1 An “expert” investigation of the Golden Gate Bridge suspension cable

At first glance, two curves that have a similar shape could be potential candidates for modeling: a parabola and a catenary. A parabola is the graphical representation of a quadratic function, and a catenary can be modeled by a hyperbolic cosine and is obtained by suspending a cable or a chain between two points. The suspension cables of bridges are largely believed to be catenaries, but the answer is far from being straightforward. Indeed, if a cable is freely suspended between two points, the curve thus formed is a catenary, but this is not the case for the suspension cable of the San Francisco Bridge. In general, when the horizontal deck of the bridge is suspended on vertical rods attached to the main suspension cable, different forces act on the suspension cable, and it will take the shape of a parabola. We are thus going to provide an analytical solution justifying a quadratic model for a suspension bridge cable supporting a uniform load.

1.1 Analytical justification of the quadratic model

The key element of the resolution lies in the vertical rods that connect the deck (support of the roadway) to the supporting cables on which the bridge is suspended (Figure 1).

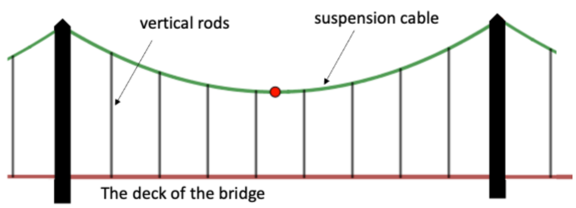

Figure 1: A simplified model of a suspension bridge

Even though the modeling of the suspension cable of a suspension bridge is much more complex and we will only consider the analytical solution for the simplest case where the cable supports a constant and uniformly distributed weight horizontally. We also assume the towers to be rigid and the cable to be flexible (can be bent without resistance) and inextensible (does not stretch).

Let us take a point P on the arc AVA’ representing the suspension cable of a suspension bridge. More precisely, as shown in Figure 2, the tension exerted by the arc A’P on the rest of the cable has the direction of the tangent at P represented by the magnitude vector T.

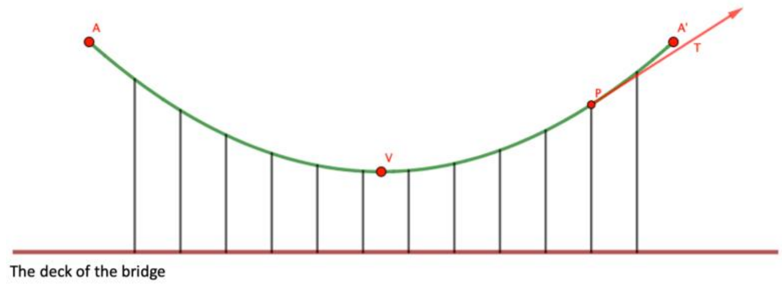

Figure 2: The tension on the suspension cable

Let us now consider any section (say the section from V to P) of the cable as shown in Figure 3. Since the traction on V from the section to the left of V is tangent to the cable, the pull is thus horizontal and directed to the left. This tension is a constant that we will call T_0. The tension exerted on VP at P by the section PA' of the cable is exerted along the tangent at P. We will refer to the magnitude of this tension T and we will call θ the angle that the direction of the tension (or tangent) at P makes with the horizontal. There is another force acting on the VP section of the cable, namely the tension of the portion of the cable, load, and pavement that extends from O to P'.

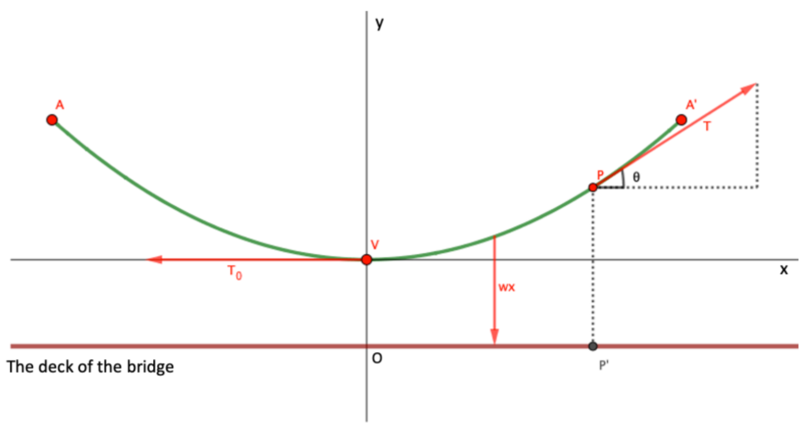

Figure 3: Distribution of the traction forces on the suspension cable of a suspension bridge

Let if w be the weight per horizontal meter. Since the total pull is assumed to be constant for each horizontal meter, the load carried by the arc VP is $w \cdot x$, where x is the abscissa of P. The pull of this load is actually distributed along VP, but all of the tractions are directed downward, therefore the total downward pull on VP is equal to $w \cdot x$.

Thus, there are three forces acting on VP: the horizontal pull T_0 to the left, the downward pull $w \cdot x$ of the total load, and the tangent pull T at P. Since the arc VP is in equilibrium, the three forces must somehow compensate for each other, because if there were a net force, the rope would bend under the action of that force. We can also apply the same reasoning to the vertical forces. The tension T is equivalent to a horizontal and a vertical force acting simultaneously, it is thus equivalent to the combined action of the horizontal component: *T·cosϑ* and the vertical component: *T·sinϑ* as shown in Figure 4.

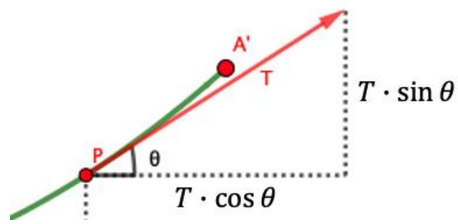

Figure 4: Components of the tension T

Hence,

$$T \cdot \cos\theta = T_0 \quad (1) \qquad \text{et} \qquad T \cdot \sin\theta = wx \quad (2).$$

If we divide (2) par (1), we get:

$$\tan\theta = \frac{w}{T_0}x \quad (3)$$

Because $\tan\theta$ est the gradient of the tangent at P, we have got:

$$\frac{dy}{dx} = \frac{w}{T_0}x \quad (4)$$

Where $\frac{dy}{dx}$ is the derivative of the function representing the shape of the suspension cable. By solving this simple differential equation, we obtain:

$$y = \frac{w}{2T_0}x^2 + C \quad (5)$$

To determine the constant *C*, we place the vertex at V=(0, 0). We have then: *y=0* when *x=0* and thus *C=0.* We obtain a particular solution for our differential equation:

$$y = \frac{w}{2T_0}x^2 \quad (6).$$

This rather simplified excursion into physics allowed us to show that under the above-mentioned circumstances a suspension cable can be modeled using a quadratic function. From now on, we just need to find another point and we can easily determine the equation of the parabola describing the shape of the suspension cable. The span between the two towers of the Golden Gate Bridge is 1280m and the height of the tower from the deck of the bridge is 152m. We place the vertex at V=(0, 0) and A=(-640, 152) at the top of the left tower and A'=(640, 152) corresponding to the top of the right tower based on the dimensions of the bridge. Using GeoGebra, we can quickly determine the equation of the parabola (Figure 5) passing through these 3 points and we obtain:

$$y = 0.00037x^2 \quad (6).$$

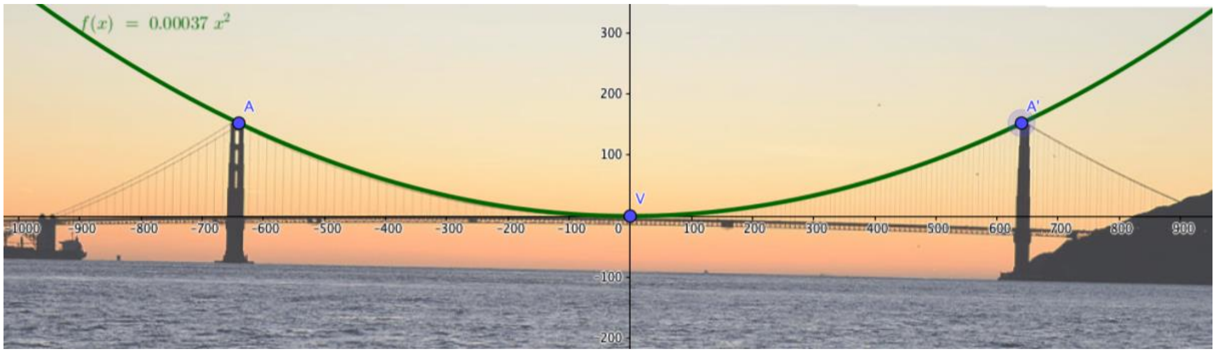

Figure 5: The parabola superposed on the Golden Gate Bridge suspension cable