

New trade models, elusive welfare gains*

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ABSTRACT: We generalize the formulae for welfare changes by Arkolakis, Costinot, and Rodríguez-Clare (2012) and Melitz and Redding (2015a) to allow for various cardinalizations of the subutility functions for varieties. Despite the same macro restrictions and the same equilibrium allocations, our new formula coincides with the original ones if and only if the number of varieties is invariant to foreign shocks. When product diversity responds to foreign shocks, different cardinalizations generate different welfare changes, thus revealing a fundamental difficulty in quantifying welfare gains implied by new trade models.

KEYWORDS: product diversity; subutility function; cardinalization; new trade models; welfare gains from trade.

JEL CLASSIFICATION: F11; F12.

1. Introduction

The gains from trade have always been central to international economics. With the advent of product differentiation (Krugman, 1980) and heterogeneous firms (Melitz, 2003), quantifying these gains has become both more important and more challenging. More important, because they stem from changes in productivity and the number of varieties consumed. More challenging, because endogenous product diversity raises conceptual issues that have to be dealt with (Feenstra, 1994; Broda and Weinstein, 2006).

Despite these new channels for gains from trade, Arkolakis, Costinot, and Rodríguez-Clare (2012; henceforth, ACR) have shown in an influential paper that – conditional on the trade elasticity and the import share of a country – many trade models deliver the same simple formula for gains from trade: $d \ln W_j = (1/\varepsilon) d \ln \lambda_{jj}$, where W_j stands for a measure of welfare in country j , $\varepsilon < 0$ is the trade elasticity, and λ_{jj} is the share of domestic expenditure. This result has stirred a debate on whether ‘new models’ have ‘old gains’, or whether they do indeed deliver ‘new gains’. ACR’s simple *sufficient statistics* for gains from trade are derived under three macro restrictions: (R1) trade in goods is balanced; (R2) aggregate profits are a constant share of GDP; and (R3) the import demand system is CES.¹ Those restrictions are satisfied in various trade models like Armington (1969) and Krugman (1980). They are also satisfied in the heterogeneous firms model by Melitz (2003) when productivity distributions are untruncated Pareto. As recently shown by Melitz and Redding (2015a), however, the ACR result is sensitive to small changes in the specification on the *technology side* of the economy. For example, when the untruncated Pareto distributions are replaced with truncated ones, the ACR

¹As pointed out by ACR, CES preferences are neither a necessary nor a sufficient condition for (R3).

result does not apply to the Melitz (2003) model: the trade elasticity and the domestic expenditure share are no longer sufficient statistics for measuring welfare changes, and firm heterogeneity provides a new channel for gains from trade.

In this paper, we show that the results of ACR and of Melitz and Redding (2015a) are both sensitive to small changes on the *preference side*. More precisely, we generalize their formulae for welfare changes to allow for various cardinalizations of the subutility functions for varieties. *Despite the same macro restrictions and the same equilibrium allocations*, our new formula coincides with the original ones if and only if the number of varieties is invariant to foreign shocks.² When product diversity varies, different cardinalizations generate different welfare changes for any given change in the trade equilibrium.

The intuition underlying this seemingly surprising result is that the change in the equilibrium allocation is preserved under different cardinalizations of the subutility functions, whereas the welfare change generally depends on the chosen cardinalization when product diversity is endogenous.³ An affine transformation of the subutility functions does not affect the first-order conditions of the utility maximization problem. Yet, it does affect the relative importance of product diversity and quantity in assessing how much income a consumer needs to be compensated with to keep her utility fixed at the initial level before foreign shocks.

Our result reveals a fundamental difficulty in quantifying welfare gains implied by new trade models, where the number of varieties consumed responds

²This is in sharp contrast to Melitz and Redding (2015a), who consider different macro restrictions and different equilibrium allocations than ACR.

³We show that the simplest Armington (1969) case analyzed by ACR is not affected by the cardinalization because each country produces a variety of a differentiated good, so that the number of varieties is fixed. Hence, it is endogenous changes in product diversity that lie at the heart of the problem.

to foreign shocks. While only aggregate data is needed in the ACR formula, Melitz and Redding (2015a) require firm-level data to cope with truncated productivity distributions. We show that even with such data, one cannot solve the cardinalization issue. Indeed, small changes on the technology side *do affect the equilibrium allocation* (and are therefore identifiable from the data), whereas the cardinalizations on the preference side *do not affect the equilibrium allocation* (and are therefore hardly identifiable from the data). Thus, there is no simple one-to-one mapping from a change in the equilibrium allocation to a change in welfare, even when using widely accepted welfare measures that are insensitive to a monotonic transformation of the upper-tier utility function. Accordingly, without knowing the ‘true’ cardinalization, any assessment of the gains from trade seems arbitrary and, therefore, debatable.

The remainder of the paper is organized as follows. In Section 2, we present the model and derive our key result, namely the generalized ACR formula. In Section 3, we apply that formula to Armington (1969), Krugman (1980), Melitz (2003), and Melitz and Redding (2015a). Finally, in Section 4 we summarize our key findings and discuss their empirical implications.

2. Model

We present a slightly modified subutility function that satisfies the same macro restrictions and generates the same equilibrium allocation than in ACR and Melitz and Redding (2015a). Following Behrens, Kanemoto, and Murata (2015), assume that the utility of a consumer in country j is given by

$$U_j = F \left(\left[\sum_{i=1}^I \int_{\Omega_{ij}} u \left(q_{ij}(\omega)^{(\sigma-1)/\sigma} \right) d\omega \right]^{\sigma/(\sigma-1)} \right) \quad (1)$$

with

$$u\left(q_{ij}(\omega)^{(\sigma-1)/\sigma}\right) = \begin{cases} a_{ij}(\omega)^{(\sigma-1)/\sigma} + b_j q_{ij}(\omega)^{(\sigma-1)/\sigma} & \text{for } q_{ij}(\omega) > 0 \\ 0 & \text{for } q_{ij}(\omega) = 0 \end{cases}, \quad (2)$$

where F is a continuously differentiable, strictly increasing and concave function; where I is the number of countries; where u is a subutility function; where $q_{ij}(\omega)$ denotes the quantity of variety ω produced in country i and consumed in country j ; and where $a_{ij}(\omega) \geq 0$, $b_j > 0$, and $\sigma > 1$ are fixed utility parameters.

We denote by $n_{ij} \equiv |\Omega_{ij}|$ the (generally endogenously determined) mass of varieties produced in i and consumed in j . Observe that: (i) the affine transformation of the CES function $q_{ij}(\omega)^{\sigma/(\sigma-1)}$ via $a_{ij}(\omega)^{(\sigma-1)/\sigma}$ and b_j is a transformation that does not affect the equilibrium allocation (see Appendix A); and (ii) when $a_{ij}(\omega) = 0$ and $b_j = 1$ for all i, j , and ω (which corresponds to a particular cardinalization of the subutility functions), and when $F(x) = x$, we obtain exactly the specification in ACR.

In what follows, we assume that $a_{ij}(\omega) > 0$ and $b_j > 0$ for all i, j , and ω . Let

$$A_j \equiv \left[\sum_{i=1}^I \int_{\Omega_{ij}} a_{ij}(\omega)^{(\sigma-1)/\sigma} d\omega \right]^{\sigma/(\sigma-1)} \quad \text{and} \quad Q_j \equiv \left[\sum_{i=1}^I \int_{\Omega_{ij}} q_{ij}(\omega)^{(\sigma-1)/\sigma} d\omega \right]^{\sigma/(\sigma-1)}. \quad (3)$$

Note that although $a_{ij}(\omega)$ is exogenous, A_j can vary since the mass n_{ij} of varieties is generally endogenous. The utility function can then be expressed as follows:

$$U_j = F\left(\left[A_j^{(\sigma-1)/\sigma} + b_j Q_j^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)}\right). \quad (4)$$

There are two different ways to obtain our main result. We first consider expenditure minimization and then show that utility maximization yields the

same result. The expenditure minimization problem is given by

$$\begin{aligned} \min \quad & \sum_{i=1}^I \int_{\Omega_{ij}} p_{ij}(\omega) q_{ij}(\omega) d\omega \\ \text{s.t.} \quad & F \left(\left[A_j^{(\sigma-1)/\sigma} + b_j Q_j^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)} \right) \geq \bar{U}_j. \end{aligned}$$

From this, we obtain the expenditure function conditional on A_j (see Appendix B):

$$e(A_j, P_j, \bar{U}_j) = P_j \left\{ \frac{[F^{-1}(\bar{U}_j)]^{(\sigma-1)/\sigma} - A_j^{(\sigma-1)/\sigma}}{b_j} \right\}^{\sigma/(\sigma-1)}, \quad (5)$$

where

$$P_j \equiv \left[\sum_{i=1}^I \int_{\Omega_{ij}} p_{ij}(\omega)^{1-\sigma} d\omega \right]^{1/(1-\sigma)} \quad (6)$$

is a price aggregate. In the initial equilibrium before foreign shocks, $e(A_j, P_j, \bar{U}_j) = w_j$ holds by definition of the expenditure function, where w_j is the wage rate in country j . Following ACR and Melitz and Redding (2015a), we assume that $w_j \equiv 1$ is the numeraire and invariant to foreign shocks.

We now define the *equivalent income* that keeps consumers at their initial utility level as $E_j \equiv w_j / e(A_j, P_j, \bar{U}_j)$.⁴ Several remarks are in order. First, since $w_j \equiv 1$, the equivalent income E_j may be viewed as the inverse of the expenditure function. Second, since $e(A_j, P_j, \bar{U}_j) = w_j$ in the initial equilibrium, $E_j = 1$ holds before foreign shocks. Third, an increase in E_j , or equivalently a decrease in $e(A_j, P_j, \bar{U}_j)$, after foreign shocks translates into welfare gains. Last, for welfare gains to be associated with a positive sign,

⁴See Mrázová and Neary (2014a, b) for an equivalent definition using the indirect utility function. The equivalent income can be viewed as a version of the Allais surplus, which is defined as the surplus of the numeraire good that is generated while keeping the utility level constant. As is well known, for a marginal change, the Marshallian consumer surplus, the compensating variation, the equivalent variation, and the Allais surplus all coincide.

we consider in what follows the rate of change in the equivalent income, $d \ln E_j = -d \ln e(A_j, P_j, \bar{U}_j)$.

Let $W_j \equiv 1/P_j$ denote the inverse of the price aggregate, which is used as a welfare measure by ACR and Melitz and Redding (2015a). Then, equation (5) can be rewritten as

$$E_j = W_j \left\{ \frac{[F^{-1}(\bar{U}_j)]^{(\sigma-1)/\sigma} - A_j^{(\sigma-1)/\sigma}}{b_j} \right\}^{-\sigma/(\sigma-1)}. \quad (7)$$

Taking the rate of change while holding \bar{U}_j constant at the initial level, we have

$$d \ln E_j = \frac{A_j^{(\sigma-1)/\sigma}}{[F^{-1}(\bar{U}_j)]^{(\sigma-1)/\sigma} - A_j^{(\sigma-1)/\sigma}} d \ln A_j + d \ln W_j. \quad (8)$$

Let

$$\theta_{A_j} \equiv \frac{A_j^{(\sigma-1)/\sigma}}{A_j^{(\sigma-1)/\sigma} + b_j Q_j^{(\sigma-1)/\sigma}} \quad \text{and} \quad \theta_{Q_j} \equiv \frac{b_j Q_j^{(\sigma-1)/\sigma}}{A_j^{(\sigma-1)/\sigma} + b_j Q_j^{(\sigma-1)/\sigma}} \quad (9)$$

be share variables such that $\theta_{A_j} + \theta_{Q_j} = 1$. Evaluating (4) at the initial utility level \bar{U}_j and plugging the resulting expression into (8), we obtain the following proposition.

Proposition (Equivalent income change due to foreign shocks) *Holding utility constant before and after foreign shocks requires the following rate of change in the equivalent income:*

$$d \ln E_j = \frac{\theta_{A_j}}{\theta_{Q_j}} d \ln A_j + d \ln W_j. \quad (10)$$

In the special case where A_j is invariant to foreign shocks, i.e., $d \ln A_j = 0$, the rate of change in the equivalent income boils down to the rate of change in the real wage in Arkolakis et al. (2012) or Melitz and Redding (2015a), i.e., $d \ln E_j = d \ln W_j$.

This proposition shows that the rate of change in the equivalent income $d \ln E_j = -d \ln e(A_j, P_j, \bar{U}_j)$ need not be the same as the rate of change in the

real wage $d \ln W_j$. Intuitively, when foreign shocks expand product diversity, domestic consumers spend less to achieve their initial utility level. This gain is captured only partially by the rate of change in $W_j = 1/P_j$ when $d \ln A_j \neq 0$. Indeed, when the mass of varieties increases, there is an additional welfare change associated with the cardinalization parameter since the value of an additional variety for a consumer depends on that parameter. Hence, omitting the first term of the right-hand side in (10) leads to an incorrect assessment of the true welfare change.

A few remarks are in order. First, the discontinuity of the subutility function u at $q_{ij}(\omega) = 0$ is not crucial for our result (see Appendix C). Our proposition holds even when u in (2) is replaced with a continuously differentiable, strictly increasing and strictly concave function that has the same properties than u in a ‘neighborhood’ of the equilibrium. This ‘neighborhood’ can be made large, and the limit case where the ‘neighborhood’ is all of \mathbb{R}_{++} is the affine transformation that we use in the main text. Second, we can obtain the same formula from utility maximization instead of expenditure minimization (see Appendix D). Last, the new formula is invariant to a monotonic transformation F of the upper-tier utility function but can vary with different cardinalizations of subutility functions. We explore this implication using workhorse trade models in what follows.

3. Examples

Unlike ACR and Melitz and Redding (2015a), how welfare changes with foreign shocks under our preference structure crucially depends on how A_j changes with those shocks. As can be seen from (3), only changes in the sets Ω_{ij} of varieties consumed can affect A_j . Thus, the way product diversity in country

j changes due to foreign shocks is crucial for assessing the equivalent income change $d \ln E_j$ in (10). We now discuss this point with reference to the existing literature.

3.1 *Armington (1969)*

Consider first the basic Armington (1969) model, where each country produces a single variety of a differentiated good, i.e., the sets Ω_{ij} of varieties are degenerated. We assume that trade costs are finite: $\tau_{ij} < \infty$ for all i and j . We then obtain the Armington specification from the one in Section 2 by letting $q_{ij}(\omega) = q_{ij}$ and $n_{ij} = 1$ for all i, j , and ω . In this case, A_j is invariant to foreign shocks, so that $d \ln A_j = 0$. Thus, the equivalent income change (10) becomes $d \ln E_j = d \ln W_j$, where the latter expression is obtained by ACR as follows:

$$d \ln W_j = \frac{1}{1 - \sigma} d \ln \lambda_{jj}. \quad (11)$$

Hence, the change in the share of domestic expenditure, λ_{jj} , and the trade elasticity $1/(1 - \sigma)$ are sufficient statistics for assessing welfare changes.

3.2 *Krugman (1980)*

We now turn to the case of monopolistic competition with homogeneous firms as in Krugman (1980). In this case, $d \ln A_j$ in (10) need not be zero because A_j , as given by (3), is endogenous and depends on the mass n_{ij} of varieties imported from i to j . For simplicity, assume that $a_{ij}(\omega) = a_{ij} > 0$ for all $\omega \in \Omega_{ij}$. Taking the log of $A^{(\sigma-1)/\sigma}$ in (3) and differentiating, we have

$$d \ln A_j = \frac{\sigma}{\sigma - 1} \sum_{i=1}^I \theta_{ij} d \ln n_{ij}, \quad (12)$$

where $\theta_{ij} \equiv (a_{ij}/A_j)^{(\sigma-1)/\sigma} n_{ij}$ are share variables with $\sum_{i=1}^I \theta_{ij} = 1$. Since $d \ln A_j$ can be different from zero, the ACR formula need not hold even with the same macro restrictions and the same equilibrium allocations as in ACR.

We need to distinguish two cases in Krugman (1980), depending on the sources of foreign shocks. First, with finite trade costs, the mass n_{ij} of varieties imported from i to j does not change with shocks to trade costs τ_{ij} . Hence, even with $a_{ij} > 0$, we have $d \ln A_j = 0$, so that the ACR formula (11) holds. Second, shocks to either population or to fixed costs of production change the mass n_{ij} of varieties and thus affect welfare depending on the chosen cardinalization a_{ij} . Specifically, n_{ij} increases with population and decreases with fixed costs in country i . Accordingly, when $a_{ij} > 0$, positive (resp., negative) shocks to population, or negative (resp., positive) shocks to fixed costs, lead to $d \ln A_j > 0$ (resp., $d \ln A_j < 0$) and, therefore, the ACR formula underestimates (resp., overestimates) the welfare changes from the foreign shocks.

Hence, in Krugman (1980) with $a_{ij} > 0$, the ACR formula: (i) holds exactly for changes in finite trade costs; (ii) provides a lower bound for possible equivalent income changes when there are positive shocks to population or negative shocks to fixed costs; and (iii) provides an upper bound for possible equivalent income changes when there are negative shocks to population or positive shocks to fixed costs.

One may argue that setting $a_{ij} = a > 0$ for all i and j yields $\theta_{ij} = n_{ij} / \sum_{i=1}^I n_{ij}$, in which case $d \ln A_j$ is independent of the cardinalization parameter a . However, there is no a priori reason to believe that this is the ‘right’ cardinalization. More importantly, even with $a_{ij} = a > 0$ for all i and j , the existing formula provides only either an upper bound or a lower bound for all possible equivalent income changes when product diversity responds to foreign shocks. Note that even with $a_{ij} = a > 0$ for all i and j , the share

variables θ_{A_j} and θ_{Q_j} in our formula (10) depend on a .

3.3 Melitz (2003)

We finally consider the case of monopolistic competition with heterogeneous firms as in Melitz (2003). We first analyze the case with an untruncated Pareto productivity distribution as in ACR. We then turn to the case of a truncated Pareto productivity distribution as in Melitz and Redding (2015a). Last, we briefly discuss the case with an arbitrary productivity distribution.

Untruncated Pareto distribution. Assuming that $A_j = 0$, and that the productivity distribution is untruncated Pareto with shape parameter $k > 1$, ACR show that the same formula as in Armington (1969) and Krugman (1980) applies to the Melitz (2003) model:

$$d \ln W_j = -\frac{1}{k} d \ln \lambda_{jj}, \quad (13)$$

which shows that the trade elasticity, $-k$, and the change in the share of domestic expenditure, λ_{jj} , are sufficient statistics for measuring welfare changes in the wake of foreign shocks.

Contrary to ACR, we consider the case where $A_j > 0$. As in the previous subsection, we assume that $a_{ij}(\omega) = a_{ij} > 0$ for all $\omega \in \Omega_{ij}$, so that equation (12) still holds. Then, foreign shocks generally affect A_j via the mass n_{ij} of varieties exported from country i to country j . Consider the productivity distribution G and its density g . Recall that in the Melitz (2003) model, the mass of firms selling from i to j is given by

$$n_{ij} = M_{ei}[1 - G(\varphi_{ij}^x)], \quad (14)$$

where M_{ei} is the mass of entrants in country i , where $1 - G(\varphi_{ij}^x)$ is the share of firms selling from i to j , and where φ_{ij}^x is the productivity cutoff for exporting

from i to j . Let $h(\varphi_{ij}^x) \equiv g(\varphi_{ij}^x)/[1 - G(\varphi_{ij}^x)]$ denote the hazard function. Since (12) holds, it follows that

$$\begin{aligned} \frac{\sigma - 1}{\sigma} d \ln A_j &= \sum_{i=1}^I \theta_{ij} [d \ln M_{ei} - h(\varphi_{ij}^x) \varphi_{ij}^x d \ln \varphi_{ij}^x] \\ &= \sum_{i \neq j}^I \theta_{ij} [d \ln M_{ei} - h(\varphi_{ij}^x) \varphi_{ij}^x d \ln \varphi_{ij}^x] \\ &\quad + \theta_{jj} [d \ln M_{ej} - h(\varphi_{jj}^x) \varphi_{jj}^x d \ln \varphi_{jj}^x]. \end{aligned} \quad (15)$$

As shown by Melitz and Redding (2015b), in the Melitz (2003) model with a Pareto productivity distribution, the mass of entrants in country i increases with a positive shock to labor supply L_i (or, equivalently, a negative shock to the sunk costs of entry F_{ei}), and it is independent of trade costs: $dM_{ei}/dL_i > 0$; $dM_{ei}/dF_{ei} < 0$; and $dM_{ei}/d\tau_{ij} = 0$.

Hence, under a shock to trade costs, equation (15) reduces to

$$\begin{aligned} \frac{\sigma - 1}{\sigma} d \ln A_j &= - \left[\sum_{i \neq j}^I \theta_{ij} h(\varphi_{ij}^x) \varphi_{ij}^x d \ln \varphi_{ij}^x + \theta_{jj} h(\varphi_{jj}^x) \varphi_{jj}^x d \ln \varphi_{jj}^x \right] \\ &= -k \left[\sum_{i \neq j}^I \theta_{ij} d \ln \varphi_{ij}^x + \theta_{jj} d \ln \varphi_{jj}^x \right], \end{aligned} \quad (16)$$

where the latter equality comes from the untruncated Pareto assumption. The sign of $d \ln A_j$ crucially depends on how the productivity cutoffs change in the wake of the trade cost shock. To our knowledge, however, there is no analytical proof for the asymmetric multi-country version of the Melitz (2003) model of how the export and domestic cutoffs, φ_{ij}^x and φ_{jj}^x , change with trade costs, especially when wages are endogenous.⁵ Similarly, not much can be

⁵The only exception is Demidova and Rodríguez-Clare (2013, Proof of Proposition 1), who show that in a two-country version of the Melitz model a fall in τ_{ij} decreases the export cutoff φ_{ij}^x , yet increases the domestic cutoff φ_{jj}^x (we thank Marc Melitz for bringing this reference to our attention). Thus, the sign of (16) is ambiguous and it is unclear whether the ACR formula over- or underestimates the welfare changes due to the trade cost shock even in the two-country case.

said about how shocks to population or to sunk costs of entry affect $d \ln A_j$. Hence, even in the basic version of the Melitz model with an untruncated Pareto productivity distribution, we can hardly assess welfare changes due to the presence of the $d \ln A_j$ term in (10). In a nutshell, a small modification on the preference side that satisfies the same macro restrictions and leaves the equilibrium allocations unchanged affects significantly the amount of information needed to measure welfare changes.

Truncated Pareto distribution. As argued by Melitz and Redding (2015a), the ACR result with heterogeneous firms crucially hinges on the assumption of an untruncated Pareto distribution with shape parameter k and lower bound φ_{\min} for firm productivities. If the underlying distribution is, for example, truncated at $\varphi_{\max} < \infty$, the trade elasticity depends on the productivity cutoff and is specific to each country. In that case, there are no simple sufficient statistics for the welfare gains from trade liberalization.

Assuming that $A_j = 0$, Melitz and Redding (2015a, eq.(107) in the online Appendix) show that with a truncated Pareto distribution the ACR formula is modified as follows:

$$d \ln W_j = \frac{1}{\vartheta_{jj}} (d \ln M_{ej} - d \ln \lambda_{jj}). \quad (17)$$

where $\vartheta_{jj} = \sigma - 1 + \gamma(\varphi_{jj}^x)$ is the domestic partial trade elasticity, and where $\gamma(\varphi_{jj}^x) = -d \ln \delta(\varphi_{jj}^x) / d \ln \varphi_{jj}^x$ is the elasticity of $\delta(\varphi_{jj}^x) = \int_{\varphi_{jj}^x}^{\varphi_{\max}^x} \varphi^{\sigma-1} dG(\varphi)$. Note that if the Pareto distribution for productivity has no upper bound, it follows that $\gamma(\varphi_{jj}^x) = k - (\sigma - 1)$. Since $d \ln M_{ej} = 0$ in the wake of a shock to trade costs, the ACR formula (13) obtains.

Our formula for equivalent income changes encompasses the case analyzed by Melitz and Redding (2015a). Indeed, plugging (17) into (10), we can

generalize their formula as follows:

$$d \ln E_j = \frac{\theta_{A_j}}{\theta_{Q_j}} d \ln A_j + \frac{1}{\sigma - 1 + \gamma(\varphi_{jj}^x)} (d \ln M_{ej} - d \ln \lambda_{jj}). \quad (18)$$

Note that (15) applies to both the untruncated and truncated Pareto versions of the model, and so does (18). In other words, in the general case the welfare changes are sensitive to modeling choices on both the technology side (truncation of the distributions) and the preference side (cardinalization parameters).

General distribution. Melitz (2003) does not impose a particular distribution for productivity draws. In general, no clear predictions can be made on whether the masses of firms and the cutoffs increase or decrease with foreign shocks. Furthermore, as shown by (15), the impact of a shock on A_j depends on a complex weighting scheme involving the cardinalization parameters. Consequently, without prior knowledge of the a_{ij} terms, nothing can be said on how foreign shocks affect A_j , except that $d \ln A_j$ is not zero generically.

4. Summary and conclusions

As shown by Melitz and Redding (2015a), the formula for welfare changes due to trade shocks put forth by ACR is sensitive to small changes on the technology side of the models. The extended ACR formula derived by Melitz and Redding (2015a) suggests that additional moments from firm-level data can be used to quantify the welfare gains. Our analysis, however, shows that things are not that simple. When product diversity responds to foreign shocks, welfare changes can be arbitrary. The reasons are that for any given change in the trade equilibrium there exists a continuum of welfare changes reflecting different cardinalizations of the subutility functions for varieties, and that the

‘true’ cardinalization is hardly identifiable. The gains from trade implied by new trade models are thus elusive.

Although this paper focuses entirely on the gains from trade formula, our result that cardinalizations matter for welfare changes can be applied to many fields of economics such as new growth theory and new economic geography, where product diversity are endogenously determined. Generalizing our result is left for future research.

Appendix

Appendix A: Affine transformation.

To establish claim (i), consider the general problem where the subutility function v is replaced by $u \circ v$, where u is an arbitrary increasing and concave function. Then, taking the ratio of the first-order conditions for the consumer problem with respect to varieties ω and ω' , sourced from countries i and k , respectively, yields

$$\left[\frac{q_{ij}(\omega)}{q_{kj}(\omega')} \right]^{-1/\sigma} \frac{u'(v(q_{ij}(\omega)))}{u'(v(q_{kj}(\omega')))} = \frac{p_{ij}(\omega)}{p_{kj}(\omega')}.$$

If u is an affine function, then u' is constant, so that $u'(v(q_{ij}(\omega))) = u'(v(q_{kj}(\omega')))$ for all varieties $\omega \in \Omega_{ij}$ and $\omega' \in \Omega_{kj}$ and for all quantities $q_{ij}(\omega) \geq 0$ and $q_{kj}(\omega') \geq 0$. Hence, the solution to this equation is identical to that in the untransformed case, the first-order conditions of which are given by

$$\left[\frac{q_{ij}(\omega)}{q_{kj}(\omega')} \right]^{-1/\sigma} = \frac{p_{ij}(\omega)}{p_{kj}(\omega')}.$$

Since the other equilibrium conditions are not affected by the affine transformation of the subutility function, the equilibrium allocation remains unchanged.

Appendix B: Expenditure function.

To obtain the expenditure function, we solve the expenditure minimization problem:

$$\begin{aligned} \min \quad & \sum_{i=1}^I \int_{\Omega_{ij}} p_{ij}(\omega) q_{ij}(\omega) d\omega \\ \text{s.t.} \quad & F \left(\left\{ \sum_{i=1}^I \int_{\Omega_{ij}} \left[a_{ij}(\omega)^{(\sigma-1)/\sigma} + b_j q_{ij}(\omega)^{(\sigma-1)/\sigma} \right] d\omega \right\}^{\sigma/(\sigma-1)} \right) \geq \bar{U}_j. \end{aligned}$$

The first-order conditions for $q_{ij}(\omega)$ and $q_{kj}(\omega')$ are given by

$$\begin{aligned} p_{ij}(\omega) &= \lambda F' \cdot \left\{ \sum_{i=1}^I \int_{\Omega_{ij}} \left[a_{ij}(\nu)^{(\sigma-1)/\sigma} + b_j q_{ij}(\nu)^{(\sigma-1)/\sigma} \right] d\nu \right\}^{1/(\sigma-1)} b_j q_{ij}(\omega)^{-1/\sigma} \\ p_{kj}(\omega') &= \lambda F' \cdot \left\{ \sum_{k=1}^I \int_{\Omega_{kj}} \left[a_{kj}(\nu)^{(\sigma-1)/\sigma} + b_j q_{kj}(\nu)^{(\sigma-1)/\sigma} \right] d\nu \right\}^{1/(\sigma-1)} b_j q_{kj}(\omega')^{-1/\sigma}, \end{aligned}$$

which implies the standard condition

$$q_{ij}(\omega)^{(\sigma-1)/\sigma} = \left[\frac{p_{ij}(\omega)}{p_{kj}(\omega')} \right]^{1-\sigma} q_{kj}(\omega')^{(\sigma-1)/\sigma}.$$

Plugging this expression into the utility constraint and using A_j from (3), we have

$$F \left(\left[A_j^{(\sigma-1)/\sigma} + b_j \frac{q_{kj}(\omega')^{(\sigma-1)/\sigma}}{p_{kj}(\omega')^{1-\sigma}} \sum_{i=1}^I \int_{\Omega_{ij}} p_{ij}(\omega)^{1-\sigma} d\omega \right]^{\sigma/(\sigma-1)} \right) = \bar{U}_j,$$

which implies

$$A_j^{(\sigma-1)/\sigma} + b_j \frac{q_{kj}(\omega')^{(\sigma-1)/\sigma}}{p_{kj}(\omega')^{1-\sigma}} \sum_{i=1}^I \int_{\Omega_{ij}} p_{ij}(\omega)^{1-\sigma} d\omega = \left[F^{-1}(\bar{U}_j) \right]^{(\sigma-1)/\sigma}.$$

Solving this equation for $q_{kj}(\omega')$ and using P_j from (6), we obtain the compensated demand function conditional on A_j as follows:

$$q_{kj}(p_{jk}(\omega'), A_j, P_j, \bar{U}_j) = p_{kj}(\omega')^{-\sigma} P_j^\sigma \left\{ \frac{\left[F^{-1}(\bar{U}_j) \right]^{(\sigma-1)/\sigma} - A_j^{(\sigma-1)/\sigma}}{b_j} \right\}^{\sigma/(\sigma-1)}.$$

Finally, the expenditure function conditional on A_j , $e(A_j, P_j, \bar{U}_j)$, can be obtained by plugging the compensated demand function into the objective function.

Appendix C: Continuous subutility function.

In the main text, we focus on the discontinuous subutility function u in (2). However, the discontinuity of u at $q_{ij}(\omega) = 0$ is not crucial for our result. Indeed, u in (2) can be replaced with a continuously differentiable, strictly increasing and strictly concave function without affecting our result.

To simplify the exposition, we omit i , j , and ω and assume that $\tilde{u}(q) = u(q^{(\sigma-1)/\sigma})$, so that

$$\tilde{u}(q) = \begin{cases} a^{(\sigma-1)/\sigma} + bq^{(\sigma-1)/\sigma} & \text{for } q > 0 \\ 0 & \text{for } q = 0 \end{cases}.$$

In what follows, we replace \tilde{u} with a function that is equivalent to \tilde{u} for all $q \geq \bar{q} > 0$, where the threshold \bar{q} can be chosen arbitrarily. To this end, let $\bar{a} = a^{(\sigma-1)/\sigma}$ and consider the following transformed utility function:

$$\bar{u}(q) = \begin{cases} \bar{a} + bq^{(\sigma-1)/\sigma} & \text{for } q \geq \bar{q} \\ \int_0^q \varphi(t) dt & \text{for } 0 \leq q < \bar{q} \end{cases},$$

where φ satisfies the following three conditions: (i) it is a strictly positive and strictly decreasing function; (ii) $\int_0^{\bar{q}} \varphi(t) dt = \bar{a} + b\bar{q}^{(\sigma-1)/\sigma}$; and (iii) $\varphi(\bar{q}) = b[(\sigma-1)/\sigma]\bar{q}^{-1/\sigma}$.

By (i), it is immediate that \bar{u} is strictly increasing and strictly concave on $[0, \bar{q})$. It is also obviously strictly increasing and strictly concave on $[\bar{q}, \infty)$. From (ii), it follows that $\lim_{q \rightarrow \bar{q}^+} \bar{a} + bq^{(\sigma-1)/\sigma} = \bar{a} + b\bar{q}^{(\sigma-1)/\sigma} = \lim_{q \rightarrow \bar{q}^-} \int_0^q \varphi(t) dt = \bar{u}(\bar{q})$, i.e., \bar{u} is continuous for all $q \geq 0$. From (iii), it follows that $\bar{u}'(\bar{q}) = \varphi(\bar{q}) = b[(\sigma-1)/\sigma]\bar{q}^{-1/\sigma}$, i.e., \bar{u} is differentiable for all $q \geq 0$.

As an illustration, assume that $\varphi(t) = \alpha - \beta t$. We can then find $\alpha > 0$ and $\beta > 0$ such that for any threshold \bar{q} the foregoing results hold. To see that φ satisfies the three conditions, let us start with condition (ii), which implies that $\alpha\bar{q} - (\beta/2)\bar{q}^2 = \bar{a} + b\bar{q}^{(\sigma-1)/\sigma}$. Condition (iii) implies that $\alpha - \beta\bar{q} = b[(\sigma - 1)/\sigma]\bar{q}^{-1/\sigma}$. These two conditions give a system of linear equations in α and β , which yields a unique solution

$$\alpha = \frac{2\bar{a}}{\bar{q}} + b\bar{q}^{-1/\sigma} \left(\frac{\sigma + 1}{\sigma} \right) > 0 \quad \text{and} \quad \beta = \frac{2}{\bar{q}} \left(\frac{\bar{a}}{\bar{q}} + \frac{b}{\sigma} \bar{q}^{-1/\sigma} \right) > 0.$$

Condition (i) then holds since φ is strictly decreasing for all $q \in [0, \bar{q})$ and

$$\varphi(\bar{q}) = \alpha - \beta\bar{q} = b\bar{q}^{-1/\sigma} \left(\frac{\sigma - 1}{\sigma} \right) > 0,$$

i.e., φ is strictly positive for all $q \in [0, \bar{q})$.

Note, finally, that there are many possible functions φ to define \bar{u} on $[0, \bar{q})$. The assumption on φ does not matter for our purpose since we can choose \bar{q} sufficiently small and need not worry about the behavior of \bar{u} close to zero.

Appendix D: Alternative derivation of formula (10).

To obtain formula (10), we may alternatively maximize utility (4) subject to the budget constraint

$$\sum_{i=1}^I \int_{\Omega_{ij}} p_{ij}(\omega) q_{ij}(\omega) d\omega = \frac{w_j}{E_j}.$$

Then, the demand functions are given by

$$q_{ij}(\omega) = p_{ij}(\omega)^{-\sigma} P_j^{\sigma-1} \frac{w_j}{E_j}.$$

Plugging this expression into (3) and setting $w_j = 1$, we obtain

$$Q_j = \frac{W_j}{E_j},$$

where $W_j = 1/P_j$ is the welfare measure used in ACR and Melitz and Redding (2015a).

Taking the log of both sides of (4) and differentiating, we obtain

$$d \ln U_j = \varepsilon_F \cdot (\theta_{A_j} d \ln A_j + \theta_{Q_j} d \ln Q_j),$$

where $\varepsilon_F \equiv (F'/F)[A_j^{(\sigma-1)/\sigma} + b_j Q_j^{(\sigma-1)/\sigma}]^{\sigma/(\sigma-1)}$ is the elasticity of the upper-tier utility, F . Noting that $d \ln Q_j = d \ln W_j - d \ln E_j$, the above expression can be rewritten as

$$d \ln U_j = \varepsilon_F \cdot [\theta_{A_j} d \ln A_j + \theta_{Q_j} (d \ln W_j - d \ln E_j)].$$

Since we consider the rate of change in the equivalent income in country j , $d \ln E_j$, that holds utility constant at the initial level, we set $d \ln U_j = 0$ to obtain the formula (10).

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