Geography, Technology, and Optimal Trade Policy with Heterogeneous Firms

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Abstract

How should multilateral trade policy be designed in a world in which countries differ in terms of technology and geography, and firms with market power differ in terms of productivity? To answer this question we develop a normative analysis of the model by Melitz and Ottaviano (2008) in the case of an arbitrary number of asymmetric countries. We first take the viewpoint of a global planner maximizing global welfare constrained only by technology and endowments. We characterize the corresponding 'first best' allocation in terms of cross-country specialization, trade patterns, firm productivity distribution and product variety. We show how the first best allocation departs from the free market equilibrium along all four dimensions, pinning down the policy tools needed for its decentralization. As these tools are hardly enshrined in any real multilateral trade agreement, we then study alternative scenarios, in which the set of available policy tools is increasingly restricted.

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1 Introduction

How should multilateral trade policy be designed in a world in which countries differ in terms of technology and geography, and firms with market power differ in terms of productivity? Should policy tools differ in kind or implementation between more and less developed countries? Should smaller less productive firms be protected against larger more productive (foreign) rivals? Should national product diversity be defended against competition from cheaper imported products? What is the optimal degree of product diversity on a global scale?

To answer these and other related questions we propose a normative analysis of the monopolistic competition model by Melitz and Ottaviano (2008). This model exhibits several features useful for our purposes. As even with heterogenous firms it is analytically solvable with all sorts of asymmetries in country size, technology and accessibility, the model allows for transparent comparative statics. As it features variable markups, it allows for firm heterogeneity to become a key source of misallocation.[1] As income effects are neutralized due to quasi-linear preferences, constant marginal utility of income allows for a consistent global welfare analysis based on a straightforward definition of global welfare for an economy with heterogeneous countries as the sum of all individuals' indirect utilities. While the absence of income effects gives our results a partial equilibrium flavor, this approach shares the focus on social surplus with mainstream policy analysis.

Melitz and Ottaviano (2008) model an economy in which countries differ in terms of market size, barriers to international trade and state of technology. Countries are active in two sectors with labor as the only production factor. A 'traditional' perfectly competitive sector supplies a freely traded homogeneous good. A 'modern' monopolistically competitive sector supplies varieties of a horizontally differentiated good. In each country the productivity of entrants in the modern sector is dispersed around a country-specific mean dictated by the national state of technology, which thus defines the country's comparative advantage in the modern sector with respect to the other countries. Moreover, in the modern sector countries face different physical transport costs for their international and domestic trade as dictated by geography. In equilibrium technology and geography endogenously determine a country's intersectoral specialization as well as its patterns of inter- and intra-industry trade. They also determine the productivity distribution of a country's modern producers as well as the number and the prices of varieties the country's consumers have access to and the dimension of the differentiated good sector in each country.

Melitz and Ottaviano (2008) do not provide any normative analysis of their model. The aim of this paper is to show that filling this gap yields new insights on optimal multilateral trade policy. In particular, we first take the viewpoint of a global planner maximizing global welfare constrained only by technology and endowments. We characterize the corresponding 'first best' allocation in terms of cross-country specialization and firm productivity distribution, trade patterns, size of the differentiated good sector and product variety. We show how the first best allocation departs from the free market equilibrium along all these dimensions, pinning down the policy tools needed for its decentralization. These include country-specific lump-sum instruments for both firms and consumers as

[1] See the discussion in Nocco, Ottaviano and Salto (2014).

well as production and consumption subsidies and taxes differentiated across firms both between and within countries.

As the wide array of policy tools needed to decentralize the first best is hardly enshrined in any real multilateral trade agreement, we then study alternative multilateral 'n-th best' scenarios, in which the set of available policy tools is increasingly restricted. In the 'second best', lump-sum instruments are still available but subsidies and taxes can not be differentiated across firms. In the 'third best', also lump-sum instruments for firms are ruled out. Finally, when also lump-sum tools for consumers are removed, the economy can not be drawn away from the free market equilibrium.

Our analysis speaks to two main literatures. First, there is the literature on optimal trade policy both under perfect competition (see, e.g., the discussion in Costinot, Donaldson, Vogel and Werning, 2014) and imperfect competition (see, e.g., Grossman, 1992). This literature does not feature firm heterogeneity with an arbitrary number of countries and firms.

Second, there is the literature on optimal product variety without firms heterogeneity (Spence, 1976, and Dixit and Stiglitz, 1977) and with firm heterogeneity (Dhingra and Morrow, 2012; Melitz and Redding, 2012).[2] This literature focuses on the closed economy or open economies in which countries are symmetric.[3]

The rest of the paper is organized as follows. Section 2 presents the market equilibrium in the model by Melitz and Ottaviano (2008) for the case of M trading open economies and derives the optimum solutions for the first best planner. Section 3 analyzes the distortions in the market equilibrium both in the case of symmetric and asymmetric countries and discusses the effects on these distortions of an increase in the number of trading countries due to 'globalization', an enlargement of the dimension of the home market size due to 'demographic growth', an improvement in the technologies of production stemming from 'development' and a reduction in trade barriers due to processes of economic 'international integration'. Section 4 presents the optimal solutions for alternative 'n-th best' planners comparing the main results they deliver in terms of selection, average firms' supply, product variety, number of domestic and exported varieties and overall size of the differentiated good sector in each country. Section 5 analyzes how these optimal solutions can be implemented by means of appropriate sets of policy instruments, and Section 6 concludes.

2 The model

In this Section we present the model with M trading countries in order to understand how the distortions in the market equilibrium with respect to that set by a first best global planner are affected by the number of trading countries and the trade barriers they face, by the dimension of the home market size of

[2] In a closed economy setup, Nocco, Ottaviano and Salto (2013) offer a systematic quantitative analysis of the impact of different degrees of firm heterogeneity on the extent of market inefficiencies, and Nocco, Ottaviano and Salto (2014) provide a discussion of decentralization of the first best outcome and of a constrained solution for a planner that can only rely on lump-sum tools on consumers and who can not impose lump-sum taxes/subsidies on firms.

[3] See Nocco, Ottaviano and Salto (2013) for an analysis of the main developments in the literature of optimum product diversity with monopolistic competition.

each country and by the level of development of the technology available within each country.

We set the analysis in a framework that is that of the traditional literature on optimum product diversity with monopolistic competition enriched by firm heterogeneity in productivity levels in which the market can potentially misallocate resources not only within the monopolistically competitive sector but also between this sector and the rest of the economy. This allows us also to use comparative static analysis to evaluate how increases in the number of trading countries ('globalization'), in the dimension of each country ('market size'), in the state of development of the level of technology available within each country ('development'), and reductions in trade barriers ('international integration') affect the intensity of market distortions.

Following Melitz and Ottaviano (2008), we consider the case in which each country, indexed by $l = 1, ..., M$, is populated by L^l consumers, each endowed with one unit of labor. Preferences of consumers in l are defined over a continuum of differentiated varieties available in l that are indexed by $i \in \Omega^l$, and a homogeneous good indexed by 0. All consumers in l indexed ε share the same utility function given by

$$U^l = q_{0l}^{\varepsilon} + \alpha \int_{i\in\Omega^l} q_l^{\varepsilon}(i)\, di - \frac{1}{2}\gamma \int_{i\in\Omega^l} \left(q_l^{\varepsilon}(i)\right)^2 di - \frac{1}{2}\eta \left(\int_{i\in\Omega^l} q_l^{\varepsilon}(i)\, di\right)^2 \quad (1)$$

with positive demand parameters α, η and γ, the latter measuring an inverse measure of the 'love for variety' as it represents the degree of product differentiation, and the others measuring the preference for the differentiated varieties with respect to the homogeneous good. More precisely, α represents the intensity of preferences for the differentiated good relative to the homogeneous good and η represents the degree of nonseparability across the differentiated varieties. The initial individual endowment $\overline{q}_0^l$ of the homogeneous good is assumed to be large enough for its consumption to be strictly positive in each country at the market equilibrium and optimal solutions.

Labor is the only factor of production. It is employed for the production of the homogeneous good under constant returns to scale with unit labor requirement equal to one. It is also employed for the production of the differentiated varieties. The technology requires a preliminary R&D effort of $f^l > 0$ units of labor in country l to design a new variety and its production process, which is also characterized by constant returns to scale. The R&D effort cannot be recovered resulting in a sunk setup ('entry') cost. The R&D effort leads to the design of a new variety with certainty whereas the unit labor requirement c of the corresponding production process is uncertain, being randomly drawn from a continuous distribution with cumulative density

$$G^l(c) = \left(\frac{c}{c_M^l}\right)^k, \; c \in [0, c_M^l] \quad (2)$$

This corresponds to the empirically relevant case in which marginal productivity $1/c$ is Pareto distributed with shape parameter $k \geq 1$ over the support $[1/c_M^l, \infty)$ for country l. Hence, as k rises, density is skewed towards the upper bound of the support of $G^l(c)$. Notice that the scale parameter c_M^l determines the level of 'richness', defined as the measure of different unit labor requirements that can be drawn within a country. Larger c_M^l leads to a rise in heterogeneity along the richness dimension and to a reduction in the comparative advantage of country l as it becomes possible to draw also larger unit labor requirements than the original ones. The shape parameter k is an inverse measure of 'evenness', that is of the similarity between the probabilities of different draws of c to happen. When $k = 1$, the unit labor requirement distribution is uniform on $[0, c_M^l]$ with maximum evenness.[4] As k decreases, the unit labor requirement distribution becomes less concentrated at higher unit labor requirements close to c_M^l and evenness increases leading to a rise in heterogeneity along this dimension making low unit labor requirements more likely. Accordingly, more richness (larger c_M^l) comes with higher average unit labor requirement ('cost-increasing richness'), more evenness (smaller k) comes with lower average unit labor requirement ('cost-decreasing evenness').

Countries are allowed to potentially differ not only in their size L^l, but also in terms of their technology with country l having a comparative advantage (disadvantage) with respect to country $h = 1, ..., M$ in the differentiated good sector if c_M^l is smaller (larger) than c_M^h, and differences allowed also in the f^l units of labor required for the preliminary R&D effort in country l. Moreover, differentiated varieties are traded at an iceberg trade cost that can differ both across and within countries. Specifically, we assume that τ^{lh} units of the good have to be produced and shipped from the production country l to sell one unit in the destination country h, and $\rho^{lh} \equiv \left(\tau^{lh}\right)^{-k} \in (0, 1]$ measures the 'freeness of trade' for exports from l to h. As within country trade may not be costless due to domestic trade costs τ^{ll}, we assume that $\rho^{ll} \equiv \left(\tau^{ll}\right)^{-k} \in (0, 1]$. The homogenous good is, instead, assumed to be freely traded.

Given that we allow for a potentially asymmetric multi-country framework, we will try to understand how the differences in terms of the dimension of the domestic market ('home market effect'), of technology and comparative advantage ('development') and market access due to trade barriers ('geography') and number of trading countries ('globalization') affect the intensity of market distortions.

2.1 Market equilibrium and first best optimum

We present now the free market equilibrium and then the allocation of the resources determined by a first best global planner maximizing global welfare constrained only by technology and endowments.

2.1.1 The market outcome

In the decentralized equilibrium, within each country, consumers maximize utility under their budget constraints, firms maximize profits given their technological constraints, and all markets clear. It is assumed that the labor market as well as the market of the homogeneous good are perfectly competitive. The homogeneous good is chosen as numeraire and this, together with its production technology and the fact that it is freely traded, implies that the wage of workers equals one in all countries. The market of differentiated varieties is,

[4]When k tends to infinity, the distribution becomes degenerate at c_M^l with all draws delivering a unit labor requirement c_M^l with probability one.

instead, monopolistically competitive with a one-to-one relation between firms and varieties and consumers buy both locally produced and imported varieties.

The first order conditions for utility maximization give individual inverse demand for variety i in l as

$$p_l\left(i\right) = \alpha - \gamma q_l^{\varepsilon}\left(i\right) - \eta Q_l^{\varepsilon} \tag{3}$$

whenever $q_l^{\varepsilon}\left(i\right) > 0$, with $p_l\left(i\right)$ denoting the price of variety i in l, and $Q_l^{\varepsilon} = \int_{i\in\Omega^l} q_l^{\varepsilon}\left(i\right) di$ representing the individual aggregate demand in l of differentiated varieties.

Aggregate demand for each variety i in country l can be derived from (3) as

$$q_l\left(i\right) \equiv L^l q_l^{\varepsilon}\left(i\right) = \frac{\alpha L^l}{\eta N_l + \gamma} - \frac{L^l}{\gamma} p_l\left(i\right) + \frac{\eta N_l}{\eta N_l + \gamma}\frac{L^l}{\gamma}\bar{p}_l,\ \forall i \in \Omega_*^l \tag{4}$$

where the set Ω_*^l is the largest subset of Ω^l such that demand in l is positive, N_l is the measure ('number') of varieties in Ω_*^l, that is given by the sum of domestic and imported varieties sold in l, and $\bar{p}_l = \left(1/N_l\right)\int_{i\in\Omega_*^l} p_l\left(i\right) di$ is their average price. Variety i belongs to this set when

$$p_l\left(i\right) \leq \frac{1}{\eta N_l + \gamma}\left(\gamma\alpha + \eta N_l \bar{p}_l\right) \equiv p_{\max}^l \tag{5}$$

where $p_{\max}^l \leq \alpha$ represents the price at which demand for a variety in l is driven to zero.[5]

The aggregate consumption in country h of a variety that is produced in l with productivity $1/c$ is

$$q_{lh}(c) = q_{lh}^{\varepsilon}(c) L^h$$

where $q_{lh}^{\varepsilon}(c)$ is the individual consumption of that variety. Note that when two suffixes are used, the first one denotes the production country while the second one the country in which the good is consumed.

The quantity $q_{lh}(c)$ is obtained by

$$\underset{q_{lh}(c)}{max}\pi_{lh}(c) = \left[p_{lh}(c) - \tau^{lh}c\right] q_{lh}(c)$$

subject to its aggregate inverse demand in h, that is

$$p_{lh}(c) = \alpha - \frac{\gamma}{L^h} q_{lh}(c) - \frac{\eta}{L^h} Q_h^m$$

where 'm' labels equilibrium variables and $Q_h^m \equiv \sum_{j=1}^{M}\left(N_E^j \int_0^{c_M^j} q_{jh}(c) dG^j(c)\right)$ represents the total supply of differentiated varieties available in the market in country h with N_E^j denoting the number of firms entering in country j.

[5] Melitz and Ottaviano (2008) show that rewriting the indirect utility function in terms of average price and price variance reveals that it decreases with average prices $\bar{p}$, but rises with the variance of prices σ_p^2 (holding $\bar{p}$ constant), as consumers then re-optimize their purchases by shifting expenditures towards lower priced varieties as well as the numeraire good. Note also that the demand system exhibits 'love of variety': holding the distribution of prices constant (namely holding the mean $\bar{p}$ and variance σ_p^2 of prices constant), utility rises with product variety N.

The corresponding first order conditions for profit maximization are satisfied by

$$q_{lh}^{m}(c)=\begin{cases}\frac{L^{h}}{2\gamma}\tau^{lh}\left(c_{lh}^{m}-c\right) & \text{if } c\leq c_{lh}^{m}\equiv\frac{p_{\max}^{h}}{\tau^{lh}}=\frac{1}{\tau^{lh}}\left(\alpha-\frac{\eta}{L^{h}}Q_{h}^{m}\right)\\ 0 & \text{if } c>c_{lh}^{m}\end{cases}\tag{6}$$

Expression (6) defines a cutoff rule as only entrants in country l that are productive enough ($c\leq c_{lh}^{m}$) sell their variety in country h. For them, the price set in h that corresponds to the profit-maximizing quantity $q_{lh}^{m}(c)$ is $p_{lh}^{m}(c)=\tau^{lh}\left(c_{lh}^{m}+c\right)/2$, implying a markup on sales in h equal to $\mu_{lh}^{m}(c)=p_{lh}^{m}(c)-\tau^{lh}c=\tau^{lh}\left(c_{lh}^{m}-c\right)/2$ and maximized profits are given by

$$\pi_{lh}(c)=\frac{L^{h}}{4\gamma}\left(\tau^{lh}\right)^{2}\left(c_{lh}^{m}-c\right)^{2}\tag{7}$$

Notice that sales in the domestic market can be obtained considering $h=l$. In this case, the domestic cutoff c_{ll}^{m} is also denoted by $c_{D^{l}}^{m}$.

Expression (6) implies that for all marginal firms selling in h we have $\tau^{lh}c_{lh}^{m}=\tau^{hh}c_{D^{h}}^{m}=p_{\max}^{h}$. Hence, the relationship among cutoffs for firms selling in h is such that

$$c_{lh}^{m}=\frac{\tau^{hh}}{\tau^{lh}}c_{D^{h}}^{m}\qquad\forall l,h=1,....,M\tag{8}$$

Due to free entry and exit condition, in equilibrium expected profit of firms producing in l is exactly offset by the sunk entry cost f^{l} and, therefore,

$$\sum_{h=1}^{M}\left\{\int_{0}^{c_{lh}^{m}}\frac{L^{h}}{4\gamma}\left(\tau^{lh}\right)^{2}\left(c_{lh}^{m}-c\right)^{2}dG^{l}(c)\right\}=f^{l}\tag{9}$$

Given (2), (7) and (8), the 'free entry condition' can be rewritten as

$$\sum_{h=1}^{M}L^{h}\rho^{lh}\ \left(\tau^{hh}c_{hh}^{m}\right)^{k+2}=2\gamma\left(k+2\right)\left(k+1\right)\left(c_{M}^{l}\right)^{k}f^{l}\tag{10}$$

This expression, together with the analogous expressions holding for all other $M-1$ countries, yields a system of M equations that, making use of Cramer's rule, can be solved for the M equilibrium domestic cutoffs given by

$$c_{D^{l}}^{m}=\frac{1}{\tau^{ll}}\left\{\frac{2\gamma\left(k+1\right)\left(k+2\right)}{L^{l}}\frac{\sum_{h=1}^{M}\left[f^{h}\left(c_{M}^{h}\right)^{k}\left|C_{hl}\right|\right]}{\left|P\right|}\right\}^{\frac{1}{k+2}}\qquad\forall l=1,...,M\tag{11}$$

where $|P|$ is the determinant of the trade freeness matrix

$$P=\begin{pmatrix}\rho^{11} & \rho^{12} & \cdots & \rho^{1M}\\ \rho^{21} & \rho^{22} & \cdots & \rho^{2M}\\ \cdots & \cdots & \ddots & \cdots\\ \rho^{M1} & \rho^{M2} & \cdots & \rho^{MM}\end{pmatrix}$$

and $|C_{hl}|$ is the cofactor of its ρ^{hl} element.[6]

In general, the total number of varieties consumed in country l is given by the sum of domestic and of imported varieties from all trading partners. Specifically, the number of varieties produced in h and consumed in l is derived from the selection conditions and is given by

$$N_{hl} = N_E^h G^h(c_{hl}) \tag{12}$$

The total number of varieties consumed in country l (N_l) is

$$N_l = \sum_{h=1}^{M} N_{hl} \tag{13}$$

Given (12), (2) and (8), product variety N_l in (13) can be rewritten in the case of the market equilibrium as follows

$$N_l^m = \left(\tau^{ll} c_{D^l}^m\right)^k \sum_{h=1}^{M} \rho^{hl} N_E^h \left(\frac{1}{c_M^h}\right)^k \tag{14}$$

Moreover, as the average price in country l is

$$\bar{p}_l^m = \frac{2k+1}{2(k+1)} \tau^{ll} c_{D^l}^m, \tag{15}$$

it can be combined with the price threshold in (5) to express the number of varieties sold in l in the market equilibrium as

$$N_l^m = \frac{2\gamma(k+1)}{\eta} \frac{\left(\alpha - \tau^{ll} c_{D^l}^m\right)}{\tau^{ll} c_{D^l}^m} \tag{16}$$

Hence, N_l^m in (16) can be equalized to N_l^m in (14) to get

$$\sum_{h=1}^{M} \left(\rho^{hl} N_E^h \frac{1}{\left(c_M^h\right)^k}\right) = \frac{2\gamma(k+1)}{\eta} \frac{\left(\alpha - \tau^{ll} c_{D^l}^m\right)}{\left(\tau^{ll} c_{D^l}^m\right)^{k+1}}$$

This gives a system of M linear equations that using Cramer's rule can be solved for the number of entrants in the M countries

$$N_E^{lm} = \frac{2\gamma(k+1)\left(c_M^l\right)^k \sum_{h=1}^{M} \left[\frac{\left(\alpha - \tau^{hh} c_{D^h}^m\right)}{\left(\tau^{hh} c_{D^h}^m\right)^{k+1}} |C_{lh}|\right]}{\eta |P|} \quad \forall l = 1, ..., M \tag{17}$$

which, as the domestic cutoffs in (11) and the cutoffs for exporting firms derived making use of (8), is affected by parameters determining the technology of production available in each country and by comparative advantage, as well as by the market size of each country, the number of trading countries and the level of trade costs.[7]

[6] Notice that $\sum_{h=1}^{M} \left[f^h \left(c_M^h\right)^k |C_{hl}|\right] > 0 \quad \forall l = 1, ..., M$ has to be satisfied also to have a positive production for domestic consumers in all countries for the market solution.

[7] Notice that in the example of two trading countries with no internal trade costs and common sunk entry cost f that will be presented later on in Section 5.1.1, the conditions that have to be satisfied to have positive values for both N_E^{1m} and N_E^{2m}, imply that $c_{12}^m < c_{D^1}^m$ and $c_{21}^m < c_{D^2}^m$, so that only a subset of relatively more productive firms export in the market.

2.1.2 The first best optimum

Let us now consider the problem faced by a social planner who can allocate the resources available within each country to design a specific number of new varieties, with the corresponding unit labor requirement c uncertain, and to the production of only a specific subset of these potentially available varieties together with the homogenous good.

The quasi-linearity of (1) implies transferable utility. Thus, social welfare may be expressed as the sum of all consumers' utilities in all M countries. This implies that the first best ('unconstrained') planner chooses the number of varieties and their output levels so as to maximize the total social welfare function given by the sum of aggregate utility of all countries obtained for each country multiplying individual utility in (1) for the number of consumers L^l. This maximization is subject to the resource constraints, the varieties' production functions and the stochastic 'innovation production function' (i.e. the mechanism that determines each variety's unit labor requirement as a random draw from $G^l(c)$ after f^l units of labor have been allocated to R&D) that hold for all countries.

In other words, the planner maximizes the utility derived by all consumers living in the M countries from the consumption of the homogeneous good and of the differentiated varieties that are both those domestically produced and those imported. Specifically, the utility of all L^l workers living in country l is given by

$$
\begin{aligned}
W^l(L^l) &= q_{0l}^{\varepsilon} L^l + \alpha \sum_{h=1}^{M} \left[N_E^h \int_0^{c_M^h} q_{hl}^{\varepsilon}(c) L^l dG^h(c) \right] + \\
&\quad - \frac{\gamma}{2} \frac{1}{L^l} \sum_{h=1}^{M} \left[N_E^h \int_0^{c_M^h} \left[q_{hl}^{\varepsilon}(c) L^l \right]^2 dG^h(c) \right] + \\
&\quad - \frac{\eta}{2} \frac{1}{L^l} \left\{ \sum_{h=1}^{M} \left[N_E^h \int_0^{c_M^h} q_{hl}^{\varepsilon}(c) L^l dG^h(c) \right] \right\}^2 ,
\end{aligned}
\tag{18}
$$

Thus, the planner maximizes the 'global' utility which is given by

$$
\begin{aligned}
W &= \sum_{l=1}^{M} W^l(L^l) = \sum_{l=1}^{M} q_{0l}^{\varepsilon} L^l + \alpha \sum_{l=1}^{M} \left\{ \sum_{h=1}^{M} \left[N_E^h \int_0^{c_M^h} q_{hl}^{\varepsilon}(c) L^l dG^h(c) \right] \right\} \\
&\quad - \frac{\gamma}{2} \sum_{l=1}^{M} \left\{ \frac{1}{L^l} \sum_{h=1}^{M} \left[N_E^h \int_0^{c_M^h} \left[q_{hl}^{\varepsilon}(c) L^l \right]^2 dG^h(c) \right] \right\} + \\
&\quad - \frac{\eta}{2} \sum_{l=1}^{M} \left\{ \frac{1}{L^l} \left[\sum_{h=1}^{M} \left(N_E^h \int_0^{c_M^h} q_{hl}^{\varepsilon}(c) L^l dG^h(c) \right) \right]^2 \right\}
\end{aligned}
\tag{19}
$$

with respect to consumption of the homogeneous goods in each country, the quantities produced for consumption in all countries by all firms in the differentiated good sector and the number of entrants in all countries, subject to the resource constraints, the firm production functions and the stochastic 'variety production function' expressed in the selection conditions (12) that hold for all M countries.

From the resource constraint of country l, the supply of the homogenous good in country l is given by

$$Q_0^l = \overline{q}_0^l L^l + L^l - f^l N_E^l - N_E^l \sum_{h=1}^{M} \left[\int_0^{c_M^l} \tau^{lh} c q_{lh}^{\varepsilon}(c) L^h dG^l(c) \right] \tag{20}$$

This implies that the supply of the homogeneous good in country l (Q_0^l) is given by the sum of total local endowments of the good ($\overline{q}_0^l L^l$) and its local production obtained employing labour units locally available (L^l) once subtracted those employed, respectively, to undertake the local R&D investment ($f^l N_E^l$) and to produce the differentiated varieties sold in the local and in the foreign countries ($N_E^l \sum_{h=1}^{M} \left[\int_0^{c_M^l} \tau^{lh} c q_{lh}^{\varepsilon}(c) L^h dG^l(c) \right]$).

The condition that the global supply of the homogenous good is equal to its global demand requires that

$$\sum_{l=1}^{M} Q_0^l = \sum_{l=1}^{M} q_{0l}^{\varepsilon} L^l \tag{21}$$

which, making use of (20), implies

$$\sum_{l=1}^{M} q_{0l}^{\varepsilon} L^l = \sum_{l=1}^{M} \left\{ \overline{q}_0^l L^l + L^l - f^l N_E^l - N_E^l \sum_{h=1}^{M} \left[\int_0^{c_M^l} \tau^{lh} c q_{lh}^{\varepsilon}(c) L^h dG^l(c) \right] \right\} \tag{22}$$

Substituting (22) into (19), and making use of $q_{lh}(c) = q_{lh}^{\varepsilon}(c) L^h$, the planner maximizes

$$\begin{aligned} W &= \sum_{l=1}^{M} W^l(L^l) = \qquad (23) \\ &= \sum_{l=1}^{M} \left\{ \overline{q}_0^l L^l + L^l - f^l N_E^l - N_E^l \sum_{h=1}^{M} \left[\int_0^{c_M^l} \tau^{lh} c q_{lh}(c) dG^l(c) \right] \right\} + \\ &\quad + \alpha \sum_{l=1}^{M} \left\{ \sum_{h=1}^{M} \left[N_E^h \int_0^{c_M^h} q_{hl}(c) dG^h(c) \right] \right\} + \\ &\quad - \frac{\gamma}{2} \sum_{l=1}^{M} \left\{ \frac{1}{L^l} \sum_{h=1}^{M} \left[N_E^h \int_0^{c_M^h} \left[q_{hl}(c) \right]^2 dG^h(c) \right] \right\} + \\ &\quad - \frac{\eta}{2} \sum_{l=1}^{M} \left\{ \frac{1}{L^l} \left[\sum_{h=1}^{M} \left(N_E^h \int_0^{c_M^h} q_{hl}(c) dG^h(c) \right) \right]^2 \right\} \end{aligned}$$

with respect to the quantities produced for all markets by all firms and the number of entrants in all countries, that is with respect to $q_{ij}(c) \forall c$ and N_E^i with $i, j = 1, ..., M$.[8]

[8] More precisely, in addition to the first order conditions written with respect to $q_{ij}(c) \forall c$ and N_E^i, the first best planner should also consider the first order condition with respect to the cutoffs. However, making use of the Pareto distribution in (2), it can be shown that the first order conditions with respect to the cutoffs becomes redundant because they identify the same cutoffs as those derived from the first order condition with respect to $q_{ij}(c)$ given in expression (24).

The first order conditions obtained for the quantity of a variety produced with productivity $1/c$ in country i and consumed in j is

$$\frac{\partial W}{\partial q_{ij}(c)} = 0 \quad \forall c$$

$$\left\{ \left(\alpha - \tau^{ij} c\right) N_E^i - \frac{\gamma}{L^j} N_E^i q_{ij}(c) - \frac{\eta}{L^j} \left[\sum_{h=1}^{M} \left(N_E^h \int_0^{c_M^h} q_{hj}(c) dG^h(c) \right) \right] N_E^i \right\} dG^i(c) = 0 \quad \forall c$$

that making use of the total quantity consumed in country j, that is $Q_j^o \equiv \sum_{h=1}^{M} \left(N_E^h \int_0^{c_M^h} q_{hj}(c) dG^h(c) \right)$, can be rewritten as

$$\alpha - \tau^{ij} c - \gamma \frac{1}{L^j} q_{ij}(c) - \eta \frac{1}{L^j} Q_j^o = 0$$

Hence, for quantities produced in l to be consumed in h the planner follows the rule

$$q_{lh}^o(c) = \begin{cases} \frac{L^h}{\gamma} \tau^{lh} \left(c_{lh}^o - c\right) & c \leq c_{lh}^o \text{ with } c_{lh}^o \equiv \frac{1}{\tau^{lh}} \left(\alpha - \eta \frac{1}{L^h} Q_h^o\right) \\ 0 & c > c_{lh}^o \end{cases} \tag{24}$$

where 'o' labels first best optimum variables. Results in (24) reveal that, just like the market, also the planner follows M cutoff rules, one for each country, allowing only for the production in country l for consumption in country h of those varieties whose unit labor requirements are low enough with $c \leq c_{lh}^o$. It can be readily verified from (24) that the first best output levels would clear the market in the decentralized scenario only if each producer in l priced the quantities sold in h at its own marginal cost, that is only if $p_{lh}(c) = \tau^{lh} c$. Indeed, from the inverse demand function, we know that $p_{lh}(c) = \alpha - \frac{\gamma}{L^h} q_{lh}(c) - \frac{\eta}{L^h} Q_h^o = \tau^{lh} c_{lh}^o - \frac{\gamma}{L^h} q_{lh}(c)$ that, together with $p_{lh}(c) = \tau^{lh} c$, implies $q_{lh}(c) = \frac{L^h}{\gamma} \tau^{lh} \left(c_{lh}^o - c\right)$.

The definitions of the cutoffs in (24) imply that the relationships between the optimal cutoffs for all marginal firms selling in h are such that

$$c_{lh}^o = \frac{\tau^{hh}}{\tau^{lh}} c_{D^h}^o \qquad \forall l, h = 1, ..., M \tag{25}$$

which are the same as those for the market equilibrium in (8), even though the cutoffs are different. Therefore, we can state that

Proposition 1 ***(Principle of no discrimination)*** *There is no discrimination of the first best planner with respect to the market equilibrium in favour of domestic or imported varieties.*

The cutoffs of the unconstrained planner are derived in Appendix A from the first order conditions for the maximization of (23) with respect to the number of entrants in each country i (that is from $\frac{\partial W}{\partial N_E^i} = 0$) and are given by

$$c_{D^l}^o = \frac{1}{\tau^{ll}} \left\{ \frac{\gamma (k+1)(k+2)}{L^l} \frac{\sum_{h=1}^{M} \left[f^h \left(c_M^h\right)^k |C_{hl}| \right]}{|P|} \right\}^{\frac{1}{k+2}} \qquad \forall l = 1, ..., M \tag{26}$$

Notice that $\sum_{h=1}^{M}\left[f^{h}\left(c_{M}^{h}\right)^{k}\left|C_{hl}\right|\right]>0 \quad \forall l=1,...,M$ has to be satisfied also to have a positive production for domestic consumers in all countries for the first best solution.

In Appendix A we derive also the optimal number of entrants in each country as a function of the cutoffs, that is

$$N_{E}^{lo}=\frac{\gamma\left(k+1\right)\left(c_{M}^{l}\right)^{k}\sum_{h=1}^{M}\left[\frac{\left(\alpha-\tau^{hh}c_{Dh}^{o}\right)}{\left(\tau^{hh}c_{Dh}^{o}\right)^{k+1}}\left|C_{lh}\right|\right]}{\eta\left|P\right|} \qquad \forall l=1,...,M \tag{27}$$

Finally, making use of (12), for each country l the optimal number of domestic varieties is $N_{Dl}^{o}=G^{l}(c_{Dl}^{o})N_{E}^{lo}$ and the optimal number of exported varieties to country h is $N_{X^{lh}}^{o}=G^{l}(c_{lh}^{o})N_{E}^{lo}$.[9]

3 Equilibrium vs. optimum

There are different dimensions along which the efficiency of the market outcome can be evaluated, such as: the (conditional) cost distribution of firms producing and exporting the different varieties as dictated by the cutoff c_{lh}^{m} and the number of varieties available for consumption in each country (product variety). In turn, the cost distributions determine the efficiency of the corresponding distributions of firm sizes, domestic production and exports. Moreover, it contributes in determining the efficiency of the overall dimension of the differentiated goods sector in the market of each country (i.e. total supply of differentiated varieties), the mass (number) of firms that undertake the initial R&D effort entering in each market, and the number of firms that survive producing the differentiated varieties for the domestic and the foreign consumers.

At the origins of the inefficiencies of the market equilibrium is the fact that consumers in each country value variety and monopolistically competitive firms heterogeneous in their productivity levels are price setters. Setting their price for each locality with a markup over their marginal cost, firms reduce the average quantity that consumers buy within each of these locations with respect to the average quantity that they would have purchased if differentiated varieties had been sold at their marginal cost.[10]

As shown in the last part of Appendix A, most of the evaluations in terms of efficiency of the market outcome can be obtained for the general case of asymmetric countries. In this Section, we use the results derived in Appendix A to compare the equilibrium outcome with the optimal unconstrained solution along different dimensions such as: selection processes that identify firms that survive producing for domestic and foreign consumers; individual and overall average size of firms in the differentiated good sector; product variety available

[9] Notice that in the case of two countries with no internal trade costs and common sunk entry cost f considered in Section 5.1.1, the conditions that have to be satisfied to have positive values for both N_{E}^{1o} and N_{E}^{2o}, imply that $c_{12}^{o}<c_{D^1}^{o}$ and $c_{21}^{o}<c_{D^2}^{o}$, so that in each country only a subset of relatively more productive firms export in the optimal solution.

[10] The tradeoffs the first best planner faces when firms are monopolistically competitive and heterogeneous are discussed in Nocco, Ottaviano and Salto (2014) for the closed economy. Specifically, see expression (8) at page 306 in Nocco, Ottaviano and Salto (2014) and the subsequent comments.

for consumption and entry of innovating firms in each country. These results are first presented in the case of M symmetric trading countries facing no internal trade costs and, then, in the case in which countries differ in terms of the size of their domestic market (L^l), technology that defines their comparative advantage (c_M^l) and the f^l units of labor required for the preliminary R&D effort, and, finally, trade barriers (τ^{lh}) that identify their accessibility in each country/locality.

3.1 The case of symmetric countries

Let us first consider the case of M perfectly symmetric trading countries without internal trade costs (that is with $\tau^{ll} = 1$ implying $\rho^{ll} = 1 \forall l = 1, ..., M$) to evaluate how the intensity of market distortions is affected by different phenomena such as: a process of 'globalization' that increases the number of trading countries M; 'demographic growth' and processes that increase the size of each country L allowing to evaluate 'home market effects'; 'international economic integration' that reduces trade barriers increasing the freeness of trade ρ; improvements in the technology available within each country ('development') that reduce the highest cost draw c_M or the entry/innovating cost f, or, eventually, increase the chances of having a high productivity draw reducing the shape parameter k. Recall that while a smaller c_M reduces heterogeneity along the cost-richness dimension implying lower average unit labor requirement, a smaller k increases heterogeneity along the evenness dimension reducing the average unit labor requirement.

Notice that all the results discussed in the case of symmetric countries are consistent with those of a closed economy, which can be obtained with $M = 1$ and/or $\rho = 0$, presented in Nocco, Ottaviano and Salto (2013, 2014).

3.1.1 Selection and firm size

In the case of perfectly symmetric countries,[11] $\tau^{lh} = \tau$ and $\rho^{lh} = \rho \forall l, h = 1, ..., M$ imply $|P| = [1 + (M-1)\rho](1-\rho)^{M-1}$ and $\sum_{h=1}^{M} [|C_{hl}|] = (1-\rho)^{M-1}$, and therefore the market equilibrium domestic cutoffs in (11) can be rewritten as

$$c_D^m = \left\{ \frac{2\gamma (k+1)(k+2) f c_M^k}{[1 + (M-1)\rho] L} \right\}^{\frac{1}{k+2}} \tag{28}$$

and the first best domestic cutoffs in (26) as

$$c_D^o = \left\{ \frac{\gamma (k+1)(k+2) f c_M^k}{[1 + (M-1)\rho] L} \right\}^{\frac{1}{k+2}} \tag{29}$$

Comparing the equilibrium domestic cutoff in (28) with the optimal one in (29) reveals that $c_D^m = 2^{1/(k+2)} c_D^o$, which implies $c_D^o < c_D^m$. Accordingly, varieties with $c \in [c_D^o, c_D^m]$ should not be supplied in the domestic market. Moreover, differences in the strength of selection translate also into the export status. In particular, comparing expressions (28) with (29) together with (8) and (25),

[11] Recall that in all this subsection, we consider the case in which there are no domestic trade costs.

reveals that the cutoffs for exporting firms (denoted as c_X when countries are symmetric) are such that $c_X^m = 2^{1/(k+2)} c_X^o$, which implies $c_X^o < c_X^m$. Accordingly, varieties with $c \in [c_X^o, c_X^m]$ should not be exported or, equivalently, looking this phenomenon from the other side of the coin, should not be imported by other countries.

The intuition behind the causes at the origin of these inefficiencies is that, in the market equilibrium, firms are price setters and a firm with marginal cost c producing in l sets the price for its variety sold in h with a markup over its marginal cost that is equal to $\mu_{lh}^m(c) = \tau^{lh} (c_{lh}^m - c)/2$. This implies that the prices of differentiated varieties are inefficiently high within each country with respect to the price of the numeraire good and, consequently, this biases the consumption in favor of the homogeneous good softening the selection process among the producers of the differentiated varieties.

The results on optimal selection processes have also implications in terms of optimality of the firm size distribution of domestic and exporting firms. Adapting to the specific case of M symmetric countries the general results derived in the final part of Appendix A for asymmetric countries, we find that, with respect to the optimum, the market equilibrium undersupplies varieties produced with marginal cost $c \in [0, \left(2 - 2^{1/(k+2)}\right) c_D^o)$, and oversupplies varieties with marginal cost $c \in (\left(2 - 2^{1/(k+2)}\right) c_D^o, c_D^m]$ in the domestic countries. Moreover, in the market equilibrium firms export a quantity that is smaller than optimal of varieties produced with marginal cost $c \in [0, \left(2 - 2^{1/(k+2)}\right) c_X^o)$, and larger than optimal of those produced with marginal cost $c \in (\left(2 - 2^{1/(k+2)}\right) c_X^o, c_X^m]$.

The intuition behind this can be explained by the fact that the markup $\mu_{lh}^m(c)$ is a decreasing function of c and this implies that more productive firms do not pass on their entire cost advantage to consumers as they absorb part of it in the markup. As a result, the price ratio of less to more productive firms is smaller than their cost ratio and thus the quantities sold by less productive firms are too large from an efficiency point of view relative to those sold by more productive firms.

Moreover, the cutoff ranking implies that in the market not only the average production of firms in each country for any of their destination country is inefficiently low (as $\overline{q}_D^m < \overline{q}_D^o$ and $\overline{q}_X^m < \overline{q}_X^o$), resulting in an inefficiently low average firm size, but also that the average quantity supplied by all, domestic and foreign, firms within each country is smaller than optimal (as $\bar{q}^m < \bar{q}^o$).[12]

The intuition behind this is that the lower cutoff implied by marginal cost pricing followed by the first best planner makes firms on average larger in the optimum than in the market equilibrium.

Finally, the implications of the results and comparative statics in Appendix A allow us to state that the percentage gap in the cutoffs, in average firm size and in the average quantity supplied within each country and the within sector misallocation that materializes as the overprovision of high cost varieties

[12] Again, these results are obtained adapting the more general ones for asymmetric countries in (69), (70) and (71) presented in the last part of Appendix A. Notice that when countries are symmetric we avoid in the variable the suffix that denotes the country (i.e. we use $\bar{q}^m$ instead of $\bar{q}_h^m$).

and underprovision of low cost varieties are not affected by any of the following processes: 'globalization' that increases the number of trading countries M; 'demographic growth' that increases the home market size L of each country; 'international integration' that reduces trade barriers increasing ρ; 'development' that improves the technology available within each country reducing c_M and/or the entry cost f. Instead, more heterogeneity due to more cost-decreasing evenness, that is a smaller k that implies a higher chance of having high productivity draws: leads to a larger percentage gap in the cutoffs between the market equilibrium and the optimum; makes the overprovision of varieties relatively more likely than its underprovision in the market equilibrium and decreases the percentage gap in the average production of firms in each country for their domestic and foreign markets and in the average quantity supplied by all firms within each country.

3.1.2 Size of the differentiated sector, product variety and entry

Adapting the general results in (72) and (73) in Appendix A, we find that, also in the specific case of M symmetric countries, $c_D^o < c_D^m$ implies that the market misallocate resources between sectors as the total supply of differentiated varieties is smaller than optimal in each country (that is, $N^m \overline{q}^m < N^o \overline{q}^o$). In this case, the percentage gap in the total output of differentiated varieties between the market equilibrium and the optimum is

$$\frac{N^o \overline{q}^o - N^m \overline{q}^m}{N^o \overline{q}^o} = \frac{c_D^m - c_D^o}{c_D^o} \frac{c_D^o}{\alpha - c_D^o}$$

where $(c_D^m - c_D^o)/c_D^o$ is only affected by k, which, together with γ, L, M, f and c_M, affects also $c_D^o / (\alpha - c_D^o)$.

Specifically, as larger values of the market size L, of the number of trading countries M and of the freeness of trade ρ, and smaller values of γ, f and c_M decrease $c_D^o / (\alpha - c_D^o)$, they imply smaller values for the ratio $\left(N^o \overline{q}^o - N^m \overline{q}^m\right) / N^o \overline{q}^o$. Moreover, we find that there can exist a threshold value of k, k^*,[13] such that more cost-decreasing evenness decreases the gap in the total output of the differentiated varieties between the market equilibrium and the optimum if $k > k^*$ and, viceversa, it increases the gap if $k < k^*$. The threshold value k^* increases with L, M, ρ and c_M, and decreases with γ and f. However, if L, M, ρ and c_M are sufficiently small and/or γ and f are sufficiently large, more cost-decreasing evenness does always decrease the gap.

Consequently, we can state what follows:

Proposition 2 *With M symmetric countries, the total supply of differentiated varieties in the market equilibrium is smaller than optimal in all locations. This gap is smaller (larger): in larger overall economies due to larger (smaller) home*

[13] More precisely, more cost-decreasing evenness increases the gap if $\left[2^{\frac{1}{k+2}} \ln 2 - (2k+3)\left(2^{\frac{1}{k+2}} - 1\right) / (k+1)\right] / \left(2^{\frac{1}{k+2}} - 1\right) + \ln\left[(k+1)(k+2)\right] > \ln\left\{\left[1 + (M-1)\rho\right] L c_M^2 / (\gamma f)\right\}$; viceversa, it decreases the gap if the opposite inequality sign holds. The threshold k^* corresponds to the value at which the function $\left[2^{\frac{1}{k+2}} \ln 2 - (2k+3)\left(2^{\frac{1}{k+2}} - 1\right) / (k+1)\right] / \left(2^{\frac{1}{k+2}} - 1\right) + \ln\left((k+1)(k+2)\right)$, which is increasing in k, crosses $\ln\left\{\left[1 + (M-1)\rho\right] L c_M^2 / (\gamma f)\right\}$, and it exists only if L, M, ρ and c_M are sufficiently large and/or γ and f are sufficiently small.

markets, more (less) integrated countries or to a larger (smaller) number of trading countries, in more (less) developed economies that benefit from better (worse) available technologies, lower (larger) entry costs and higher (smaller) chances of having low cost firms, and in countries in which varieties are closer (farther) substitutes. More cost-decreasing evenness decreases the gap if $k > k^*$ and, viceversa, it increases the gap if $k < k^*$.

Hence, increases in the overall size of the economy due to 'globalization', 'demographic growth' and 'economic integration' and processes of 'development' that reduce c_M or f, decrease the percentage gap in the total output of the differentiated varieties between the market equilibrium and the optimum.

Turning to the mass (number) of varieties supplied respectively in the domestic and in the $M-1$ foreign markets, they are, respectively, given by

$$N_D^m = \frac{2\gamma(k+1)}{\eta[1+(M-1)\rho]}\frac{(\alpha - c_D^m)}{c_D^m} \quad \text{and} \quad N_X^m = \rho\frac{2\gamma(k+1)}{\eta[1+(M-1)\rho]}\frac{(\alpha - c_D^m)}{c_D^m} \tag{30}$$

while their optimal values correspond to

$$N_D^o = \frac{\gamma(k+1)}{\eta[1+(M-1)\rho]}\frac{(\alpha - c_D^o)}{c_D^o} \quad \text{and} \quad N_X^o = \rho\frac{\gamma(k+1)}{\eta[1+(M-1)\rho]}\frac{(\alpha - c_D^o)}{c_D^o} \tag{31}$$

Comparing the number of domestic and exported varieties in (30) and (31), and overall product variety available within each country from (16) and (67), we find that $c_D^m = 2^{1/(k+2)}c_D^o$ implies $N_D^m > N_D^o$, $N_X^m > N_X^o$ and $N^m > N^o$ as long as

$$\alpha > \alpha_1 \equiv \frac{1}{2^{\frac{k+1}{k+2}}-1}c_D^o = \frac{1}{2^{\frac{k+1}{k+2}}-1}\left\{\frac{\gamma(k+1)(k+2)fc_M^k}{[1+(M-1)\rho]L}\right\}^{\frac{1}{k+2}} \tag{32}$$

which is the case when α as well as L, M and ρ are large and when γ, c_M, f and k are small. On the contrary, $N_D^m < N_D^o$, $N_X^m < N_X^o$ and $N^m < N^o$ when $\alpha < \alpha_1$.

Turning to entry, the equilibrium number of entrants in (17) can be rewritten in the case of symmetric countries as

$$N_E^m = \frac{2\gamma(k+1)c_M^k}{\eta[1+(M-1)\rho]}\frac{(\alpha - c_D^m)}{(c_D^m)^{k+1}}$$

while that for the first best planner in (27) as

$$N_E^o = \frac{\gamma(k+1)c_M^k}{\eta[1+(M-1)\rho]}\frac{(\alpha - c_D^o)}{(c_D^o)^{k+1}}$$

These can be used to show that $c_D^m = 2^{1/(k+2)}c_D^o$ implies $N_E^m > N_E^o$ as long as

$$\alpha > \alpha_2 \equiv \frac{2^{2/(k+2)}-1}{2^{1/(k+2)}-1}c_D^o = \frac{2^{2/(k+2)}-1}{2^{1/(k+2)}-1}\left\{\frac{\gamma(k+1)(k+2)fc_M^k}{[1+(M-1)\rho]L}\right\}^{\frac{1}{k+2}} > \alpha_1 \tag{33}$$

which is the case when α as well as L, M and ρ are large and when γ, c_M, f and k are small. On the contrary, $N_E^m < N_E^o$ when $\alpha < \alpha_2$.

In summary, we can state that

Proposition 3 *In the case of M symmetric countries, entry, overall product variety and the number of domestic and imported varieties are larger (smaller) in the market equilibrium than in the optimum when market size is large (small), the number of trading countries is large (small), their level of integration is high (low), varieties are close (far) substitutes, the sunk entry cost is small (large), the difference between the highest and the lowest possible cost draws is small (large), and the chances of having low cost firms are large (small).*

This has interesting implications for the impact of increases in the overall dimension of the economy driven by different channels such as: international integration processes that increases ρ, globalization processes that increase the number of trading countries and/or demographic growth or other processes that increase the home market size. Moreover, it has also interesting implications for the impact of economic development that reduce the highest cost draw c_M or the entry/innovating cost f, or, eventually, increase the chances of having a high productivity draw reducing the shape parameter k. Indeed, starting from sufficiently small levels of the overall size of the economy and/or levels of development of the technologies, increases in one or both of these levels (due to increases in ρ, L and M or to decreases in c_M, f and k) can switch the countries from a situation in which $\alpha < \alpha_1$ to one in which $\alpha > \alpha_1$, decreasing α_1. Then, this would cause the transition from a situation in which product variety, the number of domestic and exported varieties are inefficiently poor in each country ($N^m < N^o$, $N_D^m < N_D^o$ and $N_X^m < N_X^o$) to a situation in which they become inefficiently rich ($N^m > N^o$, $N_D^m > N_D^o$ and $N_X^m > N_X^o$).

As larger overall sizes of the economies and technology improvements reduce α_2, they may well cause the transition from a situation in which the resources devoted to develop new varieties are inefficiently small in each country ($N_E^m < N_E^o$) to a situation in which they are inefficiently large ($N_E^m > N_E^o$). Given that (32) and (33) imply $\alpha_1 < \alpha_2$, the market provides too little entry with too little variety in each country for $\alpha < \alpha_1$ and too much entry with too much variety in each country for $\alpha > \alpha_2$. For $\alpha_1 < \alpha < \alpha_2$ it provides, instead, too much variety and too little entry.

3.2 The case of asymmetric countries

Let us now turn to the case of M asymmetric countries to understand if the efficiency properties of the market change with respect to the case with M symmetric countries.[14]

3.2.1 Selection and firm size

The analysis in Appendix A reveals that also in the general case of M asymmetric countries, selection in the market equilibrium is weaker than optimal in each country: varieties with $c \in [c_{D^l}^o, c_{D^l}^m]$ should not be supplied in domestic markets, and varieties with $c \in [c_{lh}^o, c_{lh}^m]$ should not be exported by firms producing in l to be consumed in country h.

In turn, the cost distribution determines the efficiency of the firm size distribution and we find that resources are misallocated within the differentiated good

[14]For the analytical derivation of the results see the final part of Appendix A.

sector. Indeed, the results for the general case confirm those for symmetric countries as, with respect to the optimum, the market equilibrium undersupplies in each country h varieties produced in country l with low marginal cost, that is varieties produced with $c \in [0, \left(2 - 2^{1/(k+2)}\right) c^o_{lh})$, and oversupplies varieties with high marginal cost, that is varieties produced with $c \in (\left(2 - 2^{1/(k+2)}\right) c^o_{lh}, c^m_{lh}]$.

Finally, also in the general case of M asymmetric countries, the cutoff ranking $c^o_{lh} < c^m_{lh}$ dictates the same average output ranking with the average quantities produced in each country for each destination and the average supply of a variety in each country smaller than optimal in the market as $\overline{q}^m_{lh} < \overline{q}^o_{lh}$ and $\overline{q}^m_h < \overline{q}^o_h$.

Hence, summarizing the results discussed above for both the symmetric and asymmetric countries, we can state what follows:

Proposition 4 *In the market equilibrium with respect to the optimum: firm selection in the production status for domestic and foreign countries is weaker than optimal; high (low) cost firms oversupply (undersupply) their varieties in all countries they are serving; both the average production of firms in each country for any of their destination country and the average quantity supplied by all domestic and foreign firms within each country are smaller than optimal.*

Moreover, from comparative statics in Appendix A we find that in general for both the cases of symmetric and asymmetric countries:

Proposition 5 *The number of trading countries, the size of each economy, the technology available for the production of the modern differentiated goods as determined by the upper-bound cost and the cost of innovation, and trade barriers that affect the accessibility of other countries by exporting firms have no impact on the overprovision or on the underprovision of varieties in both domestic and foreign countries and on the percentage deviation of the market equilibrium from the optimum for domestic and export cutoffs, for the average supply of firms in each of their destination country and for the average quantity supplied by all firms within each country. However, more heterogeneity among firms due to more (less) cost-decreasing evenness that reduces (increases) k, increases (decreases) the percentage gaps in the cutoffs, makes the overprovision of high cost varieties relatively more (less) likely than its underprovision in the market equilibrium and decreases (increases) the percentage gap in average firm size and in the average quantity supplied by all firms within each country.*

3.2.2 Size of the differentiated sector, product variety and entry

The ranking of the domestic cutoffs $c^o_{D^h} < c^m_{D^h}$ is also responsible of the ranking of the overall size of the differentiated good sector that is smaller than optimal in the market in each country as $N^o_h \overline{q}^o_h > N^m_h \overline{q}^m_h$. Moreover, the ratio $(N^o_h \overline{q}^o_h - N^m_h \overline{q}^m_h)/N^o_h \overline{q}^o_h$ is affected by k as well as by the technology available in all countries for the production of the modern differentiated goods as determined by the upper-bound costs and the cost of innovation, and by the number of trading countries and their accessibility to other localities as defined by trade barriers, that is by all factors affecting the term $\sum_{h=1}^{M} \left[f^h \left(c^h_M\right)^k |C_{hl}| \right] / |P|$.

Turning to product variety, N_h^o in (67) can be used to evaluate the number of product supplied in the market in (16). In this case N_h^o and N_h^m can not be ranked unambiguously. More precisely, as $c_{D^h}^m = 2^{1/(k+2)} c_{D^h}^o$ we have that $N_h^m > N_h^o$ when

$$\alpha > \alpha_{1h} \equiv \frac{1}{2^{\frac{k+1}{k+2}} - 1} \tau^{hh} c_{D^h}^o = \frac{1}{2^{\frac{k+1}{k+2}} - 1} \left\{ \frac{\gamma (k+1)(k+2)}{L^h} \frac{\sum_{l=1}^{M} \left[f^l \left(c_M^l \right)^k |C_{lh}| \right]}{|P|} \right\}^{\frac{1}{k+2}}$$

which is the case when α as well as L^h are large and when γ and the term $\sum_{h=1}^{M} \left[f^h \left(c_M^h \right)^k |C_{hl}| \right] / |P|$ are small. On the contrary, $N_h^m < N_h^o$ when $\alpha < \alpha_{1h}$.

In the general case of asymmetric countries, the comparison between N_E^{lo} in (27) and N_E^{lm} in (17), and, consequently the comparison between exported and domestic varieties determined by the first best planner and the market, is complicated by the presence of the summation ($\sum_{h=1}^{M}$) in the numerator of the solutions. Specifically, while the terms in the summation are smaller in the market and this works in favour of having less entry in the market ($N_E^{lm} < N_E^{lo}$), the numerator of the solution for the market has a 2 in the numerator that, on the contrary, works in favour of having more entry in the market ($N_E^{lm} > N_E^{lo}$). This captures two contrasting effects present in the market, that were already described by Spence (1976). Indeed, adapting his words to this specific case, revenues in the market may fail to cover the costs of a socially desirable product because of setup costs, while they could be covered by a social planner who could allow for production at a loss. This produces a force that tends to produce in the market less entry (and product variety) than optimal. On the other hand, as firms in the market hold back output setting their price above their marginal cost, they leave more room for entry than would marginal cost pricing that would be followed by the first best planner decentralizing his/her choices, producing a business stealing effect that tends to generate more entry (and product variety) than optimal.

Finally, the number of domestic producers and of exporters in the market is clearly not optimal as $N_{D^l}^o = G^l(c_{D^l}^o) N_E^{lo}$ is different from $N_{D^l}^m = G^l(c_{D^l}^m) N_E^{lm}$ and $N_{X^{lh}}^o = G^l(c_{lh}^o) N_E^{lo}$ is different from $N_{X^{lh}}^m = G^l(c_{lh}^m) N_E^{lm}$. However, as for the case of the number of firms entering the market, the comparison between the two cases is not straightforward and requires to specify all the parameters of the model.

4 The constrained planners

We show in the next Section that the first best optimum can not be decentralized when differentiated production subsidies/taxes are not available. However, the constrained (second or third best) planner relies on a common production subsidy (eventually tax if it is negative) that in an open economy framework implies that profits of a firm producing in h for l are

$$\pi_{hl}(c) = \left[p_{hl}(c) + s^{hl}\right] q_{hl}(c) - \tau^{hl} c q_{hl}(c)$$

where s^{hl} is the specific common subsidy offered to firms producing in h to sell in l the quantity $q_{hl}(c)$.

With inverse demand $p_{hl}(c) = \alpha - \frac{\gamma}{L^l} q_{hl}(c) - \frac{\eta}{L^l} Q_l = p^l_{\max} - \frac{\gamma}{L^l} q_{hl}(c)$ and $p^l_{\max} = \alpha - \frac{\eta}{L^l} Q_l$, profit from sales in l of a good produced in h can be rewritten as

$$\pi_{hl}(c) = \left[p^l_{\max} + s^{hl} - \frac{\gamma}{L^l} q_{hl}(c) - \tau^{hl} c\right] q_{hl}(c)$$

In this case, the first order condition for profit maximization with respect to $q_{hl}(c)$ holds for

$$q_{hl}(c) = \frac{L^l}{2\gamma}\left(p^l_{\max} + s^{hl} - \tau^{hl} c\right) \tag{34}$$

showing that, for a given $p^l_{\max}$, a positive (negative) specific subsidy can be used to increase (decrease) firm sales in l.

The choke price pins down the highest marginal cost c_{hl} such that $q_{hl}(c)$ is non-negative

$$p^l_{\max} = \tau^{hl} c_{hl} - s^{hl} \tag{35}$$

so that we can rewrite the profit maximizing quantities in (36) as

$$q_{hl}(c) = \begin{cases} \frac{L^l}{2\gamma} \tau^{hl}\left(c_{hl} - c\right) & c \leq c_{hl} \\ 0 & c > c_{hl} \end{cases} \tag{36}$$

and prices as

$$p_{hl}(c) = \frac{1}{2} \tau^{hl}\left(c_{hl} + c\right) - s^{hl} \tag{37}$$

Moreover, profits are given by

$$\pi_{hl}(c) = \frac{L^l}{4\gamma}\left(\tau^{hl}\right)^2 \left(c_{hl} - c\right)^2 \tag{38}$$

Notice that, while quantities in (36) and profits in (38) have the same expression as in the market equilibrium given, respectively, in (6) and (7), prices in (37) are affected by the subsidy.

Finally, making use of the Pareto distribution in (2) and expressions (36), the average production of firms in h for consumption in l for the constrained planners is

$$\overline{q}_{hl} = \int_0^{c_{hl}} q_{hl}(c) dG^l_{hl}(c) = \frac{L^l}{2\gamma} \frac{1}{(k+1)} \tau^{hl} c_{hl} \tag{39}$$

4.1 The second best planner

The second best planner is unable to choose the output level supplied by each firm to local consumers and eventually exported, but he/she can set the minimum amount of productivity required for firms to survive in the domestic market and those required to export in each of the other countries and, finally, determine the number of firms that operate in each country controlling the number of R&D projects undertaken in each economy.

Specifically, the second best planner chooses the number of R&D projects (N_E^{ls}) and the cutoffs for domestic production and for exports (c_{lh}^s with $l, h = 1, ..., M$) for each country l so as to maximize social welfare in (23) subject to the profit maximizing quantity $q_{hl}(c)$ in (36) and the selection conditions in (12).

In Appendix B we write the first order conditions and find the solutions for the second best planner, showing that the relationship between the cutoffs for the second best planner has to be the following

$$c_{lh}^s = \frac{\tau^{hh}}{\tau^{lh}} c_{D^h}^s \qquad \forall l, h = 1, ..., M \tag{40}$$

where 's' labels second best optimum variables. Thus, as the first best planner in (25), also the second best planner does not alter the relationship between the cutoffs of the two types of marginal firms (domestic and foreign) selling in each country that characterizes the market equilibrium (8). So, also for the second best planner holds the principle of no discrimination with respect to the market equilibrium in favour of domestic or imported varieties stated in Proposition 1 for the first best planner.

Expression (40) together with (35) implies that the specific subsidy which can be used to implement the solution by the second best planner requires

$$s^{il} = s^{jl} = s^l \quad \forall i, j, l = 1, ..., M \tag{41}$$

To show this, we know from (35) that $p_{\max}^l = \tau^{il} c_{il} - s^{il} = \tau^{jl} c_{jl} - s^{jl}$ that making use of (40) can be rewritten as $\tau^{il} \frac{\tau^{ll}}{\tau^{il}} c_{D^l}^s - s^{il} = \tau^{jl} \frac{\tau^{ll}}{\tau^{jl}} c_{D^l}^s - s^{jl}$ that implies $s^{il} = s^{jl}$.

The expression for the cutoffs (78) in Appendix B, can be compared with the corresponding expression for the market (11), showing that $c_{D^l}^s = \left[\frac{2(k+1)}{2k+1}\right]^{\frac{1}{k+2}} c_{D^l}^m$, which respectively implies $c_{D^l}^s > c_{D^l}^m$ and, using (8) and (40), that $c_{hl}^s > c_{hl}^m$ given that $c_{hl}^s = \left[\frac{2(k+1)}{2k+1}\right]^{\frac{1}{k+2}} c_{hl}^m$. Accordingly, varieties with $c \in [c_{D^l}^m, c_{D^l}^s]$ are not supplied in the market, while they would be supplied by the second best planner. Furthermore, varieties with $c \in [c_{hl}^m, c_{hl}^s]$ are not exported from country h to country l in the market, while they would be exported by the second best planner.

Expressions (39) and (40) can be used to compute the average quantity supplied in each country h by all domestic and foreign firms in the case of the second best planner

$$\bar{q}_h^s = \frac{\sum_{l=1}^{M} N_{lh}^s \bar{q}_{lh}^s}{\sum_{l=1}^{M} N_{lh}^s} = \frac{L^h}{2\gamma} \frac{1}{k+1} \tau^{hh} c_{D^h}^s \tag{42}$$

with $\bar{q}_h^s > \bar{q}_h^m$ as $c_{D^h}^s > c_{D^h}^m$. Thus, we have:

Proposition 6 ***(Selection and average supply for the second best planner)*** *Firm selection that identifies both domestic producers and exporters in the market equilibrium is tougher than optimal from the point of view of the second best planner who extends the average supply of varieties in each country.*

Finally, making use of (12), the number of domestic varieties set by the second best planner for each country l is $N_{D^l}^s = G^l(c_{D^l}^s)N_E^{ls}$, while the number of exported varieties from l to country h is $N_{X^{lh}}^s = G^l(c_{lh}^s)N_E^{ls}$.[15]

4.2 The third best planner

The third best planner in the open economy can not affect the profit maximizing choices of firms in terms of quantities and prices but he/she can affect the number of firms that operate in each country in order to maximize the social welfare in (23).

Hence, the third best planner maximizes W with respect to N_E^i for each country $i = 1, ..., M$ subject to: i) the profit maximizing quantity $q_{hl}(c)$ in (36); ii) the selection conditions in (12); and, finally, the free entry conditions for each country l in (9) that, with (8) implying the same relationship between domestic and foreign cutoffs for firms selling in a country, impose to the planner the same cutoffs of the market equilibrium given in (11).

In other words, the third best planner makes use of the same relationship between c_{il} and c_{D^l} that applies between domestic and foreign cutoffs in the market equilibrium. Indeed, the third best planner is unable to set any cutoff as he/she can not rely on lump sum instruments on firms, which, instead, are available for the second best planner. However, as the second best planner, the third best planner can use a specific common production subsidy for all quantities sold in country l by firms producing in different countries h. Thus, the third best planner uses the instrument $s^{il} = s^{jl} = s^l$ given in (41) that implies $c_{il} = \tau^{ll} c_{D^l}/\tau^{il}$.[16]

In Appendix B we write the first order conditions and find the solutions for N_E^{lt} given in expression (79) with 't' labeling third best optimum variables when they differ from the corresponding market equilibrium values.[17]

Finally, comparing (79) with (17) reveals that $N_E^{lt} > N_E^{lm}$. Moreover, given that $N_{D^l}^t = G^l(c_{D^l}^m)N_E^{lt}$ and that $N_{X^{lh}}^t = G^l(c_{lh}^m)N_E^{lt}$, we find that $N_{D^l}^m < N_{D^l}^t$ and that $N_{X^{lh}}^m < N_{X^{lh}}^t$, and comparing (46) obtained in the next subsection with (16) reveals that $N_l^m < N_l^t$.

Moreover, given that the third best planner sets the same cutoffs of the market equilibrium, the average quantity supplied by firms producing in l for consumers in h in (39) and the average quantity supplied in each country h by all domestic and foreign firms $\bar{q}_h^t = \sum_{l=1}^{M} N_{lh}^t \bar{q}_{lh}^t / \sum_{l=1}^{M} N_{lh}^t$, obtained making use of (69) and (8), are the same as those of the market equilibrium, that is $\bar{q}_{hl}^t = \bar{q}_{hl}^m$ and $\bar{q}_h^t = \bar{q}_h^m$.

Therefore, we can state that

[15] Notice that in the case of two countries with no internal trade costs and common sunk entry cost f, the conditions that have to be satisfied to have positive values for both N_E^{1s} and N_E^{2s} in (76) in Appendix B, imply that $c_{12}^s < c_{D^1}^s$ and $c_{21}^s < c_{D^2}^s$, so that in each country only a subset of relatively more productive firms export in the second best case.

[16] To show this, we know from (35) that $p_{\max}^l = \tau^{il} c_{il} - s^{il} = \tau^{jl} c_{jl} - s^{jl}$ that with $s^{il} = s^{jl} = s^l$ becomes $\tau^{il} c_{il} = \tau^{jl} c_{jl}$ that with $j = l$ implies $c_{il} = \tau^{ll} c_{D^l}/\tau^{il}$.

[17] In the case of two countries with no internal trade costs and common sunk entry cost f, the solutions N_E^{1t} and N_E^{2t} are both positive when $c_{12}^t < c_{D^1}^t$ and $c_{21}^t < c_{D^2}^t$, so that, as in the market equilibrium, only a subset of relatively more productive firms export.

Proposition 7 ***(Product variety and selection for the third best planner)*** *In each country, entry, product variety, the number of locally produced varieties and of exported varieties are poorer in the market equilibrium than in the case of the third best planner, even though the cutoffs and the individual and average supply of firms are the same in the two cases.*

4.3 An overview of the results

Table 1 summarizes the main findings on selection, average firm size, product variety, the number of domestic and exported varieties, and the overall size of the differentiated good sector in each country obtained in this Section and in Appendix A for the general case of M potentially asymmetric countries.

Selection in the domestic economy:	$c^s_{D^l} > c^m_{D^l} = c^t_{D^l} > c^o_{D^l}$
Selection into exporting:	$c^s_{hl} > c^m_{hl} = c^t_{lh} > c^o_{lh}$
Average firms' supply in country h:	$\bar{q}^o_h > \bar{q}^s_h > \bar{q}^t_h = \bar{q}^m_h$
Product variety:	$N^t_h > N^m_h$ and $N^m_h \gtreqless N^o_h$ if $\alpha \gtreqless \alpha_{1h}$; $N^m_h \gtreqless N^s_h$ if $\alpha \gtreqless \alpha_{3h}$; $N^t_h \gtreqless N^s_h$ if $\alpha \gtreqless \alpha_{4h}$
Number of domestic varieties:	$N^t_{D^l} > N^m_{D^l}$
Number of exported varieties:	$N^t_{X^{lh}} > N^m_{X^{lh}}$
Differentiated sector size:	$N^o_h\bar{q}^o_h > N^s_h\bar{q}^s_h > N^t_h\bar{q}^t_h > N^m_h\bar{q}^m_h$

Table 1. An overview of the results for the differentiated good sector.

We recall that for the general case of M potentially asymmetric countries we find that $c^s_{D^l} > c^m_{D^l} = c^t_{D^l} > c^o_{D^l}$ and $c^s_{hl} > c^m_{hl} = c^t_{lh} > c^o_{lh}$ and, therefore, as already pointed out, firm selection that identifies both domestic producers and exporters in the market equilibrium is tougher than optimal from the point of view of the second best planner, while it is weaker than optimal from the point of view of the first best planner. Selection processes that take place in the market are not altered by the third best planner.

Moreover, from (42), (71), (70), making use of $c^s_{D^h} = \left[\frac{2(k+1)}{2k+1}\right]^{\frac{1}{k+2}} c^m_{D^h}$ and $c^s_{D^l} > c^m_{D^l} = c^t_{D^l} > c^o_{D^l}$, we find that

$$\bar{q}^o_h = 2^{\frac{k+1}{k+2}}\bar{q}^m_h > \bar{q}^s_h = \left[\frac{2(k+1)}{2k+1}\right]^{\frac{1}{k+2}} \bar{q}^m_h > \bar{q}^m_h = \bar{q}^t_h$$

Accordingly, we can state that:

Proposition 8 ***(Average firms supply in each country)*** *The second best planner allows the quantity supplied by firms in each country to be on average larger than in the market equilibrium, even though smaller than in the first best solution.*

Hence, the second best planner in his/her attempt to increase the average size of firms increases the average quantity supplied by firms in a market rising the cutoff with respect to that prevailing in the market. However, softening competition also for less productive firms, he/she does not allow supply of firms and thus firms' size to increase as much as in the first best case when, instead, the planner allows only more productive firms to increase their size, reducing the production of less productive firms and stopping the production of the least productive ones.

Moreover, we know that the number of domestic and exported varieties and product variety is richer in the case of the third best planner than in the market equilibrium.

Let us now compare the total consumption of differentiated varieties in a country for the second and third best planners.[18] To do this, we need to rewrite the total quantity consumed in country l, $Q_l = \sum_{h=1}^{M} \left(N_E^h \int_0^{c_M^h} q_{hl}(c) dG^h(c) \right)$, for the constrained planners making use of (36), (12), (13) and $c_{il} = \tau^{ll} c_{D^l} / \tau^{il}$ as

$$Q_l = \frac{L^l}{2\gamma} \frac{1}{(k+1)} \tau^{ll} c_{D^l} N_l \tag{43}$$

Moreover, for the second and third best planners $p_{\max}^l = \alpha - \frac{\eta}{L^l} Q_l$ with (35) imply that in both cases the cutoffs have to satisfy the following relationship

$$\alpha - \frac{\eta}{L^l} Q_l = \tau^{hl} c_{hl} - s^{hl} \tag{44}$$

Then, in the specific case of the second best planner, we can rewrite (44) making use of (43), $c_{il} = \tau^{ll} c_{D^l} / \tau^{il}$, $s^{hl} = s^l$ and s^l in (60) derived in the following Section, to find

$$N_l^s = \frac{2\gamma (k+1)}{\eta \tau^{ll} c_{D^l}^s} \left(\alpha - \frac{1}{2} \frac{2k+1}{k+1} \tau^{ll} c_{D^l}^s \right) \tag{45}$$

In the case of the third best planner, instead, $s^{hl} = \left(s^l\right)^t$ given in (63) has to be used, to get

$$N_l^t = \frac{2\gamma (k+1)}{\eta \tau^{ll} c_{D^l}^m} \left(\alpha - \frac{1}{2} \frac{2k+3}{k+2} \tau^{ll} c_{D^l}^m \right) \tag{46}$$

Hence, given (45), (46), (42) and $\bar{q}_h^t = \bar{q}_h^m$ in (70), we find that the total consumption of differentiated varieties in a country in the case of the second and of the third best planner, respectively, evaluates to $N_h^s \bar{q}_h^s = \frac{L^h}{\eta} \left\{ \alpha - \left[\frac{2k+1}{2(k+1)} \right]^{\frac{k+1}{k+2}} \tau^{hh} c_{D^h}^m \right\}$ and $N_h^t \bar{q}_h^t = \frac{L^h}{\eta} \left[\alpha - \frac{2k+3}{2(k+2)} \tau^{hh} c_{D^h}^m \right]$. Comparing the dimensions of the differentiated sector in all situations, we find that $N_h^o \bar{q}_h^o > N_h^s \bar{q}_h^s > N_h^t \bar{q}_h^t > N_h^m \bar{q}_h^m$.[19]

[18] In general, notice that $Q_h = \sum_{l=1}^{M} \left(N_E^l \int_0^{c_M^l} q_{lh}(c) dG^l(c) \right) = \sum_{l=1}^{M} \left(N_E^l G^l(c_{lh}) \int_0^{c_{lh}} q_{lh}(c) dG_{lh}^l(c) \right) = \sum_{l=1}^{M} \left(N_E^l G^l(c_{lh}) \bar{q}_{lh} \right) = \sum_{l=1}^{M} \left(N_{lh} \bar{q}_{lh} \right) = N_h \bar{q}_h$

[19] To prove that $N_h^o \bar{q}_h^o > N_h^s \bar{q}_h^s > N_h^t \bar{q}_h^t > N_h^m \bar{q}_h^m$ we consider that the following inequalities $1 > \frac{2k+3}{2(k+2)} > \left[\frac{2k+1}{2(k+1)} \right]^{\frac{k+1}{k+2}} > 2^{-1/(k+2)}$ hold for $k \in [1, \infty)$. Indeed, it is readily seen that $1 > \frac{2k+3}{2(k+2)}$. Moreover, plotting the graphics of $\frac{2k+3}{2(k+2)} - \left(\frac{2k+1}{2(k+1)} \right)^{\frac{k+1}{k+2}}$ we can see that it is positive for $k \in [1, \infty)$. Finally, for the last inequality, it holds if $\left[\frac{2k+1}{2(k+1)} \right]^{k+1} > 1/2$ which is always true as $\left[\frac{2k+1}{2(k+1)} \right]^{k+1} = 9/16$ when $k = 1$ and it is increasing in k (tending to 1 when k tends to ∞).

In summary, we can state what follows.

Proposition 9 *The larger number of available varieties in a country is responsible for the larger size of the differentiated sector in the case of the third best planner with respect to the market equilibrium, while the increase in the average supply of firms in each country (potentially also accompanied by a smaller number of supplied varieties than in the market) is responsible for the additional extension of the size of the differentiated good sector in the second best solution; the optimal level of production set for each firm according to its productivity implies the largest average supply of firms within each country and overall dimension of the differentiated good sector in the case of the first best planner.*

Comparing the number of varieties supplied in (16) and (45), making use of $c^s_{D^l} = \left[\frac{2(k+1)}{2k+1}\right]^{\frac{1}{k+2}} c^m_{D^l}$, we find that product variety is richer in the market than in the second best case ($N^m_l > N^s_l$) as long as

$$\alpha > \alpha_{3l} \equiv \frac{\left[\frac{2(k+1)}{2k+1}\right]^{\frac{1}{k+2}}}{2(k+1)\left\{\left[\frac{2(k+1)}{2k+1}\right]^{\frac{1}{k+2}} - 1\right\}} \tau^{ll} c^m_{D^l}$$

which holds when α as well as L, M and ρ are large and when γ, c_M and f are small. On the contrary, $N^m_l < N^s_l$ when $\alpha < \alpha_{3l}$.

Instead, comparing (46) and (45), yields that $N^t_l > N^s_l$ when

$$\alpha > \alpha_{4l} \equiv \frac{\left[\frac{2(k+1)}{2k+1}\right]^{\frac{1}{k+2}}}{2(k+2)(k+1)\left\{\left[\frac{2(k+1)}{2k+1}\right]^{\frac{1}{k+2}} - 1\right\}} \tau^{ll} c^m_{D^l}$$

with $\alpha_{3l} > \alpha_{4l}$, which is the case when α as well as L, M and ρ are large and when γ, c_M and f are small. On the contrary, $N^t_l < N^s_l$ when $\alpha < \alpha_{4l}$.

4.3.1 More results with symmetric countries

Finally, more results can be obtained in the case of M symmetric countries with no internal trade costs presented in the following paragraphs.

In this case, the number N^s of varieties available for consumption in each country for the second best planner is given by the sum of domestic varieties

$$N^s_D = G(c^s_D) N^s_E = \frac{2\gamma(k+1)}{\eta\left[1 + (M-1)\rho\right] c^s_D}\left(\alpha - \frac{1}{2}\frac{2k+1}{k+1} c^s_D\right) \tag{47}$$

and of the $(M-1)\, N^s_X$ varieties imported from all other countries with

$$N^s_X = G(c^s_X) N^s_E = \rho\frac{2\gamma(k+1)}{\eta\left[1 + (M-1)\rho\right] c^s_D}\left(\alpha - \frac{1}{2}\frac{2k+1}{k+1} c^s_D\right) = \rho N^s_D \tag{48}$$

Therefore, product variety is

$$N^s = N^s_D + (M-1)\, N^s_X = \frac{2\gamma(k+1)}{\eta c^s_D}\left(\alpha - \frac{1}{2}\frac{2k+1}{k+1} c^s_D\right) \tag{49}$$

For the third best planner, we find that the number of domestic varieties and exported varieties by each country are respectively given by

$$N_D^t = G(c_D^m)N_E^t = \frac{2\gamma(k+1)}{\eta[1+(M-1)\rho]c_D^m}\left(\alpha - \frac{1}{2}\frac{2k+3}{k+2}c_D^m\right) \quad \text{and} \quad N_X^t = G(c_X^m)N_E^t = \rho N_D^t \tag{50}$$

so that product variety is

$$N^t = N_D^t + (M-1)N_X^t = \frac{2\gamma(k+1)}{\eta c_D^m}\left(\alpha - \frac{1}{2}\frac{2k+3}{k+2}c_D^m\right) \tag{51}$$

Moreover, in the latter case of symmetric countries, (30), (31), (47), (48) and (50) imply that the number of domestic varieties for the market and for each of the $n-th$ planners is smaller than the number of exported varieties with

$$N_X^r = \rho N_D^r < N_D^r \quad \text{with } r \in \{m, o, s, t\}$$

However, the number of the imported varieties from the $(M-1)$ countries can in principle be larger than the number of domestic varieties.

Comparing the number of varieties supplied in (16) and (49), of domestic and exported varieties in (47), (48) and (30), making use of $c_D^s = \left[\frac{2(k+1)}{2k+1}\right]^{\frac{1}{k+2}} c_D^m$, we find the number of domestic and exported varieties and product variety is richer in the market than in the second best case with $N_D^m > N_D^s$, $N_X^m > N_X^s$ and $N^m > N^s$ as long as

$$\alpha > \alpha_3 \equiv \frac{\left[\frac{2(k+1)}{2k+1}\right]^{\frac{1}{k+2}}}{2(k+1)\left\{\left[\frac{2(k+1)}{2k+1}\right]^{\frac{1}{k+2}} - 1\right\}} c_D^m$$

which is the case when α as well as L, M and ρ are large and when γ, c_M and f are small.[20] On the contrary, $N_D^m < N_D^s$, $N_X^m < N_X^s$ and $N^m < N^s$ when $\alpha < \alpha_3$.

Finally, comparing the number of varieties supplied in (51) and (49), of domestic and exported varieties in (47), (48) and (50) making use of $c_D^s = \left[\frac{2(k+1)}{2k+1}\right]^{\frac{1}{k+2}} c_D^m$ we have that the number of domestic and exported varieties and product variety is richer in the case of the second best planner than for the third best planner with $N_D^t > N_D^s$, $N_X^t > N_X^s$ and $N^t > N^s$ when

$$\alpha > \alpha_4 \equiv \frac{\left[\frac{2(k+1)}{2k+1}\right]^{\frac{1}{k+2}}}{2(k+2)(k+1)\left\{\left[\frac{2(k+1)}{2k+1}\right]^{\frac{1}{k+2}} - 1\right\}} c_D^m$$

with $\alpha_3 > \alpha_4$, which is the case when α as well as L, M and ρ are large and when γ, c_M and f are small.[21] On the contrary, $N_D^t < N_D^s$, $N_X^t < N_X^s$ and $N^t < N^s$ when $\alpha < \alpha_4$.

[20] Notice that α_3 corresponds to α_{3l} evaluated in the specific case of symmetric countries with no internal trade costs.

[21] Notice that α_4 corresponds to α_{4l} evaluated in the specific case of symmetric countries with no internal trade costs.

5 Decentralization of the optimal solutions

5.1 Implementation of the first best solution

In the case of the open economy, the unconstrained planner has to consider for each country not only that the market equilibrium delivers a suboptimal firm production for the domestic consumers, but also that it delivers suboptimal exports for each firm that sells its production abroad. Moreover, the planner has to deal with suboptimal entry in all countries. This, in turn, implies that the number of domestic producers and exporters is not optimal.

Therefore, the first best planner uses for each country h a specific domestic production subsidy (tax) that varies across firms and destination countries. More precisely, a specific subsidy $s^{hl}(c)$ is offered to firms with productivity $1/c$ producing in h for consumers located in l to deal with their suboptimal production for each market. Moreover, the unconstrained planner uses a lump sum entry tax for each country T^h to deal with the suboptimal entry and selection in each country.

More precisely, with the specific subsidy $s^{hl}(c)$, the expression for the profit maximizing quantity sold in l by a firm producing in h is

$$q_{hl}(c) = \frac{L^l}{2\gamma}\left(p^l_{\max} + s^{hl}(c) - \tau^{hl}c\right) \tag{52}$$

showing that, given $p^l_{\max}$, a positive (negative) specific subsidy can be used to increase (decrease) firm sales in l.

In this case, the choke price that pins down the highest marginal cost c^d_{hl} such that $q^{hl}(c)$ is non-negative becomes

$$p^l_{\max} = \tau^{hl}c^d_{hl} - s^{hl}(c^d_{hl})$$

so that $q_{hl}(c)$ in (52) can be rewritten as

$$q_{hl}(c) = \frac{L^l}{2\gamma}\left\{\left[\tau^{hl}c^d_{hl} - s^{hl}(c^d_{hl})\right] - \left[\tau^{hl}c - s^{hl}(c)\right]\right\}$$

with the price set and the profits derived in country l by a firm producing in h with productivity $1/c$, respectively, given by

$$p_{hl}(c) = \frac{1}{2}\left\{\left[\tau^{hl}c^d_{hl} - s^{hl}(c^d_{hl})\right] + \left[\tau^{hl}c - s^{hl}(c)\right]\right\}$$

and

$$\pi_{hl}(c) = \frac{L^l}{4\gamma}\left\{\left[\tau^{hl}c^d_{hl} - s^{hl}(c^d_{hl})\right] - \left[\tau^{hl}c - s^{hl}(c)\right]\right\}^2 \tag{53}$$

Thus, the planner chooses the schedule $s^{hl}(c)$ that implements the first best output for country l such that $p_{hl}(c) = \tau^{hl}c$ which requires setting

$$s^{hl}(c) = -s^{hl}(c^d_{hl}) + \tau^{hl}\left(c^d_{hl} - c\right) \tag{54}$$

which is zero for $c = c^d_{hl}$, positive (a subsidy) for $c < c^d_{hl}$ and negative (a tax) for $c > c^d_{hl}$. Note that $s^{hl}(c^d_{hl}) = 0$ is the unique schedule that allows

$p_{hl}(c_{hl}^d) = \tau^{hl} c_{hl}^d$, with $c_{hl}^d = c_{hl}^o$ implemented by the planner to ensure first best selection. Hence, (54) can be rewritten as

$$s^{hl}(c) = \tau^{hl}\left(c_{hl}^o - c\right) \tag{55}$$

Note that $s^{hl}(c) = \tau^{hl}\left(c_{hl}^d - c\right) = 2\mu_{hl}^m(c)$ because $\mu_{hl}^m(c) = \tau^{hl}\left(c^{hl} - c\right)/2$: the subsidy has to be twice the markup in the country as only half of any cost cut is transferred to consumers. This leads to too much entry (and production).

The average specific subsidy received by firms producing in all countries for their sales in l is[22]

$$\bar{s}^l = \frac{2k+1}{(k+1)}\tau^{ll} c_{D^l}^o \tag{56}$$

As the first best output levels would clear the market in the decentralized scenario only if each producer in h priced the quantities sold in l at its own marginal cost $\tau^{hl}c$, with $p_{hl}(c) = \tau^{hl}c$, the average price for varieties sold in l is in this case[23]

$$\bar{p}_l^o = \frac{k}{k+1}\tau^{ll} c_{D^l}^o \tag{57}$$

Therefore, we can state that:

Proposition 10 *The optimal firm-specific per-unit subsidy to production in h for sales in l is $s^{hl}(c) = \tau^{hl}\left(c_{hl}^o - c\right)$, which implies that the production subsidy is decreasing in the marginal cost, being zero for firms with $c = c_{hl}^o$, negative ('tax') for high cost firms with $c \in (c_{hl}^o, c_M^h]$ and positive for low cost firms with $c \in [0, c_{hl}^o)$. The average specific subsidy for firms selling in a country is decreasing with the size of the same country.*

With (55), profits in (53) becomes

$$\pi_{hl}(c) = \frac{L^l}{2\gamma}\left(\tau^{hl}\right)^2\left(c_{hl}^d - c\right)^2$$

Then, the first best planner needs to complement the production subsidy with a lump-sum entry tax per entrant in each country T^h that together ensure first best selection in all countries as otherwise expected profits would be too high and lead to too much entry. Specifically, considering a per entrant lump-sum tax T^h that varies across countries, the 'free entry condition' for country h is

$$\sum_{l=1}^{M}\left[\int_0^{c_{hl}^d}\frac{L^l}{\gamma}\left(\tau^{hl}\right)^2\left(c_{hl}^d - c\right)^2 dG^h(c)\right] = f^h + T^h$$

Making use of the Pareto distribution in (2), and of (25), the free entry conditions becomes

$$\sum_{l=1}^{M}\left[\left(\tau^{hl}\right)^{-k} L^l \left(\tau^{ll} c_{D^l}^d\right)^{k+2}\right] = \frac{\gamma\left(c_M^h\right)^k (k+2)(k+1)\left(f^h + T^h\right)}{2}$$

[22] The average subsidy $\bar{s}^l$ can be computed making use of the average subsidy received by firms producing in h for sales in l, $\bar{s}^{hl} = \tau^{hl} c_{hl}^d \frac{2k+1}{(k+1)}$, (25) and (12).

[23] The average price of varieties sold in l, $\bar{p}^l$, can be computed making use of the average price for varieties produced in h and sold in l, $\bar{p}_{hl}^o = \frac{k}{k+1}\tau^{hl} c_{hl}^o$, and (25).

This yields a system of M equations that can be solved using Cramer's rule to find the M equilibrium cutoffs:

$$c^d_{D^l} = \frac{1}{\tau^{ll}} \left\{ \frac{\gamma (k+2)(k+1)}{2L^l} \frac{\sum_{h=1}^{M} \left[\left(c^h_M \right)^k \left(f^h + T^h \right) |C_{hl}| \right]}{|P|} \right\}^{\frac{1}{k+2}} \quad \forall l = 1, ..., M \tag{58}$$

The solutions in (58) can be used to set $T^h \forall h = 1, ..., M$ to implement the solutions in (26), that is $c^d_{D^l} = c^o_{D^l}$. The two systems give the same solutions for the cutoff of country l if and only if the following equation holds

$$\frac{\sum_{h=1}^{M} \left[\left(c^h_M \right)^k \left(f^h + T^h \right) |C_{hl}| \right]}{2} = \sum_{h=1}^{M} \left[f^h \left(c^h_M \right)^k |C_{hl}| \right]$$

This must be true for all M countries and is satisfied when

$$T^h = f^h \quad \forall h = 1, ..., M \tag{59}$$

Hence, the first best optimum can be decentralized by means of a production subsidy schedule financed in a non-distorting fashion (e.g. with a lump sum tax levied on consumers) and a lump sum entry tax in all countries such that $T^h = f^h$ in all countries.

In summary, the production and export subsidies/taxes for the domestic and the foreign markets together with the lump sum taxes on entries identified for the M countries, deliver the optimal number of varieties domestically produced and exported ($N^o_{D^l} = G^l(c^o_{D^l}) N^{lo}_E$ and $N^o_{X^{lh}} = G^l(c^o_{lh}) N^{lo}_E$), the optimal domestic and exported output levels with associated marginal cost prices.

Finally, we note that to describe the role of c^h_M $\forall h = 1, ..., M$ on the specification of production subsidies, we need to have more information on the international trade network, which is relevant when trade can differ in terms of accessibility, as in Behrens, Mion and Ottaviano (2011). However, in the following example, we focus on the specific case of two trading economies to identify which situations may occur in this specific case.

5.1.1 The case of two trading countries

Let us consider the case of *two countries* with no internal trade costs and equal f. In this case, only a subset of relatively more productive firms export in both the market equilibrium and in the optimal case. We focus on firms producing in country 1 with $c^m_{12} < c^m_{D^1}$ and $c^o_{12} < c^o_{D^1}$. We know that $c^o_{12} < c^m_{12}$ and $c^o_{D^1} < c^m_{D^1}$. Comparing all the cutoffs for country 1, we have that $c^o_{12} < c^m_{12} < c^m_{D^1}$ and that $c^o_{12} < c^o_{D^1} < c^m_{D^1}$. Considering these inequalities, we notice that two cases may arise that are considered in Figure 1: either $c^m_{12} < c^o_{D^1}$, or $c^o_{D^1} < c^m_{12}$. Using (8), (11), (25) and (26), we find that the first case with $c^m_{12} < c^o_{D^1}$, implying the following ranking of the cutoffs $c^o_{12} < c^m_{12} < c^o_{D^1} < c^m_{D^1}$, takes place when the relative size of country 1 is relatively small with respect to country 2 (or, alternatively, when the comparative advantage of country 2 in the differentiated good sector is relatively large) for given level of trade costs so that

$\frac{L^2}{L^1} > \lambda \equiv \frac{2}{(\tau^{12})^{k+2}}\left[\frac{(c_M^2)^k-(c_M^1)^k\rho^{21}}{(c_M^1)^k-(c_M^2)^k\rho^{12}}\right]$. Instead, the second case with $c^o_{D^1} < c^m_{12}$, that implies the following ranking of the cutoffs $c^o_{12} < c^o_{D^1} < c^m_{12} < c^m_{D^1}$, takes place only when the relative size of the country 1 is relatively large with respect to that of country 2 (or when the comparative advantage of country 2 in the differentiated good sector is relatively small) so that $\frac{L^2}{L^1} < \lambda$.

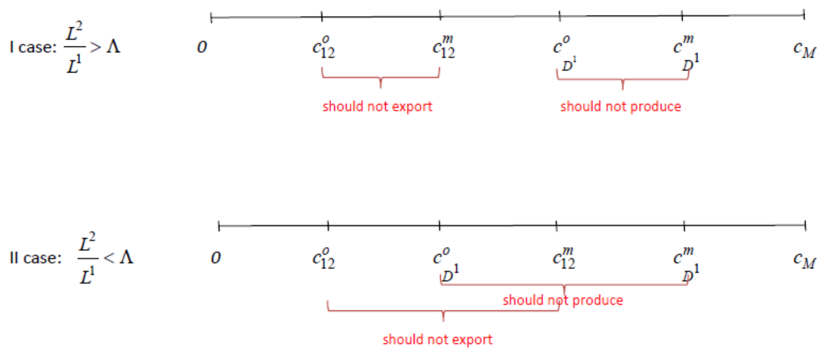

Figure 1. Cutoff rankings according to relative country size in the case of two countries.

According to these two rankings, we can identify four types of firms in both cases. The first best planner always taxes firms whose productivity is very low (that are those with $c \in (c^o_{D^1}, c^m_{D^1})$ in the first case and those with $c \in (c^m_{12}, c^m_{D^1})$ in the second case) so that they stop producing and he/she subsidizes those firms that have the highest levels of productivity in both cases (that is, firms with $c \in (0, c^o_{12})$) with a specific production subsidy that increases in their productivity level to make them expand their production in the domestic market. Moreover, export subsidies are also used in favour of these most productive firms with $c \in (0, c^o_{12})$ to allow them to expand their levels of exports and the magnitude of the specific export subsidy increases with the productivity of the firms. Besides, medium-high productive exporting firms (that are those with $c \in (c^o_{12}, c^m_{12})$ in the first case and those with $c \in (c^o_{12}, c^o_{D^1})$ in the second case) are subsidized to produce more for the domestic market, even though they are taxed on their exports to stop them selling their production abroad. Finally, firms characterized by a medium-low level of productivity are treated accordingly to the relative size of their country with respect to foreign one. Specifically, when country 1 in which they are based is sufficiently small relative to country 2, as it happens in the first case with $\frac{L^2}{L^1} > \lambda$, these firms characterized by a level of $c \in (c^m_{12}, c^o_{D^1})$ produce only for the domestic country and they are subsidized with a specific production subsidy that increases with their productivity level in order to produce more for their local market. On the contrary, when country 1 in which they are based is sufficiently large, as in the second case with $\frac{L^2}{L^1} < \lambda$, these firms characterized by a level of $c \in (c^o_{D^1}, c^m_{12})$ are taxed to stop them both producing for the local market and exporting.

5.2 Implementation of the second best solution

The second best planner cannot enforce marginal cost pricing by affecting the choices of each firm individually, but it can affect their choices, like expanding their production and exports, setting specific production and export subsidies common across all firms selling in a country. Moreover, the planner can set the optimal number of firms entering in each country with taxes that are common across all firms that undertake an R&D investment in that country.

In Appendix C, we show that the second best planner implements his/her optimal solution setting a specific subsidy per unit of quantity sold in country l given by

$$s^l = \frac{1}{2(k+1)} \tau^{ll} c^s_{D^l} \quad \forall l = 1, ..., M \tag{60}$$

and that he/she controls the number of firms that operate in each country introducing a per entrant tax in h equal to

$$\left(T^h\right)^s = \frac{f^h}{2k+1} \quad \forall h = 1, ..., M \tag{61}$$

with $\left(T^h\right)^s < T^h$.

Then, comparing (56) with (60) as in the closed economy, we find that $\bar{s}^l > s^l$. Hence, we can conclude as follows.

Proposition 11 *The second best optimal specific per-unit subsidies are common across all quantities sold in a country, smaller than those set on average by the first best planner, and, ceteris paribus, larger for quantities sold in those countries whose size is smaller.*

The average price in (82) for the second best planner can be rewritten making use of (60) as

$$\bar{p}^s_l = \frac{k}{k+1} \tau^{ll} c^s_{D^l} > \bar{p}^o_l = \frac{k}{k+1} \tau^{ll} c^o_{D^l} \tag{62}$$

where the cutoff ranking $c^s_{D^l} > c^o_{D^l}$ dictates the average price ranking $\bar{p}^s_l > \bar{p}^o_l$.

Finally, we note that to describe the role of c^h_M $\forall h = 1, ..., M$ on the specification of production subsidies, we need to have more information on the international trade network, which is relevant when countries can differ in terms of accessibility.

5.3 Implementation of the third best solution

As in the closed economy, the third best planner can not use lump-sum instruments for firms and, to implement the third best solution, he/she can only rely on lump-sum tools on consumers to finance the production subsidies to firms producing for the domestic market and, eventually, for the foreign market.

In this case, we show in Appendix C that the planner uses a specific production subsidy common across all firms that sell in country l given by

$$\left(s^l\right)^t = \frac{1}{2(k+2)} \tau^{ll} c^m_{D^l} \quad \forall l = 1, ..., M \tag{63}$$

to deal with the suboptimal number of entrants in each country.

Then, comparing the subsidy set by the second best planner s^l with $\left(s^l\right)^t$ making use of $c^s_{D^l} = \left[\frac{2(k+1)}{2k+1}\right]^{\frac{1}{k+2}} c^m_{D^l}$, we find that $s^l > \left(s^l\right)^t$, which together with $\bar{s}^l > s^l$, implies $\bar{s}^l > s^l > \left(s^l\right)^t$. Given that $\left(s^l\right)^t$ decreases with L^l, we can state what follows.

Proposition 12 *The third best planner sets a per-unit subsidy for the production of differentiated varieties that is common across all firms selling in a market, smaller than that set by the second best planner, that decreases with the size of the destination economy.*

The average price in (82) for the third best planner can be rewritten making use of (63) and $c^m_{D^l} = c^t_{D^l}$ as

$$\bar{p}^t_l = \frac{4k + 2k^2 + 1}{2\,(k+1)\,(k+2)} \tau^{ll} c^m_{D^l} < \bar{p}^m_l = \frac{2k+1}{2\,(k+1)} \tau^{ll} c^m_{D^l}$$

as $\bar{p}^t_l - \bar{p}^m_l = -1/\left[2\,(k+2)\right] \tau^{ll} c^m_{D^l} < 0$.[24] Moreover, given that $c^s_{D^l} = \left[\frac{2(k+1)}{2k+1}\right]^{\frac{1}{k+2}} c^m_{D^l}$ implies that the average price in the second best case in (62) can be rewritten as $\bar{p}^s_l = \frac{k}{k+1}\left[\frac{2(k+1)}{2k+1}\right]^{\frac{1}{k+2}} \tau^{ll} c^m_{D^l} < \bar{p}^t_l$,[25] the ranking of average prices for the differentiated good is

$$\bar{p}^m_l > \bar{p}^t_l > \bar{p}^s_l > \bar{p}^o_l$$

Finally, we note again that to describe the role of c^h_M $\forall h = 1, ..., M$ on the specification of production subsidies, we need to have more information on the international trade network, which is relevant when trade can differ in terms of accessibility.

5.4 Final remarks

Given (55) and (25), the optimal firm-specific per-unit subsidy given to firms producing in each country to sell in l can be rewritten as

$$s^{hl}(c) = \left(\tau^{ll} c^o_{D^l} - \tau^{hl} c\right) \tag{64}$$

This shows that all the firm-specific subsidies offered to firms producing in h to sell in l, that is $s^{hl}(c)$, are dependent on the variables that determine the domestic cutoff in the destination country l. Thus, observing the subsidies offered by all planners, respectively, given in (60), (63) and (64), we conclude that the specific subsidies are always set taking into account the domestic cutoff of the destination countries together with trade costs; on the contrary, the per entrant tax used by the first and the second best planner given, respectively, in (59) and (61) are determined only by the sunk entry cost of the country in which firms are located and, in addition, and only for the second best planner,

[24] The expression for $\bar{p}^m_l$ is given in (15).

[25] As $\bar{p}^t_l = \frac{4k+2k^2+1}{2(k+1)(k+2)} \tau^{ll} c^m_{D^l}$ and $\bar{p}^s_l = \frac{k}{k+1}\left[\frac{2(k+1)}{2k+1}\right]^{\frac{1}{k+2}} \tau^{ll} c^m_{D^l}$, the plot of $\frac{4k+2k^2+1}{2(k+1)(k+2)} - \frac{k}{k+1}\left[\frac{2(k+1)}{(2k+1)}\right]^{\frac{1}{k+2}}$ is positive for $k \in [1, \infty)$.

by the shape parameter k. In this case, more cost decreasing evenness (lower k) increases the per entrant tax set by the second best planner.

Moreover we can conclude observing that, starting from the market equilibrium, the availability of a common specific subsidy, which reduces the average prices of goods in the differentiated sector sold in each country, financed only by lump-sum taxes on consumers allows the third best planner to increase the number of varieties available in the country. If, in addition, a lump-sum tax on firms is available to finance a larger common specific subsidy, the second best planner is able to increase the average quantity sold by all firms in the market, further expanding the dimension of the differentiated good sector and decreasing the average price index in each country. Finally, when the planner can differentiate the specific subsidy according to the dimension of each individual firm selling in each country, the largest average specific subsidy is financed with lump-sum taxes on firms allowing the unconstrained planner to obtain the largest dimension of the differentiated good sector and average quantity sold at the lowest average price in each country.

6 Conclusion

The availability of an appropriate and parsimonious framework to deal with firm heterogeneity allows to bring back into the normative debate the full set of questions the canonical formalization of the Chamberlinian model by Spence (1976) and Dixit and Stiglitz (1977) was designed to answer and to extend it to the case of open economies. In particular, it provides a useful analytical tool to address the question whether in the market equilibrium the products are supplied and exported by the right set of firms, or there are rather 'errors' in the choice of technique.

We contribute to this debate by showing that in an open economy model with non-separable utility, variable demand elasticity and endogenous firm heterogeneity, the market outcome errs in many ways: with respect to product variety, the size and the choice of domestic producers and exporters, the overall size of the monopolistically competitive sector and entry.

We analyze how multilateral trade policy be designed in a world in which countries differ in terms of technology and geography, and firms with market power differ in terms of productivity. In this framework, we find that policy tools should not differ in kind or implementation between more and less developed countries. We show that the first best solution can be attained when it can be decentralized by means of domestic production subsidy/tax and export subsidy/tax schedules financed in a non-distorting fashion with a lump sum tax levied on consumers and a lump sum entry tax on firms in each country. In this case the planner sets the size of each firm and of exporters at their optimal levels (expanding the output and exports of more productive firms and stopping the production and eventually exports of the least productive firms). Specifically, the domestic and export specific subsidy are decreasing in the marginal cost, being zero for marginal firms producing for the domestic and the foreign market, negative ('tax') for high cost firms and positive for low cost firms. As a result, competition across firms becomes tougher than in the market in all countries and this brings prices to their lowest average levels and the size of the differentiated good sector within each country to their largest dimension.

This shows that within each country smaller less productive firms should not be protected against larger more productive (foreign) rivals and that national product diversity should not be defended against competition from cheaper imported products. We also show that the first best planner is able to implement the optimal degree of product diversity on a global scale.

However, as differentiated production and export taxes and subsidies are hardly enshrined in any real multilateral trade agreement, we show that the second best solution can be implemented by means of optimal specific per-unit subsidies that are common across all quantities sold in a country and that are larger for those quantities sold in the country whose size is smaller. In this case, the planner is only able to affect the average size of producing firms, and his/her intervention aims at increasing the scale of production and, eventually, exports of all firms selling in a country and it will result in weaker competition in both countries.

Finally, when the planner cannot use lump-sum instruments for firms, he/she can implement the optimal number of firms making use of lump-sum taxes on consumers to finance common specific subsidies to all quantities sold by domestic and foreign firms in a particular country and the magnitude of this type of subsidy is decreasing with the size of the country in which goods are sold.

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7 Appendix A. The optimal outcome in the open economy

7.1 The domestic cutoffs

The first order conditions for the maximization of W in (23) with respect to the number of entrants in country i is given by

$$\frac{\partial W}{\partial N_E^i} = 0;$$

$$\begin{aligned} & -f^i - \sum_{h=1}^{M}\left[\int_0^{c_M^i} \tau^{ih} c q_{ih}(c) dG^i(c)\right] + \alpha \sum_{l=1}^{M}\left[\int_0^{c_M^i} q_{il}(c) dG^i(c)\right] + \\ & -\frac{\gamma}{2}\sum_{l=1}^{M}\left\{\frac{1}{L^l}\left[\int_0^{c_M^i} \left[q_{il}(c)\right]^2 dG^i(c)\right]\right\} + \\ : \quad & -\eta \sum_{l=1}^{M}\left\{\frac{1}{L^l}\left[\sum_{h=1}^{M}\left(N_E^h \int_0^{c_M^h} q_{hl}(c) dG^h(c)\right)\right]\int_0^{c_M^i} q_{il}(c) dG^i(c)\right\} = 0 \end{aligned}$$

that making use of the total quantity consumed in country l

$$Q_l^o = \sum_{h=1}^{M}\left(N_E^h \int_0^{c_M^h} q_{hl}(c) dG^h(c)\right) \tag{65}$$

and of the cutoff rule (24) can be rewritten as follows

$$\begin{aligned} & -f^i + \sum_{l=1}^{M}\left[\int_0^{c_{il}^o} \left(\alpha - \tau^{il} c\right) \frac{L^l}{\gamma} \tau^{il}\left(c_{il}^o - c\right) dG^i(c)\right] + \\ & -\frac{1}{2}\frac{1}{\gamma}\sum_{l=1}^{M} L^l \left[\int_0^{c_{il}^o} \left[\tau^{il}\left(c_{il}^o - c\right)\right]^2 dG^i(c)\right] \\ : \quad & -\frac{1}{\gamma}\sum_{l=1}^{M} L^l \left\{\left[\alpha - \tau^{il} c_{il}^o\right]\int_0^{c_{il}^o} \tau^{il}\left(c_{il}^o - c\right) dG^i(c)\right\} = 0 \end{aligned}$$

Then, integrating the first order condition with respect to the number of entrants using the Pareto distribution in (2), and making use of (25), gives the following expression

$$\sum_{l=1}^{M} L^l \rho^{il} \left(\tau^{ll} c_{D^l}^o\right)^{k+2} = \gamma (k+2)(k+1) \left(c_M^i\right)^k f^i$$

that, together with the analogous expressions derived for the other $M-1$ countries, yields a system of M equations that can be solved using Cramer's rule to find the M equilibrium cutoffs in (26).

7.2 The number of entrants

Substituting Q_l^o from (65) into the cutoff c_{il}^o from (24), yields

$$\tau^{il} c_{il}^o = \alpha - \eta \frac{1}{L^l} \sum_{h=1}^{M} \left(N_E^h \int_0^{c_M^h} q^{hl}(c) dG^h(c) \right)$$

that, making use of (24) and the Pareto distribution (2) together with (25) can be rewritten as

$$\sum_{h=1}^{M} \left(N_E^h \frac{1}{\left(c_M^h\right)^k} \rho^{hl} \right) = \frac{\gamma (k+1)}{\eta} \frac{\left(\alpha - \tau^{ll} c_{D^l}^o\right)}{\left(\tau^{ll} c_{D^l}^o\right)^{k+1}}$$

This gives a system of M linear equations that can be solved using Cramer's rule for the number of entrants in the M countries given in expression (27).

Finally, the total quantity Q_h^o consumed in country h in (65) can be rewritten making use of $q_{lh}^o(c)$ in (24), $dG_{lh}^{lo}(c) = dG^l(c)/G^l\left(c_{lh}^o\right)$, $G^l\left(c_{lh}^o\right) = \left(c_{lh}^o/c_M^l\right)^k$, expression (25), $N_{hl}^o = N_E^{lo} G^l(c_{lh}^o)$ from (12), and $N_h^o = \sum_{l=1}^{M} N_{lh}^o$ from (13), as follows

$$Q_h^o = \frac{L^h}{\gamma} \frac{1}{(k+1)} \tau^{hh} c_{D^h}^o N_h^o \tag{66}$$

Then Q_h^o from (66) can be substituted into the definition of the cutoff c_{lh}^o in (24) that with (25) yields the number of varieties consumed in h in the first best solution

$$N_h^o = \frac{\gamma (k+1)}{\eta} \frac{\alpha - \tau^{hh} c_{D^h}^o}{\tau^{hh} c_{D^h}^o} \tag{67}$$

7.3 Equilibrium vs. optimum

In this subsection we compare the market equilibrium and the unconstrained optimum along different dimensions.

7.3.1 Selection and firm size

Comparing the cutoff in (11) with that in (26) reveals that $c_{D^l}^m = 2^{1/(k+2)} c_{D^l}^o$, which implies $c_{D^l}^o < c_{D^l}^m$. Accordingly, varieties with $c \in [c_{D^l}^o, c_{D^l}^m]$ should not be supplied in the domestic market. Moreover, differences in the strength of selection translate also into the export status, and comparing expressions (11) with (26) together with (8) and (25) reveals that $c^m_{lh} = 2^{1/(k+2)} c^o_{lh}$, which implies $c^o_{lh} < c^m_{lh}$. Accordingly, varieties with $c \in [c^o_{lh}, c^m_{lh}]$ should not be exported in country h from country l. Furthermore, the percentage gap between the market equilibrium and the optimum cutoffs is

$$\frac{c^m_{D^l} - c^o_{D^l}}{c^o_{D^l}} = \frac{c^m_{lh} - c^o_{lh}}{c^o_{lh}} = 2^{\frac{1}{k+2}} - 1 \tag{68}$$

which is affected only by the shape parameter k of the cost distribution, with more evenness (smaller k) leading to a larger percentage gap in the cutoffs between the market equilibrium and the optimum.

Then, making use of (6) and (24), output levels can be rewritten as

$$q^m_{lh}(c) = \frac{L^h}{2\gamma}\tau^{lh}\left(c^m_{lh} - c\right) \quad \text{and} \quad q^o_{lh}(c) = \frac{L^h}{\gamma}\tau^{lh}\left(c^o_{lh} - c\right)$$

Since $c^m_{lh} = 2^{1/(k+2)} c^o_{lh}$ implies $c^o_{lh} < c^m_{lh}$, it is readily seen that $q^m_{lh}(c) > q^o_{lh}(c)$ if and only if $c > \left(2 - 2^{1/(k+2)}\right) c^o_{lh}$, which falls in the relevant interval $[0, c^o_{lh}]$ given that $0 < \left(2 - \sqrt[3]{2}\right) < \left(2 - 2^{1/(k+2)}\right) < 1$. Hence, with respect to the optimum, the market equilibrium undersupplies in h varieties produced in l with low marginal cost, that is with $c \in [0, \left(2 - 2^{1/(k+2)}\right) c^o_{lh})$, and oversupplies varieties with high marginal cost, that is with $c \in (\left(2 - 2^{1/(k+2)}\right) c^o_{lh}, c^m_{lh}]$. Moreover, given that $\left(2 - 2^{1/(k+2)}\right) c^o_{lh}/c^m_{lh} = \left(2 - 2^{1/(k+2)}\right)/2^{1/(k+2)}$, we find that only the shape parameter k of the cost distribution affects within-sector misallocations, that is the overprovision and the underprovision of varieties in both the domestic and the foreign markets. Specifically, more (less) cost-decreasing evenness makes the overprovision of varieties relatively more (less) likely than its underprovision in the market equilibrium.

Turning to average production of firms producing in l for consumers in h computed making use of the Pareto distribution in (2), expressions (6) and (24), we find that

$$\begin{aligned} \bar{q}^m_{lh} &= \int_0^{c^m_{lh}} q^m_{lh}(c) dG^{lm}_{lh}(c) = \frac{L^h}{2\gamma}\frac{1}{k+1}\tau^{lh} c^m_{lh} \\ \bar{q}^o_{lh} &= \int_0^{c^o_{lh}} q^o_{lh}(c) dG^{lo}_{lh}(c) = \frac{L^h}{\gamma}\frac{1}{k+1}\tau^{lh} c^o_{lh} = 2^{\frac{k+1}{k+2}} \bar{q}^m_{lh} \end{aligned} \tag{69}$$

Then, the cutoff ranking $c^o_{lh} < c^m_{lh}$ dictates the average output ranking with $\bar{q}^m_{lh} < \bar{q}^o_{lh}$, and, given that $\left(\bar{q}^o_{lh} - \bar{q}^m_{lh}\right)/\bar{q}^o_{lh} = \left(2^{\frac{k+1}{k+2}} - 1\right)/2^{\frac{k+1}{k+2}}$, only the shape parameter k affects the percentage gap of the average quantities produced for each country in the market and in the optimum case, with more (less) cost-decreasing evenness decreasing (increasing) the percentage gap in average firm size between the market equilibrium and the optimum.

The average quantity sold in h by domestic and foreign firms in the market equilibrium, computed making use of (69) and (8), is

$$\bar{q}^m_h = \frac{\sum_{l=1}^{M} N^m_{lh} \bar{q}^m_{lh}}{\sum_{l=1}^{M} N^m_{lh}} = \frac{L^h}{2\gamma}\frac{1}{k+1}\tau^{hh} c^m_{D^h}, \tag{70}$$

and the average quantity of a variety available in h in the optimum case that making use of (69), (25) and (70) is

$$\bar{q}_h^o = \frac{\sum_{l=1}^{M} N_{lh}^o \bar{q}_{lh}^o}{\sum_{l=1}^{M} N_{lh}^o} = 2^{\frac{k+1}{k+2}} \bar{q}_h^m, \tag{71}$$

and it can be readily verified that $\bar{q}_h^o > \bar{q}_h^m$. The percentage gap between the average supply of a variety in a country in the market equilibrium and the optimum is

$$\frac{\bar{q}_h^o - \bar{q}_h^m}{\bar{q}_h^o} = \frac{2^{\frac{k+1}{k+2}} - 1}{2^{\frac{k+1}{k+2}}} = \frac{\bar{q}_{lh}^o - \bar{q}_{lh}^m}{\bar{q}_{lh}^o}$$

which is affected only by the shape parameter k of the cost distribution, with more (less) evenness leading to a smaller (larger) percentage gap in the average supply of varieties in each country between the market equilibrium and the optimum.

7.3.2 The total size of the differentiated sector

Then, making use of (70) and of N_h^m from (16), the total quantities sold of the differentiated varieties in h in the market evaluates to

$$N_h^m \bar{q}_h^m = \frac{L^h}{\eta} \left(\alpha - \tau^{hh} c_{D^h}^m \right) \tag{72}$$

that can be used to evaluate the efficiency of the dimension of the differentiated goods sector (that is, the total supply of differentiated varieties) in the market for each country h. Indeed, this can be compared with the corresponding dimension of the differentiated good sector for the first best planner $N_h^o \bar{q}_h^o$ obtained making use of (71), (67) and $c_{D^l}^m = 2^{1/(k+2)} c_{D^l}^o$,

$$N_h^o \bar{q}_h^o = \frac{L^h}{\eta} \left(\alpha - \tau^{hh} c_{D^h}^o \right) \tag{73}$$

Hence, $c_{D^h}^o < c_{D^h}^m$ implies $N_h^o \bar{q}_h^o > N_h^m \bar{q}_h^m$, and, therefore, in the market equilibrium the total supply of differentiated varieties is smaller than optimal in each country. Using (72) and (73), we can write that

$$\frac{N_h^o \bar{q}_h^o - N_h^m \bar{q}_h^m}{N_h^o \bar{q}_h^o} = \frac{c_{D^h}^m - c_{D^h}^o}{c_{D^h}^o} \frac{\tau^{hh} c_{D^h}^o}{\alpha - \tau^{hh} c_{D^h}^o}$$

In this expression, the percentage gap between the market equilibrium and the optimum cutoffs $(c_{D^h}^m - c_{D^h}^o)/c_{D^h}^o$ is only affected by k, while larger values of the market size L^h and smaller values of γ and of $\sum_{h=1}^{M} \left[f^h \left(c_M^h \right)^k |C_{hl}| \right] / |P|$ (which is determined by the relative value of all $f^l \left(c_M^l \right)^k$ defining the technology available in all countries and trade barriers) lead to smaller $c_{D^h}^o$ and, consequently, smaller $\tau^{hh} c_{D^h}^o / \left(\alpha - \tau^{hh} c_{D^h}^o \right)$. Accordingly, larger values of L^h and of

$\sum_{h=1}^{M}\left[f^h\left(c_M^h\right)^k|C_{hl}|\right]/|P|$ imply smaller values for the ratio $\left(N_h^o\overline{q}_h^o - N_h^m\overline{q}_h^m\right)/N_h^o\overline{q}_h^o$, while the impact of k is ambiguous.

Finally, the comparison on product variety between N_h^o in (67) and N_h^m in (16), and the comparison on entry between N_E^{lo} in (27) and N_E^{lm} in (17), are presented in the text of the paper.

8 Appendix B. The constrained planners in the open economy

The second and the third best planners can not enforce marginal cost pricing by affecting the choices of each firm individually. However, the second best planner can affect firms choices controlling the cutoff and the number of firms that operate in the markets, while the third best planner can only control the number of firms that operate in the markets.

Hence, the second best planner maximizes (23) with respect to c_{ij} and N_E^i $\forall i,j=1,....,M$, while the third best planner maximizes the same expression only with respect to N_E^i $\forall i=1,....,M$. Both maximizations are subject to: the profit maximizing quantity $q_{hl}(c)$ in (36) and the selection conditions in (12). In addition, the third best planner has to consider the additional constraint given by the free entry condition for each country l in (9).

In general, substituting quantities from (36) into (23) and making use of (2), allows to rewrite the problem of the constrained planner as the maximization of

$$\begin{aligned} W &= \sum_{l=1}^{M}U^l(L^l)=\sum_{l=1}^{M}\overline{q}_0^lL^l+\sum_{l=1}^{M}L^l-\sum_{l=1}^{M}f^lN_E^l+ \qquad (74)\\ &-\sum_{l=1}^{M}N_E^l\frac{1}{\left(c_M^l\right)^k}\sum_{h=1}^{M}\left[\frac{L^h}{2\gamma}\left(\tau^{lh}\right)^2\left(c^{lh}\right)^{k+2}\frac{k}{(k+2)(k+1)}\right]+\\ &+\alpha\sum_{l=1}^{M}\frac{L^l}{2\gamma}\left\{\sum_{h=1}^{M}\left[\tau^{hl}N_E^h\frac{1}{\left(c_M^h\right)^k}\left(c^{hl}\right)^{k+1}\frac{1}{(k+1)}\right]\right\}+\\ &-\frac{1}{4}\sum_{l=1}^{M}\left\{\frac{L^l}{\gamma}\sum_{h=1}^{M}\left[\left(\tau^{hl}\right)^2N_E^h\frac{1}{\left(c_M^h\right)^k}\left(c^{hl}\right)^{k+2}\frac{1}{(k+2)(k+1)}\right]\right\}+\\ &-\frac{\eta}{8\gamma^2}\sum_{l=1}^{M}\left\{L^l\left[\sum_{h=1}^{M}\left(\tau^{hl}N_E^h\frac{1}{\left(c_M^h\right)^k}\left(c^{hl}\right)^{k+1}\frac{1}{(k+1)}\right)\right]^2\right\} \end{aligned}$$

with respect to c_{ij} and N_E^i $\forall i,j=1,....,M$ for the second best planner, and with respect to N_E^i $\forall i=1,....,M$ for the third best planner.

8.1 The second best planner

The first order condition for the maximization of (74) with respect to c_{ij}, $\frac{\partial W}{\partial c_{ij}}=0$, is

$$-N_E^i \frac{1}{\left(c_M^i\right)^k} \frac{L^j}{2\gamma} \left(\tau^{ij}\right)^2 \left(c_{ij}^s\right)^{k+1} \frac{k}{(k+1)} +$$

$$+\alpha \frac{L^j}{2\gamma} \tau^{ij} N_E^i \frac{1}{\left(c_M^i\right)^k} \left(c_{ij}^s\right)^k +$$

$$-\frac{1}{4} \frac{L^j}{\gamma} \left(\tau^{ij}\right)^2 N_E^i \frac{1}{\left(c_M^i\right)^k} \left(c_{ij}^s\right)^{k+1} \frac{1}{(k+1)} +$$

$$-\frac{\eta}{4\gamma^2} L^j \left[\sum_{h=1}^{M} \left(\tau^{hj} N_E^h \frac{1}{\left(c_M^h\right)^k} \left(c_{hj}^s\right)^{k+1} \frac{1}{(k+1)}\right)\right] \tau^{ij} N_E^i \frac{1}{\left(c_M^i\right)^k} \left(c_{ij}^s\right)^k = 0$$

that can be simplified as follows

$$\sum_{h=1}^{M} \left(\tau^{hj} N_E^h \frac{1}{\left(c_M^h\right)^k} \left(c_{hj}^s\right)^{k+1}\right) = \frac{2\gamma (k+1)}{\eta} \left[\alpha - \frac{1}{2} \frac{2k+1}{k+1} \tau^{ij} c_{ij}^s\right] \tag{75}$$

Given that previous expression holds also for $i = j$, and we can substitute j to i to have $\tau^{jj} c_{D^j}^s$ into the right hand side at the place of $\tau^{ij} c_{ij}^s$, this implies that the relationship between the cutoffs of the two types of firms (domestic and foreign) selling in the same market has to be

$$c_{ij}^s = \frac{\tau^{jj}}{\tau^{ij}} c_{D^j}^s \qquad \forall i, j = 1, ..., M$$

Making use of $c_{ij}^s = \tau^{jj} c_{D^j}^s / \tau^{ij}$, expression (75) can be rewritten as follows

$$\sum_{h=1}^{M} \left(\rho^{hj} N_E^h \frac{1}{\left(c_M^h\right)^k}\right) = \frac{2\gamma (k+1)}{\eta} \frac{\left(\alpha - \frac{1}{2} \frac{2k+1}{k+1} \tau^{jj} c_{D^j}^s\right)}{\left(\tau^{jj} c_{D^j}^s\right)^{k+1}}$$

This gives a system of M linear equations that can be solved using Cramer's rule for the number of entrants in the M countries, with

$$N_E^{ls} = \frac{2\gamma (k+1) \left(c_M^l\right)^k \sum_{h=1}^{M} \left\{ \frac{\left(\alpha - \frac{1}{2} \frac{2k+1}{k+1} \tau^{hh} c_{D^h}^s\right)}{\left(\tau^{hh} c_{D^h}^s\right)^{k+1}} \left|C_{lh}\right| \right\}}{\eta \left|P\right|} \tag{76}$$

The first order condition for the maximization of (74) with respect to N_E^i, $\frac{\partial W}{\partial N_E^i} = 0$, is

$$-f^i - \frac{1}{\left(c_M^i\right)^k} \sum_{h=1}^{M} \left[\frac{L^h}{2\gamma} \left(\tau^{ih}\right)^2 \left(c_{ih}^s\right)^{k+2} \frac{k}{(k+2)(k+1)}\right] + \tag{77}$$

$$+\alpha \sum_{l=1}^{M} \left[\frac{L^l}{2\gamma} \tau^{il} \frac{1}{\left(c_M^i\right)^k} \left(c_{il}^s\right)^{k+1} \frac{1}{(k+1)}\right] +$$

$$-\frac{1}{4} \sum_{l=1}^{M} \left[\frac{L^l}{\gamma} \left(\tau^{il}\right)^2 \frac{1}{\left(c_M^i\right)^k} \left(c_{il}^s\right)^{k+2} \frac{1}{(k+2)(k+1)}\right] +$$

$$-\frac{\eta}{4\gamma^2} \sum_{l=1}^{M} \left\{ L^l \left[\sum_{h=1}^{M} \left(\tau^{hl} N_E^h \frac{1}{\left(c_M^h\right)^k} \left(c_{hl}^s\right)^{k+1} \frac{1}{(k+1)}\right)\right] \frac{\tau^{il} \left(c_{il}^s\right)^{k+1}}{(k+1) \left(c_M^i\right)^k} \right\} = 0$$

that making use of $c_{ij}^{s} = \tau^{jj} c_{D^j}^{s} / \tau^{ij}$ and (75) can be rewritten and simplified as follows

$$\sum_{l=1}^{M} \left[L^l \rho^{il} \left(\tau^{ll} c_{D^l}^{s} \right)^{k+2} \right] = \frac{4\gamma (k+2)(k+1)^2 \left(c_M^i \right)^k f^i}{2k+1}$$

This yields a system of M equations that can be solved to find the M equilibrium domestic cutoffs using Cramer's rule:

$$c_{D^l}^{s} = \frac{1}{\tau^{ll}} \left\{ \frac{4\gamma (k+2)(k+1)^2}{(2k+1) L^l} \frac{\sum_{h=1}^{M} \left[f^h \left(c_M^h \right)^k |C_{hl}| \right]}{|P|} \right\}^{\frac{1}{k+2}} \tag{78}$$

8.2 The third best planner

The first order condition for the maximization of (74) with respect to N_E^i, $\frac{\partial W}{\partial N_E^i} = 0$, for the third best planner corresponds to the first order condition with respect to N_E^i for the second best planner given in expression (77) where c_{il}^s is replaced by c_{il}^m, that is the cutoff for the third best planner that correspond to that of the market. Then substituting f^i from (9), making use of the Pareto distribution in (2) and of the relationship between the cutoffs in (8), and simplifying becomes

$$\sum_{l=1}^{M} \left\{ L^l \rho^{il} \left(\tau^{ll} c_{D^l}^{m} \right)^{k+1} \left[\left(\tau^{ll} c_{D^l}^{m} \right)^{k+1} \sum_{h=1}^{M} \left(\rho^{hl} N_E^h \frac{1}{\left(c_M^h \right)^k} \right) - \frac{2\gamma (k+1)}{\eta} \left(\alpha - \frac{1}{2} \frac{2k+3}{k+2} \tau^{ll} c_{D^l}^{m} \right) \right] \right\} = 0$$

Previous equation has a solution when, for all l, the expression in the square brackets is equal to zero, that is when

$$\sum_{h=1}^{M} \left(\rho^{hl} N_E^h \frac{1}{\left(c_M^h \right)^k} \right) = \frac{2\gamma (k+1)}{\eta} \frac{\left[\alpha - \frac{1}{2} \frac{2k+3}{k+2} \left(\tau^{ll} c_{D^l}^{m} \right) \right]}{\left(\tau^{ll} c_{D^l}^{m} \right)^{k+1}}$$

This gives a system of M linear equations that can be solved using Cramer's rule for the number of entrants in the M countries with

$$N_E^{lt} = \frac{2\gamma (k+1) \left(c_M^l \right)^k \sum_{h=1}^{M} \left[\frac{\left(\alpha - \frac{1}{2} \frac{2k+3}{k+2} \tau^{hh} c_{D^h}^{m} \right)}{\left(\tau^{hh} c_{D^h}^{m} \right)^{k+1}} |C_{lh}| \right]}{\eta |P|} \tag{79}$$

9 C. Implementation of the constrained solutions in the open economy

Making use of (37), the average price set in l by firms producing in h is

$$\bar{p}_{hl} = \frac{2k+1}{2(k+1)} \tau^{hl} c_{hl} - s^{hl} \tag{80}$$

Hence, making use of (80) and (12), the average price of varieties sold in l is

$$\bar{p}_l = \frac{\sum_{h=1}^{M} N_{hl}\bar{p}_{hl}}{\sum_{h=1}^{M} N_{hl}} = \frac{2k+1}{2(k+1)} \frac{\sum_{h=1}^{M} N_E^h \left(\frac{c_{hl}}{c_M^h}\right)^k \tau^{hl} c_{hl}}{\sum_{h=1}^{M} N_E^h \left(\frac{c_{hl}}{c_M^h}\right)^k} - \frac{\sum_{h=1}^{M} N_E^h \left(\frac{c_{hl}}{c_M^h}\right)^k s^{hl}}{\sum_{h=1}^{M} N_E^h \left(\frac{c_{hl}}{c_M^h}\right)^k} \tag{81}$$

Then, given that for both the constrained planners hold $s^{il} = s^{jl} = s^l$ and $c_{il} = \tau^{ll} c_{D^l}/\tau^{il}$, $\bar{p}_l$ in (81) can be rewritten as

$$\bar{p}_l = \frac{2k+1}{2(k+1)} \tau^{ll} c_{D^l} - s^l \tag{82}$$

Then, substituting $c_{hl} = \tau^{ll} c_{D^l}/\tau^{hl}$ into (35) with $s^{hl} = s^l$ gives

$$p_{\max}^l = \tau^{ll} c_{D^l} - s^l \tag{83}$$

Substituting $p_{\max}^l$ from (83) and $\bar{p}_l$ from (82) into the definition of $p_{\max}^l$ in the zero cutoff profit condition (5) yields

$$N_l = \frac{2\gamma(k+1)}{\eta} \frac{\left(\alpha - \tau^{ll} c_{D^l} + s^l\right)}{\tau^{ll} c_{D^l}} \tag{84}$$

Finally, using (12), (13) and $c_{hl} = \tau^{ll} c_{D^l}/\tau^{hl}$ gives

$$N_l = \left(\tau^{ll} c_{D^l}\right)^k \sum_{h=1}^{M} \rho^{hl} N_E^h \left(\frac{1}{c_M^h}\right)^k$$

that can be used with (84) to find

$$\sum_{h=1}^{M} \left(\rho^{hl} N_E^h \frac{1}{\left(c_M^h\right)^k}\right) = \frac{2\gamma(k+1)}{\eta} \frac{\left(\alpha - \tau^{ll} c_{D^l} + s^l\right)}{\left(\tau^{ll} c_{D^l}\right)^{k+1}}$$

This gives a system of M linear equations that can be solved using Cramer's rule for the number of entrants in the M countries

$$N_E^l = \frac{2\gamma(k+1)\left(c_M^l\right)^k \sum_{h=1}^{M} \left[\frac{\left(\alpha - \tau^{hh} c_{D^h} + s^h\right)}{\left(\tau^{hh} c_{D^h}\right)^{k+1}} |C_{lh}|\right]}{\eta |P|} \tag{85}$$

Notice that the expression for N_E^l in (85) is common for both the second and the third best planner. However, the expressions for the cutoffs c_{D^h} and the subsidies s^h differ for the two planners.

9.1 The implementation of the second best solution

The solution N_E^l for the second best planner in (76) can be implemented setting subsidies s^h for all countries in such a way that the expression for N_E^l in (76) and (85) are equal. This requires

$$\sum_{h=1}^{M} \left[\frac{\left(\alpha - \tau^{hh} c_{D^h}^s + s^h\right)}{\left(\tau^{hh} c_{D^h}^s\right)^{k+1}} |C_{lh}|\right] = \sum_{h=1}^{M} \left[\frac{\left(\alpha - \frac{1}{2}\frac{2k+1}{k+1} \tau^{hh} c_{D^h}^s\right)}{\left(\tau^{hh} c_{D^h}^s\right)^{k+1}} |C_{lh}|\right]$$

for all $l = 1, ..., M$. This implies

$$\sum_{h=1}^{M}\left[\frac{\left(s^h - \frac{1}{2(k+1)}\tau^{hh}c^s_{D^h}\right)}{\left(\tau^{hh}c^s_{D^h}\right)^{k+1}}|C_{lh}|\right] = 0$$

which is satisfied when

$$s^h = \frac{1}{2(k+1)}\tau^{hh}c^s_{D^h} \quad \forall h = 1, ..., M$$

Then, the second best planner uses lump-sum entry taxes per entrant in each country h, $\left(T^h\right)^s$, to set the cutoffs. Specifically, the 'free entry condition' for firms producing in h can be stated as

$$\sum_{l=1}^{M}\left[\int_0^{c^{hl}}\frac{L^l}{4\gamma}\left(\tau^{hl}\right)^2\left(c_{hl}-c\right)^2 dG^h(c)\right] = f^h + \left(T^h\right)^s$$

that making use of the Pareto distribution (2) and $c^s_{hl} = \tau^{ll}c^s_{D^l}/\tau^{hl}$ can be rewritten as

$$\sum_{l=1}^{M}\left[\rho^{hl}L^l\left(\tau^{ll}c^s_{D^l}\right)^{k+2}\right] = 2\gamma\left(c^h_M\right)^k(k+2)(k+1)\left[f^h + \left(T^h\right)^s\right]$$

This yields a system of M equations that can be solved using Cramer's rule to find the M equilibrium cutoffs

$$c^s_{D^l} = \frac{1}{\tau^{ll}}\left\{\frac{2\gamma(k+2)(k+1)}{L^l}\frac{\sum_{h=1}^{M}\left\{\left(c^h_M\right)^k\left[f^h + \left(T^h\right)^s\right]|C_{hl}|\right\}}{|P|}\right\}^{\frac{1}{k+2}} \tag{86}$$

Specifically, the solutions $c^s_{D^l}$ for the second best planner in (86) can be used to set $T^h \forall h = 1, ..., M$ to implement the solutions in (78). The two systems give the same solutions for the cutoff of country l if and only if the following equation holds

$$\sum_{h=1}^{M}\left[\left(c^h_M\right)^k\left(f^h + \left(T^h\right)^s\right)|C_{hl}|\right] = \frac{2(k+1)}{2k+1}\sum_{h=1}^{M}\left[f^h\left(c^h_M\right)^k|C_{hl}|\right]$$

This must be true for all M countries (so there is an equation of this type for all $l = 1, ...M$) and it can be simplified as

$$\sum_{h=1}^{M}\left[\left(c^h_M\right)^k\left(\left(T^h\right)^s - \frac{f^h}{2k+1}\right)|C_{hl}|\right] = 0$$

which is satisfied when

$$\left(T^h\right)^s = \frac{f^h}{2k+1} \quad \forall h = 1, ..., M$$

9.2 The implementation of the third best solution

We recall that the cutoffs for the third best planner are those of the market. The solution N_E^l for the third best planner in (79) can be implemented setting subsidies $\left(s^h\right)^t$ for all countries in such a way that the expression for N_E^l in (79) and (85) are equal, which requires

$$\sum_{h=1}^{M}\left\{\frac{\left[\alpha-\tau^{hh}c_{D^h}^{m}+\left(s^h\right)^t\right]}{\left(\tau^{hh}c_{D^h}^{m}\right)^{k+1}}\left|C_{lh}\right|\right\}=\sum_{h=1}^{M}\left[\frac{\left(\alpha-\frac{1}{2}\frac{2k+3}{k+2}\tau^{hh}c_{D^h}^{m}\right)}{\left(\tau^{hh}c_{D^h}^{m}\right)^{k+1}}\left|C_{lh}\right|\right]$$

for all $l=1,...,M$. This implies

$$\sum_{h=1}^{M}\left\{\frac{\left[\left(s^h\right)^t-\frac{1}{2(k+2)}\tau^{hh}c_{D^h}^{m}\right]}{\left(\tau^{hh}c_{D^h}^{m}\right)^{k+1}}\left|C_{lh}\right|\right\}=0$$

which is satisfied when

$$\left(s^h\right)^t=\frac{1}{2\left(k+2\right)}\tau^{hh}c_{D^h}^{m}\quad\forall h=1,....,M$$