\right]^{\sigma} \left[\frac{p^{\sigma-1}-\left[\frac{\gamma_{2B}}{\gamma_{1B}}\right]^{\sigma-1}\left[\frac{a_{2B}}{a_{1B}}\right]^{\sigma}}{p^{\sigma-1}-\left[\frac{\gamma_{2A}}{\gamma_{1A}}\right]^{\sigma-1}\left[\frac{a_{2A}}{a_{1A}}\right]^{\sigma}}\right]^{\frac{\sigma}{1-\sigma}} \tag{86}$$

Theorem 2. *For two competitive trading economies, with CD utility functions, (47), and CES sector technologies, (29), with the restrictions,*

$$\sigma_A = \sigma_B = \sigma \;\; ; \;\; a_{iA} = a_{iB} = a_i \, , \; i = 1,2 \; ; \;\; \gamma_{iA} \; = \gamma_{iB} \; = \gamma_i \, , \;\; i = 1,2 \qquad (87)$$

the international equilibrium terms of trade, $p = P_1/P_2$, becomes

$$\frac{a_1}{1-a_1} \frac{p^{\sigma-1} - \left[\frac{\gamma_1}{\gamma_2}\right]^{1-\sigma} \left[\frac{a_2}{a_1}\right]^{\sigma}}{p^{\sigma-1} - \left[\frac{\gamma_1}{\gamma_2}\right]^{1-\sigma} \left[\frac{1-a_2}{1-a_1}\right]^{\sigma}}$$

$$+ \left[\frac{\upsilon_A \alpha_A k_A + \upsilon_B \alpha_B k_B}{\upsilon_A \alpha_A + \upsilon_B \alpha_B} \; \frac{p^{\sigma-1} + \left[\frac{\gamma_1}{\gamma_2}\right]^{1-\sigma} \left[\frac{1-a_2}{1-a_1}\right]^{\sigma} \left[\frac{\upsilon_A k_A + \upsilon_B k_B}{\upsilon_A \alpha_A k_A + \upsilon_B \alpha_B k_B} - 1\right]}{p^{\sigma-1} + \left[\frac{\gamma_1}{\gamma_2}\right]^{1-\sigma} \left[\frac{a_2}{a_1}\right]^{\sigma} \left[\frac{1}{\upsilon_A \alpha_A + \upsilon_B \alpha_B} - 1\right]} \right]^{\frac{1-\sigma}{\sigma}} = 0 \qquad (88)$$

The relative CES commodity prices, $p_{\,ij} \; = \; c_{ij}\,(w/r)$, together with their CES pairs: $\omega_i = \frac{1-a_i}{a_i}\, k_i^{1/\sigma_i}$, $\omega_j = \frac{1-a_j}{a_j}\, k_j^{1/\sigma_j}$, cf. (31), are shown in Fig. 1.

The limits - for $\omega \, = \, w/r$, going to zero and infinity - of the relative price (cost) functions, $p_{\,ij} = c_{\,ij}\,(w/r)$, become either

$$p^{\,*}_{\,ij} \equiv \frac{\gamma_j}{\gamma_i} \frac{a_j^{\sigma_j/(\sigma_j-1)}}{a_i^{\sigma_i/(\sigma_i-1)}} \;\; ; \quad p^{\,**}_{\,ij} \equiv \frac{\gamma_j}{\gamma_i} \frac{(1-a_j)^{\sigma_j/(\sigma_j-1)}}{(1-a_i)^{\sigma_i/(\sigma_i-1)}}. \qquad (89)$$

and (89) is depicted in Figure 1.

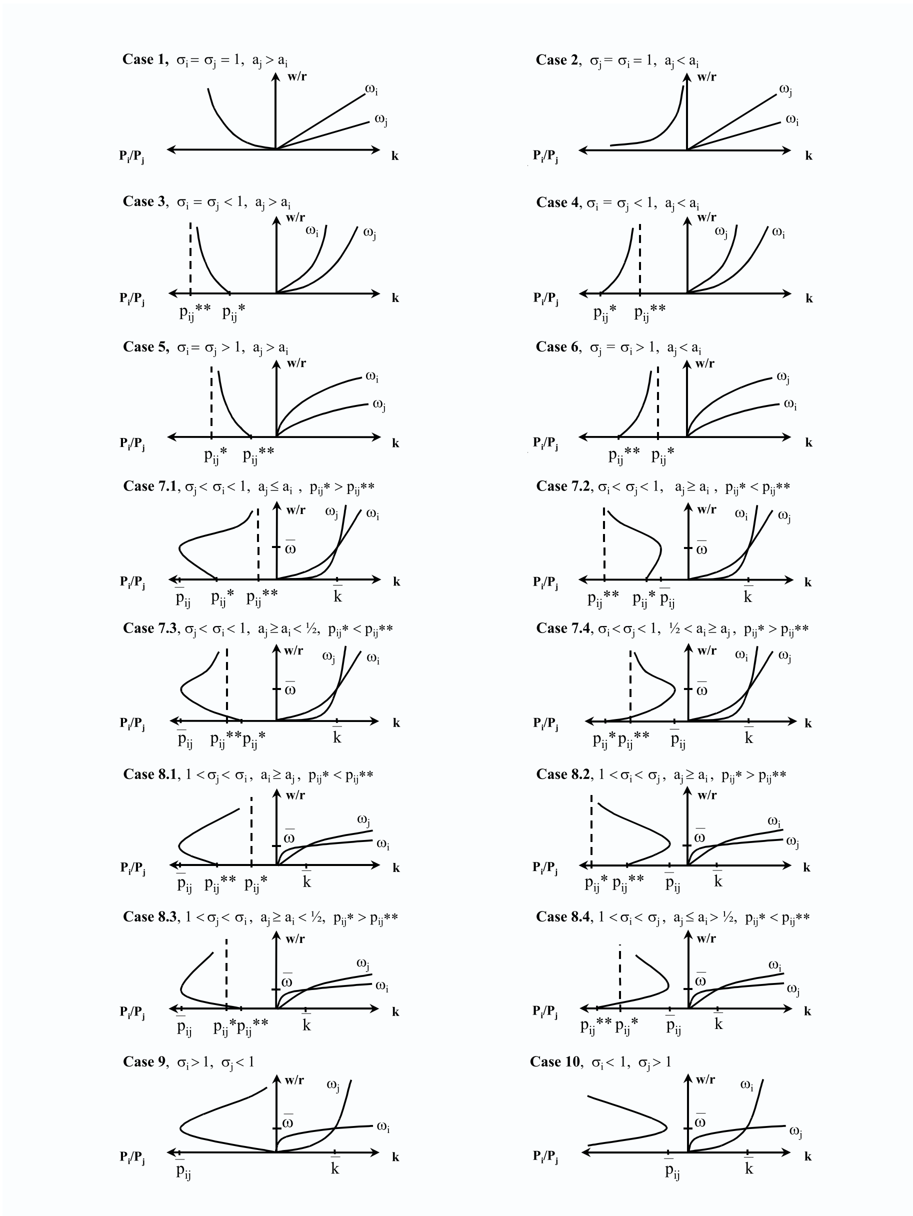

Figure 1: The relative prices $P_{iJ}/P_{jJ} = c_{iJ}(w/r)/c_{jJ}(w/r)$, $i = 1, j = 2$, with CD, (19), and CES, (34). The isoelastic pair: $\omega_i(k_i)$, $\omega_j(k_j)$
, $i = 1, j = 2$, (16), (31), of CD and of CES.

5 Results

The general model of international trade with two country and two goods contains a series of simpler models, which can be obtained as single special cases of the broader model. In Table 1, we provide an overview of these sub-models. They are ranked from the simplest one (model I, in the second column from left) to the most complex one (model IX in the last column right). The degree of generality increases with the number of parameters that differ across the two countries. Note that the list of parameters in the first column is exhaustive. In Table 1 only we assume that both countries are equally sized.

	No-Trade Model	Linder Model	Ricardo Model	Ricardo-Mill Model	H-O-S Model	H-O Model	H-O-L Model	Basic Model	General Model
	I	II	III	IV	V	VI	VII	VIII	IX
endowments	=	=	=	=	≠	≠	≠	≠	≠
technology	=	=	≠	≠	=	≠	=	≠	≠
preferences	=	≠	=	≠	=	=	≠	≠	≠
country size	=	=	=	=	=	=	=	=	≠

Table 1

In the CD-case, the international differences in technologies are simply obtained by assuming unequal sectorial productivities in the two countries, but assuming equal sectorial production elasticities throughout the world. In the CES-case, ...

Lemma 2. The international equilibrium terms of trade in the sub-models I-VIII, with CD technology

Here, I would insert equations (71)-(80) of the current version of our paper, although I am unsure whether we need to provide the solution for each sub-model

Lemma 3. The international equilibrium terms of trade in the sub-models I-VIII, with CES technology

Here, I would insert equation (88) of the current version of our paper plus the solutions for the other simpler cases, although (as above) I am unsure whether we need to provide the solution for each sub-model

5.1 The standard models

In this section we study the models with internationally equal consumer preferences. These are

1. The No-Trade Model (Model I)
2. The Ricardo Model (Model III)
3. The H-O-S Model (Model V)

We first prove the existence

Proposition 1: Existence of a free trade equilibrium in the standard models
If countries differ either in technologies or in endowments, a free trade equilibrium always exists.
Proof: The proof distinguishes the two cases of either different technologies or different endowments.
Case 1: different technologies

$$\frac{P_{1A}}{P_{2A}} > p > \frac{P_{1B}}{P_{2B}} \tag{90}$$

Corollary 1: No trade between identical countries If countries are identical, there is no international trade, and the international terms of trade equate the autarky relative price. Proof: The analytical proof is trivial and I have it.
The autarky relative price is

$$\frac{P_1}{P_2} = \frac{1}{\bar{a}} \frac{\gamma_2}{\gamma_1} \left(\frac{1-\bar{\alpha}}{\bar{\alpha}} k \right)^{a_2 - a_1} \tag{91}$$

which is identical to the international terms of trade (80).

Proposition 2: technologies and trade patterns
If countries differ only in their total factor productivities, the country will export the good which it can produce with the highest relative productivity.

Proof: Without loss of generality, imagine that country A is relatively more productive in sector 2

$$\frac{\gamma_{2A}}{\gamma_{1A}} > \frac{\gamma_{2B}}{\gamma_{1B}} \tag{92}$$

$$\frac{\gamma_{2A}}{\gamma_{1A}} \frac{1}{\bar{a}} \left(\frac{1-\bar{\alpha}}{\bar{\alpha}} \frac{K}{L} \right)^{a_2 - a_1} > \frac{\gamma_{2B}}{\gamma_{1B}} \frac{1}{\bar{a}} \left(\frac{1-\bar{\alpha}}{\bar{\alpha}} \frac{K}{L} \right)^{a_2 - a_1} \tag{93}$$

$$\frac{P_{1A}}{P_{2A}} > \frac{P_{1B}}{P_{2B}} \tag{94}$$

Proposition 3: endowments and trade patterns
If countries differ only in their endowments, the relative capital abundant country will export the capital-intensive commodity.
Proof: Without loss of generality, imagine that $k_A > k_B$ and that sector 1 is capital-intensive, i.e. $a_2 < a_1$. Then:

$$(k_A)^{a_2 - a_1} < (k_B)^{a_2 - a_1} \tag{95}$$

$$\frac{\gamma_2 (a_2)^{a_2} (1-a_2)^{1-a_2}}{\gamma_1 (a_1)^{a_1} (1-a_1)^{1-a_1}} \left(\frac{1-\rho}{\rho} k_A \right)^{a_2 - a_1} < \frac{\gamma_2 (a_2)^{a_2} (1-a_2)^{1-a_2}}{\gamma_1 (a_1)^{a_1} (1-a_1)^{1-a_1}} \left(\frac{1-\rho}{\rho} k_B \right)^{a_2 - a_1} \tag{96}$$

$$\frac{P_{1A}}{P_{2A}} < \frac{P_{1B}}{P_{2B}} \tag{97}$$

Country A exports good 1.

5.2 The role of preferences

Proposition 4: preferences and trade patterns
If countries differ only in preferences, and $\alpha_A > \alpha_B$, country A is a net exporter of good 2, i.e. $p_A > p_B$.
Proof: Without loss of generality, assume that $a_1 > a_2$. Extend this to:

$$\left[(1-\alpha_B) - (1-\alpha_A) \right] a_2 < (\alpha_A - \alpha_B)\, a_1 \tag{98}$$

and obtain

$$\frac{1-\bar{\alpha}_A}{\bar{\alpha}_A} < \frac{1-\bar{\alpha}_B}{\bar{\alpha}_B} \tag{99}$$

which is equivalent to $p_A > p_B$.

Table 2 illustrates Proposition 4.

α_A	α_B	p	ω_A	ω_B	k_{1A}	k_{1B}	k_{2A}	k_{2B}	$\lambda_{L(1A)}$	$\lambda_{L(1B)}$	$\lambda_{K(1A)}$	$\lambda_{K(1B)}$	$Y_{1A} - Q_{1A}$
0,60	0,50	1,111	3,161	3,161	3,161	3,161	1,054	1,054	0,449	0,449	0,710	0,710	-72,572
0,70	0,50	1,126	3,000	3,000	3,000	3,000	1,000	1,000	0,500	0,500	0,750	0,750	-144,338
0,80	0,50	1,141	2,848	2,848	2,848	2,848	0,949	0,949	0,553	0,553	0,788	0,788	-215,457
0,60	0,40	1,097	3,333	3,333	3,333	3,333	1,111	1,111	0,400	0,400	0,667	0,667	-146,059
0,70	0,30	1,097	3,333	3,333	3,333	3,333	1,111	1,111	0,400	0,400	0,667	0,667	-292,119
0,80	0,20	1,097	3,333	3,333	3,333	3,333	1,111	1,111	0,400	0,400	0,667	0,667	-438,178

Table 2a $k_J = 2;\ a_{1J} = 0.5;\ a_{2J} = 0.25;\ \gamma_{1J} = 2;\ \gamma_{2J} = 2.6;$

α_A	α_B	p	ω_A	ω_B	k_{1A}	k_{1B}	k_{2A}	k_{2B}	$\lambda_{L(1A)}$	$\lambda_{L(1B)}$	$\lambda_{K(1A)}$	$\lambda_{K(1B)}$	$Y_{1A} - Q_{1A}$
0,60	0,50	1,562	3,517	3,517	1,172	1,172	3,517	3,517	0,647	0,647	0,379	0,379	-61,210
0,70	0,50	1,583	3,714	3,714	1,238	1,238	3,714	3,714	0,692	0,692	0,429	0,429	-121,713
0,80	0,50	1,606	3,926	3,926	1,309	1,309	3,926	3,926	0,736	0,736	0,481	0,481	-181,623
0,60	0,40	1,541	3,333	3,333	1,111	1,111	3,333	3,333	0,600	0,600	0,333	0,333	-123,203
0,70	0,30	1,541	3,333	3,333	1,111	1,111	3,333	3,333	0,600	0,600	0,333	0,333	-246,406
0,80	0,20	1,541	3,333	3,333	1,111	1,111	3,333	3,333	0,600	0,600	0,333	0,333	-369,608

Table 2b $k_J = 2;\ a_{1J} = 0.25;\ a_{2J} = 0.5;\ \gamma_{1J} = 2;\ \gamma_{2J} = 2.6;$

Proposition 5: technologies, preferences and trade patterns

If countries differ in technologies and preferences, the trade patterns follow from the interplay between both aspects.

α_A	α_B	p	ω_A	ω_B	k_{1A}	k_{1B}	k_{2A}	k_{2B}	$\lambda_{L(1A)}$	$\lambda_{L(1B)}$	$\lambda_{K(1A)}$	$\lambda_{K(1B)}$	$Y_{1A} - Q_{1A}$
0,20	0,80	0,90	3,60	3,02	3,60	3,02	1,20	1,01	0,33	0,49	0,60	0,74	404,54
0,30	0,70	0,90	3,62	3,04	3,62	3,04	1,21	1,01	0,33	0,49	0,60	0,74	219,26
0,40	0,60	0,89	3,64	3,05	3,64	3,05	1,21	1,02	0,32	0,48	0,59	0,74	33,74
0,60	0,40	0,89	3,67	3,09	3,67	3,09	1,22	1,03	0,32	0,47	0,58	0,73	-338,02
0,70	0,30	0,89	3,69	3,10	3,69	3,10	1,23	1,03	0,31	0,47	0,58	0,72	-524,27
0,80	0,20	0,89	3,71	3,12	3,71	3,12	1,24	1,04	0,31	0,46	0,57	0,72	-710,75
0,50	0,20	0,86	4,34	3,64	4,34	3,64	1,45	1,21	0,19	0,32	0,42	0,59	-434,35
0,50	0,30	0,87	4,09	3,44	4,09	3,44	1,36	1,15	0,23	0,37	0,48	0,64	-338,61
0,50	0,40	0,88	3,87	3,25	3,87	3,25	1,29	1,08	0,28	0,42	0,53	0,69	-244,59
0,50	0,60	0,91	3,46	2,90	3,46	2,90	1,15	0,97	0,37	0,53	0,64	0,77	-60,66
0,50	0,70	0,92	3,27	2,75	3,27	2,75	1,09	0,92	0,42	0,59	0,68	0,81	29,73
0,50	0,80	0,93	3,10	2,60	3,10	2,60	1,03	0,87	0,47	0,65	0,73	0,85	119,37

Table 3 $k_J = 2;\ a_{1J} = 0.5;\ a_{2J} = 0.25;\ \gamma_{1A} = 2.4;\ \gamma_{1B} = 2.7;\ \gamma_{2A} = 2.6;\ \gamma_{2B} = 2.8$

Proposition 6: endowments, preferences and trade patterns
If countries differ in relative endowments and preferences, the trade patterns follow from the interplay between both aspects.

α_A	α_B	p	ω_A	ω_B	k_{1A}	k_{1B}	k_{2A}	k_{2B}	$\lambda_{L(1A)}$	$\lambda_{L(1B)}$	$\lambda_{K(1A)}$	$\lambda_{K(1B)}$	Y_{1A} - Q_{1A}
0,8	0,2	0,561	3,373	3,373	3,373	3,373	1,124	1,124	0,300	0,478	0,563	0,733	-862,433
0,7	0,3	0,561	3,360	3,360	3,360	3,360	1,120	1,120	0,304	0,482	0,567	0,736	-643,197
0,6	0,4	0,562	3,347	3,347	3,347	3,347	1,116	1,116	0,307	0,486	0,570	0,739	-424,185
0,4	0,6	0,563	3,320	3,320	3,320	3,320	1,107	1,107	0,313	0,494	0,578	0,745	13,172
0,3	0,7	0,563	3,307	3,307	3,307	3,307	1,102	1,102	0,317	0,498	0,581	0,748	231,519
0,2	0,8	0,564	3,293	3,293	3,293	3,293	1,098	1,098	0,320	0,502	0,585	0,752	449,648
0,5	0,2	0,539	3,948	3,948	3,948	3,948	1,316	1,316	0,184	0,336	0,403	0,603	-536,824
0,5	0,3	0,547	3,729	3,729	3,729	3,729	1,243	1,243	0,224	0,385	0,464	0,653	-424,476
0,5	0,4	0,555	3,524	3,524	3,524	3,524	1,175	1,175	0,266	0,436	0,521	0,699	-314,103
0,5	0,6	0,570	3,155	3,155	3,155	3,155	1,052	1,052	0,356	0,546	0,624	0,783	-98,072
0,5	0,7	0,578	2,987	2,987	2,987	2,987	0,996	0,996	0,404	0,605	0,670	0,821	8,136
0,5	0,8	0,586	2,830	2,830	2,830	2,830	0,943	0,943	0,454	0,666	0,714	0,857	113,478

Table 4 $k_A = 1.8$; $k_B = 2.2$; $a_{1J} = 0.5$; $a_{2J} = 0.25$; $\gamma_{1J} = 3$; $\gamma_{2J} = 2$

Propositions 4-6 yield the following

Theorem 3: trade patterns in the general two-country, two-good model of international trade
The production side and the consumption side are equally relevant for trade patterns.

Corollary 3: technologies, relative endowments, preferences and trade patterns
If countries differ in technologies, relative endowments and preferences, the trade patterns follow from the interplay between all these three features.

5.3 The role of countries' relative size

Contrarily to the common neoclassical wisdom, we show in this section that the relative country size indeed plays a crucial role for the existence of a free trade equilibrium. This holds true for the cases of countries with unequal relative endowments or technologies or both of them. In the case of countries, which differ only their absolute sizes or in their preferences, the relative country size is irrelevant. These results are provided by the following two propositions.

	No-Trade Model	**Linder Model**	**Ricardo Model**	**Ricardo-Mill Model**	**H-O-S Model**	**H-O Model**	**H-O-L Model**
	I	**II**	**III**	**IV**	**V**	**VI**	**VII**
endowments	=	=	=	=	≠	≠	≠
technology	=	=	≠	≠	=	≠	=
preferences	=	≠	=	≠	=	=	≠
country size	≠	≠	≠	≠	≠	≠	≠

Proposition 7: Existence of a free trade equilibrium for countries of unequal size
If countries differ in relative endowments, in technologies or in both of them, the existence of an international trade equilibrium is subject to the country relative size.
Corollary 1: If countries differ only in their relative sizes, there is no free trade equilibrium **Proof:** trivial
Corollary 2: if countries differ only in preferences, a free trade equilibrium always exists (the relative country size is irrelevant for the free trade equilibrium).
Proof: (to be completed)

$$\frac{P_{1A}}{P_{2A}} > p > \frac{P_{1B}}{P_{2B}} \tag{100}$$

$$\frac{1}{\bar{a}}\frac{\gamma_2}{\gamma_1}\left(\frac{1-\bar{\alpha}_A}{\bar{\alpha}_A}k\right)^{a_2-a_1} > p > \frac{1}{\bar{a}}\frac{\gamma_2}{\gamma_1}\left(\frac{1-\bar{\alpha}_B}{\bar{\alpha}_B}k\right)^{a_2-a_1} \tag{101}$$

$$\frac{1}{\bar{a}}\frac{\gamma_2}{\gamma_1}\left(\frac{1-\bar{\alpha}_A}{\bar{\alpha}_A}k\right)^{a_2-a_1} > \frac{1}{\bar{a}}\frac{\gamma_2}{\gamma_1}\left[\frac{\upsilon_A\left(1-\bar{\alpha}_A\right)+\upsilon_B\left(1-\bar{\alpha}_B\right)}{\upsilon_A\cdot\bar{\alpha}_A+\upsilon_B\cdot\bar{\alpha}_B}k\right]^{a_2-a_1} > \frac{1}{\bar{a}}\frac{\gamma_2}{\gamma_1}\left(\frac{1-\bar{\alpha}_B}{\bar{\alpha}_B}k\right)^{a_2-a_1} \tag{102}$$

Proposition 8: relative country size and trade patterns
The relative country size is irrelevant for the trade patterns.
Proof: This is based on the observation that the relative price in autarky does not depend on the country size (this is indeed a basic feature of any neoclassical model)

H-O-S (cases 3a and 3b)

υ_A	k_A	k_B	a_{1A}	a_{2A}	a_{1B}	a_{2B}	γ_{1A}	γ_{2A}	γ_{1B}	γ_{2B}	α_A	α_B	p	ω_A	ω_B	k_{1A}	k_{1B}	k_{2A}	k_{2B}	$\lambda_{L(1A)}$	$\lambda_{L(1B)}$	$\lambda_{K(1A)}$	$\lambda_{K(1B)}$	Y_{1A} - Q_{1A}
0.25	1.8	2.2	0.5	0.25	0.5	0.25	2	2.6	2	2.6	0.2	0.2	1.00	4.90	4.90	4.90	4.90	1.63	1.63	0.05	0.17	0.14	0.39	-94.87
0.25	1.8	2.2	0.5	0.25	0.5	0.25	2	2.6	2	2.6	0.3	0.3	1.03	4.36	4.36	4.36	4.36	1.45	1.45	0.12	0.26	0.29	0.51	-96.96
0.25	1.8	2.2	0.5	0.25	0.5	0.25	2	2.6	2	2.6	0.5	0.5	1.08	3.50	3.50	3.50	3.50	1.17	1.17	0.27	0.44	0.53	0.70	-100.22
0.25	1.8	2.2	0.5	0.25	0.5	0.25	2	2.6	2	2.6	0.7	0.7	1.14	2.84	2.84	2.84	2.84	0.95	0.95	0.45	0.66	0.71	0.85	-102.34
0.25	1.8	2.2	0.5	0.25	0.5	0.25	2	2.6	2	2.6	0.8	0.8	1.17	2.57	2.57	2.57	2.57	0.86	0.86	0.55	0.79	0.79	0.92	-102.99
0.25	1.8	2.2	0.35	0.25	0.35	0.25	2	2.6	2	2.6	0.3	0.3	1.20	5.40	5.40	2.91	2.91	1.80	1.80	0.00	0.36	0.00	0.48	-188.88
0.25	1.8	2.2	0.35	0.25	0.35	0.25	2	2.6	2	2.6	0.4	0.4	1.20	5.14	5.14	2.77	2.77	1.71	1.71	0.08	0.46	0.13	0.58	-192.29
0.25	1.8	2.2	0.35	0.25	0.35	0.25	2	2.6	2	2.6	0.5	0.5	1.21	4.90	4.90	2.64	2.64	1.63	1.63	0.17	0.56	0.24	0.68	-195.60
0.25	1.8	2.2	0.35	0.25	0.35	0.25	2	2.6	2	2.6	0.6	0.6	1.21	4.67	4.67	2.52	2.52	1.56	1.56	0.25	0.67	0.35	0.77	-198.81
0.25	1.8	2.2	0.35	0.25	0.35	0.25	2	2.6	2	2.6	0.7	0.7	1.22	4.46	4.46	2.40	2.40	1.49	1.49	0.34	0.78	0.46	0.85	-201.93
0.5	1.8	2.2	0.5	0.25	0.5	0.25	2	2.6	2	2.6	0.2	0.2	1.01	4.67	4.67	4.67	4.67	1.56	1.56	0.08	0.21	0.20	0.44	-129.61
0.5	1.8	2.2	0.5	0.25	0.5	0.25	2	2.6	2	2.6	0.3	0.3	1.04	4.15	4.15	4.15	4.15	1.38	1.38	0.15	0.29	0.35	0.56	-132.48
0.5	1.8	2.2	0.5	0.25	0.5	0.25	2	2.6	2	2.6	0.5	0.5	1.10	3.33	3.33	3.33	3.33	1.11	1.11	0.31	0.49	0.57	0.74	-136.93
0.5	1.8	2.2	0.5	0.25	0.5	0.25	2	2.6	2	2.6	0.7	0.7	1.16	2.71	2.71	2.71	2.71	0.90	0.90	0.50	0.72	0.75	0.89	-139.82
0.5	1.8	2.2	0.5	0.25	0.5	0.25	2	2.6	2	2.6	0.8	0.8	1.18	2.44	2.44	2.44	2.44	0.81	0.81	0.60	0.85	0.82	0.94	-140.71
0.5	1.8	2.2	0.35	0.25	0.35	0.25	2	2.6	2	2.6	0.3	0.3	1.20	5.14	5.14	2.77	2.77	1.71	1.71	0.08	0.46	0.13	0.58	-259.95
0.5	1.8	2.2	0.35	0.25	0.35	0.25	2	2.6	2	2.6	0.4	0.4	1.21	4.90	4.90	2.64	2.64	1.63	1.63	0.17	0.57	0.24	0.68	-264.65
0.5	1.8	2.2	0.35	0.25	0.35	0.25	2	2.6	2	2.6	0.5	0.5	1.21	4.67	4.67	2.51	2.51	1.56	1.56	0.26	0.67	0.36	0.77	-269.21
0.5	1.8	2.2	0.35	0.25	0.35	0.25	2	2.6	2	2.6	0.6	0.6	1.22	4.45	4.45	2.40	2.40	1.48	1.48	0.35	0.78	0.46	0.85	-273.63
0.5	1.8	2.2	0.35	0.25	0.35	0.25	2	2.6	2	2.6	0.7	0.7	1.22	4.25	4.25	2.29	2.29	1.42	1.42	0.44	0.90	0.56	0.93	-277.91
0.692	1.8	2.2	0.5	0.25	0.5	0.25	2	2.6	2	2.6	0.2	0.2	1.02	4.49	4.49	4.49	4.49	1.50	1.50	0.10	0.24	0.25	0.48	-112.69
0.692	1.8	2.2	0.5	0.25	0.5	0.25	2	2.6	2	2.6	0.3	0.3	1.05	3.99	3.99	3.99	3.99	1.33	1.33	0.18	0.33	0.39	0.59	-115.17
0.692	1.8	2.2	0.5	0.25	0.5	0.25	2	2.6	2	2.6	0.5	0.5	1.11	3.21	3.21	3.21	3.21	1.07	1.07	0.34	0.53	0.61	0.77	-119.05
0.692	1.8	2.2	0.5	0.25	0.5	0.25	2	2.6	2	2.6	0.7	0.7	1.17	2.60	2.60	2.60	2.60	0.87	0.87	0.54	0.77	0.78	0.91	-121.56
0.692	1.8	2.2	0.5	0.25	0.5	0.25	2	2.6	2	2.6	0.8	0.8	1.20	2.35	2.35	2.35	2.35	0.78	0.78	0.65	0.90	0.85	0.97	-122.34

6 Conclusions

7 References

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