Intersectoral markup divergence and welfare

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Abstract

We develop a general equilibrium model of monopolistic competition with a traded and a non-traded sector. Using preferences that generate variable markups and pro-competitive effects, we show how trade liberalization reduces markups in the traded sector and increases markups in the non-traded sector. Hence, while markups in the traded sector converge across countries due to trade liberalization, they diverge across sectors within countries. The welfare effects of trade liberalization are therefore ambiguous: the direct positive effects in the traded sector may be dominated by the indirect negative effects in the non-traded sector, especially when trade costs are large and the non-traded sector is small.

Keywords: monopolistic competition; implicitly additive preferences; variable markups; trade liberalization; non-traded goods.

JEL Classification: F12, F15.

1 Introduction

The remainder of the paper is organized as follows. Section 2 lays out the preference structure that we build upon and describes the trade model. Section 3 studies the equilibrium in the non-traded sector, whereas Section 4 studies the equilibrium in the traded sector. Section 5 explores the consequences of trade liberalization in both sectors. Finally, Section 6 concludes.

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2 The model

We develop a general equilibrium model of monopolistic competition with costly trade between two countries, 'home', H, and 'foreign', F. For simplicity, countries are assumed to have the same population, L. We relax that assumption in Appendix and show that our key results extend to the case of asymmetric population sizes. There are two sectors in each country. The first one produces a continuum of horizontally differentiated varieties of a traded good ('manufacturing'), whereas the second one produces a continuum of varieties of a non-traded good ('services'). There is a single production factor, labor. Countries have identical technologies.

2.1 Preferences

We work with homothetic preferences that are not exactly CES but that lie in a 'small neighborhood' of CES preferences – in a mathematical sense that will become clear below. There are two main reasons for using these preferences. First, homotheticity implies that (ideal) price indices are well-defined, which simplifies the analysis considerably. Second, that class of preferences retains some desirable properties of the CES and nests them as a special case. Since CES preferences are the most popular ones in the international trade literature (e.g., Krugman, 1980; Eaton and Kortum, 2002; Melitz, 2003), staying close to them does not require us to go into completely new modeling directions. It also allows us to strike a balance between generality and tractability. As we show later, using such 'ε-CES preferences' makes our analysis of the interactions between the traded and the non-traded sectors fairly tractable even when allowing for empirically relevant features like variable markups and varying firm sizes, which generally make the analysis fairly involved. now define in a rigorous way what we mean by 'being in a small neighborhood of CES preferences'. To this end, we proceed in two steps:

Step 1: Implicitly additive homothetic preferences. As shown by Zhelobodko *et al.* (2012), under symmetric additive preferences the elasticity of substitution between varieties i and j, $\overline{\sigma}(x_i, x_j, \mathbf{x})$, is independent of the remaining consumption pattern $\mathbf{x}$, given that both varieties are consumed in equal quantities. Put differently, if $x_i = x_j = x$, we have $\overline{\sigma}(x, x, \mathbf{x}) = \sigma(x)$. Thus, additive preferences have a simple behavior with respect to the elasticity of substitution. Since the substitutability across varieties is a key feature of the demand side in models of imperfect competition, the aforementioned property explains, at least partly, why additive preferences are so popular.[1] However, additive non-CES preferences are well-known to be non-homothetic, i.e., they do not have a well-defined (ideal) price index.

To combine both desirable properties mentioned above – additivity and the existence of an (ideal) price index – we focus on homothetic preferences for which there exists a function $s(x, u)$ such that

$$\overline{\sigma}(x, x, \mathbf{x}) = s(x, u(\mathbf{x})). \tag{1}$$

In expression (1), $u(\mathbf{x})$ is the consumption index, which is increasing, strictly quasi-concave, and positive homogeneous of degree 1 in $\mathbf{x}$. Moreover, observe that at a symmetric consumption pattern, given by $\mathbf{x} = x\,\mathbf{1}_{[0,N]}$, where N is the mass of available varieties, we have

$$u(\mathbf{x}) = x\nu(N), \quad \text{so that} \quad s(x, u(\mathbf{x})) = s(x, x\nu(N)). \tag{2}$$

As shown by Parenti *et al.* (2014), when preferences are homothetic the elasticity of substitution σ depends solely on N at a symmetric consumption pattern. Combining this fact with (2) shows that $s(x, u)$ is positively homogeneous of degree 0. Hence, expression (1) boils down to

$$\overline{\sigma}(x, x, \mathbf{x}) = \sigma(x/u(\mathbf{x})). \tag{3}$$

The intuition behind expression (3) is as follows. If $\sigma(z)$ is a decreasing function, then a higher consumption level of both varieties makes them worse substitutes, while an increase in the overall level of consumption captured by $u(\mathbf{x})$ does the opposite.[2] However, a *proportional change in the consumption of all varieties* leaves the degree of substitutability between any two of them unchanged.

The following proposition provides an alternative characterization of the preferences satisfying (3).

1 *[Implicitly additive homothetic preferences] A symmetric homothetic preference relationship satisfies (3) if and only if the utility function u is described by Kimball's (1995) flexible aggregator*

$$\int_0^N \theta\left(\frac{x_i}{u}\right) \mathrm{d}i = 1, \tag{4}$$

where $\theta(\cdot)$ is an arbitrary non-negative, increasing, concave, and twice continuously differentiable function.

[1]Examples include, among others, Krugman (1979, 1980); Eaton and Kortum (2002); Melitz (2003); Behrens and Murata (2007); Simonovska (2010); and Zhelobodko *et al.* (2012). Most of these papers stick to CES preferences, though non-CES preferences have attracted more attention lately.

[2]Of course, if $\sigma(z)$ is decreasing then the opposite properties hold.

Proof. See Appendix A.1. ■

Note that Kimball (1995) uses (4) to represent a production function for a fixed range of varieties. Dotsey and King (2005) use the same approach for modeling preferences in a macroeconomic context, having again a fixed range of varieties, while Barde (2008) does the same in the context of a new economic geography model with a variable range of varieties.

Step 2: Neighborhood of the ces (ε-ces). We now consider preferences of the implicitly additive class defined above, with

$$\theta(z) = z^{\rho} \exp\big(\varphi(z)\big), \quad \rho \in (0,1), \tag{5}$$

where $\varphi(\cdot)$ is a twice continuously differentiable function that is sufficiently small in a mathematical sense. To be precise, we assume that $\varphi \in C^2(\mathbb{R}_+)$ and $||\varphi||_{C^2} < \varepsilon$, where $||\cdot||_{C^2}$ is the standard norm in the space of twice continuously differentiable functions:

$$||\varphi||_{C^2} \equiv ||\varphi||_C + ||\varphi'||_C + ||\varphi''||_C,$$

with $||\cdot||_C$ being the norm of uniform convergence defined as $||\varphi||_C \equiv \sup_{z\geq 0} |\varphi(z)|$. To use (5) when solving the equilibrium conditions of the model, we need the expressions for $\theta'(\cdot)$ and $r_\theta(\cdot)$, which are given by

$$\theta'(z) = z^{\rho-1} \exp\left[\varphi(z)\right] \left[\rho + z\varphi'(z)\right], \tag{6}$$

and

$$r_\theta(z) = 1 - \rho - \xi(z), \tag{7}$$

respectively. The residual term, $\xi(\cdot)$, is given by

$$\xi(z) \equiv \frac{z\varphi'(z)}{\rho + z\varphi'(z)} \left[1 + \rho + z\varphi'(z)\right] + \frac{z^2\varphi''(z)}{\rho + z\varphi'(z)}. \tag{8}$$

To guarantee that $||\xi||_C \to 0$ as $||\varphi||_{C^2} \to 0$, we need that $z\varphi'(z)$ and $z^2\varphi''(z)$ uniformly converge to zero. Therefore, we restrict ourselves to perturbations $\varphi \in \mathcal{F}$, where $\mathcal{F}$ is the closed subspace of $C^2(\mathbb{R}_+)$ defined as

$$\mathcal{F} \equiv \left\{\varphi \in C^2(\mathbb{R}_+) \mid \lim_{z\to\infty} z^2\varphi'(z) = \lim_{z\to\infty} z^3\varphi''(z) = 0\right\}. \tag{9}$$

The second restriction in (9) guarantees that $||\xi||_C \to 0$, so that we can make (5) arbitrarily close to the standard ces case.

2.2 Consumers

Each country has L consumers, who share identical preferences given by

$$\mathcal{U}^k = U(u(\mathbf{x}^{kk}, \mathbf{x}^{lk}), v(\mathbf{y}^k)), \quad k, l \in \{H, F\}, \quad k \neq l, \tag{10}$$

where U is a homothetic upper-tier utility; u and v are lower-tier utilities for the consumption of, respectively, traded and non-traded goods; and where $\mathbf{x}^{kk}$, $\mathbf{x}^{lk}$, and $\mathbf{y}^k$ are vectors of individual consumption of, respectively, varieties of the traded good produced and consumed in country k, varieties of the traded good imported from country l to country k, and varieties of the non-traded good.

We assume that U, u, and v are strictly increasing, strictly quasi-concave and positively homogeneous of degree 1. We also assume that the lower-tier utilities are implicitly additive, as defined in Section **??**, i.e., there exist increasing and concave functions θ and ψ, such that for any $\mathbf{x}^{kk}$, $\mathbf{x}^{lk}$, and $\mathbf{y}^k$, the lower-tier utilities u and v satisfy

$$\int_0^{N^k} \theta\left(\frac{x_i^{kk}}{u^k}\right) \mathrm{d}i + \int_0^{N^l} \theta\left(\frac{x_j^{lk}}{u^k}\right) \mathrm{d}j = 1, \quad \int_0^{M^k} \psi\left(\frac{y_i^k}{v^k}\right) \mathrm{d}i = 1, \quad k, l \in \{H, F\}, \tag{11}$$

where $u^k \equiv u(\mathbf{x}^{kk}, \mathbf{x}^{lk})$, $v^k \equiv v(\mathbf{y}^k)$; where N^k and N^l are the (endogenously determined) masses of firms in the traded sector in countries k and l, respectively; and where M^k is the number of firms in the non-traded sector in country k.

Letting the wage $w = 1$ by choice of numéraire – and recalling that countries are symmetric and thus have the same equilibrium wage – consumers maximize their utility (10) subject to the budget constraint

$$\int_0^{N^k} p_i^{kk} x_i^{kk} \mathrm{d}i + \int_0^{N^l} p_i^{lk} x_i^{lk} \mathrm{d}i + \int_0^{M^k} \wp_i^k y_i^k \mathrm{d}i = 1,$$

where p_i^{kk} and p_i^{lk} are the prices of domestic and imported tradable varieties in country k; and where $\wp_i^k$ are the prices of non-tradable varieties.

Let P^k and $\mathcal{P}^k$ denote the price indices in the traded and in the non-traded sectors in country k, respectively. Let also $\alpha(P^k/\mathcal{P}^k)$ denote the share of consumer expenditure for traded goods, which depends on the ratio of the traded and the non-traded sectoral price indices. The functional form of $\alpha(\cdot)$ is determined by the upper-tier utility, while the value of $\alpha(P/\mathcal{P})$ is perceived by consumers as a constant (though it changes with the price aggregates in the two sectors).

The inverse demands can then be derived as follows. First, note that the consumers' subproblem for the consumption of the traded good is given by:[3]

$$\begin{aligned} \max_{u^k, \mathbf{x}^{kk}, \mathbf{x}^{lk}} \quad & u^k \\ \text{s.t.} \quad & \int_0^{N^k} p_i^{kk} x_i^{kk} \mathrm{d}i + \int_0^{N^l} p_i^{lk} x_i^{lk} \mathrm{d}i = \alpha \left(\frac{P}{\mathcal{P}} \right) \\ & \int_0^{N^k} \theta \left(\frac{x_i^{kk}}{u^k} \right) \mathrm{d}i + \int_0^{N^l} \theta \left(\frac{x_j^{lk}}{u^k} \right) \mathrm{d}j = 1. \end{aligned} \tag{12}$$

Setting $\mathbf{z}^{kk} \equiv \mathbf{x}^{kk}/u^k$ and $\mathbf{z}^{lk} \equiv \mathbf{x}^{lk}/u^k$ for notational convenience, we reformulate the constraints as

$$\int_0^{N^k} p_i^{kk} z_i^{kk} \mathrm{d}i + \int_0^{N^l} p_i^{lk} z_i^{lk} \mathrm{d}i = \frac{\alpha(P/\mathcal{P})}{u}, \quad \text{and} \quad \int_0^{N^k} \theta \left(z_i^{kk} \right) \mathrm{d}i + \int_0^{N^l} \theta \left(z_i^{lk} \right) \mathrm{d}j = 1.$$

Since maximizing u^k is equivalent to minimizing $1/u^k$, we have the following equivalent reformulation of (12) as an expenditure minimization problem:

$$\min_{\mathbf{z}} \quad \int_0^{N^k} p_i^{kk} z_i^{kk} \mathrm{d}i + \int_0^{N^l} p_i^{lk} z_i^{lk} \mathrm{d}i \quad \text{s.t.} \quad \int_0^{N^k} \theta \left(z_i^{kk} \right) \mathrm{d}i + \int_0^{N^l} \theta \left(z_i^{lk} \right) \mathrm{d}j = 1. \tag{13}$$

The first-order conditions of (13) are given by $p_i = \lambda \theta'(z_i)$, which implies that

$$\frac{\theta'(z_i^{kk})}{\mu^k} = p_i^{kk}, \quad \frac{\theta'(z_j^{lk})}{\mu^k} = p_j^{kl}, \tag{14}$$

where μ^k is a sectoral market aggregate that involves the Lagrange multiplier of the budget constraint as well as the marginal utilities of consuming tradable varieties. Using the subproblem for the non-traded sector, we can obtain the inverse demands for the non-traded good as follows:

$$\frac{\psi' \left(\frac{y_i^k}{v} \right)}{\lambda^k} = \wp_i^k, \tag{15}$$

where λ^k is sectoral market aggregate for non-tradable varieties.

2.3 Firms

Firms in both sectors incur a constant fixed cost, f, and a constant marginal cost, c, both paid in terms of labor.[4] The costs for shipping goods between countries in the traded

[3]Because of homothetic preferences in both the upper- and the lower-tier of the utility functions, be can apply standard two-stage budgeting techniques.

[4]Introducing technological differences between sectors does not add anything substantial to the analysis but makes the algebra more complicated.

sector are of the standard 'iceberg form': $\tau \geq 1$ units of the good have to be shipped for one unit to arrive. We now turn to the profits of firms in country k. The profits of firm i in the traded sector and of firm j in the non-traded sector in country k are given by

$$\pi_i^k = (p_i^{kk} - c)Lx_i^{kk} + (p_i^{kl} - c)Lx_i^{kl} - f \quad \text{and} \quad \pi_j^k = (\wp_j^k - c)Ly_j^k - f,$$

respectively. Because countries are symmetric, we naturally focus on a symmetric outcome. In what follows, we use the following notation, where d denotes domestic and m denotes import values of variables:

$$x^d \equiv x^{HH} = x^{FF},\ x^m \equiv x^{FH} = x^{HF},\ y \equiv y^H = y^F,$$

$$p^d \equiv p^{HH} = p^{FF},\ p^m \equiv p^{FH} = p^{HF},\ \wp \equiv \wp^H = \wp^F,$$

$$\lambda \equiv \lambda^H = \lambda^F,\ \mu \equiv \mu^H = \mu^F,$$

$$N \equiv N^H = N^F,\ M \equiv M^H = M^F.$$

$$u \equiv u^H = u^F,\ v \equiv v^H = v^F.$$

Because we work with a continuum of monopolistically competitive firms, no single firm has any impact on the market aggregates μ and λ. Thus, at a symmetric outcome, the profit-maximizing prices are given by

$$p^d = \frac{c}{1 - r_\theta(z^d)}, \quad p^m = \frac{c\tau}{1 - r_\theta(z^m)}, \quad \wp = \frac{c}{1 - r_\psi(y/v)}, \tag{16}$$

where $r_\theta(z^d)$ is the markup for domestically produced varieties of the non-traded good; where $r_\theta(z^m)$ is the markup for imported varieties; and where $r_\psi(y/v)$ is the markup for non-traded varieties.

Combining (14) and (16), we readily obtain

$$\theta'(z^d)\left[1 - r_\theta(z^d)\right] = \mu c, \quad \theta'(z^m)\left[1 - r_\theta(z^m)\right] = \mu c\tau, \quad \theta'\left(\frac{y}{v}\right)\left[1 - r_\psi\left(\frac{y}{v}\right)\right] = \lambda c. \tag{17}$$

The conditions (17), which equate marginal revenue and marginal cost, allow us to pin down the quantities z^d, z^m, and y/v as functions of the market aggregates μ and λ only.

3 Equilibrium in the non-traded sector

We now study how the equilibrium in the non-traded sector changes in response to trade liberalization, as measured by a decrease in trade costs τ. The key finding is that the impacts of trade liberalization on the non-traded sector are entirely channeled through changes in the price index for traded varieties.

(11) and (16) for a given mass, M, of non-traded varieties, the symmetric equilibrium conditions are given by

$$\mathcal{P}v = M\frac{cy}{1 - r_{\psi}\left(\frac{y}{v}\right)} = 1 - \alpha\left(\frac{P}{\mathcal{P}}\right), \tag{18}$$

and

$$M\psi\left(\frac{y}{v}\right) = 1. \tag{19}$$

Solving (19) for the ratio y/v yields

$$\frac{y}{v} = \psi^{-1}\left(\frac{1}{M}\right). \tag{20}$$

Since ψ is an increasing function, the relationship (20) implies that the relative consumption of each variety of the non-traded good decreases with an expanding range of varieties in the non-traded sector. (16) and (20), the equilibrium markup for a variety of the non-traded good is thus given by $r_{\psi}\big(\psi^{-1}(1/M)\big)$. the definition of $r_{\psi}(\cdot)$ in (A-1) in Appendix A, this implies varieties decrease in response to entry if and only if $r_{\psi}(\cdot)$ is increasing or, equivalently, if the elasticity of substitution $\sigma(\cdot)$ is decreasing (see Zhelobodko *et al.*, 2012).[5]

We next analyze how the price index $\mathcal{P}$ varies with entry. Dividing (18) by v and using (20), we can express the price index as a function of the mass of firms as follows:

$$\mathcal{P} = \frac{cM\psi^{-1}(1/M)}{1 - r_{\psi}\big(\psi^{-1}(1/M)\big)}. \tag{21}$$

As can be seen from (21), if $r_{\psi}(\cdot)$ is increasing, additional entry leads to a fall in prices. In other words, in that case more firms means tougher competition.[6] The total expenditure for the varieties of the non-traded good is given by

$$E(M, P) = L\left[1 - \alpha(P/\mathcal{P})\right], \tag{22}$$

where we use the fact that the price index in the non-traded sector depends on M only by (21). Combining (21) and (22) shows that E increases in response to an expansion of product diversity because consumers value variety. Furthermore, E naturally decreases when

[5]The latter condition may be interpreted as follows: an increase in the consumption index $v(\mathbf{y})$ makes varieties more differentiated or, conversely, a decrease in $v(\mathbf{y})$ makes varieties closer substitutes.

[6]Actually, $r_{\psi}(\cdot)$ could also be 'moderately decreasing' without changing that result. For simplicity, we henceforth consider only the case where it is increasing, as this seems to be empirically more relevant.

traded goods become cheaper due to the substitution effect between tradables and non-tradables. This effect will be key in what follows to understand how trade liberalization affects the economy as a whole.

Finally, using (22) and the expressions for markups imply that the equilibrium firm size, q_n, and operating profit, π_n, in the non-traded sector are given by

$$q_n(M, P/\mathcal{P}) = \frac{cL}{M} \frac{1-\alpha(P/\mathcal{P})}{1-r_\psi\big(\psi^{-1}(1/M)\big)}, \tag{23}$$

and

$$\pi_n(M, P/\mathcal{P}) = \frac{L}{M}(1-\alpha(P/\mathcal{P}))r_\psi\big(\psi^{-1}(1/M)\big), \tag{24}$$

respectively. These expressions will be helpful to analyze how firms' sizes change in response to trade liberalization.

We now need to determine the equilibrium mass, M, of firms. To this end, we assume that there is free-entry so that profits are zero. The zero-profit condition is given by

$$\frac{L}{M}\big(1-\alpha(P/\mathcal{P})\big)r_\psi\big(\psi^{-1}(1/M)\big) = F. \tag{25}$$

Condition (25) allows us to pin down the mass of firms in the non-traded sector as a function of the price index, P, of traded varieties. We can show the following result.

2 *[Traded prices and the mass of non-traded firms] Assume that $r_\psi(\cdot)$ is increasing and that the budget share of non-traded goods, $\alpha(P/\mathcal{P})$, decreases not too fast. Then there exists a unique symmetric free-entry equilibrium in the non-traded sector.*

Proof. See Appendix A.2. ■

What is the economic contents of the assumptions underlying Proposition 2? As discussed above, the first assumption $r'_\psi > 0$ is a necessary and sufficient condition for entry to generate pro-competitive effects. We believe that this is the empirically plausible case. Moreover, this condition has a purely demand side-related interpretation: a higher consumption index is equivalent to more product differentiation. The second assumption, namely that $\alpha(P/\mathcal{P})$ is a moderately decreasing function, means that traded and non-traded goods are poor substitutes for consumers. If we think about the former as including mostly manufactured goods, whereas the latter consist mostly of services, this seems to be a fairly natural assumption to make.[7] In what follows, we take $r'_\psi(\cdot) > 0$ and $\alpha(P/\mathcal{P})$ decreases not too fast as our benchmark assumptions.

To illustrate the latter assumption by means of a simple example, consider the following CES upper-tier utility:

$$\mathcal{U} = \left[\beta u^{(\sigma-1)/\sigma} + (1-\beta)v^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}, \quad 0 < \beta < 1 < \sigma.$$

In that case, we have

$$\alpha\left(\frac{P}{\mathcal{P}}\right) = \frac{(P/\mathcal{P})^{1-\sigma}}{[(1-\beta)/\beta]^\sigma + (P/\mathcal{P})^{1-\sigma}}. \tag{26}$$

From equation (26) we see that when $\sigma \to 1$ (i.e., goods become less and less substitutable), the slope of $\alpha(\cdot)$ is less and less steep. Hence, the operating profit (24) is more likely to decrease in M since tradables and non-tradables are poor substitutes.

Equations (21) and (25) uniquely pin down the equilibrium price index, $\mathcal{P}^*(P)$, and the equilibrium number of firms, $M^*(P)$. More precisely, we can solve (25) for M and plug the solution into (21). We then obtain a decreasing function $\mathcal{P} = \mathcal{P}^*(P)$. allows us to express the equilibrium expenditure share for non-traded goods as a function of P only: $a(P) \equiv \alpha(P/\mathcal{P}^*(P))$. Since the expenditure share is the only channel through which the traded sector impacts on the non-traded sector, *the impact of a fall in trade costs on the non-traded sector is fully captured by changes in the price index in the traded sector.*

How does a change in P affect the consumers' expenditure shares? Two effects are at work. First, there is the standard substitution effect between traded and non-traded goods, which increases α when tradables become cheaper. Second, there is the effect that arises because an increase in α leads to more demand for traded goods, thus enticing more firms to enter the tradable sector. This makes competition tougher in that sector and leads to an additional fall in P. both effects work in the same direction, we have that $a(P)$ unambiguously decreases with P. In other words, if trade liberalization reduces the price index P for traded goods, expenditures on non-traded goods always decrease. We now show that trade liberalization indeed has this effect on both sectors under fairly plausible assumptions.

[7] When these two conditions do not jointly hold, there could be multiple free-entry equilibria. In that case, we naturally focus on *stable* equilibria, i.e. those where

$$\left.\frac{\partial \pi}{\partial M}\right|_{M=M^*} < 0.$$

4 Equilibrium in the traded sector

We now look at the impacts of freer trade on the traded sector. We first examine the general case, and then discuss in detail the ε-CES case.

To simplify notation, let $z^d \equiv x^d/u$ and $z^m \equiv x^m/u$. A symmetric equilibrium in the traded sector then satisfies the following four equilibrium conditions:

(i) zero profit condition:

$$x^d \frac{r_\theta\left(z^d\right)}{1-r_\theta\left(z^d\right)} + \tau x^m \frac{r_\theta\left(z^m\right)}{1-r_\theta\left(z^m\right)} = \frac{F}{cL}; \tag{27}$$

(ii) Kimball's (1995) flexible aggregator condition:

$$N\left[\theta\left(z^d\right)+\theta\left(z^m\right)\right]=1; \tag{28}$$

(iii) profit maximization:

$$\frac{\theta'\left(z^d\right)\left(1-r_\theta\left(z^d\right)\right)}{\theta'\left(z^m\right)\left(1-r_\theta\left(z^m\right)\right)} = \frac{1}{\tau}; \tag{29}$$

(iv) sectoral budget constraint:

$$Pu = a(P) = Nu\left[\frac{cz^d}{1-r_\theta\left(z^d\right)} + \frac{c\tau z^m}{1-r_\theta\left(z^m\right)}\right]. \tag{30}$$

We first solve (28)–(29) for the relative consumption levels, z^d and z^m. Given the mass of firms, N, in each country, the locus in (28) is a downward-sloping curve in (z^H, z^F)-space, because $\theta(\cdot)$ is an increasing function. The slope of the locus in (29) is positive, which comes from the second-order condition for profit maximization (that condition is given by $r_{\theta'} < 2$; see Zhelobodko *et al.*, 2012). As a consequence, the two curves have a unique intersection, which we denote by $\left(\overline{z}^d(N, \tau), \overline{z}^m(N, \tau)\right)$.

How do $\overline{z}^d(N, \tau)$ and $\overline{z}^m(N, \tau)$ vary with the mass of firms, N, and trade costs, τ? When N increases, the locus in (28) is shifts downwards, while the locus in (29) remains unchanged. Thus, both $\overline{z}^d$ and $\overline{z}^m$ decrease in the mass of firms. An intuitive explanation for this result is 'love for variety': additional entry leads consumers' to fragment their budget over a broader range of varieties. Higher trade costs, τ, lead to a downward shift of the locus in (29), while the locus in (28) remains unchanged. Thus, $\overline{z}^d$ increases with trade costs, while $\overline{z}^m$ decreases with trade cost. Intuitively, freer trade shifts relative demands away from domestically produced varieties and towards imported varieties.

Observe that domestic markups are given by $r_\theta^d(N, \tau) \equiv r_\theta\left(\overline{z}^d(N, \tau)\right)$, while foreign markups equal $r_\theta^m(N, \tau) \equiv r_\theta\left(\overline{z}^m(N, \tau)\right)$. Entry drives both markups downward if and only if $r_\theta(z)$ is an increasing function. In other words, whether the effect of entry is pro- or anti-competitive is fully determined by the nature of consumers' variety-loving behavior. Note that for a given mass of firms, trade liberalization always shifts domestic and foreign markups in opposite directions. More precisely, a decrease in τ drives domestic markups downwards – and foreign markups upwards – if and only if $r_\theta(z)$ is an increasing function of z. Otherwise, the result is reversed. Hence, in the empirically plausible case, domestic markups and foreign markups converge as trade becomes freer (in the limit, when trade is costless, domestic and export markups are the same). We will see later that while markups *converge across countries* in the traded sector, markups *diverge within countries* between the traded and the non-traded sector.

We summarize the foregoing results in the following proposition:

3 *[Equilibrium in the traded sector] Assume that $r_\theta'(z) > 0$. Then: (i) there exists a unique symmetric equilibrium for any given N; (ii) the equilibrium markups $r_\theta^d(N, \tau)$ and $r_\theta^m(N, \tau)$ both decrease with N; and (iii) $r_\theta^d(N, \tau)$ decreases with τ, while $r_\theta^m(N, \tau)$ increases.*

Proof. In the text. ■

How do the price index, P, and the consumption index, u, vary with the mass of firms N and trade cost τ? We first rewrite (30) as follows:

$$P = \overline{P}(N, \tau) \equiv cN\left[\frac{\overline{z}^d(N, \tau)}{1 - r_\theta(\overline{z}^d(N, \tau))} + \frac{\tau\overline{z}^m(N, \tau)}{1 - r_\theta(\overline{z}^m(N, \tau))}\right],$$

and use the definition of homothetic preferences to obtain

$$u = \overline{u}(N, \tau) \equiv \frac{a\left(\overline{P}(N, \tau)\right)}{\overline{P}(N, \tau)}.$$

We still need to pin down N, which is endogenously determined by free entry. To this end, we use the zero profit condition (27), which now takes the following form:

$$\overline{u}(N, \tau)\left[\overline{z}^d(N, \tau)\frac{r_\theta(\overline{z}^d(N, \tau))}{1 - r_\theta(\overline{z}^d(N, \tau))} + \tau\overline{z}^m(N, \tau)\frac{r_\theta(\overline{z}^m(N, \tau))}{1 - r_\theta(\overline{z}^m(N, \tau))}\right] = \frac{F}{cL}. \tag{31}$$

How the left-hand side of (31) varies in general with N is ambiguous, so that multiple equilibria may a priori arise. Since the left-hand side of (31) is the traded good producers' operating profit $\pi(N, \tau)$, we focus in what follows on "stable" equilibria, i.e., those where

$\partial\pi/\partial N < 0$ at $N = N^*$ (where N^* denotes a solution to (31)). By doing so, we do of course not rule out the possibility of multiple stable equilibria. We will do this in the next subsection where we focus on a specific instance of preferences that lead to a unique stable equilibrium.

The impact of variations in τ on firms' profits is twofold: (i) trade liberalization reduces costs, which leads to rising profits; and (ii) trade liberalization shifts the price index $\overline{P}(N, \tau)$, which may result in tougher competition, hence in lower profits. In any case, it is implied by (31) that at any stable equilibrium trade liberalization leads to an increase in the equilibrium mass of firms N^* if and only if the former effect dominates the latter, i.e., when $\partial\pi/\partial\tau < 0$. If, in addition, $\overline{P}(N, \tau)$ decreases in N and increases in τ, then trade liberalization also drives down the equilibrium value of the price index $P^* \equiv \overline{P}(N^*, \tau)$. Thus, we have the following result.

4 *[Decreasing price index] A sufficient condition for* $\mathrm{d}P^*/\mathrm{d}\tau > 0$ *is given by: (i)* $\partial\overline{P}/\partial N < 0$*; and (ii)* $\mathrm{d}N^*/\mathrm{d}\tau < 0$*.*

Proof. In the text. ■

Can we explain the assumptions underlying Proposition 4 in terms of the model's primitives? The answer is 'yes' when preferences are "not too far from the CES", i.e., when they are ϵ-CES. This is why we now perturb the CES equilibrium conditions to focus on ε-CES equilibria. Doing so will allow us to derive clear-cut results while making use of homothetic non-CES preferences. We will derive most of our analytical results by continuity in the neighborhood of CES preferences. We show in Section **??** below – using numerical illustrations – that these results extend to larger 'neighborhoods' of the CES.

Plugging expressions (6)–(7) into the equilibrium conditions (27)–(30) yields

$$z^d \frac{1-\rho-\xi^d}{\rho+\xi^d} + \tau z^m \frac{1-\rho-\xi^m}{\rho+\xi^m} = \frac{F}{cL}\frac{P}{a(P)}, \tag{32}$$

$$N\left[(z^d)^\rho \exp(\varphi^d) + (z^m)^\rho \exp(\varphi^m)\right] = 1, \tag{33}$$

$$\frac{\left(z^d\right)^{\rho-1}\exp(\varphi^d)(\rho+\xi^d)\left[\rho+z^d\varphi'(z^d)\right]}{\left(z^m\right)^{\rho-1}\exp(\varphi^m)(\rho+\xi^m)\left[\rho+z^m\varphi'(z^m)\right]} = \frac{1}{\tau}, \tag{34}$$

$$P = cN\left(\frac{z^d}{\rho+\xi^d} + \tau\frac{z^m}{\rho+\xi^m}\right), \tag{35}$$

where $\varphi^k \equiv \varphi(z^k)$ and $\xi^k \equiv \xi(z^k)$ for $k \in \{d, m\}$ for notational convenience. Clearly, when $\varphi \in \mathcal{F}$, we have $\xi^k \to 0$ when $||\varphi||_{C^2} \to 0$ (refer to Section **??** for details). In the limiting case we have $\varphi^d = \varphi^m = 0$ and $\xi^d = \xi^m = 0$, so that equations (32)–(35) boil down to the standard equilibrium conditions under CES preferences (see Appendix B for these expressions).

Plugging (35) into (32), we obtain

$$(1-\rho)\frac{P}{cN} - \zeta = \frac{F}{cL}\frac{P}{a(P)}, \quad \text{where} \quad \zeta(\xi^d, \xi^m) \equiv \frac{z^d \xi^d}{\rho + \xi^d} + \tau \frac{z^m \xi^m}{\rho + \xi^m}, \tag{36}$$

is a residual term. Since $\varphi \in \mathcal{F}$, we have $||\zeta||_C \to 0$ when $||\varphi||_{C^2} \to 0$. Solving the first equation in (36) for N, we obtain

$$N = (1-\rho)\frac{La(P)}{F + cL\frac{a(P)}{P}\zeta}. \tag{37}$$

Solving (33)–(34) for z^d and z^m, we obtain

$$\overline{z}^m(N, \tau) = \left[N\left(\tau^{\rho/(1-\rho)}\mathcal{A}^\rho \exp(\varphi^d) + \exp(\varphi^m)\right)\right]^{-1/\rho}, \tag{38}$$

$$\overline{z}^d(N, \tau) = \tau^{1/(1-\rho)}\mathcal{A}\overline{z}^m(N, \tau), \tag{39}$$

where

$$\mathcal{A}(\varphi^d, \varphi^m, \xi^d, \xi^m) \equiv \left[\frac{\exp(\varphi^d)(\rho + \xi^d)\left(\rho + z^d\varphi'(z^d)\right)}{\exp(\varphi^m)(\rho + \xi^m)\left(\rho + z^m\varphi'(z^m)\right)}\right]^{1/(\rho-1)}.$$

Note that $\mathcal{A} \to 1$ as $||\varphi||_{C_0^2} \to 0$. Finally, plugging (38)–(39) into (35) yields the expression for the price index:

$$P = cN^{-(1-\rho)/\rho}\left[\mathcal{A}^\rho \exp(\varphi^d) + \tau^{-\rho/(1-\rho)}\exp(\varphi^m)\right]^{-1/\rho}\left(\frac{\mathcal{A}}{\rho + \xi^d} + \frac{\tau^{-\rho/(1-\rho)}}{\rho + \xi^m}\right). \tag{40}$$

We can now establish the following result.

5 *[Existence of an ε-CES equilibrium] For ε-CES preferences, with ε 'sufficiently small', an equilibrium: (i) exists; and (ii) is a small perturbation of the CES equilibrium.*

Proof. See Appendix A.3. ■

5 The impacts of trade liberalization

We are now equipped to investigate how trade liberalization, i.e., a decrease in τ, influences the equilibrium in the traded and in the non-traded sectors. First, we look at the traded sector. We then look at the indirect effects of trade liberalization on the non-traded sector. As shown in Section 3, those effects stem entirely from changes in the price index associated with the traded sector.

5.1 Traded sector

Using the implicit function theorem, we show in Appendix A.4 that the following comparative statics hold in the ε-CES case when ε is small and when $a(\cdot)$ decreases not too quickly:

$$\frac{\mathrm{d}z^{d*}}{\mathrm{d}\tau} > 0 > \frac{\mathrm{d}z^{m*}}{\mathrm{d}\tau}, \quad \frac{\mathrm{d}P^{*}}{\mathrm{d}\tau} > 0, \quad \text{and} \quad \frac{\mathrm{d}N^{*}}{\mathrm{d}\tau} < 0. \tag{41}$$

These comparative statics are important for assessing how markups and firm sizes in the traded sector react to trade liberalization in the traded sector.

Markups. We first look at how domestic and export markups in the traded sector change in response to trade liberalization. Recall that in the CES case, markups are invariant. This is, however, no longer the case under ε-CES preferences, even when ε is small. Put differently, even small departures from the CES lead to variable markups, and we can investigate their behavior in response to freer trade.

In what follows, we focus on perturbations, φ, that generate $r'_\theta(z) > 0$ whenever $||\varphi||_{C^2} > 0$.[8] In that case, which features pro-competitive effects, $\mathrm{d}z^{d*}/\mathrm{d}\tau > 0 > \mathrm{d}z^{m*}/\mathrm{d}\tau$ directly implies

$$\frac{\mathrm{d}r_\theta^{m*}}{\mathrm{d}\tau} > 0 > \frac{\mathrm{d}r_\theta^{d*}}{\mathrm{d}\tau}. \tag{42}$$

In words, trade liberalization leads to a reduction in markups for locally produced varieties, and to an increase in the markups of imported varieties. Since domestic markups exceed export markups, this means that freer trade leads to the convergence of markups across countries. Yet, as we show in the next subsection, it also leads to a divergence of markups across industries within countries.

Firm size. We now study how trade liberalization affects firm size, $q \equiv L(x^d + \tau x^m)$. In the CES case, it is well known that trade liberalization does not affect firm size but leads to a decrease in domestic sales, x^d, and an increase in exports, τx^m. Perturbing the CES preferences slightly reveals additional effects of trade liberalization on firm size. To see this, we rewrite (32) as follows, using (7):

$$q\frac{r_\theta(z^d)}{1-r_\theta(z^d)} + L\frac{a(P)}{P}\tau z^m \left[\frac{r_\theta(z^m)}{1-r_\theta(z^m)} - \frac{r_\theta(z^d)}{1-r_\theta(z^d)}\right] = \frac{F}{c}. \tag{43}$$

[8] It is worth noting that the method of establishing comparative statics "by continuity" like in Appendix A.4 does not work with markups, for under the CES we have $\partial r_\theta^{d*}/\partial\tau = \partial r_\theta^{m*}/\partial\tau = 0$. Hence, we can say nothing using continuity arguments.

As we have shown before, a reduction in trade costs τ leads to: (i) a decrease in the price index, P, and an increase in the expenditure share, $a(P)$, associated with tradables, i.e., $a(P)/P$ increases unambiguously; (ii) an increase in output for the foreign market, τx^m; and (iii) an increase in the markups of imported varieties, $r_\theta(z^m)$, and a decrease in the markup of domestic varieties, $r_\theta(z^d)$. Note that the opposite effect of a fall in τ on the markups implies that the expression in square brackets in (43) increases. Thus, the second term on the left-hand side of (43) increases with trade liberalization. This implies that the first term must decrease for the zero-profit condition to hold. Since domestic markups, $r_\theta(z^d)$, decrease with a fall in trade costs, we can conclude that firm output q must hence increase with trade liberalization.

The following proposition summarizes our key results concerning the impacts of trade liberalization on firms in the traded sector.

6 *[Effects of trade liberalization on the traded sector] Consider the case of ε-CES preference with $r'_\theta > 0$. Then there exists $\overline{\varepsilon}$ such that for every $0 < \varepsilon < \overline{\varepsilon}$, trade liberalization leads to: (i) lower markups for domestically produced traded varieties; (ii) higher markups for imported traded varieties; (iii) more firms in the traded sector; (iv) larger firms, as measured by their total output, in the traded sector.*

Proof. In the text. ■

Observe that the results of Proposition 6 are guaranteed to hold in a 'small neighborhood' of the CES by continuity. Furthermore, the same qualitative properties also hold in other models of monopolistic competition, such as those by Krugman (1979), Behrens and Murata (2007, 2012), Zhelobodko *et al.* (2012), and Kichko *et al.* (2014). Those models feature pro-competitive effects, although they do not rely on homothetic preferences. We hence do not claim that our results are particularly novel, but we show that they hold more generically for a potentially large class of homothetic preferences derived from implicitly additive relationships. We further show in Section **??** below that the results of Proposition 6 may also continue to hold outside of a 'small neighborhood', i.e., when we move farther away from the CES case.

5.2 Non-traded sector

We now turn to the non-traded sector. As shown in Section 3, the impacts of trade costs on the non-traded sector are fully captured by the changes in the price index associated with the traded sector. Since we have established the impact of trade costs on the price index in the foregoing subsection, the analysis is now straightforward.

Markups. Because trade liberalization makes competition in the traded sector tougher, as shown in Section 5.1, the price index for tradables falls as trade gets freer. Furthermore, when traded goods get relatively cheaper, operating profits in the non-traded sector are shifted downwards because of tougher competition from the traded sector, which reduces the expenditure share on non-tradables. As a consequence, if $r'_\psi > 0$ and $\alpha(\cdot)$ is a slowly decreasing function, a decrease in P reduces the equilibrium mass of firms in the non-traded sector, M^*. This, in turn, leads to an increase in both the price index and the markups in that sector, as shown in Section 3. Put differently, trade liberalization in the traded sector has 'anti-competitive effects' in the non-traded sector, whereas it has pro-competitive effects in the traded sector. It is worth pointing out here that this effect is entirely driven by the general equilibrium demand nature of the model, which leads to a reallocation of expenditure across industries in the wake of trade liberalization. There is no 'anti-competitive behavior' of firms, although such behavioral reactions of firms to trade liberalization have often been put forward to explain the increase in markups in service industries in the wake of deeper European integration (see Badinger, 2007). With trade liberalization, the *markups in the traded and non-traded sectors move in opposite directions*, and this is simply a market reaction to a change in relative prices and the associated changes in expenditures, entry and, ultimately, toughness of competition.

Firm size. Recall that firm size in non-traded sector is given by (23), i.e.,

$$q_n(M, P) = \frac{cL}{M} \frac{1 - a(P)}{1 - r_\psi[\psi^{-1}(1/M)]}.$$

Since we assume that $a(P)$ decreases slowly with the price index, a change in the denominator of q_n, $M\left[1 - r_\psi\left(\psi^{-1}(1/M)\right)\right]$, provides the first-order effect for changes in q_n. Because M^* increases with τ, so does the denominator in (23), and hence the whole fraction decreases.

The following proposition summarizes our key results concerning the impacts of trade liberalization on firms in the non-traded sector.

7 *[Effects of trade liberalization on the non-traded sector] Consider the case of ε-CES preference with $r'_\psi > 0$. Then trade liberalization leads to: (i) an increase in the markups for non-traded* *varieties; (ii) an increase in the prices of non-traded varieties; (iii) fewer firms in the non-traded sector; and (iv) larger firm sizes in the non-traded sector.*

Proof. In the text. ∎

5.3 Welfare

We now turn to the welfare effects of trade liberalization. As we have shown in Propositions 6 and 7, a decrease in τ increases market power and reduces product diversity in the non-traded sector, whereas the opposite holds in the traded sector. Hence, the welfare effects of trade liberalization are a priori ambiguous. With an unspecified upper-tier utility function, consumers may gain or loose from freer trade, depending crucially on the cross-sectoral versus the within-sectoral elasticities of substitution.

To make our point most clearly and to show that the impacts of freer trade are a priori ambiguous, we consider in what follows the simplest possible instance of our framework. Assuming that traded and non-traded goods are relatively poor substitutes, we now look at the impacts of trade liberalization on welfare in a setup where both the upper- and the lower-tier utilities are CES, with the upper-tier elasticity, σ, being smaller than the lower-tier elasticity, γ (formally, we assume that $1 < \sigma < \gamma$). simplicity, we assume that the lower-tier elasticity γ is the same for traded and for non-traded goods.

Due to homotheticity, the following relationships:

$$u = \frac{a(P)}{P} \quad \text{and} \quad v = \frac{1-a(P)}{\mathcal{P}}.$$

Assuming CES upper-tier preferences we then have:

$$\mathcal{U} = \left[\beta u^{\frac{\sigma-1}{\sigma}} + (1-\beta) v^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} = \left[\beta \left(\frac{a(P)}{P}\right)^{\frac{\sigma-1}{\sigma}} + (1-\beta)\left(\frac{1-a(P)}{\mathcal{P}}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

where $0 < \beta < 1$ are weights, and where

$$P = \frac{c\gamma}{\gamma-1} N^{\frac{1}{1-\gamma}} \left(1+\tau^{1-\gamma}\right)^{\frac{1}{1-\gamma}} \quad \text{and} \quad \mathcal{P} = \frac{c\gamma}{\gamma-1} M^{\frac{1}{1-\gamma}}.$$

are the CES price aggregates in the symmetric case for the traded and the non-traded sectors, respectively.

As shown in Appendix C, the indirect utility is given as follows:

$$\mathcal{U} = \frac{\beta^{\frac{\sigma}{\sigma-1}}(\gamma-1)}{c\gamma^{\frac{\gamma}{\gamma-1}}} \left(\frac{L}{f}\right)^{\frac{1}{\gamma-1}} \frac{\left(\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+1\right)^{\frac{\sigma}{\sigma-1}}}{\left(\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+B\right)^{\frac{\gamma}{\gamma-1}}}. \tag{44}$$

Welfare changes in response to freer trade are thus given by

$$\begin{aligned}\frac{d\mathcal{U}}{d\varphi} &= \left(\frac{L}{f}\right)^{\frac{1}{\gamma-1}} \frac{\beta^{\frac{\sigma}{\sigma-1}} \varphi^{\frac{\sigma-1}{\gamma-\sigma}-1} \left(\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+1\right)^{\frac{1}{\sigma-1}} \left(\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+B\right)^{\frac{1}{\gamma-1}}}{c\gamma^{\frac{\gamma}{\gamma-1}}(\gamma-\sigma)} \\ &\quad \times \frac{\sigma(\gamma-1)\left(\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+B\right)-\gamma(\sigma-1)\left(\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+1\right)}{\left(\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+B\right)^{\frac{2\gamma}{\gamma-1}}}.\end{aligned}$$

The sign of this derivative depends only on the sign of the numerator of the second term, i.e., on the sign of

$$\sigma(\gamma-1)\left(\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+B\right)-\gamma(\sigma-1)\left(\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+1\right).$$

Using the definition of B, there are gains from freer trade if and only if

$$\sigma(\gamma-1)\left[\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+\left(\frac{1-\beta}{\beta}\right)^{\frac{\sigma(\gamma-1)}{\gamma-\sigma}}\right]-\gamma(\sigma-1)\left(\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+1\right)>0. \tag{45}$$

Condition (45) naturally depends on the relative sectoral weights, β, the two elasticities of substitution, γ and σ, as well as trade costs φ (which contains τ). Since $\varphi \in [1,2]$ there are gains from trade irrespective of the value of τ if and only if the foregoing condition holds for $\varphi = 1$. This yields,

$$\sigma(\gamma-1)+\left(\frac{1-\beta}{\beta}\right)^{\frac{\sigma(\gamma-1)}{\gamma-\sigma}}\sigma(\gamma-1)-2\gamma(\sigma-1)>0$$

or, equivalently,

$$\beta<\frac{1}{\left[\frac{2\gamma(\sigma-1)}{\sigma(\gamma-1)}-1\right]^{\frac{\gamma-\sigma}{\sigma(\gamma-1)}}+1}. \tag{46}$$

The right-hand side of (46) is between $1/2$ and 1. Stated differently, there are gains from freer trade in the traded sector for all values of trade costs if and only if the weight β of the traded sector in the utility is sufficiently small.

Several comments are in order. First, we can show that $\frac{2\gamma(\sigma-1)}{\sigma(\gamma-1)}>1$ for all values of $\gamma>\sigma$. One can check that as $\beta\to 0$, $d\mathcal{U}/d\varphi$ goes to zero as well. In a nutshell, if the traded sector is small, there are gains from trade in that sector, though the gains to the overall economy are small too. The gains in the traded sector are a direct effect of trade liberalization on welfare. That effect is always positive. The indirect effect of the traded sector on the non-traded sector is very small when β is small (since the price index barely changes). In that case, the direct effect in the traded sector dominates the indirect effect in the non-traded sector, and there are gains from trade. Note also that gains from trade are more likely to arise if trade costs are low. Indeed, we can express condition (45) in term of φ as follows:

$$\varphi > \left[\frac{\gamma(\sigma-1) - \sigma(\gamma-1)\left((1-\beta)/\beta\right)^{\frac{\sigma(\gamma-1)}{\gamma-\sigma}}}{\gamma-\sigma} \right]^{\frac{\gamma-\sigma}{\sigma-1}},$$

which shows that gains are more likely to arise of φ is large, i.e., trade is sufficiently free. This result is similar to the result in Brander and Krugman (1983), that trade integration may actually reduce welfare if trade costs are sufficiently large, whereas it may yield gains once trade is sufficiently free. The reason in our model is, however, very different from the reciprocal dumping of Brander and Krugman (1983). It is in our case fully driven by the effect of trade on the non-traded sector via the traded price index. If trade costs are high, a decrease in φ has a large impact on the price index for traded goods and, therefore, on the non-traded sector. In that case, the direct effect may be dominated by the indirect effect and losses from trade may occur. When trade costs are low, however, the direct may dominate the indirect effect and gains from trade may occur.

It should be clear from the foregoing discussion that losses from trade may arise for a broad class of utility functions and model specifications if there are: (i) pro-competitive effects in the traded and in the non-traded sectors; (ii) if the traded sector is relatively large, so that the indirect price effect on the non-traded sector is large; and (iii) if trade costs are relatively high. Since trade costs are relatively low nowadays, whereas the traded sector represents only a small share of most developed economies, gains from trade in those economies seem the more likely outcome.

Last, it is also worth emphasizing that our welfare results are a simple illustration of the general theory of the second-best (see Lipsey and Lancaster, 1956). As is well known, when markets are not perfectly competitive, correcting the distortions in one market (the traded good sector in our case, where freer trade reduces monopoly distortions) may exacerbate the distortions in another market (the non-traded good sector in our case, where monoply distortions increase). The resulting welfare effects are ambiguous in that case, and we have used the simplest possible example to illustrate that they can go either way and that no pathological example is required for this.

6 Conclusion

In this paper, we have developed a two-country trade model with a traded and a non-traded sector in each country. Both sectors are monopolistically competitive. Consumers have generally non-CES preferences, so that markups and firm sizes are variable. Using preferences described by Kimball's (1995) flexible aggregator, these preferences remain homothetic, which simplifies the analysis significantly. Furthermore, these preferences encompass the CES as a special case, which allows us to derive a number of results for non-CES preferences in a 'small neighborhood' of the CES.

Our two key results relate to markups and welfare. First, we show that while trade liberalization promotes the convergence of markups across countries, it leads to the divergence of markups across sectors within countries. This finding may explain the empirical evidence that suggests that markups for traded manufacturing goods has decreased in the wake of the Single Market Program of the European Union in the 1990s, where markups for non-traded services has increased (Badinger, 2007). Although anti-competitive practices have been put forward as a possible explanation, our model shows that simple market-driven general equilibrium effects are enough to generate that outcome.

Second, as markups fall and product diversity rises in the traded sector, yet markups rise and product diversity shrinks in the non-traded sector, the welfare impacts of trade integration are a priori ambiguous. In particular, if trade costs are high, if preferences are such that pro-competitive effects arise in both industries, and if the traded sector is large, losses from trade may arise. Even when all preferences are CES this may occur, which suggest that this finding is not a theoretical curiosum. However, since trade costs are relatively low nowadays, whereas the traded sector represents only a small share of most developed economies, gains from trade in those economies seem to be the empirically more plausible outcome.

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Globalization. All errors are ours.

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Appendix

A. Proofs of Propositions

A.1. Proof of Proposition 1 Given $\theta(z)$, the proof of the "if" part essentially boils down to choosing a $\sigma(z)$ which satisfies

$$\frac{1}{\sigma(z)} = r_\theta(z) \equiv -\frac{z\theta''(z)}{\theta'(z)}. \tag{A-1}$$

Conversely, to prove the "only if" part, we choose $\theta(z)$ satisfying (A-1) for a given $\sigma(z)$.

Assume that (4) holds. Then, since the inverse demands are given by (14), the elasticity η_i of the inverse demand for variety i is given by

$$\eta_i = r_\theta\left(\frac{x_i}{u(\mathbf{x})}\right), \tag{A-2}$$

where $r_\theta(\cdot)$ is defined by (A-1). As shown by Parenti *et al.* (2014), for any symmetric preferences defined over a continuum of goods, the following equality holds:

$$\overline{\sigma}(x_i, x_j, \mathbf{x})\big|_{x_i=x_j} = \frac{1}{\eta_i}. \tag{A-3}$$

Combining (A-2) with (A-3) and setting $\sigma(z) \equiv 1/r_\theta(z)$, we obtain (3). This proves the "if" part of Proposition 1.

To prove the "only if" part, assume that (3) holds for a given $\sigma(z)$. It is straightforward to verify that implicitly additive preferences with $\theta(\cdot)$ defined by

$$\theta(z) \equiv \int_0^z \exp\left(-\int_1^\varsigma \frac{\mathrm{d}\xi}{\xi\sigma(\xi)}\right)\mathrm{d}\zeta$$

also satisfy (3) for the same $\sigma(z)$. Since a preference relationship is uniquely determined by its elasticity of substitution, this completes the proof. □

A.2. Proof of Proposition 2 When $r'_\psi(\cdot) > 0$, and $\alpha(P/\mathcal{P})$ decreases not too fast, then the operating profit (24) is a decreasing function of M. In this case, (25) has a unique solution M^*. Plugging M^* into equations (22) and (23) and using the definition of the markups then determines a unique symmetric free-entry equilibrium. □

A.3. Proof of Proposition 5 Denote by $\varepsilon \equiv (\varphi^d, \varphi^m, \xi^d, \xi^m, \mathcal{A}, \zeta)$ the vector of 'shocks' entering (37) and (40). We first show that equations (37) and (40) have a unique solution (P^*_{CES}, N^*_{CES}) when $\varepsilon = \mathbf{0}$.

To see this, observe that when $\varepsilon = \mathbf{0}$, equations (37) and (40) boil down to

$$N = (1-\rho)\frac{L}{F}a(P), \tag{A-4}$$

and

$$P = N^{1/(1-\sigma)}\left(\frac{c\sigma}{\sigma-1}\right)\left(1+\tau^{1-\sigma}\right)^{1/(1-\sigma)}, \tag{A-5}$$

which are the standard expressions for the CES case. Plugging (A-5) into (A-4) yields an equation which uniquely pins down the equilibrium number of firms, N^*_{CES}, provided that $a(\cdot)$ is a sufficiently slowly decreasing function (which we assume, as explained before). Substituting N^*_{CES} back into (A-5), we then obtain the unique equilibrium value of the price index for traded varieties, P^*_{CES}.

It then follows from the implicit function theorem that there exists an open set $V \subseteq \mathbb{R}^6$, such that: (i) $\mathbf{0} \in V$; (ii) (37) and (40) have a solution $(P^*(\varepsilon), N^*(\varepsilon))$ for any $\varepsilon \in V$; and (iii) this solution converges to (P^*_{CES}, N^*_{CES}) as $\varepsilon \to \mathbf{0}$. □

A.4. Expressions for comparative statics The implicit function theorem implies that, for a small enough vector ε, the solution $\left(z^{d*}, z^{m*}, P^*, N^*\right)$ of equations (32)–(35) is twice continuously differentiable in (ε, τ). This implies then that $\partial z^{d*}/\partial\tau$, $\partial z^{m*}/\partial\tau$, $\partial P^*/\partial\tau$, and $\partial N^*/\partial\tau$ are all continuous in ε. We may thus conclude that if, for example, $\partial z^{d*}/\partial\tau > 0$ in the CES case, this result will be preserved by continuity for the ε-CES case when ε is small. Thus, it suffices to conduct comparative statics of $\left(z^{d*}, z^{m*}, P^*, N^*\right)$ for the CES case, and all results for the ϵ-CES case are then obtained by continuity.

The expressions for the CES case are given in Appendix A.3. Denote the right-hand side of (A-5) by $\overline{P}(N, \tau)$. We have

$$\frac{\partial\overline{P}}{\partial N} < 0 \quad \text{and} \quad \frac{\partial\overline{P}}{\partial\tau} > 0. \tag{A-6}$$

Totally differentiating (A-4) with respect to τ yields

$$\frac{\mathrm{d}N^*}{\mathrm{d}\tau} = (1-\rho)\frac{L}{F}a'\left[\overline{P}(N^*,\tau)\right]\left(\frac{\partial\overline{P}}{\partial N}\frac{\mathrm{d}N^*}{\mathrm{d}\tau} + \frac{\partial\overline{P}}{\partial\tau}\right),$$

so that

$$\frac{\mathrm{d}N^*}{\mathrm{d}\tau} = \frac{(1-\rho)La'\left[\overline{P}(N^*,\tau)\right]}{F-(1-\rho)La'\left(\overline{P}(N^*,\tau)\right)\frac{\partial\overline{P}}{\partial N}}\frac{\partial\overline{P}}{\partial\tau}. \tag{A-7}$$

Since we assume that $a'(\cdot)$ is small enough in absolute value, (A-7) immediately implies

$$\frac{\mathrm{d}N^*}{\mathrm{d}\tau} < 0. \tag{A-8}$$

It is worth pointing out, however, that when $a'(\cdot)$ is sufficiently small, $\mathrm{d}N^*/\mathrm{d}\tau$ is also small in absolute value.

Turning to the relative consumptions $\overline{z}^d(N, \tau)$ and $\overline{z}^m(N, \tau)$, they are determined from expressions (32)–(33) and are given by

$$\overline{z}^m(N, \tau) = \left[N\left(1 + \tau^{\rho/(1-\rho)}\right)\right]^{-\rho}, \quad \text{and} \quad \overline{z}^d(N, \tau) = \tau^{1/(1-\rho)}\left[N\left(1 + \tau^{\rho/(1-\rho)}\right)\right]^{-\rho}.$$

Plugging $N = N^*$ into the foregoing expression and totally differentiating it

$$\frac{\mathrm{d}z^{d*}}{\mathrm{d}\tau} = \left(\frac{\partial \overline{z}^d}{\partial N}\frac{\mathrm{d}N^*}{\mathrm{d}\tau} + \frac{\partial \overline{z}^d}{\partial \tau}\right)\bigg|_{N=N^*}, \qquad \frac{\mathrm{d}z^{m*}}{\mathrm{d}\tau} = \left(\frac{\partial \overline{z}^m}{\partial N}\frac{\mathrm{d}N^*}{\mathrm{d}\tau} + \frac{\partial \overline{z}^m}{\partial \tau}\right)\bigg|_{N=N^*}. \tag{A-9}$$

Because, as discussed above, the magnitude of $\mathrm{d}N^*/\mathrm{d}\tau$ is small, (A-9) implies

$$\frac{\mathrm{d}z^{d*}}{\mathrm{d}\tau} > 0 > \frac{\mathrm{d}z^{m*}}{\mathrm{d}\tau}.$$

We next study how the price index P^* varies with trade costs τ. Totally differentiating $P^* = \overline{P}(N^*, \tau)$ with respect to τ yields

$$\frac{\mathrm{d}P^*}{\mathrm{d}\tau} = \left(\frac{\partial \overline{P}}{\partial \tau} + \frac{\partial \overline{P}}{\partial N}\cdot\frac{\mathrm{d}N^*}{\mathrm{d}\tau}\right)\bigg|_{N=N^*}.$$

Combining this expression with (A-6) and (A-8), we finally obtain

$$\frac{\mathrm{d}P^*}{\mathrm{d}\tau} > 0.$$

B. Expressions for the ces case

The equilibrium conditions for the traded sector under CES preferences are given by:

$$z^H + \tau z^F = \frac{\rho}{1-\rho}\frac{F}{cL}\frac{P}{a(P)}, \tag{B-1}$$

$$N\left[\left(z^H\right)^{\rho} + \left(z^F\right)^{\rho}\right] = 1, \tag{B-2}$$

$$\left(\frac{z^H}{z^F}\right)^{\rho-1} = \frac{1}{\tau}, \tag{B-3}$$

$$P = \frac{c}{\rho}N\cdot\left(z^H + \tau z^F\right). \tag{B-4}$$

C. Welfare in the ces-ces case

In this appendix, we derive the expression for the indirect utility in the case where both the upper- and the lower-tier utilities are of the CES form.

The zero-profit conditions in the two sectors take the following form:

$$(\wp - c)Ly = f \quad \text{and} \quad (p^d - c)Lx^d + (p^m - \tau c)Lx^m = f.$$

Given constant markups in each sector, we then have the following quantities:

$$y = \frac{(\gamma - 1)}{cL} f \quad \text{and} \quad x^d + \tau x^m = \frac{(\gamma - 1)}{cL} f.$$

Using the budget constraints $M\wp y = 1 - \alpha$, and $Np^d x^d + Np^m x^m = \alpha$, we can then derive the equilibrium masses of firms in both sectors:

$$M = \frac{L}{f\gamma}(1 - \alpha) \quad \text{and} \quad N = \frac{L}{f\gamma}\alpha. \tag{C-1}$$

Plugging (C-1) into the price indices, the ratio $P/\mathcal{P}$ of the price indices is given by:

$$\frac{P}{\mathcal{P}} = \frac{\frac{c\gamma^{\frac{\gamma}{\gamma-1}}}{\gamma-1} \left(\frac{L}{f}\alpha\right)^{\frac{1}{1-\gamma}} \left(1 + \tau^{1-\gamma}\right)^{\frac{1}{1-\gamma}}}{\frac{c\gamma^{\frac{\gamma}{\gamma-1}}}{\gamma-1} \left[\frac{L}{f}(1-\alpha)\right]^{\frac{1}{1-\gamma}}} = \frac{\alpha^{\frac{1}{1-\gamma}} \left(1 + \tau^{1-\gamma}\right)^{\frac{1}{1-\gamma}}}{(1-\alpha)^{\frac{1}{1-\gamma}}}.$$

The expenditure share for the tradables takes the following form

$$\begin{aligned} \alpha\left(\frac{P}{\mathcal{P}}\right) &= \frac{(P/\mathcal{P})^{1-\sigma}}{[(1-\beta)/\beta]^{\sigma} + (P/\mathcal{P})^{1-\sigma}} \\ &= \frac{\alpha^{\frac{\sigma-1}{\gamma-1}} \left(1 + \tau^{1-\gamma}\right)^{\frac{\sigma-1}{\gamma-1}}}{[(1-\beta)/\beta]^{\sigma} (1-\alpha)^{\frac{\sigma-1}{\gamma-1}} + \alpha^{\frac{\sigma-1}{\gamma-1}} \left(1 + \tau^{1-\gamma}\right)^{\frac{\sigma-1}{\gamma-1}}}. \end{aligned}$$

Solving this equation for α we obtain

$$\alpha = \frac{\left(1 + \tau^{1-\gamma}\right)^{\frac{\sigma-1}{\gamma-\sigma}}}{\left(1 + \tau^{1-\gamma}\right)^{\frac{\sigma-1}{\gamma-\sigma}} + [(1-\beta)/\beta]^{\frac{\sigma(\gamma-1)}{\gamma-\sigma}}}.$$

For notational convenience, let $B = [(1-\beta)/\beta]^{\frac{\sigma(\gamma-1)}{\gamma-\sigma}} > 0$ and $\varphi = 1 + \tau^{1-\gamma} \in (1, 2)$, so that the expenditure shares can be expressed as follows:

$$\alpha = \frac{\varphi^{\frac{\sigma-1}{\gamma-\sigma}}}{\varphi^{\frac{\sigma-1}{\gamma-\sigma}} + B} \quad \text{and} \quad 1 - \alpha = \frac{B}{\varphi^{\frac{\sigma-1}{\gamma-\sigma}} + B}. \tag{C-2}$$

Using (C-2), we first rewrite the price indices as follows:

$$
\begin{aligned}
P &= \frac{c\gamma^{\frac{\gamma}{\gamma-1}}}{\gamma-1}\left(\frac{L}{f}\alpha\right)^{\frac{1}{1-\gamma}}\left(1+\tau^{1-\gamma}\right)^{\frac{1}{1-\gamma}} = \frac{c\gamma^{\frac{\gamma}{\gamma-1}}}{\gamma-1}\left(\frac{L}{f}\right)^{\frac{1}{1-\gamma}}\left(\frac{\varphi^{\frac{\sigma-1}{\gamma-\sigma}}}{\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+B}\right)^{\frac{1}{1-\gamma}}\varphi^{\frac{1}{1-\gamma}} \\
&= \frac{c\gamma^{\frac{\gamma}{\gamma-1}}}{\gamma-1}\left(\frac{L}{f}\right)^{\frac{1}{1-\gamma}}\left(\frac{\varphi^{\frac{\gamma-1}{\gamma-\sigma}}}{\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+B}\right)^{\frac{1}{1-\gamma}}
\end{aligned}
$$

so that

$$
\begin{aligned}
\frac{\alpha}{P} &= \frac{\gamma-1}{c\gamma^{\frac{\gamma}{\gamma-1}}}\left(\frac{L}{f}\right)^{\frac{1}{\gamma-1}}\left(\frac{\varphi^{\frac{\gamma-1}{\gamma-\sigma}}}{\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+B}\right)^{\frac{1}{\gamma-1}}\frac{\varphi^{\frac{\sigma-1}{\gamma-\sigma}}}{\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+B} \\
&= \frac{\gamma-1}{c\gamma^{\frac{\gamma}{\gamma-1}}}\left(\frac{L}{f}\right)^{\frac{1}{\gamma-1}}\left(\frac{\varphi^{\frac{\gamma-1}{\gamma-\sigma}}}{\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+B}\right)^{\frac{\gamma}{\gamma-1}}\frac{1}{\varphi} = \frac{\gamma-1}{c\gamma^{\frac{\gamma}{\gamma-1}}}\left(\frac{L}{f}\right)^{\frac{1}{\gamma-1}}\left(\frac{\varphi^{\frac{\sigma-1}{\gamma-\sigma}}}{\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+B}\right)^{\frac{\gamma}{\gamma-1}}.
\end{aligned}
$$

Turning to the non-traded sector, we have:

$$
\mathcal{P} = \frac{c\gamma^{\frac{\gamma}{\gamma-1}}}{\gamma-1}\left(\frac{L}{f}(1-\alpha)\right)^{\frac{1}{1-\gamma}} = \frac{c\gamma^{\frac{\gamma}{\gamma-1}}}{\gamma-1}\left(\frac{L}{f}\right)^{\frac{1}{1-\gamma}}\left(\frac{B}{\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+B}\right)^{\frac{1}{1-\gamma}}.
$$

We can now derive the expression for the indirect utility as follows:

$$
\begin{aligned}
\mathcal{U} &= \left(\beta\left(\frac{\alpha}{P}\right)^{(\sigma-1)/\sigma} + (1-\beta)\left(\frac{1-\alpha}{\mathcal{P}}\right)^{(\sigma-1)/\sigma}\right)^{\sigma/(\sigma-1)} \\
&= \left(\beta\left[\frac{\gamma-1}{c\gamma^{\frac{\gamma}{\gamma-1}}}\left(\frac{L}{f}\right)^{\frac{1}{\gamma-1}}\left(\frac{\varphi^{\frac{\gamma-1}{\gamma-\sigma}}}{\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+B}\right)^{\frac{\gamma}{\gamma-1}}\frac{1}{\varphi}\right]^{\frac{\sigma-1}{\sigma}} + (1-\beta)\left[\frac{\gamma-1}{c\gamma^{\frac{\gamma}{\gamma-1}}}\left(\frac{L}{f}\right)^{\frac{1}{\gamma-1}}\left(\frac{B}{\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+B}\right)^{\frac{\gamma}{\gamma-1}}\right]^{\frac{\sigma-1}{\sigma}}\right) \\
&= \frac{\gamma-1}{c\gamma^{\frac{\gamma}{\gamma-1}}}\left(\frac{L}{f}\right)^{\frac{1}{\gamma-1}}\left(\beta\left(\frac{\varphi^{\frac{\gamma-1}{\gamma-\sigma}}}{\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+B}\right)^{\frac{\gamma(\sigma-1)}{\sigma(\gamma-1)}}\left(\frac{1}{\varphi}\right)^{(\sigma-1)/\sigma} + (1-\beta)\left(\frac{B}{\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+B}\right)^{\frac{\gamma(\sigma-1)}{\sigma(\gamma-1)}}\right)^{\sigma/(\sigma-1)} \\
&= \frac{\gamma-1}{c\gamma^{\frac{\gamma}{\gamma-1}}}\left(\frac{L}{f}\right)^{\frac{1}{\gamma-1}}\left(\frac{\beta\varphi^{\frac{\sigma-1}{\gamma-\sigma}} + (1-\beta)B^{\frac{\gamma(\sigma-1)}{\sigma(\gamma-1)}}}{\left(\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+B\right)^{\frac{\gamma(\sigma-1)}{\sigma(\gamma-1)}}}\right)^{\frac{\sigma}{\sigma-1}} = \frac{\beta^{\frac{\sigma}{\sigma-1}}(\gamma-1)}{c\gamma^{\frac{\gamma}{\gamma-1}}}\left(\frac{L}{f}\right)^{\frac{1}{\gamma-1}}\left(\frac{\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+1}{\left(\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+B\right)^{\frac{\gamma(\sigma-1)}{\sigma(\gamma-1)}}}\right)^{\frac{\sigma}{\sigma-1}} \\
&= \frac{\beta^{\frac{\sigma}{\sigma-1}}(\gamma-1)}{c\gamma^{\frac{\gamma}{\gamma-1}}}\left(\frac{L}{f}\right)^{\frac{1}{\gamma-1}}\frac{\left(\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+1\right)^{\frac{\sigma}{\sigma-1}}}{\left(\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+B\right)^{\frac{\gamma}{\gamma-1}}}.
\end{aligned}
$$

Hence, the indirect utility is given as follows:

$$\mathcal{U} = \frac{\beta^{\frac{\sigma}{\sigma-1}}(\gamma-1)}{c\gamma^{\frac{\gamma}{\gamma-1}}}\left(\frac{L}{f}\right)^{\frac{1}{\gamma-1}} \frac{\left(\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+1\right)^{\frac{\sigma}{\sigma-1}}}{\left(\varphi^{\frac{\sigma-1}{\gamma-\sigma}}+B\right)^{\frac{\gamma}{\gamma-1}}}. \tag{C-3}$$