More than just the mean : Modeling intra-individual variability with the mixed-effects location scale model

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ISSBD workshop – Geneva University

September 3-5th, 2015

A fruitful convergence

From a methodological point of view : “Variances are not always nuisance parameters”

Understanding the structure of variability and estimating its different components is as central as understanding the mean structure.

From a psychological point of view : short-term intra-individual variability can be a conceptual tool

We, as psychologists, use quantifications and models of intra-individual variability to measure and describe human functional and dynamic characteristics.

This reinforces the need to a deeper investigation on intra-individual variability above and beyond the normative view of mean values and heterogeneity among subjects.

Plan

1. Quantifying intra-individual variability
2. Modeling between-subject and within-subject variances using mixed-effects location scale models
3. A Bayesian model for estimating intra-individual variability as a predictor

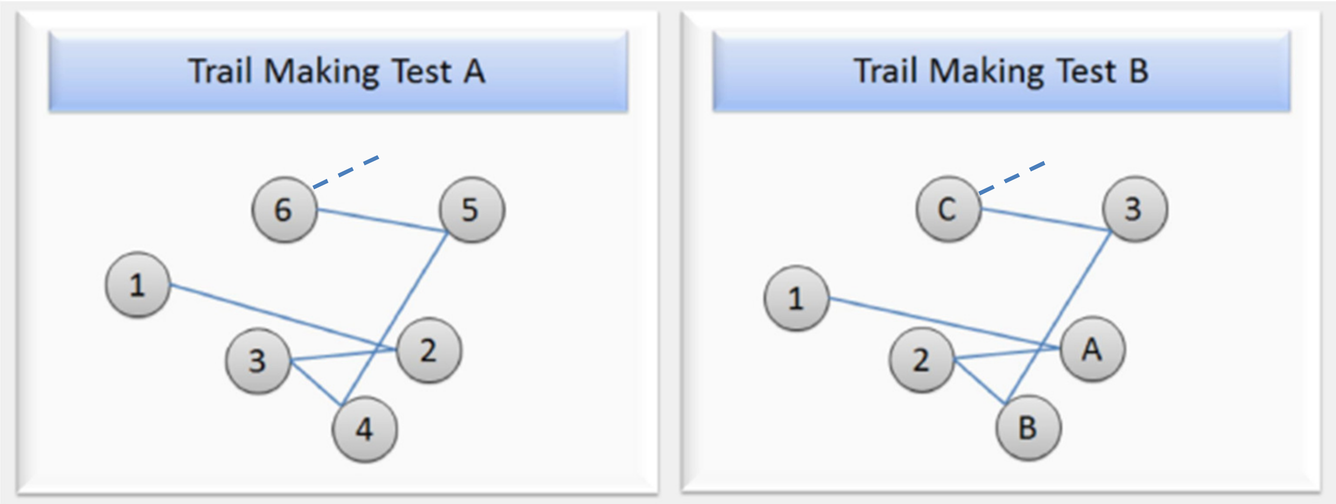
The analyzed data

Participants and procedure :

Thirty-five adults aged 61-97 years (mean age 71 years), MMSE > 25. Eight measurement occasions, every two weeks (data collected by J. Lebahar, Phd).

At each measurement occasion :

Trail Making Test

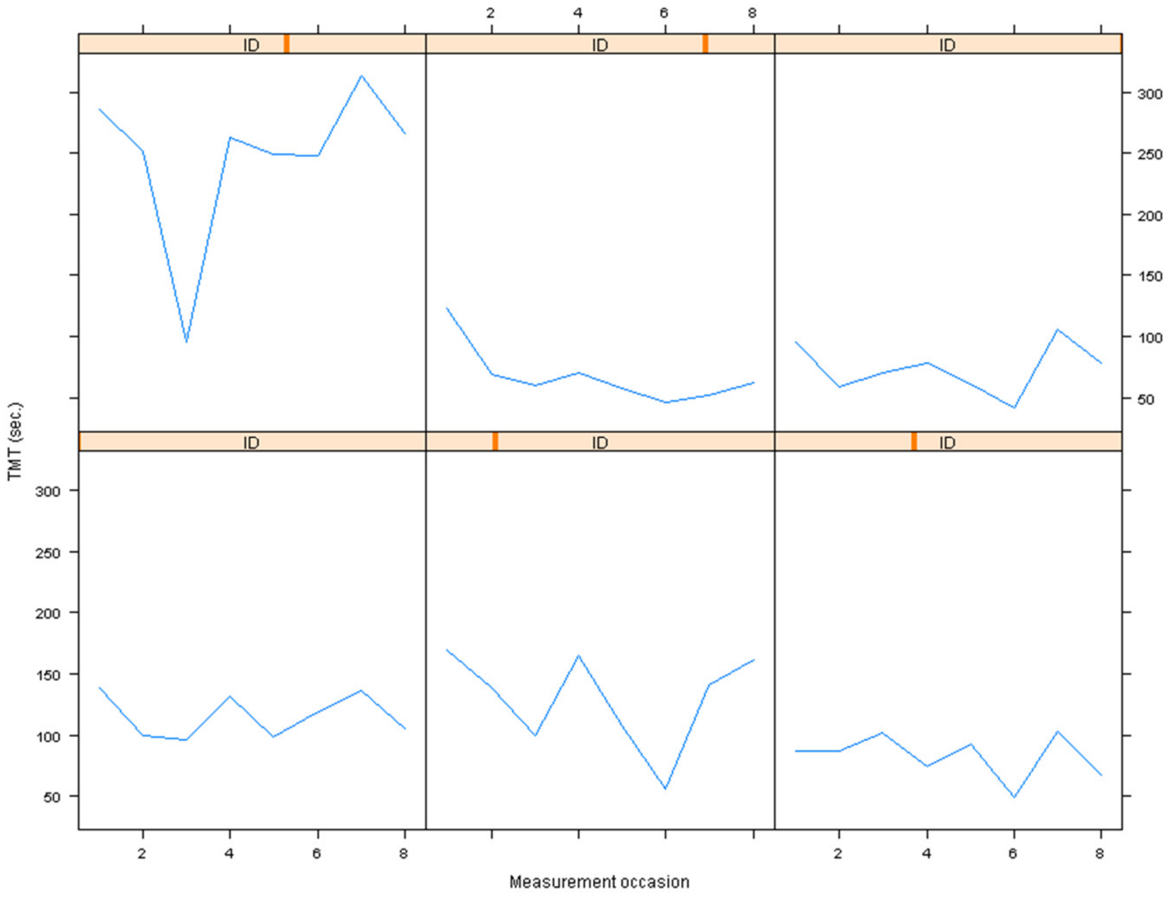


Realization time in sec.

The analyzed data

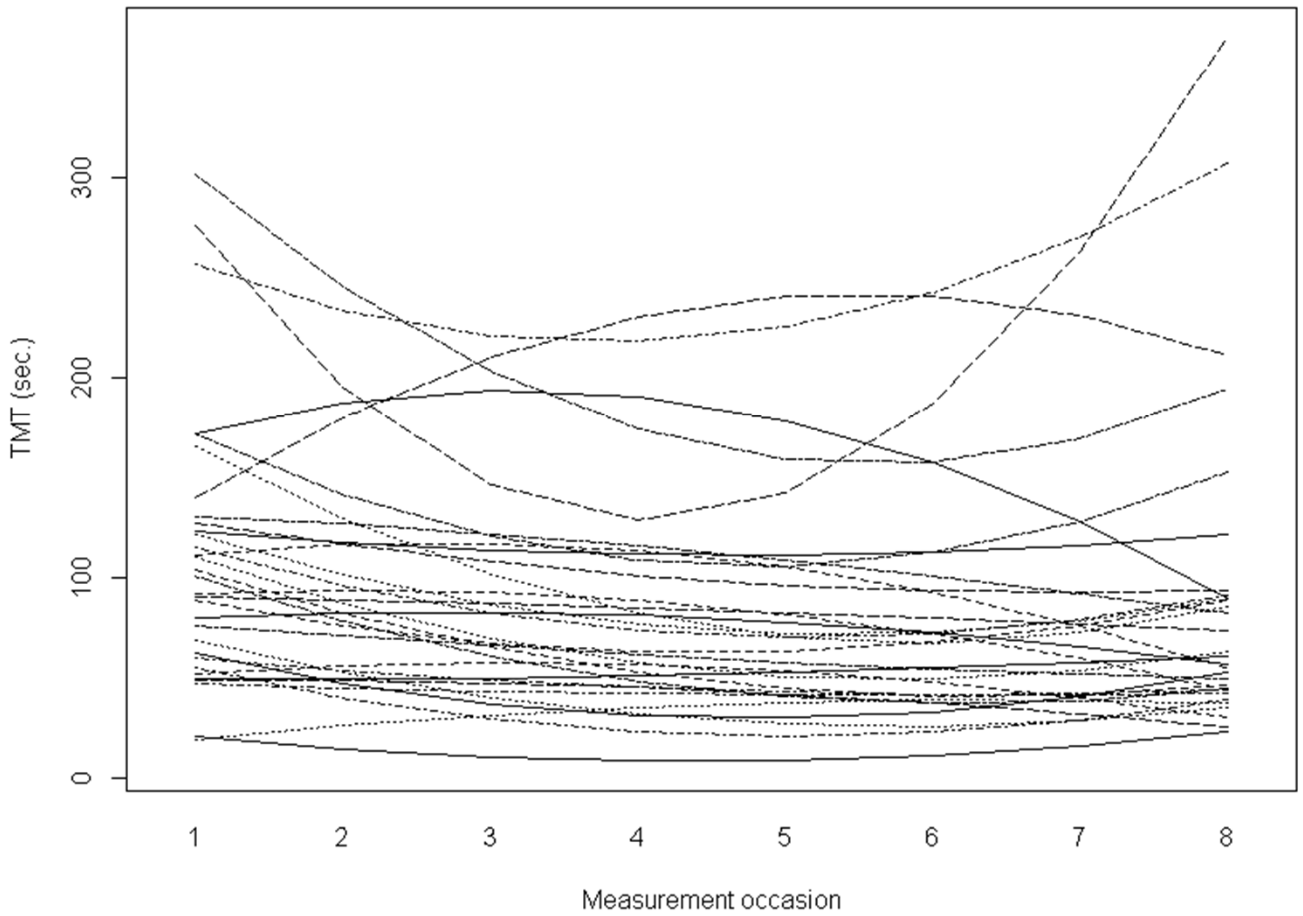
Intra-individual variability of Realization Time for the Trail Making Test

The first 6 participants



The analyzed data

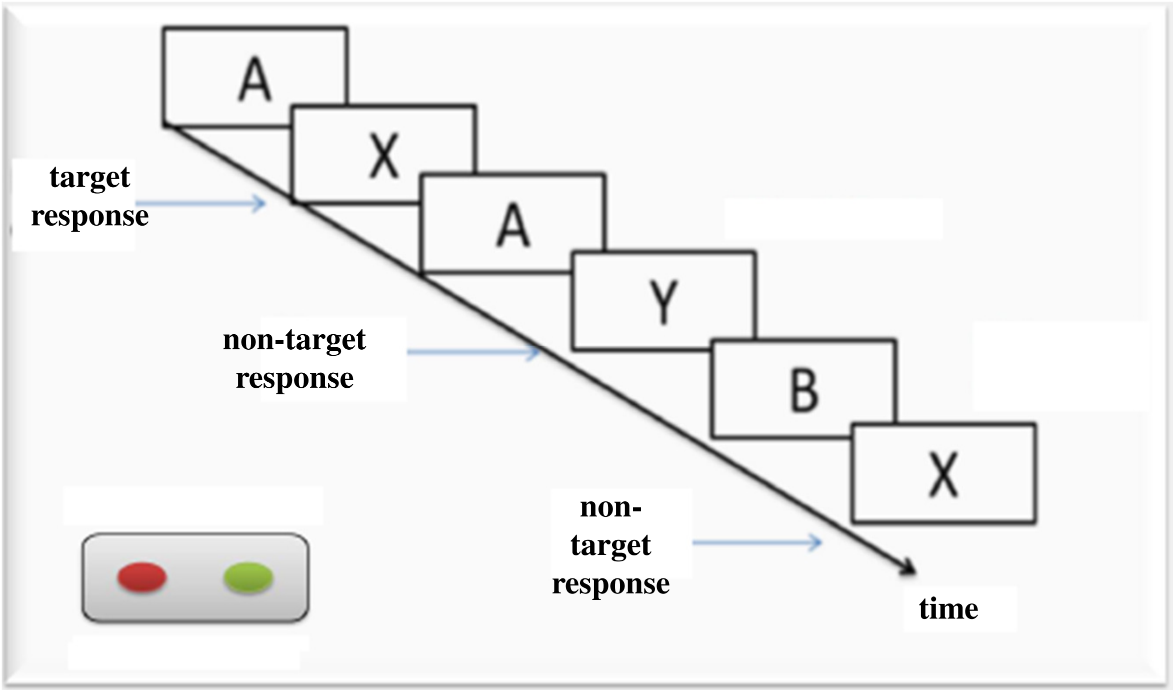
IDs in IIV in Realization Time for the Trail Making Test



The analyzed data

At each measurement occasion :

The AX-Continuous Performance Test : 100 trials (40 AX, 20 AY, 20 BX, 20 BY) : RT for correct responses.

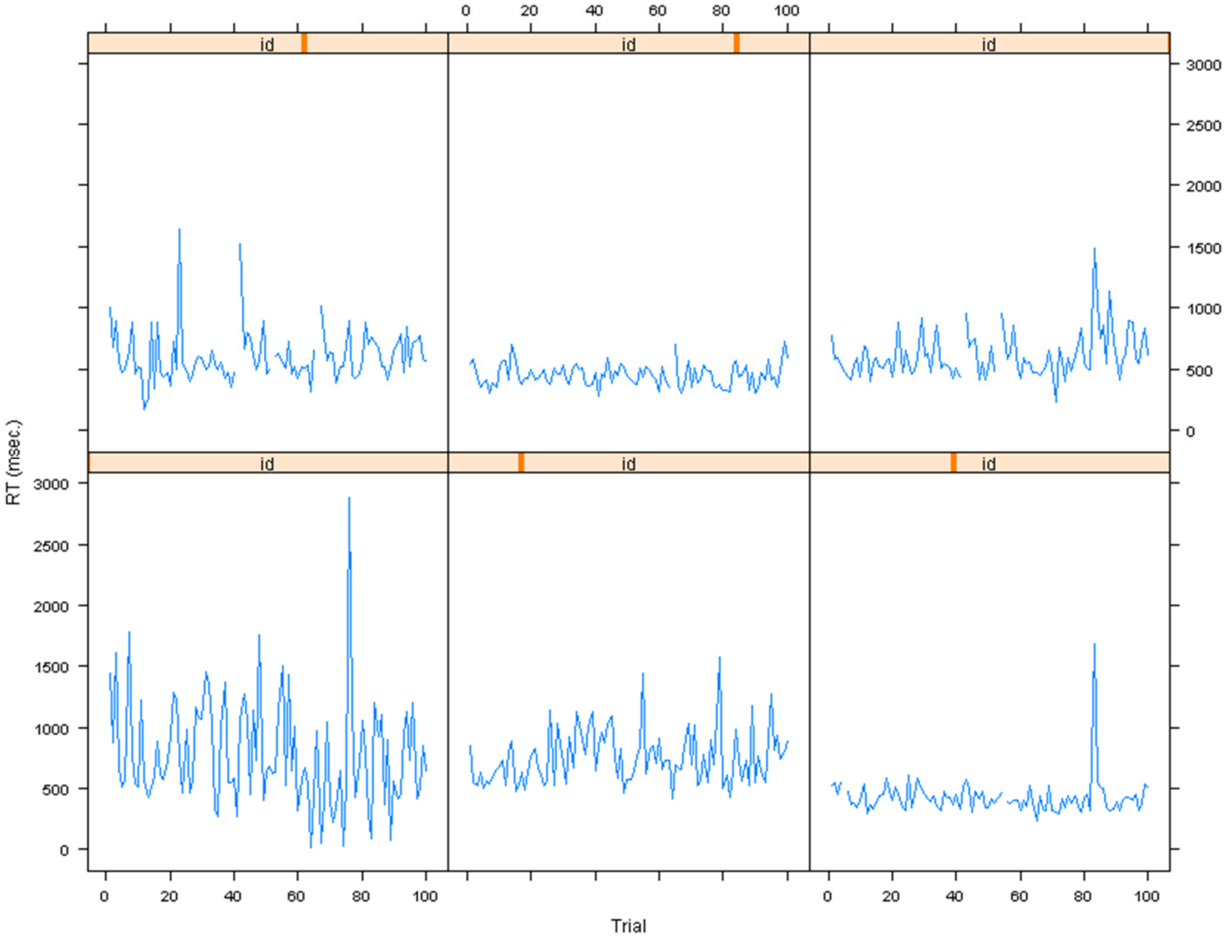


Reaction time (RT) in sec.

Data from an illustrative example

Intra-individual variability of Reaction Time for AX-CPT (session 3)

The first 6 participants



Quantifying intra-individual variability

A two-stage approach

Stage 1

Estimation of one or another index of IIV on the basis of observations at the individual level.

→ Each estimate has its own error of estimation.

Stage 2

IIV as an outcome :

- regression of IIV on one or several covariates of interest.

IIV as a predictor :

- regression of one or several outcomes of interest on IIV.

→ More often, the SE of estimation will be downwardly biased and the probability of rejecting a null hypothesis although it is true (Type I error) will be inflated.

Some conventional methods for quantifying intra-individual variability

The amplitude of intra-individual variability (*ISD*)

$Y_{t,i}$: the observed score of individual *i* at occasion *t* ;

T_i : the number of measurement occasions for individual *i* ;

$$\text{raw-score } ISD_i = \sqrt{\frac{\Sigma(y_{t,i} - y_{i.})^2}{T_i - 1}} \text{ and raw-score } ISD_i^2.$$

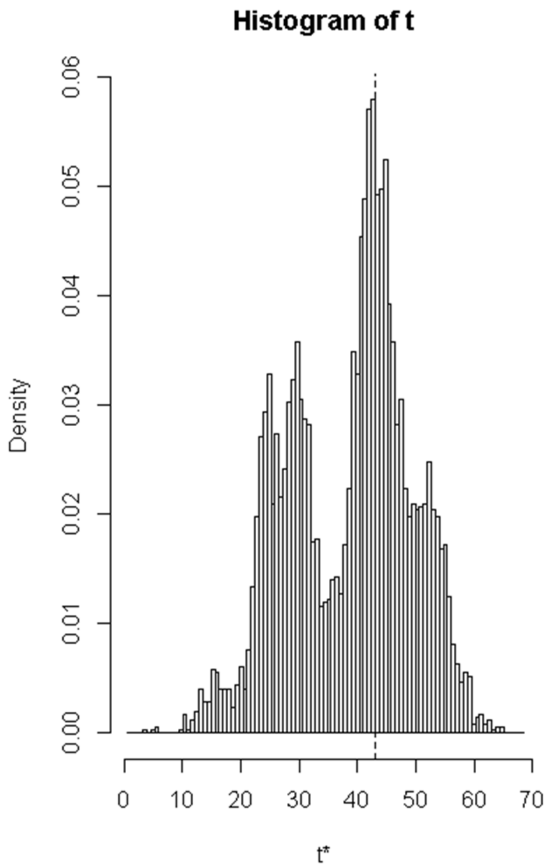
No consideration of temporal order or serial correlation.

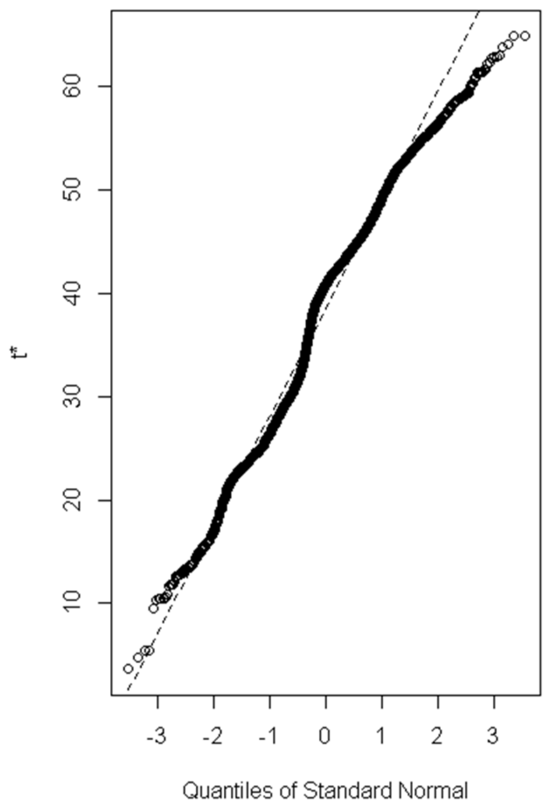
Practical

→ An application with R

Some conventional methods for quantifying intra-individual variability

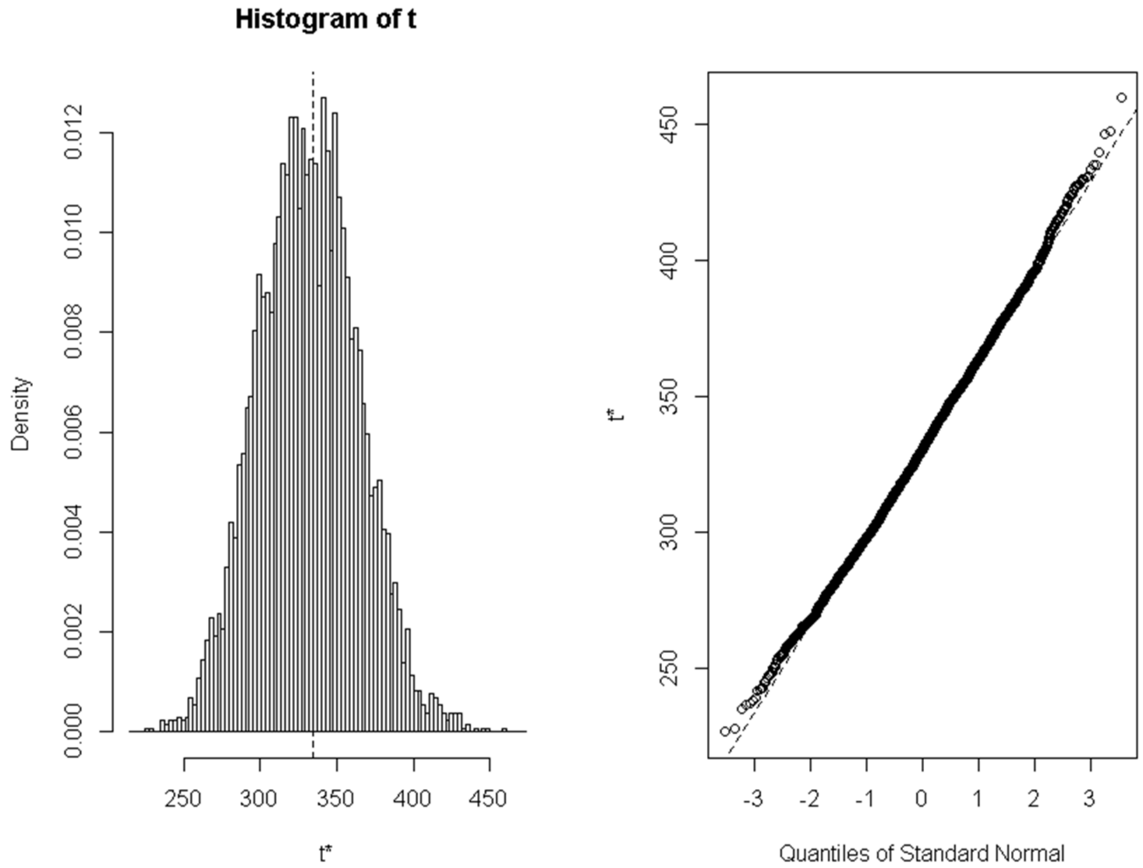
TMTb : empirical distribution of *ISD* for subject 2 (5000 bootstrap samples); ISD = 43.04, 95% IC [17.63, 56.19].





Some conventional methods for quantifying intra-individual variability

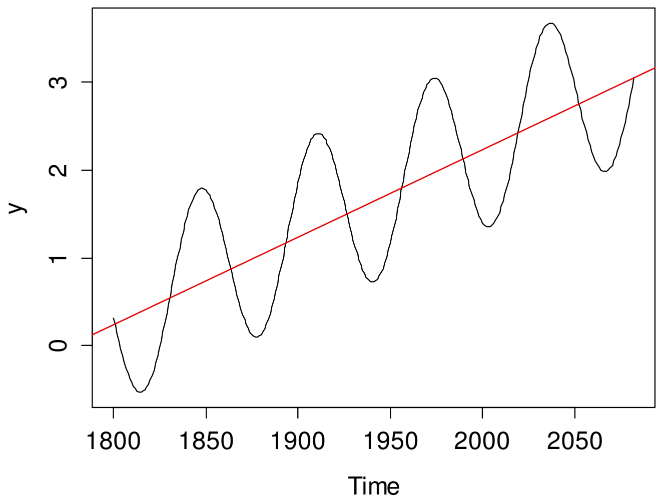
AX-CPT Reaction Time : empirical distribution of *ISD* for subject 1 (5000 bootstrap samples); ISD = 334, 95% IC [269, 396].



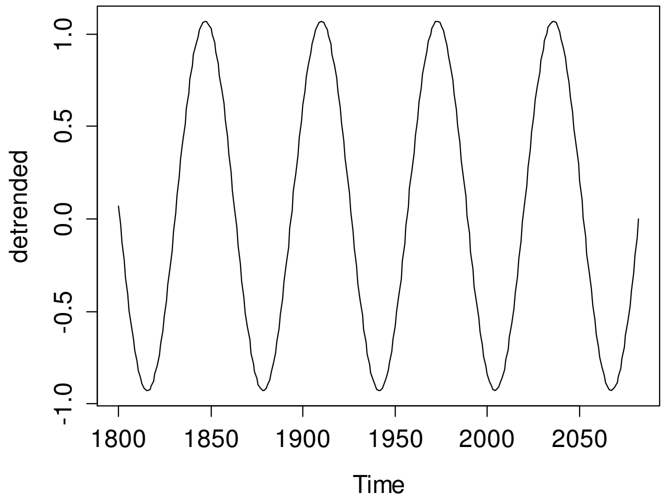
Some conventional methods for quantifying intra-individual variability

ISD and ISD^2 are sensitive to systematic intra-individual change (trends).

→ detrending the data :



$ISD_y = 1.079$



detrended $ISD_y = 0.704$

Raw-score *ISD* and ISD^2 are likely to be correlated with individual mean scores.

Raw-score Individual Coefficient of Variation (*ICV*) and detrended *ICV*.

Some conventional methods for quantifying intra-individual variability

The temporal dependency in the data

The autocorrelation or the degree to which current observations are correlated with previous observations.

The autocorrelations can be obtained for different lags.

$\rho_i(\tau)$ is the autocorrelation at lag τ ($\tau = 1, 2, \ldots$) with :

$$\hat{\rho}_i(\tau) = \frac{\hat{\gamma}_i(\tau)}{ISD_i^2}$$

$\hat{\gamma}_i(\tau)$ is the auto-covariance at lag τ defined by the covariance between pairs of observations that are formed by $y_{t,i}$ and $y_{t+\tau,i}$.

Practical

→ An application with R

Some conventional methods for quantifying intra-individual variability

The degree of temporal instability

Refers to occasion-to-occasion squared differences and combines both the amplitude of intra-individual variability and the temporal dependency.

The mean square successive difference (*MSSD*) :

$$MSSD_i = \frac{1}{T_i - 1} \sum_{t=1}^{T-1_i} (y_{t+1,i} - y_{t,i})^2$$

Given stationary data, $MSSD_i = 2ISD_i^2(1 - \hat{\rho}_i(1))$.

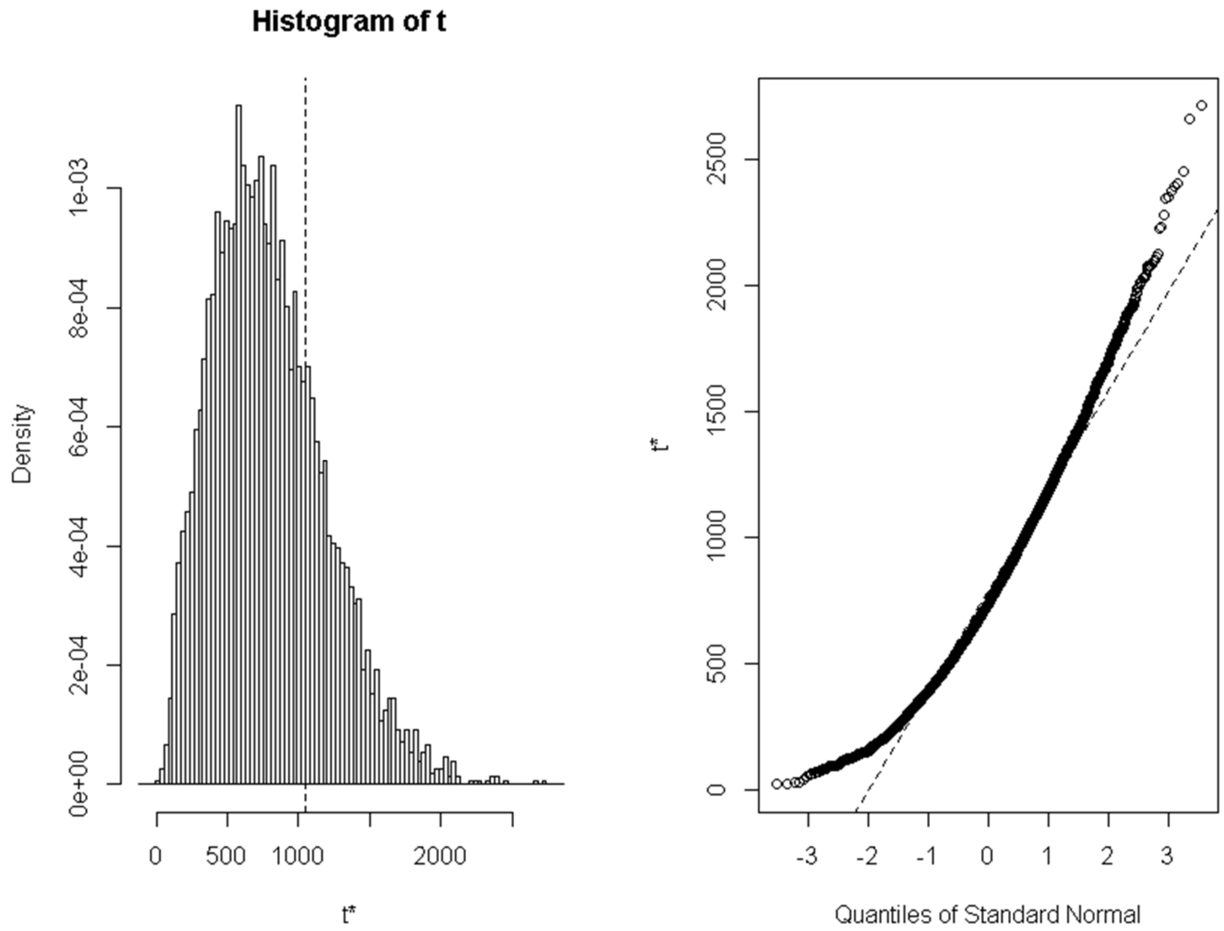
The MSSD is a lag-dependent measure.

Practical

→ An application with R

Some conventional methods for quantifying intra-individual variability

TMTb : empirical distribution of mssd for subject 1 (5000 bootstrap samples); mssd = 1050, 95% IC [166, 1683].



Modeling between-subject and within-subject variances using mixed-effects location scale models

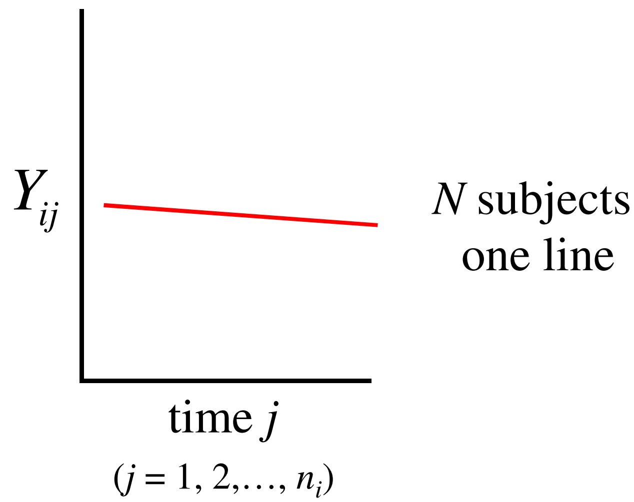
Simple linear regression on time

Measurement y of subject i (i = 1, 2,…, N) on occasion j (j = 1, 2,…, n_i)

$$Y_{ij} = \underbrace{\beta_0 + \beta_1 \times time_j}_{\text{fixed part}} + \underbrace{e_i}_{\text{random part}}$$

where

$$e_i \sim \mathcal{N}(0, \sigma^2)$$

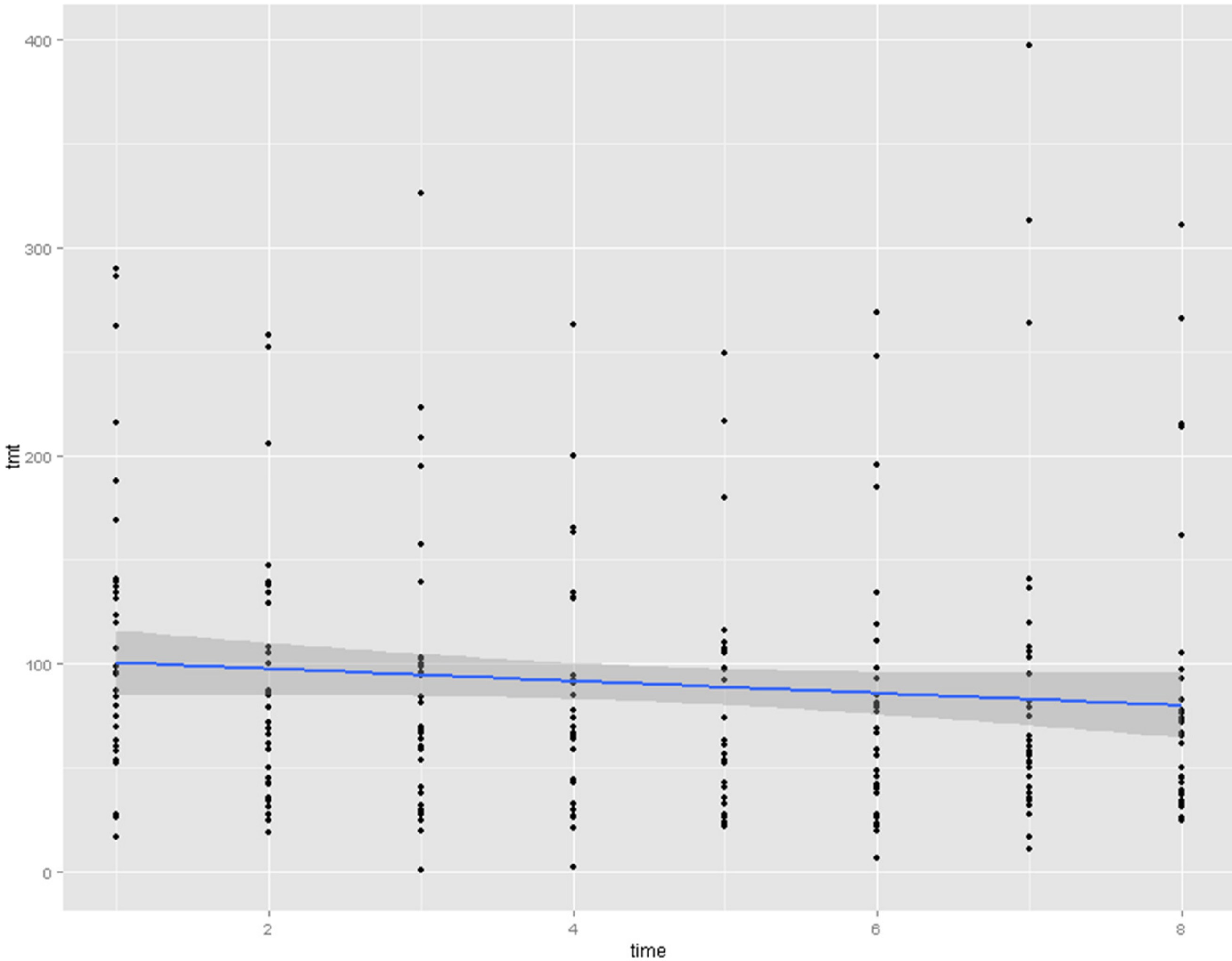


Practical

→ An application with R

Simple linear regression on time

TMT Realization Time : Evolution of performance over time : linear trend



The mixed-effects regression model as a primary method for analysis of repeated and longitudinal data

MRMs account for the influence of subjects on their repeated observations

Random subject effects reflect each subject's performance or development across time.

Random subject effects = between-subjects (BS) differences in intercept, linear slope, quadratic slope, etc. of the regression function.

Differences between subjects are measured by the variance of the random effects :

- the BS (inter-individual) variance,
- the within-subjects (WS) or intra-individual variance.

The mixed-effects regression model as a primary method for analysis of repeated and longitudinal data

The two-level random intercept model

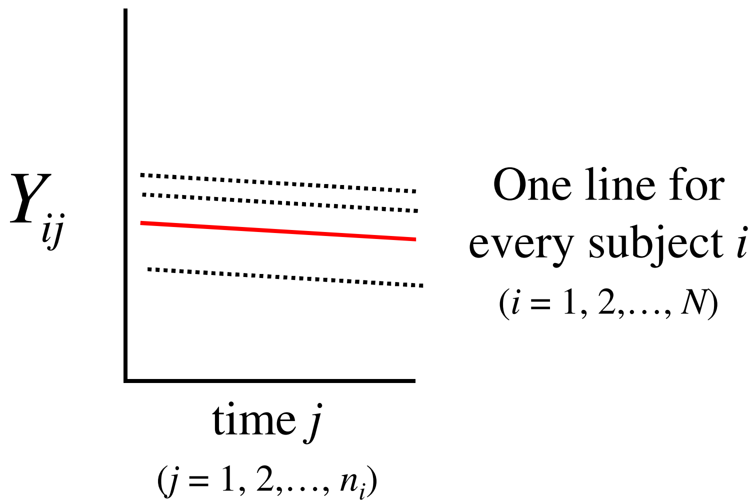
Measurement y of subject i ($i = 1, 2, \ldots, N$) on occasion j ($j = 1, 2, \ldots, n_i$)

$$Y_{ij} = \underbrace{\beta_0 + \beta_1 \times time_j}_{\text{fixed part}} + \underbrace{\upsilon_i + \varepsilon_{ij}}_{\text{random part}},$$

where

$\upsilon_i \sim \mathcal{N}(0, \sigma_\upsilon^2)$ → BS variance

$\varepsilon_{ij} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ → WS variance



The mixed-effects regression model as a primary method for analysis of repeated and longitudinal data

The two-level random intercept model

Measurement y of subject i (i = 1, 2,…, N) on occasion j (j = 1, 2,…, n_i) :

$$y_{ij} = x_{ij}^{'}\beta + \upsilon_i + \varepsilon_{ij} \qquad (1)$$

x_{ij} is the $p \times 1$ vector of regressors ;

β is the $p \times 1$ vector of regression coefficients ;

υ_i : random effects for intercepts,

$$\upsilon_i \sim \mathcal{N}(0, \sigma_\upsilon^2),\ \sigma_\upsilon^2 \text{ represents the BS variance ;}$$

ε_{ij} : residual "random" effects that are assumed to be independent of $\upsilon_{i,}$

$$\varepsilon_{ij} \sim \mathcal{N}(0, \sigma_\varepsilon^2),\ \sigma_\varepsilon^2 \text{ represents the WS variance.}$$

σ_υ^2 and σ_ε^2 are supposed to be homogeneous across subject groups or levels of covariates.

The mixed-effects regression model as a primary method for analysis of repeated and longitudinal data

In the long format, each row is one time point per subject

The stacked data set :

ID	time	age	fab	tmta	tmtb	tmt
1	1	68	18	60	199	139
1	2	68	18	41	141	100
1	3	68	18	56	152	96
1	4	68	18	46	178	132
1	5	68	18	42	140	98
1	6	68	18	43	162	119
1	7	68	18	49	185	136
1	8	68	18	53	158	105
2	1	88	16	37	206	169
2	2	88	16	34	173	139
2	3	88	16	39	139	100
2	4	88	16	35	200	165
2	5	88	16	37	144	107
2	6	88	16	29	85	56
2	7	88	16	39	180	141
2	8	88	16	51	213	162
...	...	...	...	...	...	...

Practical

→ An application with R

The mixed-effects regression model as a primary method for analysis of repeated and longitudinal data

Evolution with time and effect of age on the trail making test performance.

Results of the Chi-square test used to compare mrm1 and mrm2 :

```
Models:
mrm1: tmt ~ time + (1 | ID)
mrm2: tmt ~ time + age + (1 | ID)
     Df    AIC    BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
mrm1  4 2589.2 2603.3 -1290.6   2581.2
mrm2  5 2581.7 2599.2 -1285.8   2571.7 9.6003      1   0.001945 **
```

Smaller is better !

The RSS reduction is statistically significant

The mixed-effects regression rodels as a primary method for analysis of repeated and longitudinal data

TMT performance : Evolution with time and effect of age

```
Linear mixed model fit by REML ['lmerMod']
Formula: tmt ~ time + age + (1 | ID)
   Data: tmt31

Random effects:
 Groups   Name        Variance Std.Dev.
 ID       (Intercept) 2505     50.05
 Residual             1336     36.55
Number of obs: 248, groups:  ID, 31

Fixed effects:
            Estimate Std. Error t value
(Intercept) -115.6516   68.3211  -1.693
time          -2.9228    1.0130  -2.885
age            2.8758    0.8863   3.245

Correlation of Fixed Effects:
     (Intr) time
time -0.067
age  -0.988  0.000
```

$\rightarrow$ BS variance : σ_{υ}^2

$\rightarrow$ WS variance : σ_{ε}^2

β_0 : constant term

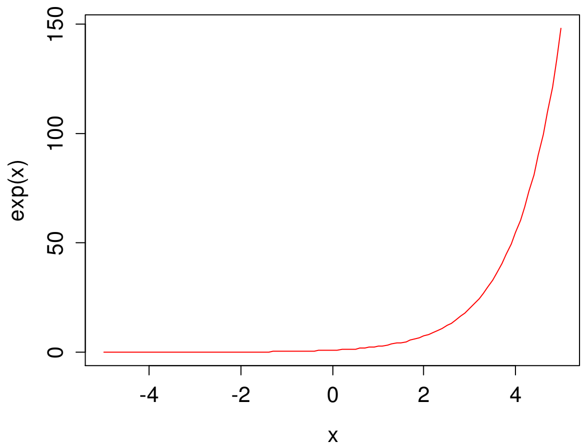
β_1 : time $\rightarrow$ tmt

β_2 : age $\rightarrow$ tmt

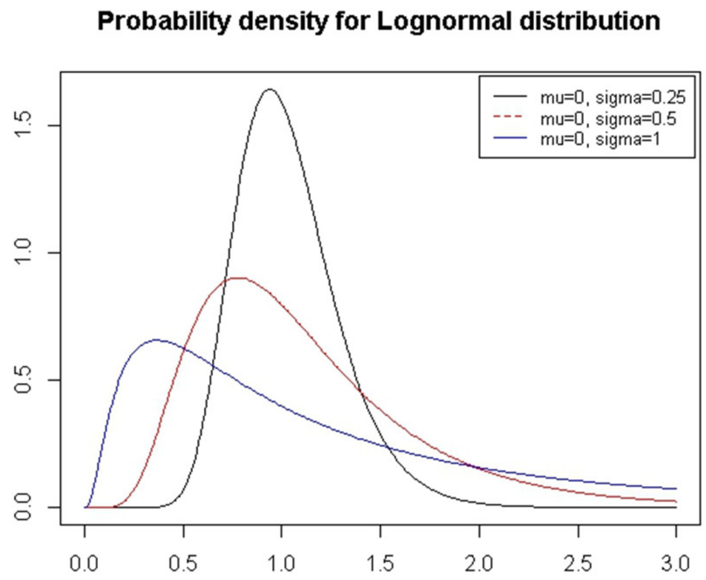
The mixed-effects location scale model

Assumptions that σ_υ^2 and σ_ε^2 are homogeneous across subject groups or levels of covariates can be relaxed.

→ *Using a log-linear representation for variances* (*to ensure positive variances*)



variance = exp(linear predictor)



log(variance) = linear predictor

The mixed-effects location scale model

$$y_{ij} = x_{ij}^{'}\beta + \upsilon_i + \varepsilon_{ij} \qquad (1)$$

BS variance : $\sigma_{\upsilon_i}^2 = \exp(\boldsymbol{u}_i^{'}\boldsymbol{\alpha})$ or $\log(\sigma_{\upsilon_i}^2) = \boldsymbol{u}_i^{'}\boldsymbol{\alpha}$ (2)

fixed part + random part

WS variance : $\sigma_{\varepsilon_{ij}}^2 = \exp(\boldsymbol{w}_{ij}^{'}\boldsymbol{\tau})$ or $\log(\sigma_{\varepsilon_{ij}}^2) = \boldsymbol{w}_{ij}^{'}\boldsymbol{\tau}$ (3)

fixed part + random part

(Hedeker, Mermelstein & Demirtas, 2008, 2012)

The mixed-effects location scale model

→ *Some covariates can influence the BS and WS variances.*

BS variance : - subject-level covariates $\rightarrow \log(\sigma^2_{\upsilon_i})$

WS variance : - subject-varying covariates $\rightarrow \log(\sigma^2_{\varepsilon_{ij}})$

- time-varying covariates $\rightarrow \log(\sigma^2_{\varepsilon_{ij}})$

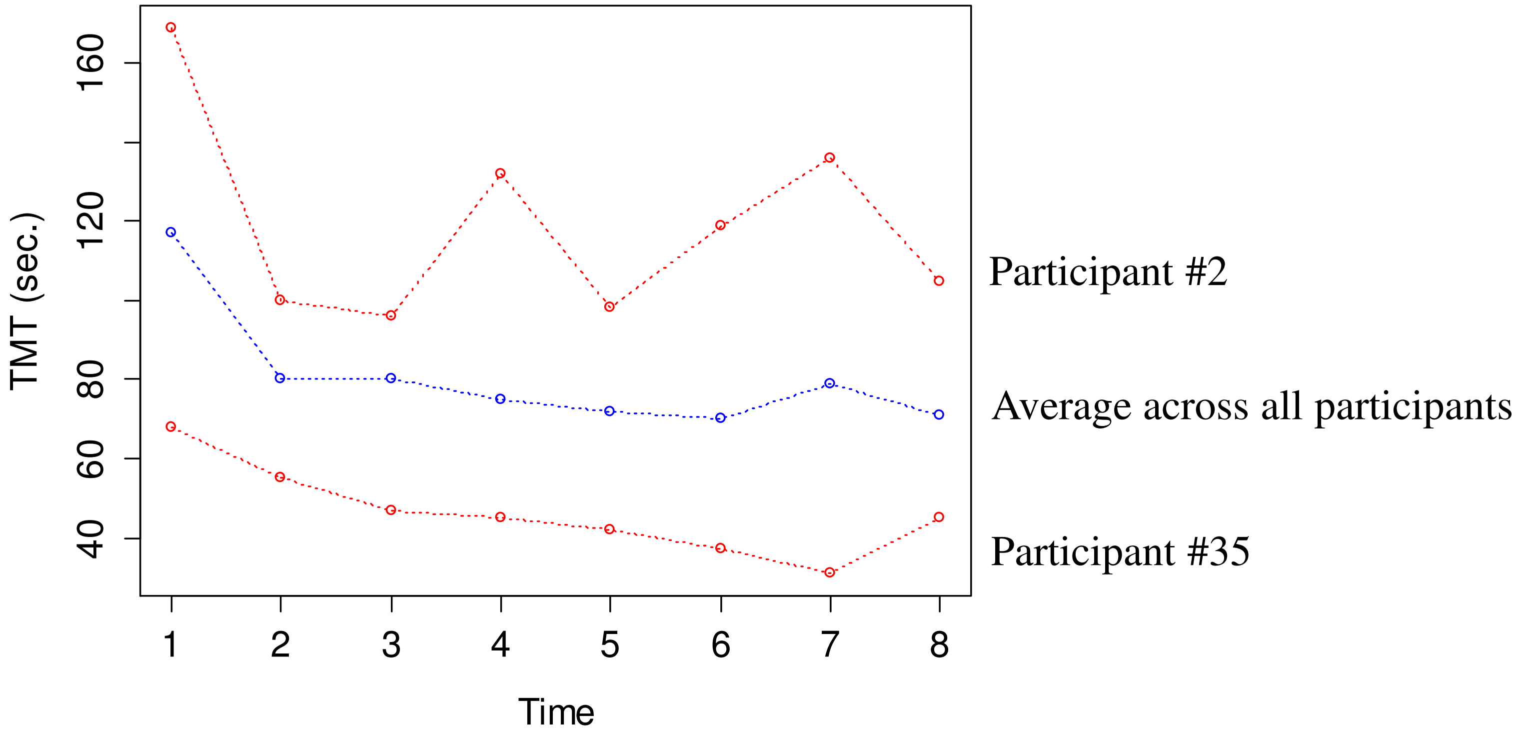
- residual variation across subjects : $\log(\sigma^2_{\varepsilon_{ij}}) = \boldsymbol{w}'_{ij}\boldsymbol{\tau} + \omega_i$

$\omega_i \sim N(0, \sigma^2_{\omega})$ ↑ random subject (scale) effects

(Hedeker, Mermelstein & Demirtas, 2008, 2012)

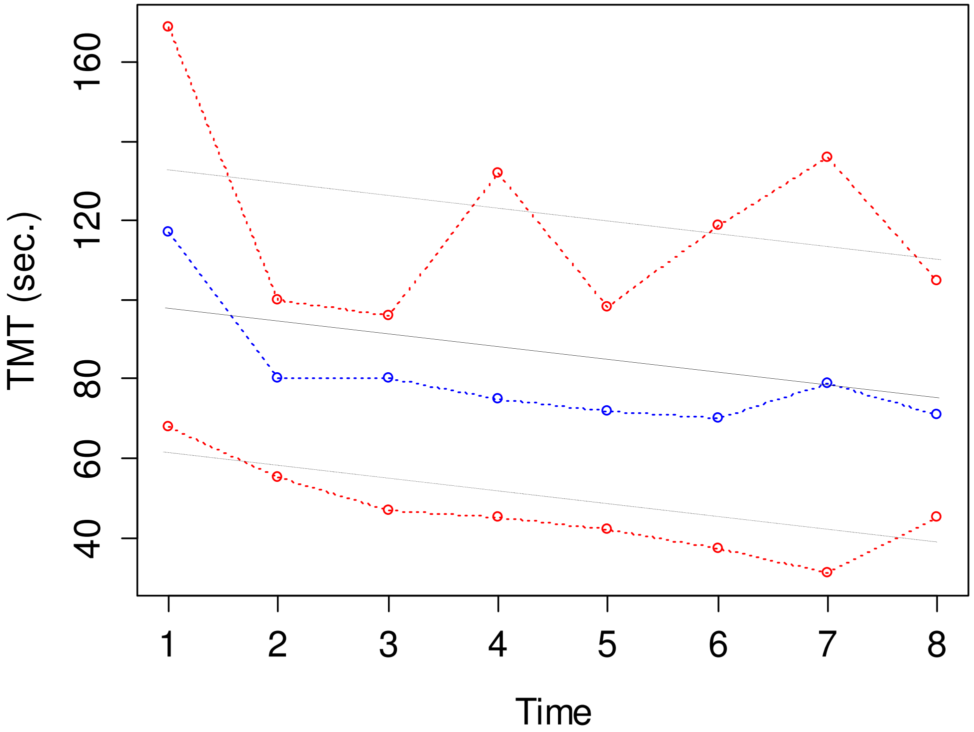
The mixed-effects location scale model

Measurement y of subject i (i = 1, 2,…, N) on occasion j (j = 1, 2,…, n_i)



The mixed-effects location scale model

Measurement y of subject i (i = 1, 2,…, N) on occasion j (j = 1, 2,…, n_i)

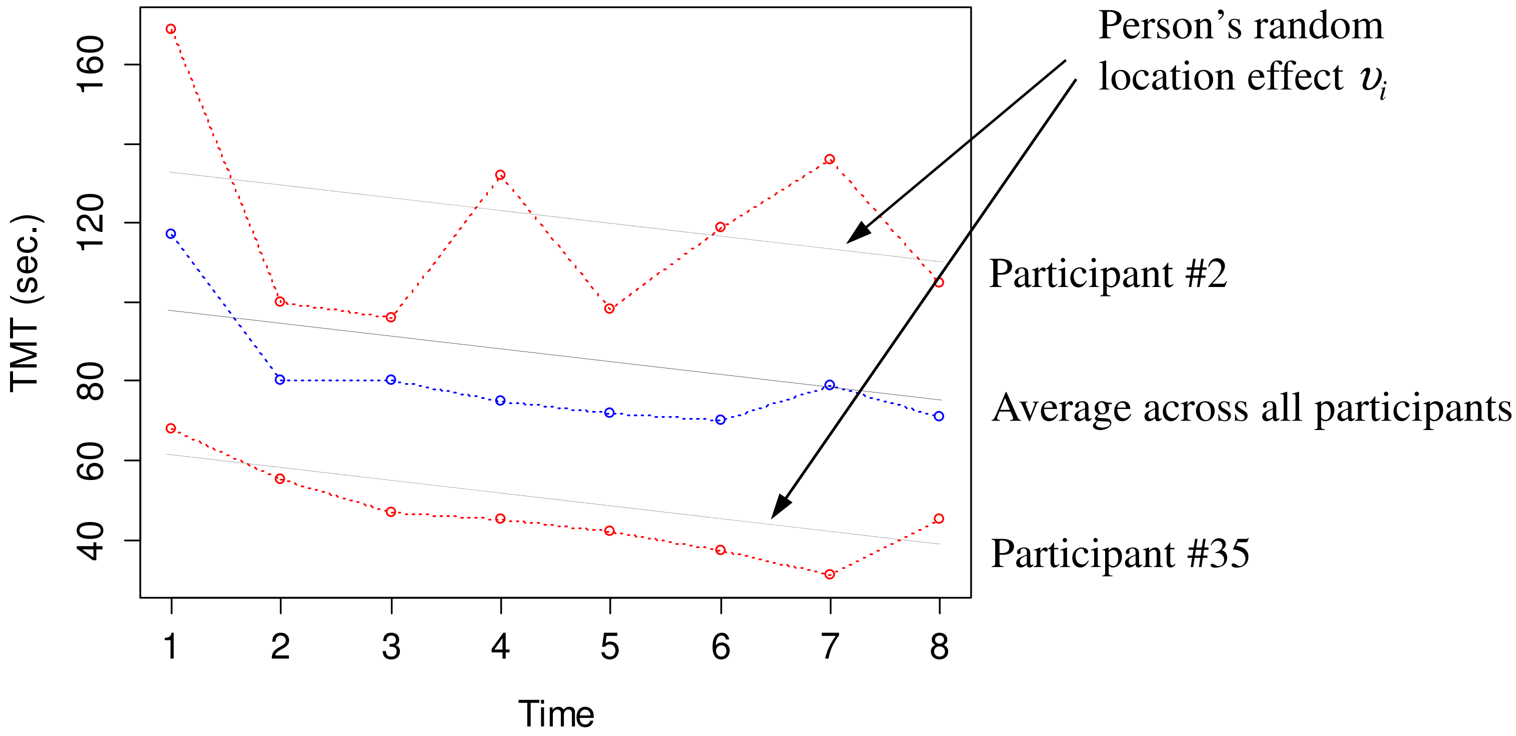


Average across all participants

β : effect of covariates

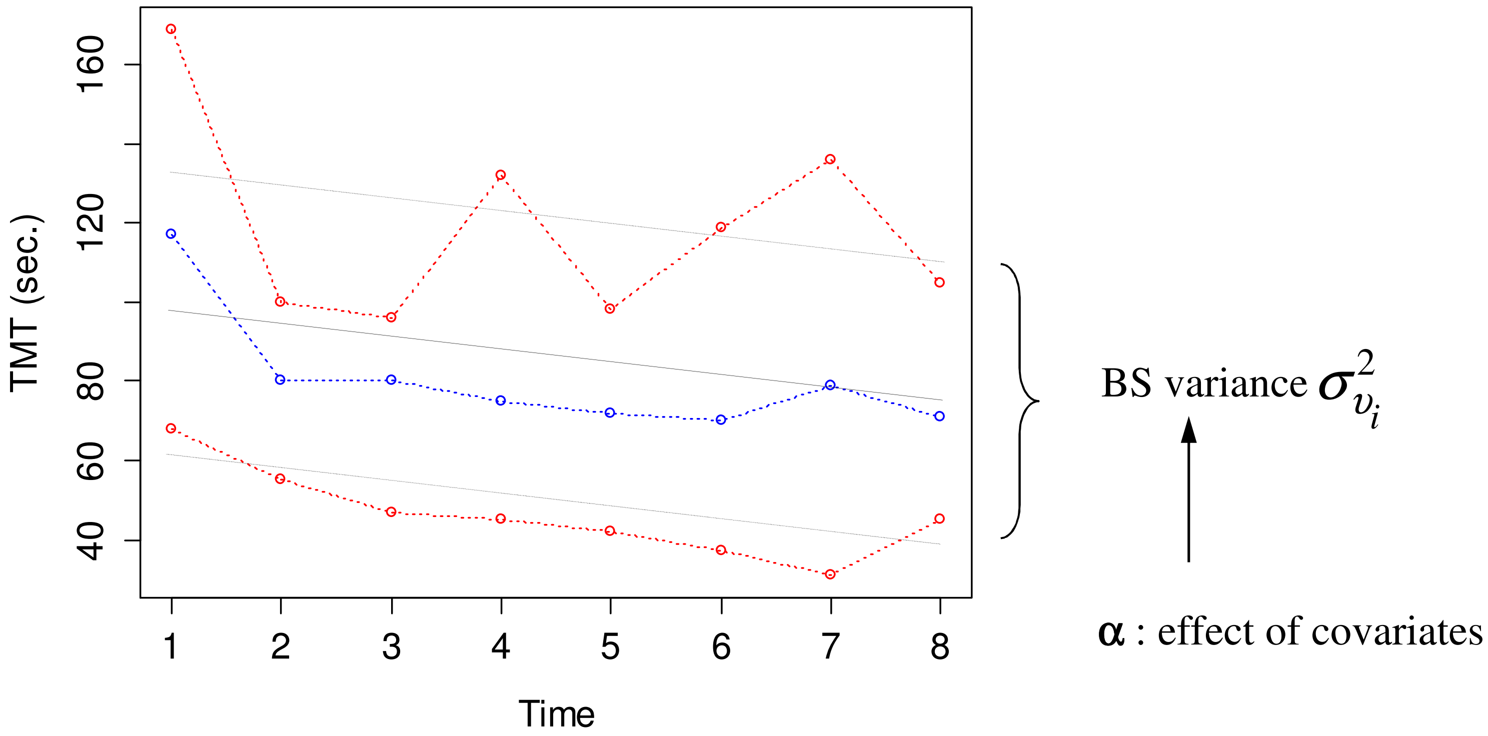
The mixed-effects location scale model

Measurement y of subject i (i = 1, 2,…, N) on occasion j (j = 1, 2,…, n_i)



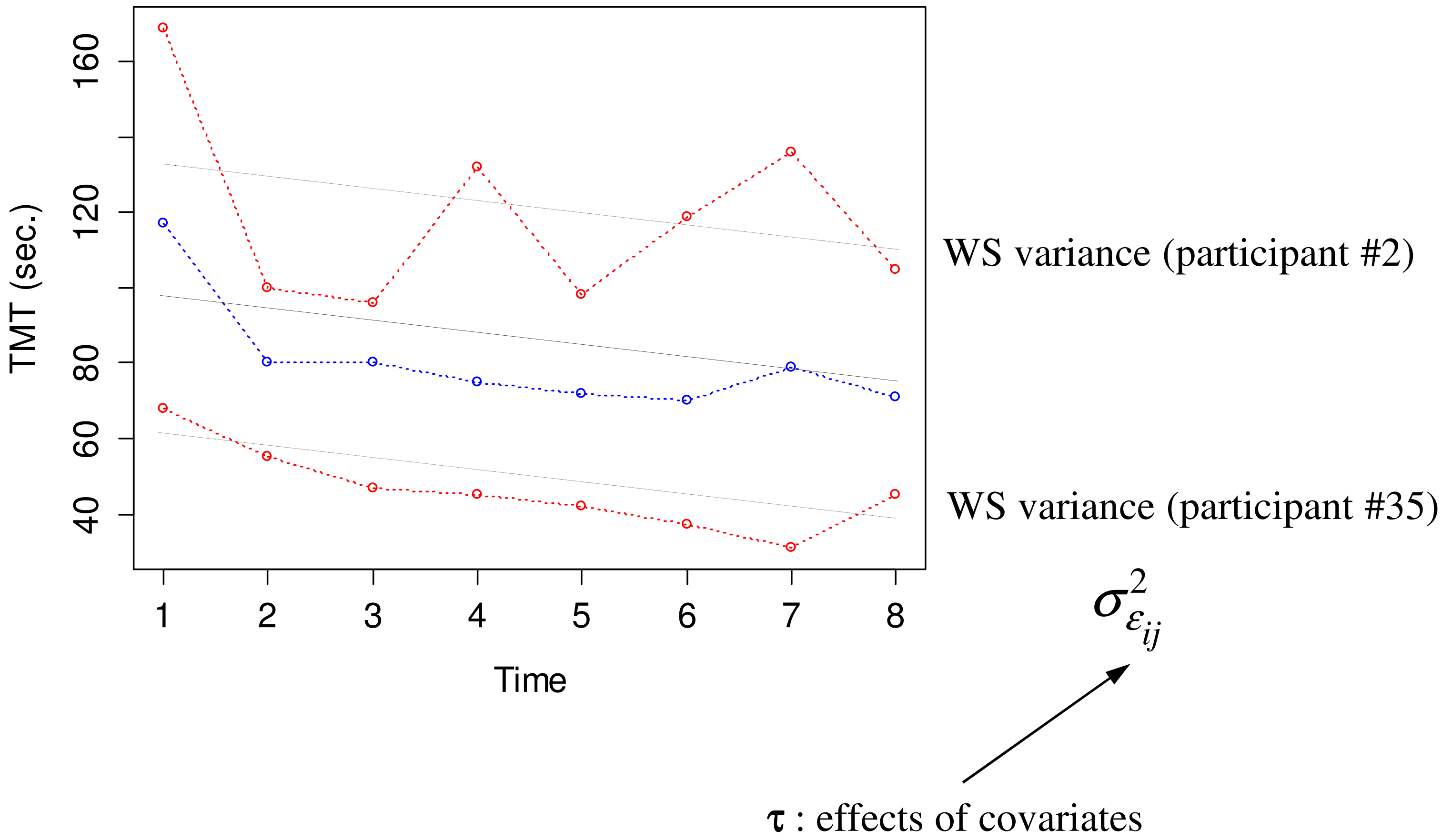
The mixed-effects location scale model

Measurement y of subject i (i = 1, 2,…, N) on occasion j (j = 1, 2,…, n_i)



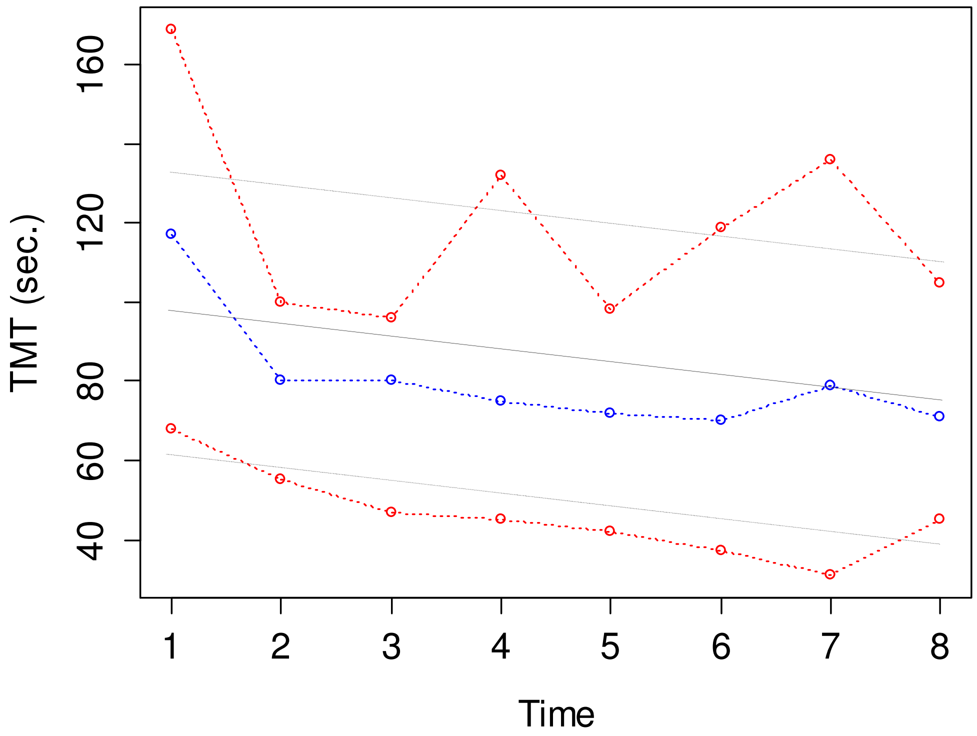
The mixed-effects location scale model

Measurement *y* of subject *i* ($i = 1, 2, \ldots, N$) on occasion *j* ($j = 1, 2, \ldots, n_i$)



The mixed-effects location scale model

Measurement y of subject i (i = 1, 2,…, N) on occasion j (j = 1, 2,…, n_i)



WS variance (participant #2)

WS variance (participant #35)

$\sigma^2_{\varepsilon_{ij}}$

Covariates do not explain all of WS variance : ω_i

The mixed-effects location scale model

EXAMPLE - Analysis of the trail making test performance with random location and scale effects using MIXREGLS (Hedeker & Nordgren, 2013)

Model : tmt ~ 1+cov. | 1+cov. | 1+cov., id="ID"

The mean submodel :

Reference mean : β_0

+

$\boldsymbol{\beta}$* for covariate $\boldsymbol{u}$*

The mixed-effects location scale model

EXAMPLE - Analysis of the trail making test performance with random location and scale effects using MIXREGLS (Hedeker & Nordgren, 2013)

Model : tmt ~ 1+cov. | 1+cov. | 1+cov. , id="ID"

The BS variance submodel :

Reference variance : $\exp(\alpha_0)$ when $\boldsymbol{u}_j = 0$

$+ \exp(\alpha^*)$ for covariate $\boldsymbol{u}^*$

The mixed-effects location scale model

EXAMPLE - Analysis of the trail making test performance with random location and scale effects using MIXREGLS (Hedeker & Nordgren, 2013)

Model : tmt ~ 1+cov. | 1+cov. | 1+cov. , id="ID"

The WS variance submodel :

Reference variance : exp(τ_0) when w_{ij} = 0

+ exp(τ*) for covariate w*

The mixed-effects location scale model

EXAMPLE - Analysis of the trail making test (part B) performance with random location and scale effects using MIXREGLS

Results :

MODEL : tmtb ~1 | 1 | 1, id="id"				
-2 ln L: 451.560				
Variable	Estimate	AsymStdError	z-value	p-value
BETA (regression coefficients)				
Intercept (β_0)	2.232	0.2171	10.283	0.000
ALPHA (BS variance parameters: log-linear model)				
Intercept (α_0)	0.336	0.2926	1.149	0.251
TAU (WS variance parameters: log-linear model)				
Intercept (τ_0)	-1.576	0.226	-6.965	0.000
Influence of the location random effect on the (log of the) WS variance				
τ_{linear}	1.056	0.190	5.553	0.000
Standard deviation of the random subject scale effect				
σ_ω	0.347	0.189	1.833	0.067
BS variance : exp(α_0)	1.399			
WS variance : exp(τ_0+0.5* $(\tau_{linear}^2 + \sigma_\omega^2)$)	0.369			
ICC = BS/(BS + WS)	0.791			

The mixed-effects location scale model

EXAMPLE - Analysis of the trail making test (part B) performance with random location and scale effects using MIXREGLS

Results :

MODEL : tmtb ~ time | time | 1, id="id"

-2 ln L: 413.792 ; Likelihood ratio test : $\Delta\chi^2 = 451.60-413.79 = 37.81$, $\Delta df=2$

Variable	Estimate	AsymStdError	z-value	p-value
BETA (regression coefficients)				
Intercept (β_0)	2.489	0.237	10.504	0.000
time (β_1)	-0.080	0.016	-5.147	0.000
ALPHA (BS variance parameters: log-linear model)				
Intercept (α_0)	0.499	0.301	1.661	0.097
time (α_1)	-0.056	0.031	-1.776	0.076
TAU (WS variance parameters: log-linear model)				
Intercept (τ_0)	-1.727	0.231	-7.466	0.000
Influence of the location random effect on the (log of the) WS variance				
τ_{linear}	1.138	0.184	6.202	0.000
Standard deviation of the random subject scale effect				
σ_ω	0.101	0.427	0.237	0.813

The mixed-effects location scale model

EXAMPLE - Analysis of the trail making test (part B) performance with random location and scale effects using MIXREGLS

Results :

MODEL : tmtb ~ time + tmta + age | 1 | age, id="id"

-2 ln L: 392.580 ; Likelihood ratio test : $\Delta\chi^2 = 413.79-392.58 = 21.21$, $\Delta df=2$

Variable	Estimate	AsymStdError	z-value	p-value
BETA (regression coefficients)				
Intercept (β_0)	1.893	0.209	9.053	0.000
time (β_1)	-0.052	0.010	-5.111	0.000
tmta (β_2)	0.693	0.172	4.035	0.000
age (β_3)	0.508	0.157	3.238	0.000
ALPHA (BS variance parameters: log-linear model)				
Intercept (α_0)	-0.370	0.305	-1.212	0.226
TAU (WS variance parameters: log-linear model)				
Intercept (τ_0)	-1.710	0.197	-8.671	0.000
age (τ_3)	0.504	0.198	2.547	0.011
Influence of the location random effect on the (log of the) WS variance				
τ_{linear}	0.955	0.161	5.918	0.000
Standard deviation of the random subject scale effect				
σ_ω	0.000	0.215	0.000	1.000

The mixed-effects location scale model

EXAMPLE - Analysis of AX-CPT reaction time with random location and scale effects using MIXREGLS

Results :

MODEL WITHOUT ANY COVARIATES	: tr ~ 1 | 1 | 1, id="id"			
-2 ln L: -167.264				
Variable	Estimate	AsymStdError	z-value	p-value
BETA (regression coefficients)				
Intercept (β_0)	0.629	0.025	25.190	0.000
ALPHA (BS variance parameters: log-linear model)				
Intercept (α_0)	-3.889	0.252	-15.424	0.000
TAU (WS variance parameters: log-linear model)				
Intercept (τ_0)	-3.029	0.177	-17.093	0.000
Influence of the location random effect on the (log of the) WS variance				
τ_{linear}	0.708	0.158	4.494	0.000
Standard deviation of the random subject scale effect				
σ_ω	0.851	0.099	8.540	0.000
BS variance : exp(α_0)	0.020			
WS variance : exp(τ_0+0.5*($\tau_{linear}^2 + \sigma_\omega^2$))	0.089			
ICC = BS/(BS + WS)	0.187			

The mixed-effects location scale model

EXAMPLE - Analysis of AX-CPT reaction time with random location and scale effects using MIXREGLS

Results :

MODEL WITH RANDOM SCALE : tr ~ age | 1 | age, id="id"

-2 ln L: -178.394, Likelihood ratio test : $\chi^2_2 = -167.264 + 178.394 = 11.131$

Variable	Estimate	AsymStdEr r	z-value	p-value
BETA (regression coefficients)				
Intercept (β_0)	0.048	0.152	0.317	0.751
age (β_1)	0.008	0.002	3.863	0.000
ALPHA (BS variance parameters: log-linear model)				
Intercept (α_0)	-4.276	0.259	-16.536	0.000
TAU (WS variance parameters: log-linear model)				
Intercept (τ_0)	-5.472	1.295	-4.225	0.000
age (τ_1)	0.032	0.016	1.909	0.006
Influence of the location random effect on the (log of the) WS variance				
τ_{linear}	0.632	0.144	4.386	0.000
Standard deviation of the random subject scale effect				
σ_ω	0.826	0.089	9.232	0.000

A Bayesian model for estimating intra-individual variability as a predictor

Statistics from a Bayesian perspective, in a (very) few words…

Bayesian statistics starts by using (prior) probabilities to describe the current state of knowledge

The prior distribution represents a specific assumption about a model parameter.

It is a distribution of credibility across parameter values θ of the model that expresses previous knowledge about the parameter values without the newly collected data :

the prior density $p(\theta)$

The specification of a very uncertain prior implies that the prior has minimal influence on the estimates of the parameters.

Statistics from a Bayesian perspective, in a (very) few words…

Bayesian statistics uses the sampling distribution $p(y \mid \theta)$ *of the data y as a function of a model with its parameters* θ

The likelihood function $p(y \mid \theta)$ is the same as the sampling distribution of the observed data y but read in the opposite way.

The value $\hat{\theta}$ which yields the maximum of the likelihood function for the observed data y is called the maximum of the likelihood estimate of the parameter θ.

Statistics from a Bayesian perspective, in a (very) few words…

Bayesian statistics uses Bayes rule and incorporates information through the collected data

Bayes rule describes the relationship between the two conditional probabilities $p(\mathrm{A|B})$ and $p(\mathrm{B|A})$:

$$p(\mathrm{A} \mid \mathrm{B}) = \frac{p(\mathrm{B} \mid \mathrm{A})p(\mathrm{A})}{p(\mathrm{B})}.$$

Bayes rule is used to derive the probability of the parameters θ given the data y, that is the posterior distribution $p(\theta \mid y)$:

$$p(\theta \mid y) = \frac{p(y \mid \theta)p(\theta)}{p(y)}.$$

The marginal density $p(y)$ is a constant that contains high-dimensional integrals which are often impossible to compute analytically.

Statistics from a Bayesian perspective, in a (very) few words…

Bayesian statistics relies on computer simulations that draw samples from the posterior distribution given a model, a likelihood $p(\theta \mid y)$, *and data y*

Integration is typically performed by computer simulations :

Let the data be y and a vector $\theta = (\theta_1, \theta_2, ..., \theta_k)$ of k unknown parameters.

1. Choose initial values $\theta_1^0, \theta_2^0, ..., \theta_k^0$

2. Sample θ_1^1 from $p(\theta_1 \mid \theta_2^0, \theta_3^0, ..., \theta_k^0, y)$

   Sample θ_2^1 from $p(\theta_2 \mid \theta_1^0, \theta_3^0, ..., \theta_k^0, y)$

   ....

   Sample θ_k^1 from $p(\theta_k \mid \theta_1^0, \theta_2^0, ..., \theta_{k-1}^0, y)$

3. repeat step 2 very many times (e.g. 50000)

The sequence of random draws for each of k parameters resulting from step 3 forms a Markov Chain Monte Carlo (MCMC) sample.

If chains are converged, the sample that approximates the posterior is summarized for inference : mean, mode, median, variance, probability interval (e.g., 95% PI) for every parameter.

A Bayesian model for estimating intra-individual variability

The Bayesian variability model (Wiley, 2015) :

1. offers unbiased, correct estimates ;
2. gives more effective and more power results with smaller sample sizes ;
3. accounts for systematic changes ;
4. allows for some missing data.

A Bayesian model for estimating intra-individual variability

A two-stage modeling

1. Estimating intra-individual variability of X_{ij} (subject i, occasion j) in a mixed-effects regression framework :

$$X_{ij} \sim N(\mu_i, \sigma_i^2).$$

Prior distribution on μ_i : $\mu_i \sim N(\mu_\mu, \sigma_\mu^2)$;

Reference prior on σ_i^2 that is independent of μ_i : $\sigma_i^2 \sim \Gamma(\alpha, \beta)$,

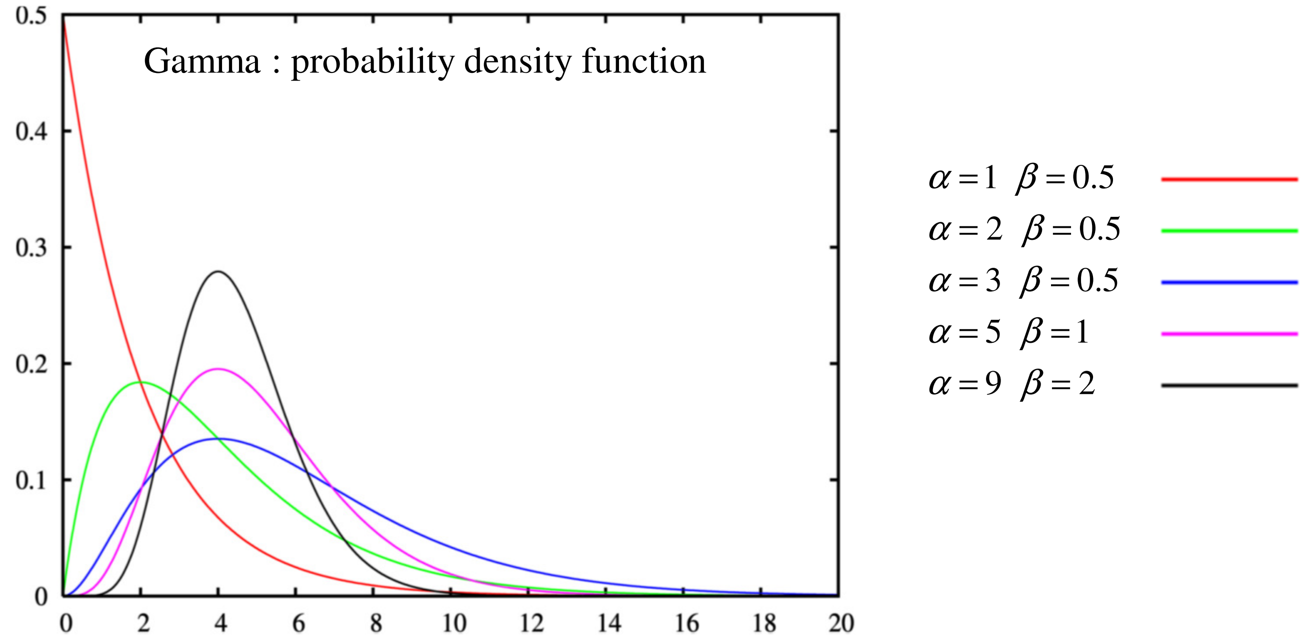
with shape and (inverse) scale parameters α and β.

A Bayesian model for estimating intra-individual variability

A two-stage modeling

1. Estimating intra-individual variability of X_{ij} (subject i, occasion j) in a mixed-effects regression framework :

$$\sigma_i^2 \sim \Gamma(\alpha, \beta), \text{ with shape and (inverse) scale parameters } \alpha \text{ and } \beta.$$



A Bayesian model for estimating intra-individual variability

A two-stage modeling

2. Using the estimate of intra-individual variability, accounting for measurement error, as a predictor in a multiple regression framework :

$$Y_i \sim N(\mu_{2i}, \sigma_2),$$

where:

$$\mu_{2i} = \beta_0 + \beta_k \, Covariates_k + \underbrace{\alpha_1 \sigma_i + \alpha_2 \mu_i}_{\text{latent (estimated) variables from step 1}}.$$

A Bayesian model for estimating intra-individual variability

EXAMPLE - Analysis of the effect of intra-individual variability (*TMT part B*) *on performance on the Frontal Assessment Battery* (*FAB*) *using* 'varian'

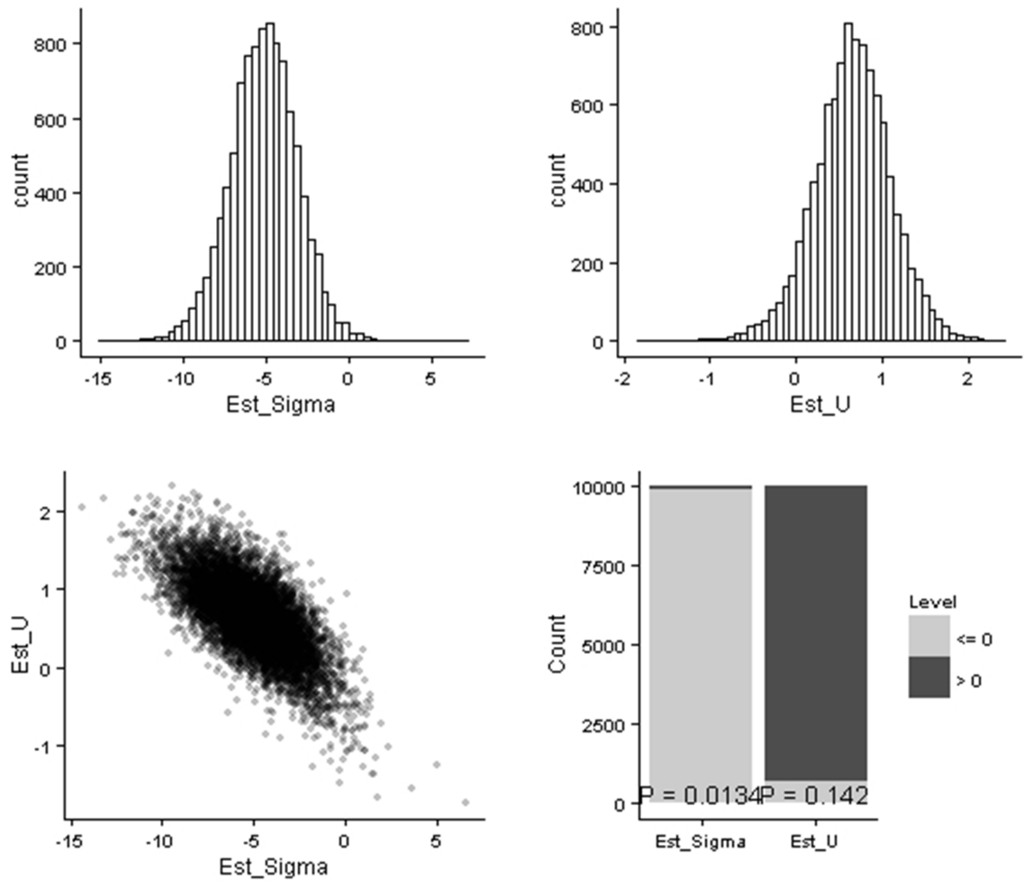
Results :

MODEL			
tmb ~time + tmta + age | ID (linear time detrending)			
var_intra(tmtb) + mean(tmtb) → FAB			
VARIABILITY ANALYSIS			
	Estimate	95% CI	p-value
TMT.B			
intercept	2.01	[1.84, 2.52]	0.000
time	-0.05	[-0.07, -0.03]	0.000
tmta	0.54	[0.39, 0.99]	0.013
age	0.55	[0.42, 0.92]	0.001
σ_{μ}	1.01	[0.72, 1.38]	0.000
gamma shape	3.91	[2.11, 6.60]	0.000
gamma rate	8.01	[4.02,13.92]	0.000
average ISD (α/β)	0.49		
FAB			
intercept	18.96	[16.92, 21.01]	0.000
α_1 : var_intra(tmtb)	-5.12	[-9.20, -1.19]	0.013
α_2 : mean(tmtb)	0.65	[-0.29, 1.53]	0.142
residual	1.35	[0.67, 1.99]	0.000

A Bayesian model for estimating intra-individual variability

EXAMPLE - Analysis of the effect of intra-individual variability (*TMT part B)* *on performance on the Frontal Assessment Battery* (*FAB*) *using* 'varian'

Results :



A Bayesian model for estimating intra-individual variability

EXAMPLE - Analysis of the effect of intra-individual variability (*TMT part B*) *on performance on the Frontal Assessment Battery* (*FAB*) *using* 'varian'

Results :

random effects

μ_i

ID	est.	2.5% 97.5%
1	0.88	[0.38 , 1.40]
2	0.01	[-0.66 , 0.68]
3	-0.46	[-0.97 , 0.06]
4	2.10	[1.31 , 2.89]
5	0.04	[-0.51 , 0.59]
6	-0.09	[-0.62 ,-0.04]
...	...	...

σ_i

ID	est.	2.5% 97.5%
1	0.38	[0.24 , 0.59]
2	0.63	[0.44 , 0.91]
3	0.41	[0.22 , 0.69]
4	0.85	[0.62 , 1.19]
5	0.39	[0.24 , 0.63]
6	0.50	[0.27 , 0.83]
...	...	...

Raw ISD
0.35
0.72
0.31
1.19
0.45
0.30
...

A Bayesian model for estimating intra-individual variability

EXAMPLE - Analysis of the effect of intra-individual reaction time variability (*AX-CPT paradigm*) *on performance on the Frontal Assessment Battery* (*FAB*) *using* ‘varian’

Results :

MODEL			
axcpt ~1| ID			
axcpt_var+axcpt_mean → FAB			
VARIABILITY ANALYSIS			
	Estimate	95% CI	p-value
AX-CPT			
intercept	0.62	[0.57, 0.67]	0.000
σ_μ	0.14	[0.11, 0.19]	0.000
gamma shape	3.18	[1.93, 4.77]	0.000
gamma rate	12.24	[7.07, 18.98]	0.000
average ISD (α/β)	0.26		
FAB			
intercept	16.84	[15.46, 18.21]	0.000
α_1 : var_intra(tmtb)	-1.98	[-6.60, 2.62]	0.387
α_2 : mean(tmtb)	-1.55	[-7.30, 4.23]	0.594
residual	1.87	[1.46, 2.44]	0.000

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