

# Visuo-Manual Pursuit Tracking of Human Two-Dimensional Movements

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We investigated visuo-manual pursuit tracking in 10 adult subjects. The main features of the experiments were as follows: (a) The targets to be pursued were planar (two-dimensional) trajectories. (b) Targets were based on actual human hand movements. A set of five different extemporaneous movements (scribbles) were rear-projected on a digitizing table with their original time-scale. (c) Targets were presented both as originally recorded (Condition N) and after a numerical manipulation that made the tangential velocity constant throughout the motion (Condition T). After a set of descriptors of the performance suitable for the two-dimensional case was introduced, individual performances were analyzed to characterize both the features common to all subjects and the idiosyncratic differences. The main results of the study were as follows: (a) Performances are extremely consistent across repetitions. Differences among subjects are mostly confined to the value of the descriptors. (b) The single most characteristic descriptor is the instantaneous delay between target and pursuit. The position error depends jointly on this delay and the instantaneous target velocity. (c) The operating strategy is significantly modified in Condition T. (d) A simple formal scheme based on the notion of a delayed velocity feed-back accounts quite accurately for the experimental results. This is in contrast with most classical models of pursuit tracking.

The ability of human operators to track the variations in time of physical stimuli has attracted considerable interest since the early fifties (Adams, 1961; Conklin, 1957; Elkind, 1953; Garvey & Mitnick, 1957; Hartman & Fitts, 1955; Holding, 1959; Licklider, 1960; Mather & Putschat, 1983; McRuer & Krendel, 1959a, 1959b; Noble, Fitts, & Warren, 1955; Notterman & Tufano, 1980; Poulton, 1952a, 1952b, 1957; Stark, 1972). Besides the obvious implications that the study of this skill has for the design of ergonomically efficient control implements (for a review see Poulton, 1974), a general consensus has emerged that important aspects of both the motor control system and the sensori-motor interface can be elucidated by a quantitative analysis of the tracking task. Several important pathways from a sensory input to a motor response have been investigated, but eye and visuo-manual tracking have consistently had the lion's share and still provide prototypical examples of the approaches and problems specific to this field.

Two general remarks apply to almost all tracking studies in these two systems. First, despite early warnings against this tendency (Adams, 1961), the conceptual framework adopted to de-

sign experiments and formalize results still draws quite heavily from the panoply of system control theory. Consequently, the stimuli are usually selected from the traditional repertoire of driving inputs for the analysis of man-made systems (sinewaves, ramps, and pseudorandom combinations of sinewaves). Input-output relations have been proposed (cf. Stark, 1972) in which perceptuo-motor transcoding is characterized in purely physical terms, neglecting in general both the processes whereby perceived stimuli and appropriate responses are represented centrally and some specific properties of the motor control system whose relevance is being increasingly acknowledged. Second, in the vast majority of cases, the targets are one dimensional: Only one aspect of the stimulus varies in time, and, whatever the biomechanical complexity of the response system, end-point control is exerted only on one parameter of the motor response. We do not consider dual-pursuit tasks (e.g., Adams & Xhignesse, 1960; Fitts & Simon, 1952) as two-dimensional pursuit.

It can be argued that both remarks, taken together, imply a substantial limitation in scope and depth of tracking research. Indeed, developmental studies (Gachoud, Mounoud, Hauert, & Viviani, 1983; Hay, 1979; Mounoud, Viviani, Hauert, & Guyon, 1985; Piaget, 1946; Pick, 1970) have suggested that the motor performance of young children is best understood if one postulates a representational stage between sensory input and motor output whereby the general properties of the stimulus are apprehended. This process should not be confused with perceptual anticipation through which forthcoming changes of the target are guessed (Conklin, 1957; Notterman & Tufano, 1980; Poulton, 1952a, 1952b, 1957; cf. Vince, 1953, 1955). More spe-

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This research was supported by Consiglio Nazionale delle Ricerche and by Research Grant 1.073.0.79 of the Fonds National Suisse de la Recherche to Paolo Viviani and Pierre Mounoud.

We gratefully acknowledge the contributions of C. A. Fowler, R. Proctor, and an anonymous reviewer for improving the first version of this article.

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cifically, the classical concepts of system analysis alone seem to be inadequate for interpreting certain types of unsuccessful performances in sine-wave tracking. For instance, 5-year-old children occasionally produce low-distortion sinusoidal movements that match the amplitude but not the frequency of the target (Mounoud et al., 1985). Their movements appear to be controlled not by the actual time course of the stimulus but rather by an abstract representation of the target that does not yet contain frequency as a qualifying attribute. This basic inadequacy may be concealed in adults when predictable stimuli are used that are relatively simple and do not tax the representational processes of adults as much as those of children. Pseudo-random stimuli escape this criticism but, in the one-dimensional case, are not ideally suited for verifying the putative role of perceptual and motor representation because hand movements normally take place in three dimensions, and whatever intrinsic properties the input and output systems might have are likely to emerge only under these normal operating conditions. Constraining the tracking response to just one dimension not only limits the variety of possible stimuli but also, more importantly, makes it difficult even in principle to probe the possible role of these properties in the overall performance.

This article reports a study of visuo-manual tracking in conditions that partly remove the limitations discussed above. We will consider hand tracking of a target moving along planar (two-dimensional) trajectories. Although these trajectories are but one special case among all possible hand movements, there is evidence (Soechting & Terzuolo, 1986) that many complex tridimensional trajectories can be decomposed into sequences of such special cases. Thus, considering two-dimensional trajectories represents a significant generalization in the study of visuo-manual tracking. The other distinctive feature of this study is that the movements used as targets were produced extemporaneously by a human operator. It has been demonstrated (Lacquaniti, Terzuolo, & Viviani, 1983, 1984; Viviani & Cenzato, 1985; Viviani & McCollum, 1983; Viviani & Terzuolo, 1982) that functionally significant relations exist between the geometrical and kinematic parameters of planar man-made trajectories, which are likely to express specific properties of the motor control system. Therefore, by using this type of planar target, one can investigate the extent to which visuo-manual tracking is affected by the fact that the stimuli exhibit the same form-kinematics covariation that is characteristic of spontaneous motor productions.

In consideration of the fact that most usual measures of tracking performance (time on target, gain attenuation, harmonic distortion, root mean square error—cf. Poulton, 1974; Stark, 1972) are not directly applicable in the two-dimensional case, we will give considerable attention to the methodological problem of defining a set of descriptors suitable for this more general condition. The formal development to be presented will represent an attempt to extract from the wealth of raw data a minimal subset of descriptors sufficient for capturing the major individual differences.

## Method

### Subjects

Ten subjects (5 male, 5 female) between 22 and 43 years old volunteered for the experiment. They all were right-handed and had normal or corrected-to-normal vision.

### Apparatus

The visual targets to be tracked were produced by rear-projecting a laser beam (Helium-Neon red,  $\phi$ : 0.3 cm) on a transparent digitizing table (Calcomp 9240RP) mounted horizontally at 90 cm from the floor. The position of the target on the table was controlled by two galvanometric mirrors interposed at 90° on the beam path and driven by the computer input/output interface. The bandwidth of the system formed by the computer's digital-to-analog converter and the galvanometers exceeded 400 Hz. Because of the geometry of the optical setup, pin-cushion deformations occur in the transduction from the electrical output signal to the target position. These deformations were calculated and corrected. The tracking implement was the standard cursor of the digitizing table. It consists of a lightweight 7 × 12-cm metal tablet with a circular opening of 6 cm in radius. The center of the opening is marked by a crosshair. The cursor could be held comfortably in the palm and moved around with no appreciable effort. Frictional resistive forces were negligible. The instantaneous position of the crosshair is sensed by the table with an accuracy of 0.025 mm. Synchronization between stimulus and response sample was obtained by triggering the DAC operation with the table's gating signal. The sampling rate was 60 Hz. Subjects were free to choose between a standing or seated posture.

### Targets

The complete set of two-dimensional targets used for these experiments was created with a two-step procedure. First, one of the authors used the digitizing table to record five extemporaneous scribbles. The only explicit constraint was that the trajectory lie entirely within a 25 × 25-cm frame. However, the experimenter intentionally changed the average speed of execution from scribble to scribble. In all cases 1,000 samples of the movement were recorded at 60 Hz, which corresponds to a total target duration of 17.1 s. An example of scribble trajectory is shown in Figure 1, Panel A. These five scribbles (heretofore referred to as N-type targets) formed the first half of the experimental set. As shown previously (Viviani & Terzuolo, 1982), the tangential velocity of these natural movements is a function of the curvature of the trajectory and varies continuously during the motion. In the second step of the procedure, five additional targets (to be called T-type targets) were obtained from the original scribbles by manipulating numerically their law of motion. Each N-type target yielded a corresponding T-type, one which had the same duration and the same trajectory but whose tangential velocity was constant throughout the motion. The complete set of targets, then, consisted of five different pairs of trajectories, the only difference within a pair being the time course of the tangential velocity.

### Procedure

In one experimental session each N- and T-type target was presented 10 times. The order of presentation of the 100 items was randomized and different for each subject. Three of the subjects were aware of the experimental design. Of the 7 other subjects to whom no information was provided, some realized in the course of the experiment that the same target was presented several times. However, the average interval between repetitions was too large and irregular for any motor learning to occur. Instead, a general familiarization effect was conspicuous in the course of 10 practice trials, which always preceded the experiment. The instructions were introduced verbally and occasionally clarified during practice. At the beginning of each trial the subject had to position the cursor crosshair on the initial point of the target. A few seconds after an acoustic warning signal, the target spot started moving, and the subject's task was to track the spot as carefully as he or she could during the entire motion. The pace of the presentations was controlled by the experimenter, and, on average, trials were 10 s apart. However, short periods of rest were inserted at subjects' request. Subjects could also ask to repeat a

trial if they felt that during the movement they had lost the necessary concentration.

Results

The Results section is organized as follows. First, we will provide the operational definition of the main parameters used to describe the performances. Then we will describe the major qualitative findings of the experiments. Two subsections will be devoted to the quantitative analysis of the relation between the velocity of the movement and the pursuit error. Evidence is presented that this relation is mediated by a time delay that is specified by the subject's control strategy. In a successive subsection we capitalize on these findings to elaborate on a simple formal scheme for interpreting individual control strategies. Finally, the last subsection describes those idiosyncratic differences that cannot be accounted for by the formal scheme.

Descriptive Parameters

A full description of the tracking performance is provided by the position error vector,  $\Delta S(t) = T(t) - P(t)$ , joining the instantaneous positions of target and pursuit (cf. Figure 1, Panel B.) Various combinations of amplitude and phase of the error vector  $\Delta S(t)$  result in a variety of qualitatively different performances. For instance, one can imagine a subject following exactly the target trajectory with a relatively long time delay. Conversely, one can conceive of a subject who goes through all the geometrical features of the target trajectory with little or no delay but whose trajectory is shifted sidewise with respect to the target. Thus, to characterize individual performances quantitatively, it is convenient to distinguish between geometrical parameters, which describe the difference between target and pursuit trajectories, and kinematic parameters, which relate to the corresponding laws of motion. Among the many possible choices, the set of parameters to be described below appeared both parsimonious and sufficient for our purposes.

The shapes of the target and pursuit trajectories are uniquely defined by the respective curvatures  $C_T(t)$  and  $C_P(t)$ . To measure the similarity between trajectories, we have first defined an instantaneous delay  $\delta(t)$  as the time shift along the target trajectory for which the average distance between target and pursuit attains a minimum value (see Figure 1, Panel D). This distance is defined as

$$X^2 = \frac{1}{2\epsilon} \int_{-c}^{+c} |P(t+x) - T(t-\delta(t)+x)| dx.$$

Qualitatively speaking, for each point of the target trajectory,  $\delta(t)$  indicates the delay after which the pursuing point goes through the geometrically equivalent point along its own trajectory. Then, a global measure of the geometrical distortion of the pursuit with respect to the target is calculated as the mean distance of equivalent points over the entire duration  $T$  of the pursuit:

$$\sigma = \frac{1}{T} \int_0^T |P(t) - T(t - \delta(t))| dt.$$

As for the kinematic description of the movement, two quantities have been considered: the instantaneous difference of the velocity vectors,

$$\Delta V_i(t) = \dot{T}(t) - \dot{P}(t) = V_T(t) - V_P(t),$$

and the delayed velocity difference,

$$\Delta V_\delta(t) = \dot{T}(t - \delta(t)) - \dot{P}(t) = V_T(t - \delta(t)) - V_P(t).$$

The first parameter provides a real-time estimate of the velocity mismatch. From the point of view of the control strategy used to pursue the targets,  $\Delta V_i$  can be construed as a dynamic error signal. The second parameter estimates the effectiveness with which the subject has processed the target input and reproduced its velocity profile. Because the presence of a temporal delay seems inevitable in a tracking task, it can be assumed that the best possible performance that subjects can aspire to is achieved when  $\Delta V_\delta$  vanishes.

Analytic Description of the Performance

Figure 2 illustrates qualitatively the performance of 2 subjects for the same target (Condition N). Left panels show the

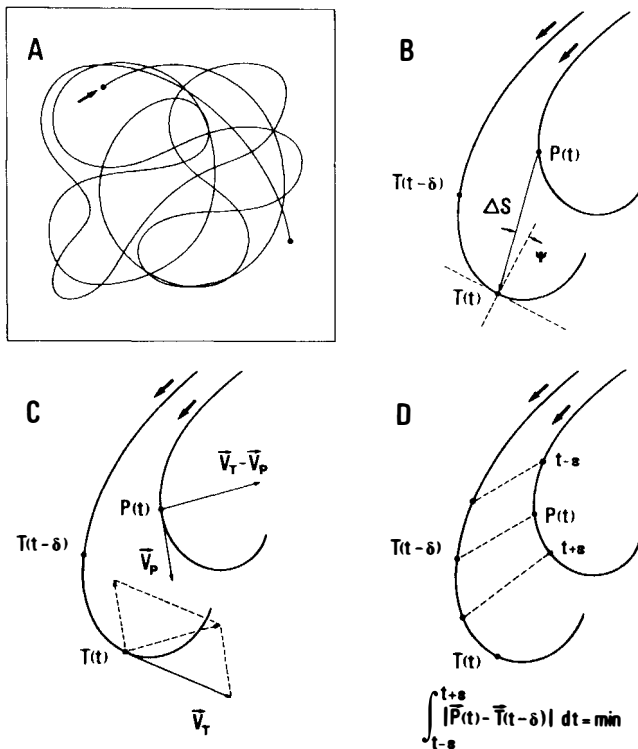


Figure 1. Characterizing a pursuit trajectory. Panel A: One of the five target trajectories. (The arrow indicates the starting point. The side of the square frame is 25 cm.) The other three panels illustrate schematically the main parameters used to characterize the pursuit performance.  $T(t)$  and  $P(t)$  indicate the instantaneous position of target and pursuit, respectively. Panel B: Position error vector  $\Delta S = T - P$ . (The vector phase angle  $\psi$  is calculated with respect to a moving frame of reference centered on  $T$ . Conventionally  $\psi$  is negative when the vectors  $\Delta S$  and  $V_T$  lie within the same half plane of the reference system [as in the example] and is calculated modulo  $\pi/2$ .) Panel C: Instantaneous difference between the tangential velocity vectors of target and pursuit. Panel D:  $T(t - \delta)$  is the point of the target trajectory that is most similar (in the mean-square sense) to the point  $P(t)$  on the pursuit trajectory. (The instantaneous delay  $\delta$  is the value that minimizes the expression shown inset. In general,  $\delta$  varies continuously during the pursuit [see Figure 3].)

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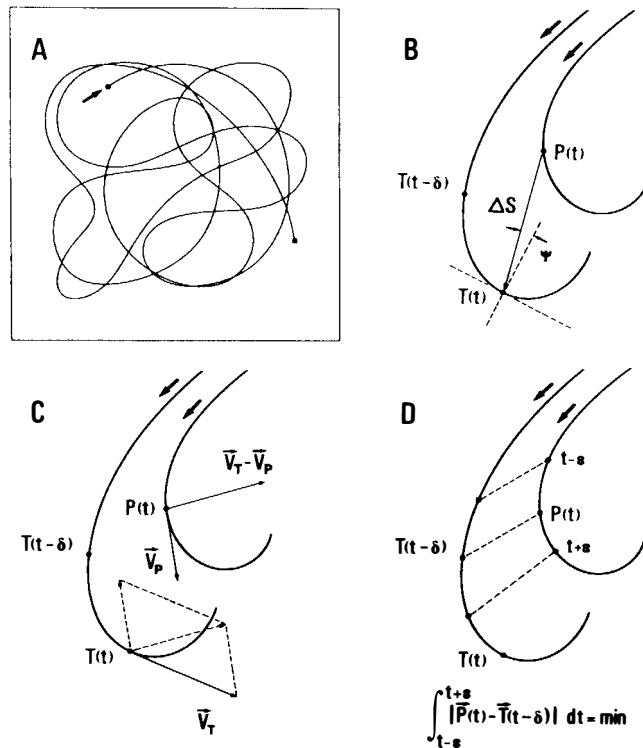


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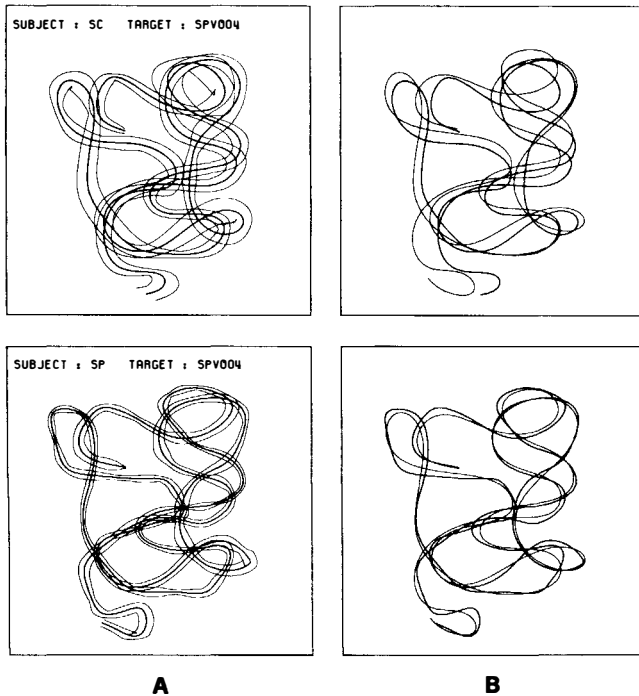


Figure 2. Typical examples of pursuit trajectories: data for the two most dissimilar subjects (SC and SP). Panel A: Heavy lines are average pursuit trajectories over all trials; light line bands around averages are the envelopes of the confidence ellipses for each point of the trajectory and estimate the trial-by-trial geometrical variability. Panel B: Comparison between average pursuit trajectory (heavy lines) and target (light line). (Over and above the considerable individual differences both in mean accuracy and variability, the trial-by-trial consistency of all subjects is quite remarkable.)

average pursuit trajectory (heavy line) and the geometrical variability across 10 pursuits of that particular target. Right panels compare the target trajectory (light line) with the average pursuit (heavy line). From the point of view of the trajectories, these 2 subjects represent the extreme cases in our experimental sample. Subject SC is the most variable across trials and the least accurate. Subject SP is both quite consistent and accurate.

Each panel in Figure 3 shows representative examples for 1 subject and one target pair of the time course of some of the descriptive parameters defined in the previous section. Upper and lower panels are relative to N- and T-type targets, respectively (cf. Method section). Corresponding panels in Figure 4 summarize the average performance of all subjects for a different target pair. In Figure 3 standard deviation bands indicate the variability across repetitions for some of the parameters; in Figure 4 they indicate the variability of the individual means.

In both figures the first three rows in each panel represent the modulus and the phase  $\psi$  with respect to the moving reference of the position error  $\Delta S$  (cf. Figure 1, Panel B), and the modulus of the pursuit velocity  $V_P$ . The fourth row shows the modulus of the instantaneous velocity difference  $\Delta V_i$ . Fifth and sixth rows report the instantaneous time delay  $\delta$  and the modulus of the delayed velocity difference  $\Delta V_d$ . Finally, the bottom row compares the curvatures of the target (heavy line) with that of the average pursuit. To allow a better comparison, the pursuit curve

has been uniformly shifted leftward by a time interval that maximizes the cross correlation of the two curves (Crossmann, 1960). The last four parameters have been calculated on the average performance across 10 trials in Figure 3 and over all subjects and all trials in Figure 4. For this reason there are no standard deviation bands for these curves.

Several qualitative conclusions can be derived from these typical results. First, the within-subjects variability across successive trials is strikingly low. This is sharply in contrast with the subjective awareness of the difficulty of the task. All reported, in fact, that their motor behavior during the task was quite erratic. Second, the interindividual variability, as measured at this coarse-grained level of analysis, is also rather low. In particular, the relative variability across subjects of the pursuit velocity (third row) is about 0.10. Thus, the obvious differences in the time course of the kinematic parameters are, to a large extent, target specific. However, in a later section, a closer analysis of the data will permit us to separate out the dependence of the performance on the target from the idiosyncratic characteristics of individual subjects. Third, all parameters of the performance vary systematically in the course of the movement. A comparison of the pursuit parameters with the curvature (bottom row) suggests a major influence of the trajectory shape in determining these variations. This is particularly evident for the pursuit velocity and geometrical error (first row). The phase angle of the error vector (second row) shows that at all points of small curvature, pursuit trails behind the target, the two trajectories being almost aligned. As curvature gets larger, the pursuit path tends to lie sideways with respect to the target and occasionally may lead it. As a fourth and final point, we consider the difference between Conditions N and T. A comparison between the two sets of curves cannot be done on an instant-by-instant basis. In fact, to obtain a constant tangential velocity in Condition T, the time scale had to be modified so that corresponding figural elements in the two trajectories occurred at different absolute times. However, by comparing equivalent features, it is possible to conclude from these typical examples that the difference between the performances in the two conditions depends upon the target. In the case of the target analyzed in Figure 3, both geometrical and kinematic features are quite similar. Conspicuous differences are instead present in the kinematic parameters of the target analyzed in Figure 4. For all targets, the shape of the pursuit trajectory, as revealed by the curvature profile, departs from the target trajectory in a quite similar way for N and T trials. The major individual differences are in the delay  $\delta(t)$  with which the pursuit reaches equivalent positions on the trajectory with respect to the target. This point is analyzed in detail in the next paragraph.

#### Average Delay and Average Position Error

Intuitively, the position error vector  $\Delta S(t)$ , the delay  $\delta(t)$ , and the pursuit velocity vector  $V_P(t)$  are mutually dependent. In the simplest possible case (target and pursuit trajectories coincide,  $V_P(t) \approx \text{constant}$ ), the relation among these three quantities is

$$\Delta S(t) = \delta(t) \cdot V_P(t).$$

In fact, the traces in Figure 2 show that  $\Delta S(t)$ ,  $\delta(t)$ , and  $V_P$  covary in qualitative agreement with this simple relation. One can, however, pose the question of what causal relation should

SUBJECT : SG TARGET : SPV001

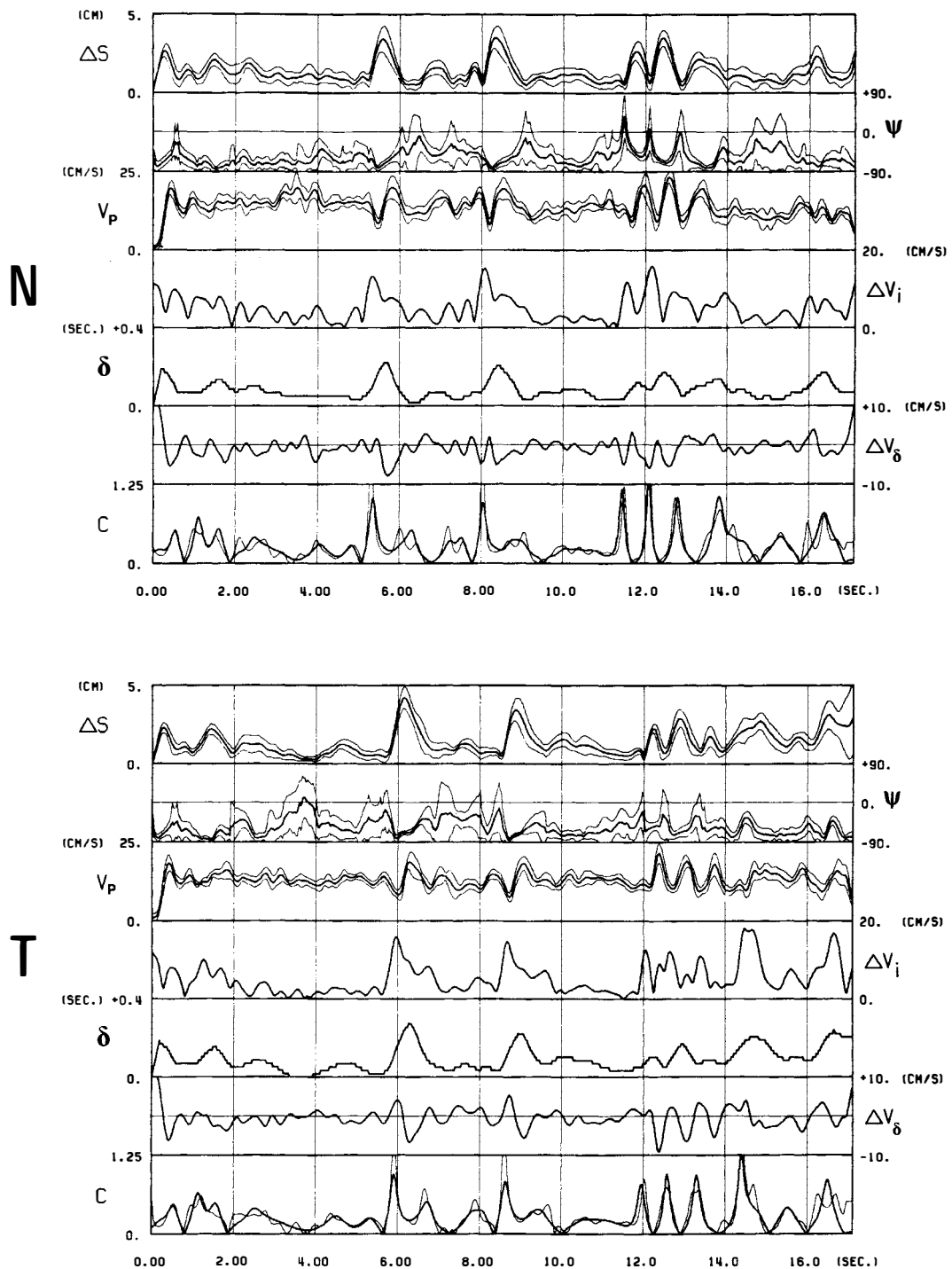


Figure 3. Descriptive pursuit parameters: data for 1 representative subject (SG) and one target. (N = results for N-type targets; T = results for T-type targets [constant tangential velocity]). Each panel reports the time course of several pursuit parameters chosen to characterize the performance. The first three curves show the averages and standard deviations bands of the indicated parameters across all repetitions:  $\Delta S$  = modulus of position error vector;  $\psi$  = phase of position error vector [cf. Figure 1];  $V_p$  = modulus of tangential pursuit velocity. The instantaneous velocity error [ $\Delta V_i$ ], the instantaneous delay  $\delta$ , and the delayed velocity error [ $\Delta V_\delta$ ] have been calculated on the average performance across all trials. Bottom line compares target [heavy lines] and pursuit [light lines] curvatures. The latter has been shifted leftward by an amount equal to the average delay.)

SUBJECT : AV TARGET : SPV005

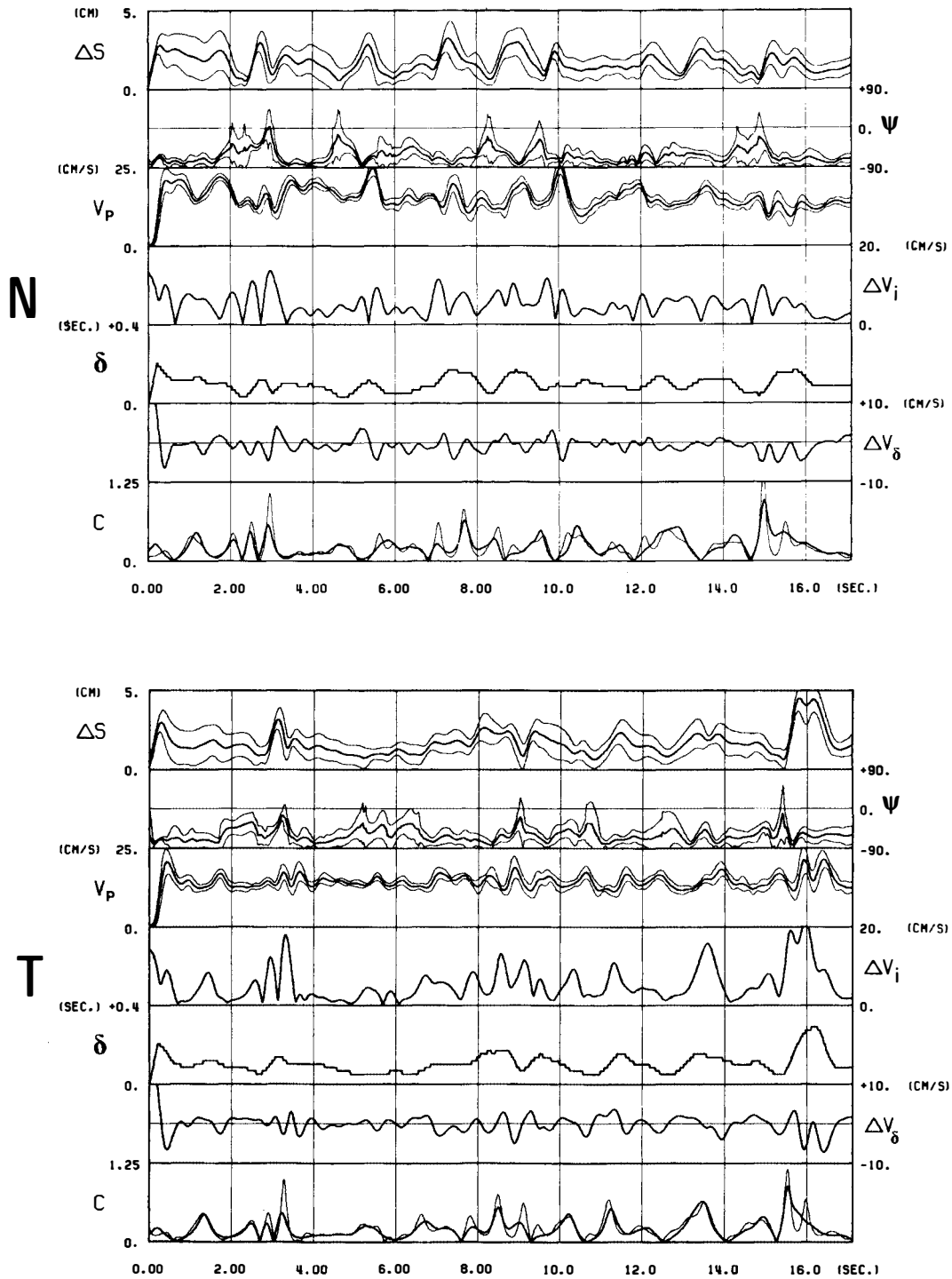


Figure 4. Descriptive pursuit parameters: average results for all subjects. (Same conventions as in Figure 3.)

be inferred from these data. More specifically, two simple hypotheses can be entertained: Either the time delay is somehow specified by the subject's control strategy, and the position error  $\Delta S$  results simply from kinematics, or the converse is true. A three-way analysis of variance (10 Subjects  $\times$  5 Trajectories  $\times$

2 Dynamic conditions, 10 repetitions for each cell) of the average of  $\delta$  and  $\Delta S$  over each trial provides a clue to answer this question. All three factors affect both  $\delta$  and  $\Delta S$  significantly (see Table 1). However, the target component of  $\delta$  is far smaller than the subject component (the percentage of variance accounted



Table 1  
Temporal Delay, Modulus of Position Error, and Geometrical Distortion

Modality	Targets									
	1	2	3	4	5					
Average of instantaneous delay $\delta$ (ms)										
N	104	112	87	102	98					
T	111	118	93	106	107					
Average position error $\Delta S$ (cm)										
N	1.64	2.18	1.65	1.39	1.78					
T	1.63	2.04	1.63	1.35	1.85					
Geometrical distortion $\sigma$ (cm)										
N	0.72	0.96	0.74	0.64	0.81					
T	0.69	0.88	0.69	0.62	0.81					
Modality	Subjects									
	SV	SC	SG	SA	SP	SF	SS	SX	SM	SZ
Average of instantaneous delay $\delta$ (ms)										
N	52	185	74	82	56	114	86	127	116	114
T	53	194	78	89	59	128	90	134	127	120
Average of position error $\Delta S$ (cm)										
N	1.14	2.78	1.34	1.48	1.04	1.96	1.59	2.08	1.98	1.90
T	1.10	2.64	1.44	1.47	1.01	1.96	1.53	2.03	1.93	1.85
Geometrical distortion $\sigma$ (cm)										
N	0.68	0.93	0.68	0.70	0.57	0.91	0.79	0.86	0.83	0.78
T	0.65	0.86	0.74	0.69	0.54	0.84	0.74	0.81	0.76	0.74

Note. Averages across all trials of the indicated quantities are collapsed over targets and subjects for both normal (N) and transformed (T) modalities. All experimental factors affect significantly both the instantaneous temporal delay  $\delta$  and the position error  $\Delta S$ . For targets,  $\delta$ :  $F(4, 900) = 53.20, p \ll .001$ ;  $\Delta S$ :  $F(4, 900) = 152.35, p \ll .001$ ;  $\sigma$ :  $F(4, 900) = 57.68, p \ll .001$ . For subjects,  $\delta$ :  $F(9, 900) = 544.45, p \ll .001$ ;  $\Delta S$ :  $F(9, 900) = 257.02, p \ll .001$ ;  $\sigma$ :  $F(9, 900) = 25.72, p \ll .001$ . For modality,  $\delta$ :  $F(1, 900) = 34.85, p \ll .001$ ;  $\Delta S$ :  $F(1, 900) = 2.59, p = .104$ ;  $\sigma$ :  $F(1, 900) = 8.14, p = .010$ . The analysis of the effect sizes suggests that the time delay is mostly subject specific and that the position error results from the interaction of this delay with the average target velocity.

for by the two factors is 3.4% and 79.5%, respectively). The size of the effect of the target and of the subject on the geometrical error  $\Delta S$  is also different (15.1% and 57.6% of the total variance, respectively). Thus, to a first approximation, the time delay  $\delta$  can be assumed to be largely subject specific, and the results may be taken to suggest a causal sequence in which the stimulus-driven pursuit velocity interacts with the delay to produce a position error.

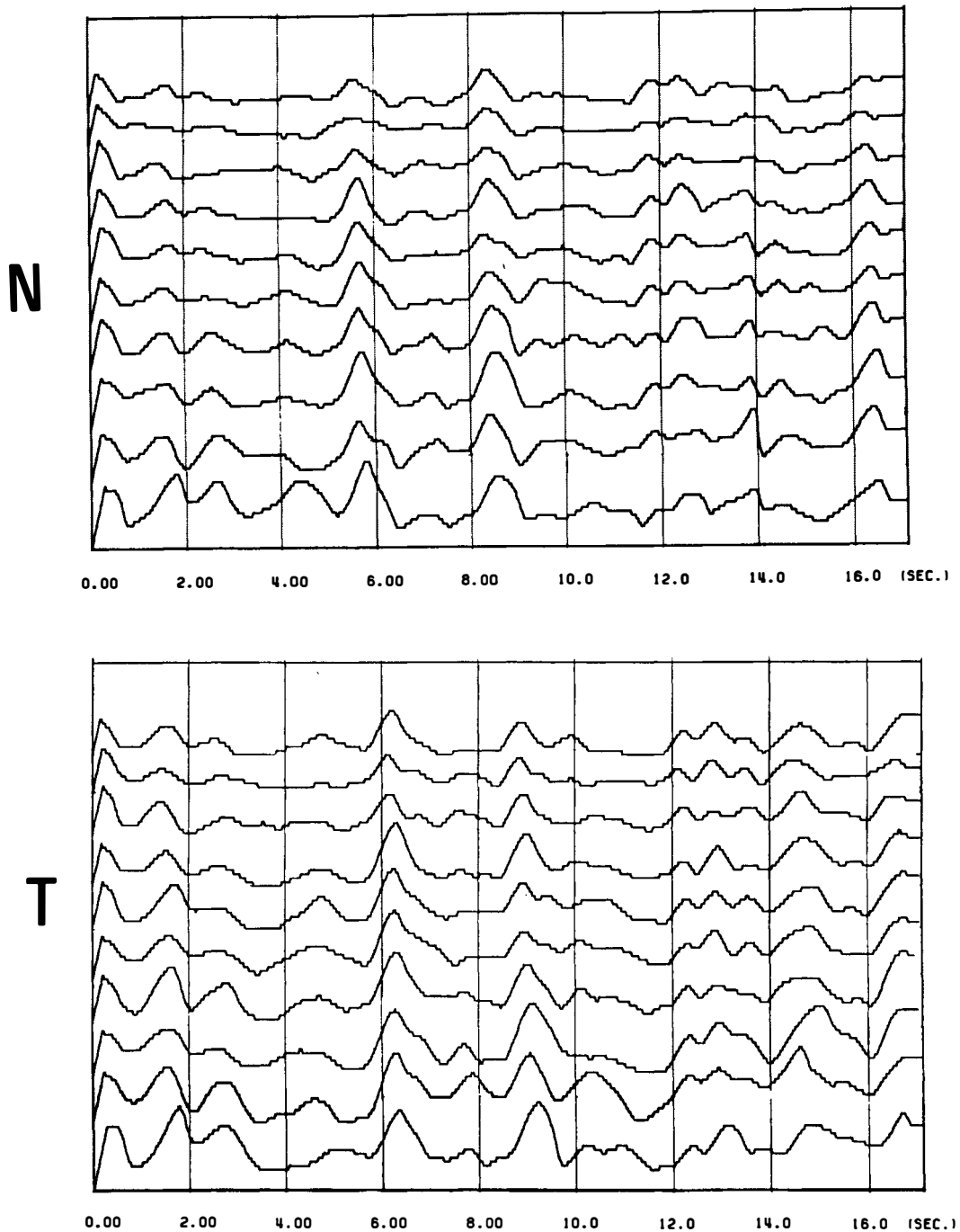
The above conclusions, however, are not fully satisfactory on at least three counts: (a) A significant target effect does exist; (b) a significant condition effect (normal vs. transformed) can also be demonstrated; (c) the instantaneous delays are far from constant; thus, the analysis of the average values tells only one part of the story. Ideally, one would wish to single out a small set of subject-specific parameters that, by interacting with the experimental conditions, account for the points mentioned above. However, before attempting to do so, we shall analyze in more detail the fluctuations of the instantaneous delays and their relation with pursuit and target velocity.

#### Instantaneous Delays and Velocities

Whereas the average time delay  $\delta$  varies considerably across subjects and, to a lesser extent, across targets and modalities (cf.

previous section), its instantaneous fluctuations are basically target dependent. This is shown in Figure 5, where the time course of the delay for one target in both modalities is compared for all subjects. In this figure the zero lines have been displaced arbitrarily to emphasize graphically the fact that all major features of the delay curves (both in Modality N and T) are present in all subjects. As already shown in Figures 3 and 4, these features differ from target to target. However, the peak-to-peak amplitudes of the delay oscillations are proportional to their average values. Intuitively, the delay increases when target velocity exceeds pursuit velocity and decreases when target velocity falls behind pursuit velocity. Thus, delay fluctuations must reflect the relation between the instantaneous velocities. To investigate this point further, we must first consider the relation between target velocity and the shape of its trajectory.

Previous researchers (Viviani & Terzuolo, 1982) have shown that, at all points sufficiently removed from inflections, the tangential velocity of spontaneous movements (as the N-type targets) is proportional to the one third power of the radius of curvature  $R_T$  of the trajectory. This is illustrated, for the fastest N-type target, in the upper left panel of Figure 6, where the instantaneous values of  $V_T$  are plotted as a function of corresponding values of  $R_T^{1/3}$ . The other two upper panels of the fig-



*Figure 5.* Instantaneous delay of the pursuit. (Each panel compares the delay profiles of all subjects for one target in normal [N] and transformed [T] conditions. Each profile has been calculated on the average performance of the subject over 10 trials. Individual records have been stacked from top to bottom according to the average delay. Zero lines have been shifted arbitrarily to emphasize graphically the presence in all subjects of the major curve features. It can be shown that the amplitude of peak-to-peak oscillations of  $\delta$  is positively correlated with the average delay and that these oscillations occur at the same points on the trajectory irrespective of the modality. Notice that curves in N and T cannot be compared on an instant-by-instant basis because of the time scale deformation introduced by the transformation procedure [see Method section].)

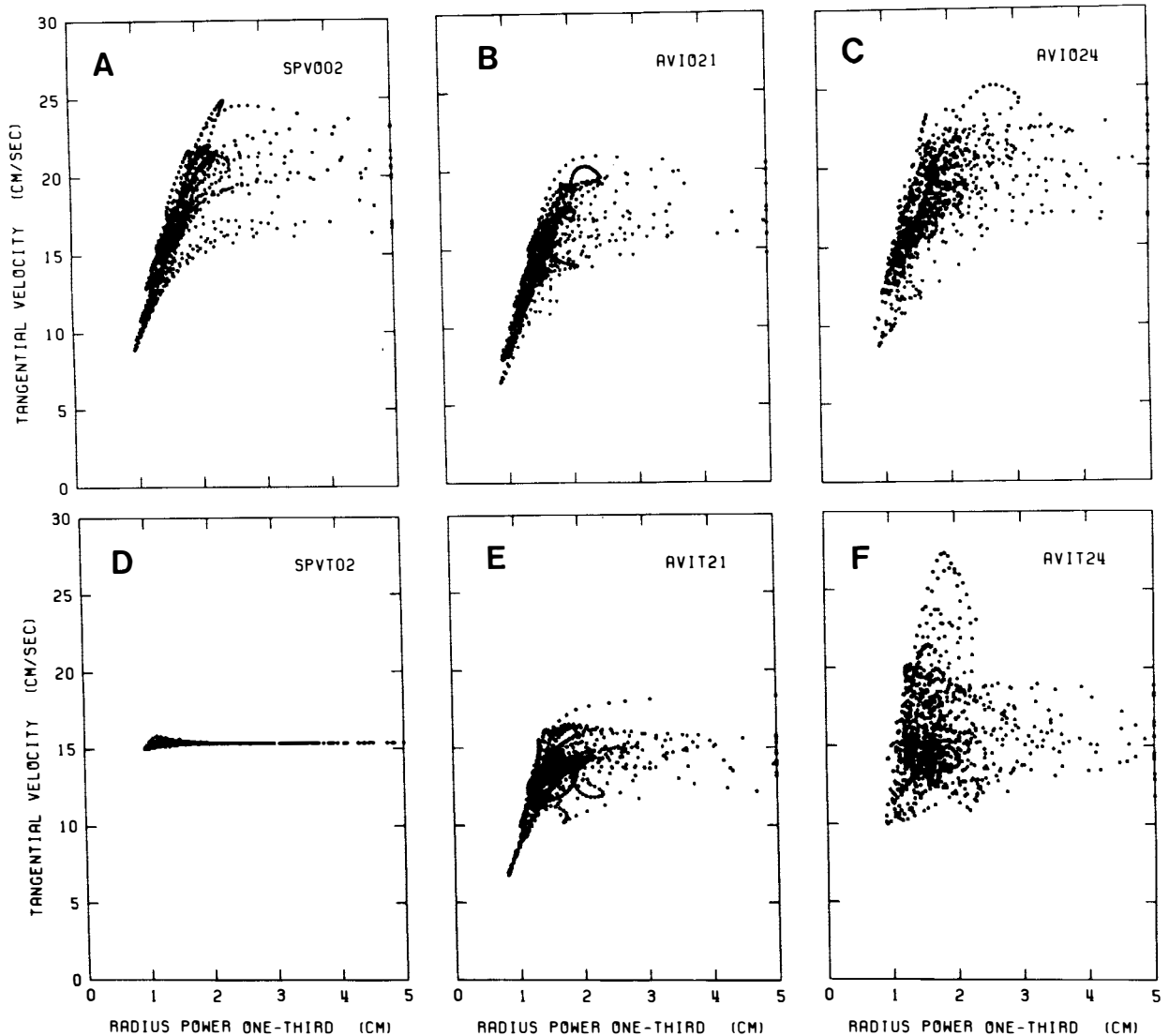


Figure 6. Relation between curvature and tangential velocity. Panel A: tangential velocity at each point of the movement as a function of the corresponding one-third power of the radius of curvature for an N-type target. (At all points of the trajectory sufficiently removed from inflections the two quantities are almost proportional. The tangential velocity levels off for large radii of curvature.) Panels B and C: same relation as in Panel A for pursuit data in the 2 most dissimilar subjects (also shown in Figures 2 and 7). Panels D, E, and F: same as in Panels A, B, and C for a T-type target.

ure describe the relation  $R_p^{1/3} - V_p$  for the average pursuit data in Condition N from the 2 subjects (SC and SP) with the longest (middle) and shortest (right) average delay, respectively. High average values of  $\delta$  result in a performance that largely preserves the covariation between radius of curvature and tangential velocity. Small average values of  $\delta$  somehow disrupt this relation. In Condition T (lower panels of Figure 6) no subject was ever able to reproduce the constant tangential velocity of the target (cf. Figures 3 and 4). Subjects with long delays preserve some amount of correlation between velocity and radius of curvature. As the average delay gets shorter, the correlation disappears altogether.

These data permit us to account partly for the fact that delay fluctuations are basically target specific. To the extent that tar-

get and pursuit trajectories are similar (cf. Figure 2) and that an average delay is introduced as part of the individual strategies (cf. section on average delay and position error), fluctuations of  $\delta(t)$  in Condition N can be due to the delay with which the variations of the target velocity are replicated by the pursuit. In Condition T,  $|V_T|$  is constant, but, as shown above,  $V_p$  is nevertheless modulated by changes in pursuit curvature. Thus, also in this case, the difference between  $V_p$  and  $V_T$  is related to the shape of the trajectory.

#### A Virtual Mechanical Model

In this section we present a simple scheme to account for individual control strategies in terms of few subject-specific pa-

rameters. At any time instant,  $t$ , target and pursuit differ both in their values  $\mathbf{T}(t)$  and  $\mathbf{P}(t)$  and in the values of their time derivatives. In principle, all these quantities may be used by the subject as input error signal in a classical feedback control scheme (Gottsdanker, 1955, 1956). Let us suppose, however, that only displacement ( $\Delta\mathbf{S}$ ) and velocity ( $\Delta\mathbf{V}_i$ ) differences are actually taken into account. Moreover, by analogy with the behavior of a mass-spring system, let us assume that the force signal that drives the subject movement results from a linear combination of these two differences. Finally, we also suppose that the processing of the error signals takes a constant time  $\Delta t$ , which may vary from subject to subject. In summary, the equation of the pursuit movement can be written as

$$\ddot{\mathbf{P}}(t + \Delta t) = \alpha(\mathbf{T}(t) - \mathbf{P}(t)) + \beta(\dot{\mathbf{T}}(t) - \dot{\mathbf{P}}(t)) + \mu, \quad (1)$$

where the values of the coefficients  $\alpha$ ,  $\beta$ , and  $\mu$  are supposed to be characteristic of each subject. The position and velocity error coefficients  $\alpha$  and  $\beta$  can be conceptualized as a virtual stiffness and as a virtual viscosity, respectively.

The adequacy of this model has been tested using multiple regression analysis. For each trial and each fixed value of the time constant,  $\Delta t$ , this analysis yields both the least square estimates of  $\alpha$ ,  $\beta$ , and  $\mu$  as well as the residual quadratic error. The time constant,  $\Delta t$ , for which this residual takes the absolute minimum value, and the corresponding triples  $\alpha$ ,  $\beta$ ,  $\mu$  are taken to represent the pursuit tracking performance of 1 subject. The results of this validation of the model are extremely satisfactory inasmuch as the average and standard deviations of the multiple correlation coefficient are .924 and .24, respectively. Figure 7 compares for 2 subjects and one target in Conditions N and T the actual profiles of the pursuit acceleration with the prediction of the linear regression model.

The velocity error coefficient  $\beta$  accounts for most of the variance as demonstrated by the fact that the average ratio of the partial correlation coefficient to the multiple correlation coefficient is .992. The intercept parameter  $\mu$  does not turn out to be significantly different from zero and can be neglected. Thus, the pursuit motion is described quite accurately by the system of delayed differential equations:

$$\begin{aligned} \ddot{X}_p(t) &= \alpha(X_T(t - \Delta t) - X_p(t - \Delta t)) \\ &\quad + \beta(\dot{X}_T(t - \Delta t) - \dot{X}_p(t - \Delta t)) \\ \ddot{Y}_p(t) &= \alpha(Y_T(t - \Delta t) - Y_p(t - \Delta t)) \\ &\quad + \beta(\dot{Y}_T(t - \Delta t) - \dot{Y}_p(t - \Delta t)). \quad (2) \end{aligned}$$

Table 2 reports the average across all repetitions of the least square estimate of  $\alpha$ ,  $\beta$ , and  $\Delta t$  for all subjects and all targets. In the same table are reported the results of a three-way analysis of variance (10 Subjects  $\times$  5 Targets  $\times$  2 Modalities, 10 repetitions for each cell) of these data. The "viscous" parameter  $\beta$  is highly variable across subjects ( $p \ll .01$ ) and for each subject depends on the modality of the target (N vs. T;  $p \ll .01$ ). In contrast, it is almost independent of the actual trajectory of the movement ( $p \simeq .1$ ). The delay  $\Delta t$  is again subject and modality specific. A significant effect of the target trajectory can also be demonstrated (in all three cases  $p \ll .01$ ). Finally, a number of two- and three way interactions turn out to be significant both for  $\beta$  and  $\Delta t$ . Over and above these statistical findings, the row and column averages in Table 2 permit us to summarize the results qualitatively as follows: (a) The velocity error coefficient

is the main factor for distinguishing subjects. (b) The processing delay,  $\Delta t$ , is far less variable across subjects than the average pursuit delay  $\delta$ . Thus, one would be inclined to interpret  $\beta$  as a strategic parameter that characterizes the subject's approach to the task, and  $\Delta t$  as an intrinsic property of the visuo-motor loop.

### Patterns of Behavior

Because the velocity error coefficient,  $\beta$ , is the one target-independent parameter that discriminates most effectively among subjects, it should correlate with other measurable aspects of individual performances. Figure 8 illustrates the direct relations that indeed exist between the inverse of the velocity error coefficient and three such aspects: the average position error  $\Delta\mathbf{S}$  (A), the average instantaneous delay  $\delta$  (B), and the average geometrical distortion  $\sigma$  (C). In all these cases, linear correlation with  $1/\beta$  is highly significant (pooling the two conditions, for  $\Delta\mathbf{S}$ ,  $r = .94$ ; for  $\delta$ ,  $r = .91$ ; for  $\sigma$ ,  $r = .74$ ). Thus, describing the tracking control strategy by a delayed-velocity feedback scheme allows one to represent the experimental population in a simple one-dimensional continuum, at least as far as some global descriptors of the behavior are concerned. However, when one analyzes the pursuit velocity in more detail, individual patterns of behavior emerge that cannot be reduced to differences in  $\beta$  values.

A finer distinction among subjects is obtained by evaluating the spectral components of the delayed velocity difference,  $\Delta\mathbf{V}_\delta$ , which estimate the effectiveness of the velocity matching at corresponding points on the trajectories (see section on descriptive parameters). Figure 9 compares the spectra (average and standard deviations bands for 10 repetitions) of  $\Delta\mathbf{V}_\delta$  for all N-type targets in 2 subjects. In all cases, the spectral components of the velocity mismatch cluster within the 0.4–0.8 Hz frequency range. However, Subject SF also has a prominent secondary narrow-band component centered at about 1.7 Hz, which is hardly perceptible in Subject SP. Some of the subjects behave like SF and others like SP, but the presence or absence of an additional spectral peak does not correlate with the corresponding  $\beta$  values.

Figure 10 compares the velocity mismatch spectra of all subjects for Targets 1 and 2 both in Conditions N and T. In the leftmost column, spectra have been ranked from top to bottom, using as a criterion the frequency of the secondary narrow-band component (arrows). As mentioned, this component is barely perceptible in some subjects. Records in all other columns have been arranged in the same order as in the first one. Vertical lines have been traced to emphasize that the dominant spectral component occurs at the same frequency in all subjects despite the large differences that exist in many other measurable aspects of the performance. In particular, the dominant frequency is independent of the value of the velocity error gain  $\beta$ . When present, the secondary control mode revealed by the peak in the spectrum tends to involve higher frequencies in subjects with large values of  $\beta$ , but this correlation disappears altogether for the very fast Target 2. Finally, the spectra of  $\Delta\mathbf{V}_\delta$  show that pursuit strategies in N and T trials differ systematically. In the latter, the main component is always sharply defined, but the secondary mode is never very obvious.

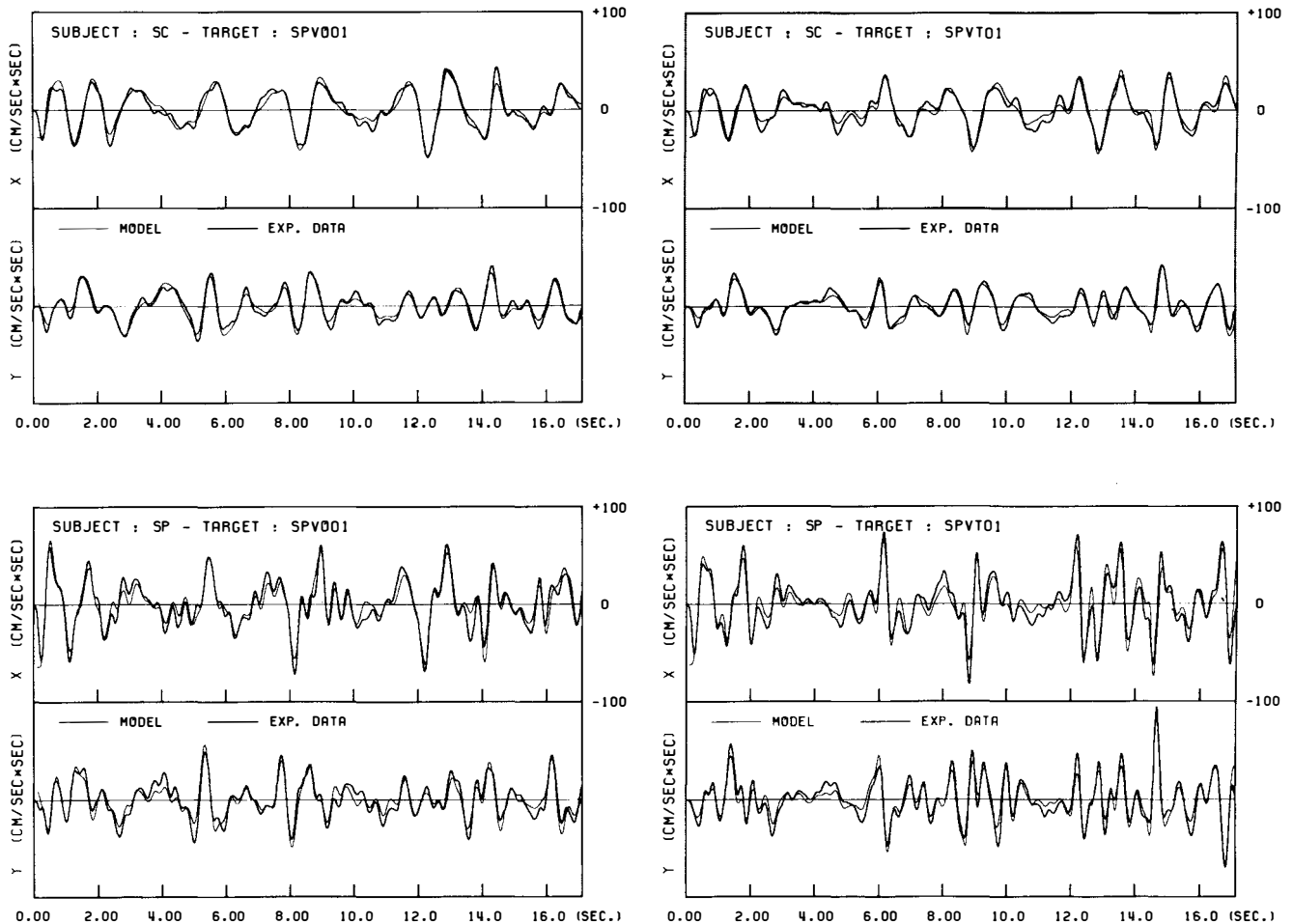


Figure 7. Theoretical analysis of the performances. Each panel compares the X and Y components of the pursuit acceleration (heavy lines) with the values predicted by Equation 1 in the text (light lines). (Upper panel is relative to 1 subject [SC] and one target in both modalities. Results for another subject [SP] and the same target are shown in the lower panel. Notice the large difference in frequency components between Subject SC, who has a long average delay, and Subject SP, who has the shortest one. In both cases the theoretical predictions are quite accurate.)

### Discussion

The experiments provide the first description of visuo-manual pursuit tracking of two-dimensional human movements. The methodological, factual, and theoretical contributions of the study will be discussed in this order.

One-dimensional pursuit tracking is completely described by the scalar difference  $T(t) - P(t)$  between the instantaneous values of the target and the pursuit. In the two-dimensional case, using two scalar differences, one for each coordinate of T and P, is formally correct. However, it may not provide the most relevant representation of the dynamic relation of the pursuit to the target because both the visual error input and the motor response are likely to be coded directly in vectorial terms. From this point of view, using the modulus and phase of the position error vector  $\Delta S$  seems much more satisfactory. Moreover, measuring the phase angle of  $\Delta S$  with respect to a moving frame of reference centered on the target (Figure 1, Panel B) is preferable to measuring it in an absolute frame because the error vector

direction is then independent of the actual shape of the target. In particular, the transition from a geometrical lag to a lead (Figure 1) is indicated unambiguously by a sign reversal of  $\psi$  (cf. Results). The instantaneous temporal delay has been cited as a possible measure of the performance in the one-dimensional case (Poulton, 1974), but, to our knowledge, no systematic attempt has been made to provide this parameter with an operational definition. The technique proposed here to evaluate  $\delta(t)$  is new and could also be applied in one dimension. Finally, the instantaneous and delayed velocity differences proved effective for modeling the performance and for distinguishing individual motor styles, respectively.

A first substantial issue that has been addressed by the experiments is the nature of the lag that always exists between target and pursuit. The classical scheme for describing pursuit tracking (Elkind, 1956; Krendel & McRuer, 1960; McRuer & Krendel, 1957, 1959a, 1959b; Stark, 1972) assumes that a position mismatch signal is sensed by the visual system and fed back to the movement-producing motor plant. If so, the instantaneous

Table 2  
*Synthetic Description of Individual Performances*

Modality	Targets									
	1	2	3	4	5					
Displacement error coefficient $\alpha$ (1/s <sup>2</sup> )										
N	0.57	-2.22	1.14	1.14	1.72					
T	1.53	-0.31	0.87	1.16	2.17					
Velocity error coefficient $\beta$ (1/s)										
N	7.63	7.66	7.48	7.53	7.65					
T	6.89	6.87	6.97	6.76	6.63					
Time constant $\Delta t$ (ms)										
N	127	128	133	130	133					
T	126	124	126	126	128					
Modality	Subjects									
	SV	SC	SG	SA	SP	SF	SS	SX	SM	SZ
Displacement error coefficient $\alpha$ (1/s <sup>2</sup> )										
N	4.40	-1.86	2.19	-1.38	3.68	0.45	1.34	-1.82	-2.02	-0.28
T	5.66	-1.60	2.25	-0.68	3.89	0.29	2.30	-1.09	-0.88	0.71
Velocity error coefficient $\beta$ (1/s)										
N	9.19	4.86	8.75	8.07	9.93	7.11	7.83	6.73	6.39	7.03
T	8.35	4.38	7.56	6.78	9.05	5.58	7.21	6.21	5.76	6.37
Time constant $\Delta t$ (ms)										
N	114	144	122	124	110	135	127	142	149	136
T	106	147	119	122	103	134	120	139	140	129

*Note.* Averages across all trials of model parameters are collapsed over the main experimental factors for both normal (N) and transformed (T) modalities. For targets,  $\alpha$ :  $F(4, 900) = 34.64$ ,  $p \ll .001$ ;  $\beta$ :  $F(4, 900) = 2.02$ ,  $p = .090$ ;  $\Delta t$ :  $F(4, 900) = 10.17$ ,  $p \ll .001$ . For subjects,  $\alpha$ :  $F(9, 900) = 64.86$ ,  $p \ll .001$ ;  $\beta$ :  $F(9, 900) = 550.00$ ,  $p \ll .001$ ;  $\Delta t$ :  $F(9, 900) = 256.87$ ,  $p \ll .001$ . For modality,  $\alpha$ :  $F(1, 900) = 10.00$ ,  $p = .001$ ;  $\beta$ :  $F(1, 900) = 410.66$ ,  $p \ll .001$ ;  $\Delta t$ :  $F(1, 900) = 62.92$ ,  $p \ll .001$ . The velocity error term alone accounts for most of the observed variance. The corresponding coefficient  $\beta$  depends mainly on the subject and the modality and is almost independent of the shape of the target trajectory.

lag would depend mainly on the time constants of the motor plant and on the frequency components of the target. (In this context, the perceptual system may be assumed to have no dynamic properties; supposedly, only pure delays are introduced by the visual system and the central transmission, which have been estimated at 40 ms and 15 ms, respectively [Stark, 1972].) In fact, we have shown that average lags vary considerably across individuals and far less across targets. For instance, despite the fact that Targets 2 and 4 differ substantially in average speed (16.67 cm/s vs. 10.42 cm/s) and that the former contains components with much higher frequency, the corresponding average  $\delta$  for all subjects is almost equal to that for Target 4. By contrast, the longest lag among all subjects is more than three times the value of the shortest one. All this seems incompatible with the hypothesis that the lag is due to only relatively constant factors such as the motor plant dynamics or the visual processing and muscle contraction delays. It suggests instead that each subject spontaneously develops an idiosyncratic strategy for coping with the task, which results in a specific temporal lag of the pursuit movement relative to the target. Individual strategies, however, do not differ qualitatively, as demonstrated by the fact that one simple scheme can encompass a large array of performances. Actually, one single parameter in this scheme, the

velocity error coefficient  $\beta$ , accounts for most of the individual differences. Thus, the high covariation of  $\beta$  with the average delay suggests that the operating point selected spontaneously by each subject is mainly defined in terms of this coefficient. In other words, subjects adjust the degree of coupling between the perceptual process of perceiving the velocity mismatch and the motor process that reacts to such mismatch. Because subjects with long lags are also those whose pursuit track is most distorted with respect to the target (Figure 8), this process of adjustment may conceivably be dictated by a trade-off between the level of commitment to perform well and the attentional load required for operating under conditions of high visuo-motor coupling. To examine this point adequately, however, these two variables should be manipulated experimentally.

The second issue directly addressed by our study concerns the possible interaction between the motor requirements posed by the tracking task and the intrinsic properties of spontaneous movements. If indeed geometrical and kinematical aspects of voluntary, unimpeded movements are intrinsically related (Lacquaniti, Terzuolo, & Viviani, 1983; Viviani & Cenzato, 1985; Viviani & Terzuolo, 1982), many conceivable tracking strategies would conflict with this relation. In particular, the hypothetical presence of a velocity feedback loop (cf. Equation 1)

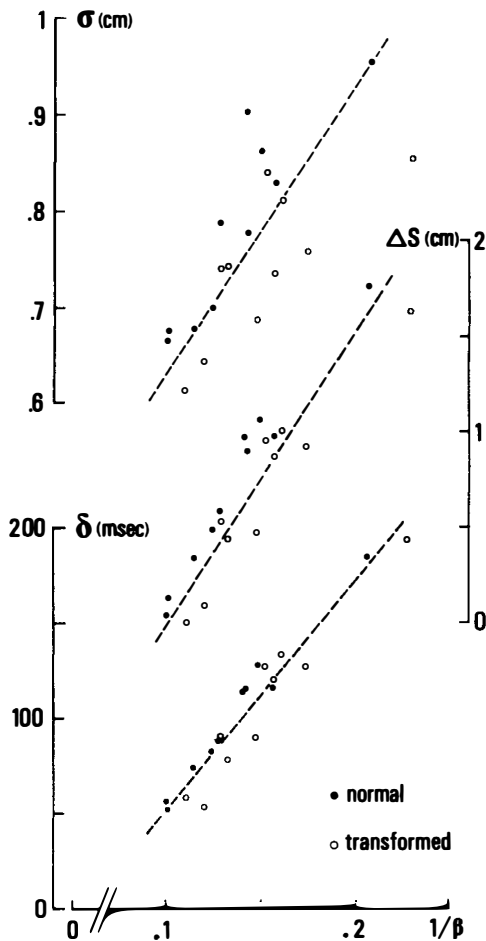


Figure 8. The velocity error coefficient characterizes pursuit behavior. (From top to bottom: mean values of the geometrical distortion  $\sigma$ , of the position error  $\Delta S$ , and of the instantaneous delay  $\delta$  plotted as a function of the inverse of the velocity error coefficient  $\beta$ . Each data point represents the average performance of 1 subject for all trials and all targets in one of the two modalities, normal [N] and transformed [T] [N = filled circles; T = empty circles]. Linear interpolations [dashed lines] are fitted by eye. The results demonstrate that several aspects of individual performances covary with the velocity error coefficient.)

would force the instantaneous pursuit velocity to depart from the value that is appropriate according to the empirical power law illustrated in Figure 6, Panel A. The stronger the coupling, the more severe we expect this departure to be. The data of Figure 6 support this line of reasoning by showing that the subject with the smallest  $\beta$  value (Subject SC, Panel B) preserves a fair amount of correlation between the velocity and curvature of her pursuit, but this is not the case for the subject with the highest  $\beta$  value (Subject SP, Panel C). Several differences have been noted between the pursuit of N- and T-type targets: In Condition T both the geometrical distortion  $\sigma$  and the response time constant  $\Delta t$  are slightly but significantly smaller, while the position error is higher than in Condition N. Moreover, the phase angle of the error vector is qualitatively different, and so are the spectra of the delayed velocity difference. All these effects, however, cannot obscure the fact that, qualitatively speaking, the overt motor responses in the two conditions are

quite similar. A very significant and consistent difference appears instead in the velocity error parameter  $\beta$ , which represents the degree of coupling. Within the framework of the proposed scheme, this implies that all subjects adjust this strategic parameter when switching from one condition to the other and that such adjustment is instrumental in maintaining overt behavior almost invariant. As shown in Figure 6, Panel E, under conditions of weak coupling, pursuit movements of T-type targets also exhibit a strong correlation between velocity and radius of curvature. In other words, the tracking mechanism shapes the response kinematics mainly on the basis of target trajectory, and the responses are compatible with the relational properties of spontaneous movements mentioned above. As the coupling gets tighter, such a compatibility remains (to some extent) only for N-type targets. Instead, with T-type targets a conflict arises between the velocity profile induced by the act of pursuing and the motor system's own peculiar mode of operation. As a result, response velocity is neither constant as in the target nor related to curvature (Figure 6, Panel F). Moreover, the pursuit acceleration is far more irregular (Figure 7). If we interpret the reaction of each subject to N-type targets in analogy with the interpretation given above of individual differences, we can surmise that slackening the coupling in Condition T represents a simple solution for reducing the mismatch between the natural mode of operation of the motor system and the requirements of the task.

To summarize the comparison between N and T conditions, we might say that pursuit tracking movements are not intrinsically different from spontaneous ones. As long as the input has those relational properties possessed by natural gestures, pursuit mechanisms tend to produce responses that have similar properties. Conversely, the fact that even a relatively mild manipulation of the target's law of motion results in a measurable increase in this lag stresses again the necessity of taking into account the properties of the motor control system in the design of effective tracking devices.

As a third and final point, we discuss briefly the import of the formal scheme expressed by Equation 1. It should be clear that this scheme is inappropriate as a model of the actual operations involved in tracking pursuit. Our aim was simply to isolate the minimal set of assumptions necessary to capture the salient features of the performances. Accordingly, no attempt was made to develop block diagrams such as those proposed, for instance, to describe the operation of the ocular pursuit system (cf. Carpenter, 1977). The predictions of the scheme, however, turned out to be surprisingly accurate, and this is somewhat embarrassing. On the one hand, it is difficult to justify any further refinement of Equation 1, and, on the other hand, it forces one to interpret the results within a very streamlined framework. In fact, from Equation 2 it follows that each pursuit component is driven by the corresponding target component via the same transfer function:

$$Z(s) = \frac{\beta s + \alpha}{e^{\Delta t s} s^2 + \beta s + \alpha} \quad (3)$$

By inserting in this expression the individual means of  $\alpha$ ,  $\beta$ , and  $\Delta t$  in Condition N (Table 2), the gain and phase characteristics of the transfer function can be displayed as in Figure 11 (analogous curves are obtained for Condition T). It then appears that

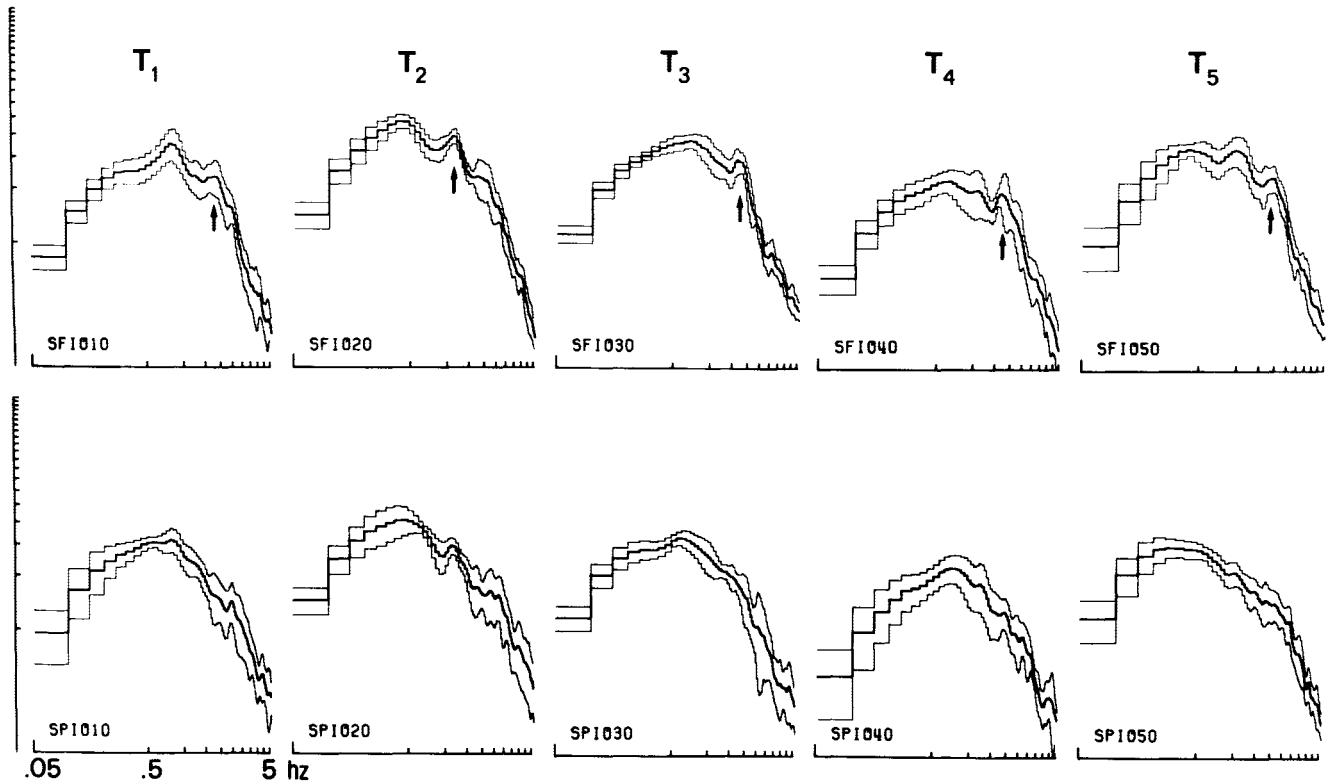


Figure 9. Individual patterns of behavior. (Amplitude Fourier spectra of the delayed velocity difference  $\Delta V_s$  for 2 representative subjects [upper and lower row] and all N-type targets [ $T_1$  to  $T_5$ ]. Averages [heavy lines] and standard deviation bands were calculated from 10 trials. Vertical scales are arbitrary but identical in all cases. In all subjects the main spectral component of the velocity mismatch is at about 0.5 Hz. A harmonic component at higher frequency [arrows] is clearly distinguishable in the performances of some subjects [such as SF] and almost nonexistent in some others.)

the behavior of all subjects is qualitatively similar to that of a second-order underdamped system (notice that the values of the "viscous" and "elastic" parameters  $\beta$  and  $\alpha$  would make the system overdamped were it not for the central processing delay).

To what extent does Equation 3 represent the properties of the behaving subject rather than the physics of the apparatus and of the moving masses? To address this question, let us note that although the experimental condition has been described as a pursuit-tracking one, it can also be construed as an instance of compensatory tracking. Following Powers (1978), one could, in fact, argue that the subject's primary goal is not to produce a specified overt behavior (i.e., a specific trajectory) but rather to minimize the discrepancy between the actual and ideal value of the position error. Thus, within the general framework proposed by Powers for describing purposive systems, we would identify the system input with the position error vector  $\Delta S$ , the system output with the pursuit position; the target position takes the role of an external disturbance that moves the input away from its ideal internal reference ( $\Delta S = 0$ , under the stated assignments). The system function (forward branch of the loop) is described by the operator  $[\alpha/s^2 + \beta/s]e^{-\Delta t s}$  and the feedback function is just  $-1$ . The loop gain is such that the system is stable. However, as one can see by substituting the values of Table 2 in the system function, its modulus in the frequency domain of interest is not very large. Therefore, the relation be-

tween  $P(t)$  and  $T(t)$  expressed by the functional (Equation 3) does indeed characterize some aspects of the subject's performance.

Despite the simplicity of the assumptions, two such aspects brought to the fore by the proposed scheme are worth emphasizing: (a) The velocity error is the principal factor for driving the response in the course of the movement; (b) a long central processing delay exists, which appears to be fairly constant across individuals.

Point 1 is in direct conflict with the view already mentioned above, that is, in the case of unpredictable inputs, the human tracking system acts as a position feedback control system (Stark, 1972). Because position errors in pointing tasks are limited to a very narrow dead zone, first-order control is certainly available and may be operating also in pursuit tracking. However, our analysis indicates that its contribution is marginal and fluctuates considerably not only from subject to subject but also between targets. In 4 out of 10 subjects, and for Target 2, the position feedback actually becomes positive (i.e., error-increasing instead of error-decreasing). This model-based inference can be made less paradoxical if one considers that, for all but 1 subject,  $\Delta t$  is longer than  $\delta$ . Thus, in most cases, a point on the pursuit trajectory is reached (say, at time  $t$ ) before the mismatch between the geometrically corresponding target point (at time  $t - \delta$ ) and the simultaneous pursuit position has had a chance



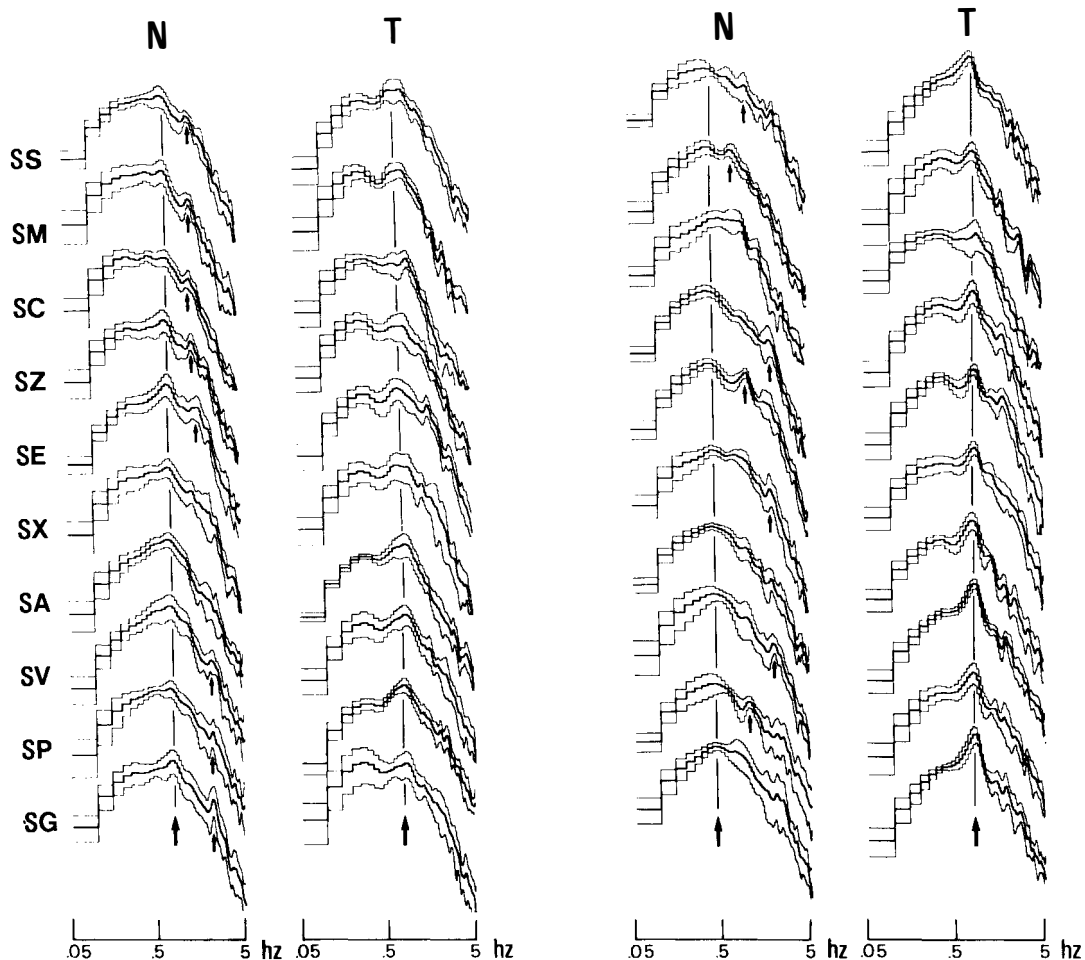


Figure 10. Comparison of individual behaviors. (Amplitude Fourier spectra [in arbitrary units] of the delayed velocity difference  $\Delta V_s$  for all subjects and two targets in both modalities. Spectra in the leftmost column have been ranked from Subject SS to Subject SG according to the estimated frequency of the secondary harmonic component [arrows] which, in some cases, is barely perceptible. Spectra in the other three columns are arranged accordingly. Zero values have been displaced arbitrarily to emphasize the fact that the frequency of the main spectral component [large arrows and vertical lines] depends on the target and on the modality [N = normal; T = transformed] but is remarkably constant across subjects. Notice the absence of a secondary component in Modality T.)

to be processed (the situation is illustrated in schematic inset in Figure 11). Because velocity feedback dominates, subjects are temporally closer to the target than the error processing time. Obviously, this is all the more risky when the target is erratic and fast. It can then be argued that positive position feedback is used (by some subjects for all targets and by all subjects for the faster target) as a way to maintain a security distance from the target.

The processing time  $\Delta t$  largely exceeds any reasonable estimate of central transmission delays (see above) and, at least in 1 subject, becomes a sizable fraction of reaction times to external stimuli in a pursuit tracking situation (Angel & Higgins, 1969). Its average value is much smaller than the early estimates of the minimum time necessary to process visual feedback (190–260 ms, according to Keele & Posner, 1968; 285–295 ms, according to Beggs & Howarth, 1970). It is in keeping, however, with more recent estimates of this time obtained by withdrawing visual

feedback information from the initial portions of aiming responses (Smith & Bowen, 1980:  $100 \leq \Delta t \leq 150$  ms; Carlton, 1981:  $\Delta t \approx 135$  ms; Zelaznik, Hawkins, & Kisselburgh, 1983:  $100 \leq \Delta t \leq 200$  ms). One should be aware, however, that the minimum time to use vision may be task specific (Elliott & Allard, 1985), and generalizing from single aiming movements to continuous pursuit may be unwarranted. Moreover, feedback information need not be exclusively visual. In particular, there is evidence that proprioceptive afferences are adequately fast to contribute to the accuracy of rapid positioning movements (Adams, 1977). Actually, it has been claimed (Gibbs, 1965) that proprioceptive monitoring provides faster corrective inputs than does vision (cf. Poulton, 1957). Alternatively, one may choose to emphasize the possible role of the central representation of the intended movement. If, indeed, the geometrical regularities of the target were to provide the subject with a continuously updated guess on the future course of the movement, the

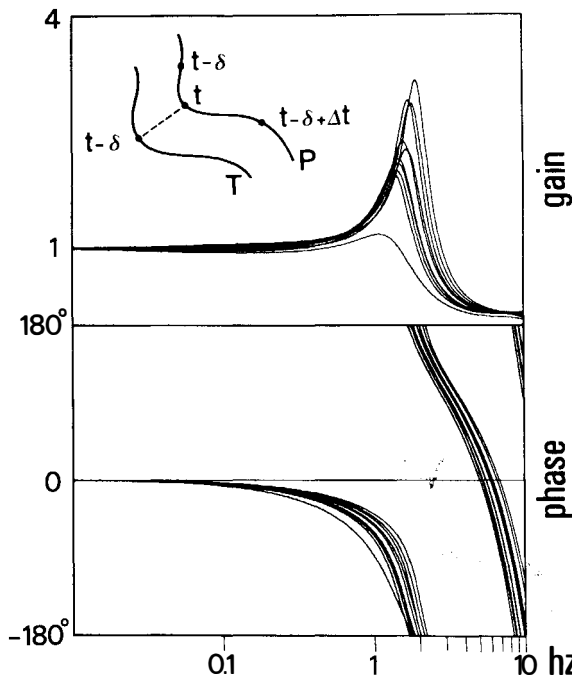


Figure 11. Transfer function for Condition N (normal). (Gain and phase characteristics for all subjects were derived from Equation 3 in the text by inserting the individual values of  $\alpha$ ,  $\beta$ , and  $\Delta t$  from Table 2. Frequency scale is logarithmic. Gain and phase scale are linear. The time interval between the moment when position and velocity errors are perceived and that when pursuit acceleration is affected [schematic diagram inset] is responsible for the resonant behavior of the transfer function. The longer this interval, the smaller the resonance and its frequency.)

appropriate motor responses might be prepared by specifying the values of a smaller number of parameters (Requin, 1980). The hypothesis should be scrutinized by manipulating experimentally the predictability of the target trajectory.

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Received March 21, 1986

Revision received August 22, 1986 ■