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Adults and 5-year-old children draw rectangles and triangles around a prototype but not in the golden ratio

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This study uses a production task to probe the representations of two geometrical shape categories (rectangles and triangles) in adults and children before the onset of geometry instruction. We specifically assessed whether drawings of these shapes would average around a prototype and whether the prototypical side-length ratio of the shapes would be situated in the range of the 'golden ratio', as it has been reported in the perception domain. We asked 78 adults and 68 five-year-old children to draw one rectangle and one triangle. In both populations, the prototypical rectangle was horizontally oriented with a ratio between sides superior to the 'golden' value of 1.62. For the triangle, both children and adults tended to produce horizontal acute isosceles triangles with a ratio inferior to the golden value. These findings suggest that adults' and children's shape categories of triangles and rectangles are organized around a prototypical shape, but the characteristics of this prototype may differ to a certain extent with the ones observed in previous perceptual tasks. Implications of this perception/production dissociation for length concept development, as well as the potential origins of these prototypes are discussed.

Geometrical shapes can be considered as categories including an infinite number of particular shapes sharing common properties. Categories are classically regarded as graded structures built around a central prototype, or some set of highly typical exemplars, with less-typical members inhabiting the category's periphery (Medin & Schaffer, 1978; Nosofsky, 1988; Rosch, 1975, Rosch & Mervis, 1975). In the case of geometrical shapes, however, mathematical definitions are at odds with the presence of

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prototypes: for example, shapes verify the definition of a rectangle as long as they have two pairs of parallel sides and right angles, but they may vary drastically in aspect.

The general goal of the present study was to examine the representation of two geometrical shape categories (rectangles and triangles) in adults and children before the onset of geometry instruction. Abstract knowledge of the properties defining these geometrical shape categories allows humans to organize their geometrical knowledge and successfully process some new particular shapes (Neisser, 1976). Yet, even though adults are able to categorize shape correctly, according to abstract geometrical rules, they show a bias towards prototypical shapes both in perception and production tasks (Feldman, 2000). In Feldman's first experiment, participants saw 800 shapes (generated by a random procedure) and were instructed to rate the 'typicality' of the shapes (on a subjective scale), regarded as members of their respective categories. In the second experiment, a different group of participants saw a single generic (not regular or symmetric) triangle or quadrilateral as an example and then were asked to draw a number of different exemplars of the category (with triangles, 6; with quadrilateral, 10). The distribution of the rated typicality of a particular shape and probability that this shape was generated by adults both aggregated around certain particular shapes, and deviated from neutral distributions generated from random shapes. In particular, both in perception and production, participants showed a marked bias towards an equilateral triangle and a square. However, this study did not investigate adult's prototypes for finer shape categories (e.g., rectangles, parallelograms), nor did it analyse whether the drawings and judgements of typicality showed a bias towards a preferred orientation since Feldman chose descriptors that were invariant by rotation. Thus, although there seems to be a bias towards a certain shape in adults' geometrical shape representations, the exact characteristics of these prototypes are still largely unknown.

In children, knowledge of the abstract rules defining shapes is not trivial and depends of the particular shape (Clements & Battista, 1992; Clements, Swaminathan, Hannibal, & Sarama, 1999). Q3 Young children seem to be progressing from an early phase, where they categorize shapes according to their resemblance to a prototype and neglect obvious violations of the geometric definition, to the adult level of knowledge where they use abstract rules for categorization (Satlow & Newcombe, 1998). Thus, 5-year olds consider that a shape constructed by cutting out the vertex of an isosceles triangle makes a good exemplar of 'triangle', although they refuse to include irregular triangles in this category. Nonetheless, these studies better inform about the development of shape concept content (i.e., from visual to abstract) than about the characteristics and evolution of shape prototypes. Previous work from our laboratories (Kalénine, Pinet, & Gentaz, 2010; Pinet & Gentaz, 2007, 2008) showed that at the age of 5, the visual recognition of rectangles or triangles is best for some particular shapes in each category, such as (a) horizontal and vertical rectangles with a 1.5 ratio between the small and large sides, (b) equilateral triangles with a horizontal base aligned with the horizontal reference, and (c) vertical isosceles triangle with a ratio of 1.5 between the longest sides and the shorter base aligned with the horizontal reference. These particular shapes may already constitute prototypes of each category in children (Kalénine *et al.*, 2010; Pinet & Gentaz, 2007, 2008; Rosch, 1973). Q4

The extent to which young children's geometrical shape prototypes, before the onset formal geometry learning, are similar to the ones exhibited by adults is an open question. Moreover, geometrical shape representation development has been mostly studied using perceptual tasks. Performance in perceptual tasks such as identification of geometrical shapes among distractors is influenced by the number and type of distractors presented, Q5

making the assessment of individuals' typical representations challenging. Production tasks show the advantage of bypassing this issue, but are more complex to analyse, and thus more rarely used. A drawing task was introduced in a clinical study conducted by Burger and Shaughnessy (1986) in a very small sample of children. Despite the richness of the information collected, only individual behavioural response patterns were reported, preventing generalization to the typical representations of a given age group. Here, we aimed at evaluating geometrical shape representations in both adults and 5-year olds using a production task in order to avoid biases inherent from the choice of particular exemplars. Moreover, we developed a systematic method for describing and analysing the shapes produced, and thus were able to include a large number of participants both adults and children.

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One interesting characteristic of children's prototypical shapes in previous perceptual studies from our labs was the value of the ratio between the shape sides. The empirical ratio of 1.5 observed is not far from the 'golden ratio' (1.618 . . .), a number that would confer special mathematical properties to these rectangular and triangular shapes (cf. Cleyet-Michaud, 1978; Green, 1996; Livio, 2002). If the golden ratio is respected, it ensures that the ratio between the two lengths of the rectangle is equal to the ratio between the sum of both lengths and the length of the longer side $(A + B)/A = A/B$. If the ratio between the sides is different, it is mathematically impossible for this property to be verified. At least since the Renaissance, many artists and architects have proportioned their works to approximate the golden ratio, believing this proportion to be aesthetically pleasing. Artists particularly value two special figures: golden rectangles, where the ratio between the length of the longer and shorter sides is equal to the golden ratio, and golden triangles, which are acute isosceles triangles where the ratio between the longer sides and the shorter base is equal to the golden ratio. Fechner (1876) investigated whether figures instantiating the golden ratio were judged more pleasing, and found a preference for rectangle centred on the golden ratio in perception, although this result has since been debated (Green, 1995; Markowski, 1992). Among many criticisms, it has been argued that bias towards exemplars with golden proportions in perceptual tasks would highly depend on the number and variety of the exemplars to choose from. For example, individuals may tend to prefer the 'golden' rectangle among a set of rectangle shapes when it corresponds to the exemplar with proportions in the middle of the range proposed in the experiment, regardless of its absolute proportions (Markowski, 1992). Thus, although children's geometrical shape recognition has been shown to be more accurate for exemplars respecting the golden value, it is unclear whether the golden ratio drives their shape representations. The use of a production task further enables us testing more directly the importance of the golden ratio in geometrical shape representations.

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In the present study, we investigated drawings of rectangles and triangles produced by adults and 5-year-old children. Drawing provides a good tool to investigate the nature of prototypes for geometrical shapes, and can be asked of adults as well as young children (e.g., Karmiloff-Smith, 1990; Lange-Kuttner & Vinter, 2007; Piaget & Inhelder, 1947; Picard & Durand, 2005; van Sommers, 1984). Nevertheless, production tasks can be challenging for young children. Thus, we only proposed a simple drawing task and focused on two shape categories, that is, triangle and rectangle, which are familiar to children of this age. We hypothesized that even if adults know that geometric shapes are defined by abstract rules, their drawings might still show a convergence towards a prototype and, similarly, if the children's representations of shapes are organized around a prototypical shape, their drawings should reproduce this prototype. Our first goal was to examine whether the drawing of adults and young children (tested before the onset of formal instruction in geometry) would be distributed around a prototypical shape. Second, for children, we assessed whether these prototypical shapes are similar to the prototypes we identified in a perceptual task (Kalénine *et al.*, 2010; Pinet & Gentaz, 2007, 2008). Our third and last goal was to investigate whether the size proportions of the shapes drawn by adults and/or young children would correspond to the 'golden ratio'.

Method

Participants

Seventy-eight adults with a mean age of 29 years 9 months (ranged from 24 to 42 years) and 68 kindergarten children with a mean age of 5 years 6 months (ranged from 5 to 6 years 5 months) took part in this study.[1] Adults were students from University of Grenoble and children were attending preschool in classrooms in Grenoble. Written consents were obtained from the participants (or a parent for children).

Drawing analysis

We asked each participant to draw one rectangle on a circular sheet and one triangle on another circular sheet with a pen and without a ruler. The circular sheet had a diameter of 21 cm. The order of the shape requests was counterbalanced between participants. Both the orientation of the shape and ratios between the different sides of the shape were analysed. Figure 1 shows four shapes drawn by children and explains the parameters used in the drawing analysis.

In all drawings, the apexes could be identified very easily as the sides crossed. When the sides were not totally joined ($<$3 mm; 5% of drawings), we decided to complete and accept the drawing (about 5% of drawings of children). Once the apexes were defined, we used their position to compute the length of the different sides; this way a consistent length measure could be obtained even when the sides were not perfectly straight. Lengths were measured in centimetre with a ruler.

Rectangles were considered rectangle if the angles between the perpendicular sides did not deviate from 90° by more than 4°, and if the deviations in length between the parallel sides did not exceed 6% of the average side length. Then, we extracted the ratio between the average of the lengths of the two longest sides and the average of the smallest sides. We analysed the distribution of this ratio using the dip test (Hartigan & Hartigan, 1985). The dip statistic is the maximum distance between the empirical distribution and the best fitting unimodal distribution. The dip test allowed us to test the uni- versus multi-modality of the distribution and provided the corresponding modal interval. Q8 Q9

For triangles, we assessed the length of the three sides of the triangle and determined the two sides that have the closest length (AB, AC). Triangles were considered regular, that is, at least isosceles, if the deviation between AB and AC did not exceed 6% of the average side length (corresponding to a difference of about 3 mm for a triangle of 5 cm). The rest of the drawings were categorized as scalene triangles. For equilateral

[1] *We did not observe any gender difference in the present study. Thus, performance of males and females will not be distinguished throughout the paper.*

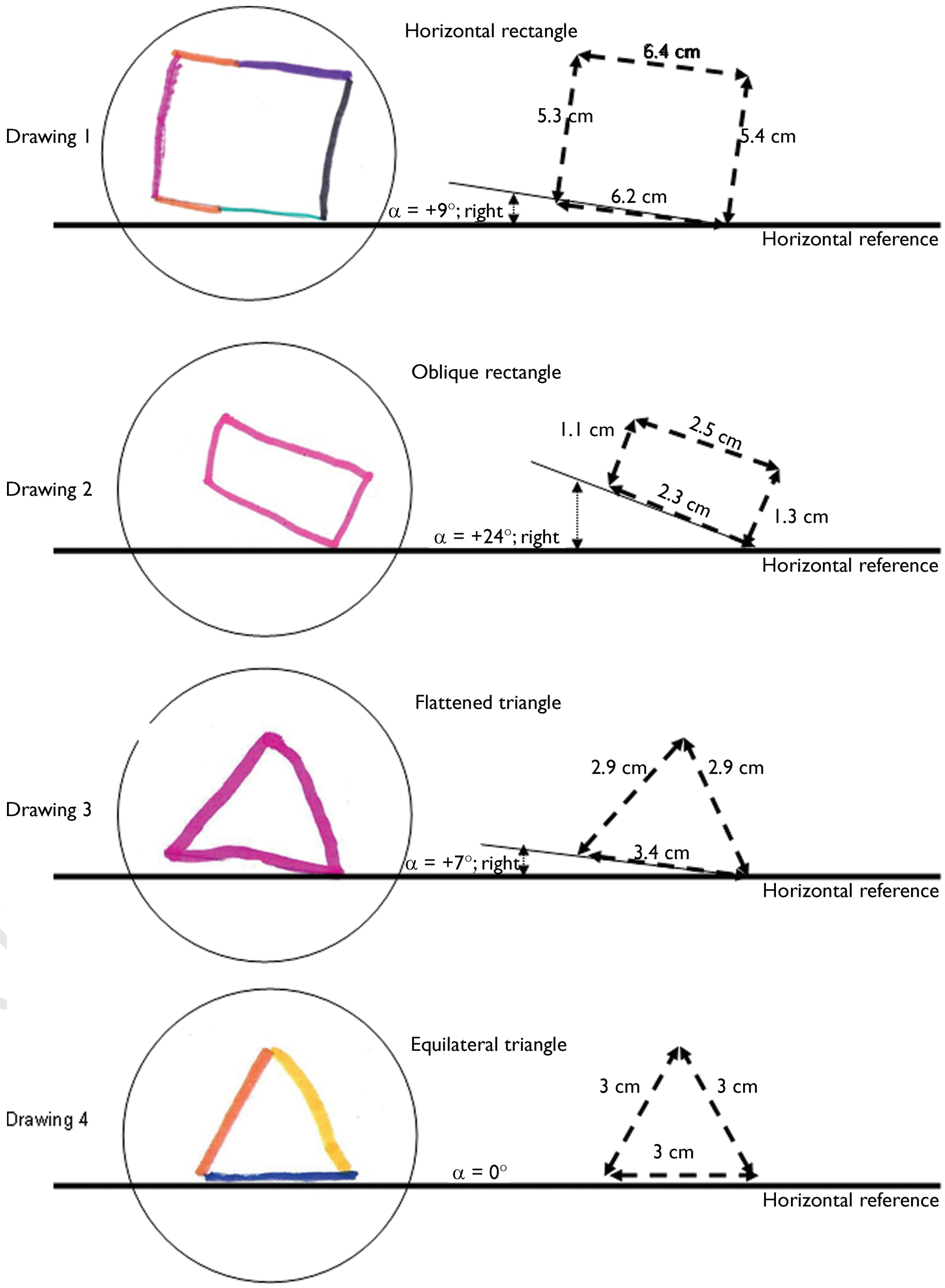

Figure 1. Examples of four shapes drawn by children on a circular sheet (with a diameter of 21 cm) with a pen, without a ruler, and illustrations of the drawing analysis. The orientation of the rectangle by calculating the angular deviation in degrees (α°) between the longest side of the rectangle and the horizontal reference (the edge of the table) and those of the triangle by measuring the angular deviation in degrees between the sides drawn the closest to the edge of the table and the horizontal reference. To assess the length of each 'side', which was most of the times not a straight line especially in children, we chose to plot a line between the two apexes and we measured (in centimetre) its length with a ruler.

Table 1. Mean in centimetre (with standard deviation) of the sides of the rectangles drawn by adults and children and the corresponding ratio as a function of rectangle orientation

	N	Longest sides	Shortest sides	Ratio
Adults				
Horizontal rectangles	72	6.83 (3.63)	3.23 (2.07)	2.31 (0.79)
Vertical rectangles	1	4.05 (—)	1.45 (—)	2.79 (—)
Oblique rectangles	5	6.93 (2.60)	3.13 (0.95)	2.20 (0.35)
Children				
Horizontal rectangles	48	5.14 (3.81)	2.16 (1.96)	2.95 (1.67)
Vertical rectangles	0	— (—)	— (—)	— (—)
Oblique rectangles	1	2.55 (—)	0.8 (—)	3.18 (—)

or isosceles triangles, we further computed the ratio between the average length of AB and AC and the length of the third side (BC). A ratio of 1 corresponds to an equilateral triangle, a ratio superior to 1 to an acute isosceles triangle and a ratio inferior to 1 to a flattened isosceles triangle. The unimodality of the distribution of regular triangle ratios was assessed with the dip test.

After verifying that the ratio distribution across rectangles or triangles were unimodal, we compared the empirical ratio to the golden value of 1.62. Logarithmic transformation was applied to the ratio to ensure normality of distributions before comparing the mean ratio to the golden ratio.

Finally, we determined the orientation of the rectangle by calculating the angular deviation in degrees (α°) between the longest sides of the rectangle and the horizontal reference (the edge of the table) and those of the triangle by measuring the angular deviation in degrees between the side drawn the closest to the edge of the table and the horizontal reference. Rectangles and triangles were assigned to one of three main orientation categories: horizontal/0° [−22.5; + 22.5], oblique/ + 45° [+ 22.5; 67.5], and vertical/ + 90° [+ 67.5°; 112.5]. In brief, drawings were considered oblique when they were rotated by more than 22° from the horizontal or vertical axis. We further evaluated whether the three orientation proportions significantly differ using the Chi-square test in order to specify the prototypical rectangle and triangle orientation.

Results

The mean and standard deviations of the ratio for each rectangle orientation and each triangle category are presented, respectively, in Tables 1 and 2.

All 78 adults were able to draw a rectangle as requested. Seventy-two adults (92%) drew a rectangle with the longest side almost parallel to the horizontal orientation (range 0–21°). There were only five oblique rectangles (range 25–57°) and one vertical (68°) rectangle. The analysis of the different orientations confirmed that the proportion of horizontal, vertical, and diagonal rectangles was not equivalent ($\chi^2 = 122$, ddl = 2, $p < .001$). An analysis of the distribution of the rectangle ratios indicates that the distribution is unimodal (dip = 0.0285, $p = .90$) with a modal interval between corresponding to ratios 2.0 and 2.05 (Figure 2A). Two rectangles with a ratio above 2 *SD* (*SD* = 0.76) of the mean (*M* = 2.31) of this distribution were considered outliers and excluded from subsequent analyses. The final distribution included 76 rectangles

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Table 2. Mean in centimetre (standard deviation) of the sides of the triangles drawn by adults and children and the corresponding ratio as a function of triangle category

	N	Longest sides	Shortest sides	Ratio
Adults				
Acute triangles	43	5.41 (3.98)	4.39 (3.16)	1.23 (0.13)
Flattened triangles	10	7.37 (3.70)	6.35 (2.98)	0.87 (0.06)
Equilateral triangles	14	5.55 (2.86)	5.55 (2.86)	1
Children				
Acute triangles	29	3.67 (4.10)	3.07 (3.38)	1.22 (0.15)
Flattened triangles	10	2.9 (2.15)	2.25 (1.62)	0.79 (0.08)
Equilateral triangles	12	2.20 (0.96)	2.20 (0.96)	1

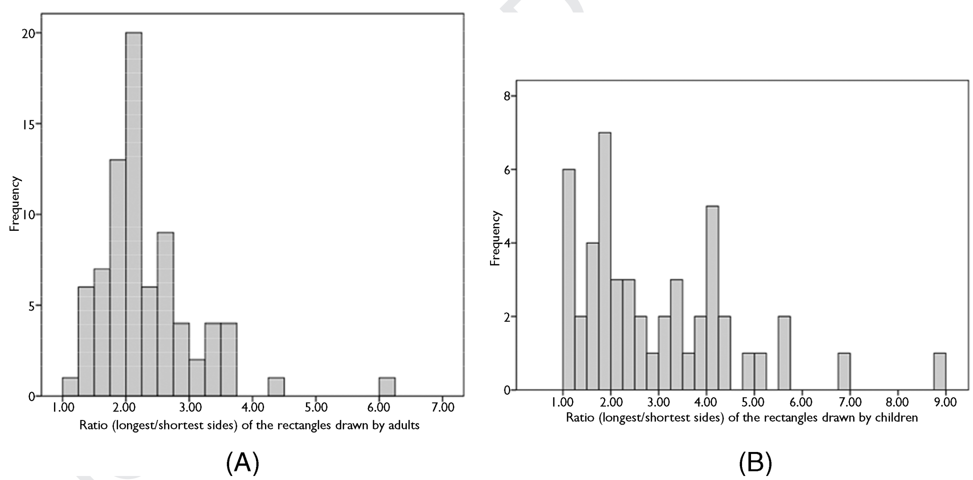
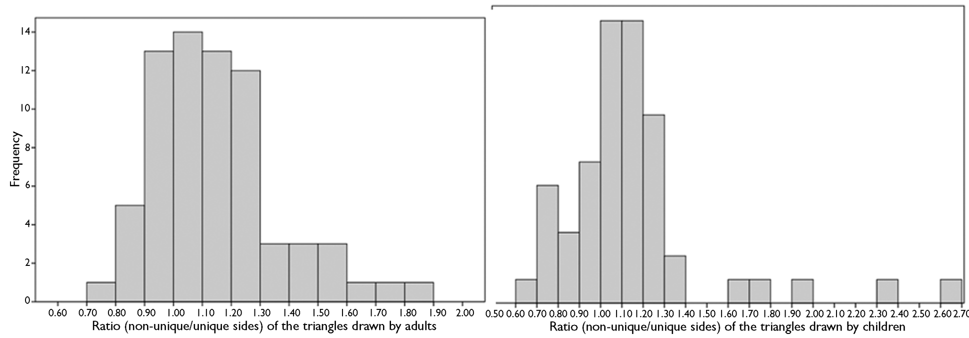


Figure 2. Ratio between the longest and shortest sides of (A) the 78 rectangles drawn by adults and of (B) the 49 rectangles drawn by children.

(97%) with a mean ratio of 2.24 ($SD = 0.61$), which significantly differed from the 1.62 golden ratio ($t = 9.08$, ddl = 75, $p < .001$).

All 78 adults drew a triangle as requested. Most triangles were aligned with the horizontal orientation as defined by the edge of the table ($\chi^2 = 133$, ddl = 2, $p < .001$): there were 74 horizontal triangles (range 0–21°) and only four oblique triangles (range 34–46°). Seventy triangles (90%) were classified as equilateral or isosceles with a mean AB/AC ratio of 1.00 ($SD = 0.02$). The final ratio between the average length of AB and AC and the length of the third side (BC) was computed for these 70 triangles. The analysis of the final ratio distribution on these 70 triangles indicates that the distribution is unimodal (dip = 0.0286, $p = .90$) with a modal interval between 1.08 and 1.09 (Figure 3A). Three triangles with a ratio above 2 *SD*s ($SD = 0.214$) of the mean ($M = 1.15$) of this distribution were considered outliers and were excluded from subsequent analyses. The final distribution involved 67 (86%) triangles with a mean ratio of 1.13 ($SD = 0.187$), which significantly differed from the 1.62 golden ratio ($t = -19.48$ ddl = 66, $p < .001$). Furthermore, the mean ratio also departed significantly from the equilateral ratio of 1 ($t = 5.69$, ddl = 66, $p < .001$).



(A) (B)

Figure 3. Ratio between the unique and the non-unique sides of (A) the 70 isosceles and equilateral triangles drawn by adults and of (B) the 54 isosceles and equilateral triangles drawn by children.

In order to ensure that this difference from the golden ratio was not due to the presence of equilateral and flattened triangles with a ratio by definition inferior or equal to 1 in this triangle set, we then divided these 67 triangles in three sub-categories. There were 14 equilateral triangles with the length of AB, AC, and BC deviating from less than 6% of each other. The remaining triangles were classified as acute (vs. flattened) isosceles triangles when the average of AB and AC was superior (vs. inferior) to BC. When the analysis was restricted to the 43 acute triangles that show, by definition, a mean ratio superior to 1, the empirical ratio of 1.23 was still largely inferior to the golden ratio ($t = -17.40$ ddl $= 42$, $p < .001$).

Out of 68 five-year olds, 49 (72%) children were able to draw a rectangle as requested. Note that quadrilaterals that show the necessary and sufficient properties of rectangles (two pairs of parallel sides and right angles) fell in this category, including particular rectangles (i.e., squares). Other quadrilaterals (e.g., parallelograms, rhombuses) and non-quadrilateral shapes (e.g., triangles, circles) were excluded. Nineteen children (28%) drew a rhombus ($N = 6$), a triangle ($N = 7$), a circle ($N = 1$). and the last five children said that they did not know what a rectangle is (they drew a star [1], a sun [1], nothing [1], or a pentagon [2]). Forty-eight of the 49 rectangles were aligned with the edge of the table (range 0–21°), and last one was classified oblique (24°). Thus, the proportion of horizontal, vertical, and oblique rectangles largely differed in children as well ($\chi^2 = 92.15$, ddl $= 2$, $p < .001$). The ratio distribution on the 49 rectangles drawn by children was unimodal, as indicated by the dip test (dip $= 0.0383$ $p = .80$) with a modal interval between 1.67 and 1.86 (Figure 2B). Two rectangles with a ratio above 2 *SD* ($SD = 1.65$) of the mean ($M = 2.95$) of this distribution were considered outliers and were excluded from the subsequent analyses. The final distribution included 47 rectangles with a mean ratio of 2.74 ($SD = 1.30$), which significantly differed from the 1.62 golden ratio ($t = 5.72$ ddl $= 46$, $p < .001$). Note that although the mean ratio of rectangles was greater than the golden ratio, children most frequently drew rectangles with size proportions just superior to the golden value, as indicated by the range of the most frequent ratio (interval 1.67–1.86 corresponding to 10 children).

Sixty children (90%) of 68 five-year olds produced a triangle as requested. The remaining eight children drew a square ($N = 3$) or nothing ($N = 5$). As in adults, the proportion of horizontal triangles was largely prevailing ($\chi^2 = 119$, ddl $= 2$, $p < .001$)

with 57 horizontal triangles (range 0–21°) and only three oblique triangles. Fifty-four children drew these triangles with the AB/AC ratio within 6%. The distribution of the ratio between the average length of AB and AC and the length of the third side (BC) for these 54 triangles was unimodal, as indicated by the dip test (dip $= 0.044$, $p = .50$ – modal interval: 1.15–1.20; Figure 3B). Three triangles with a ratio above 2 *SD* ($SD = 0.36$) of the mean ($M = 1.15$) of this distribution were considered outliers and were excluded for the following analysis. The final distribution included 51 (75%) triangles with a mean ratio of 1.08 ($SD = 0.21$), which significantly differed from the 1.62 golden ratio ($t = -15.31$, ddl $= 50$, $p < .001$). As with adults, this ratio also departed significantly from the equilateral ratio of 1 ($t = 2.70$, ddl $= 50$, $p < .005$).

Again, to verify that the deviation from the golden ratio was not due to the presence of equilateral and flattened triangles, we divided these 51 triangles in three sub-categories: there were 12 equilateral triangles, 29 acute, and 10 flattened isosceles triangles (maximum deviation of 6% for same length sides). Even when the analysis was restricted to the 29 acute triangles drawn by children, the average empirical ratio of 1.22 was still largely inferior to the golden ratio ($t = -14.19$, ddl $= 28$, $p < .001$).

Finally, we compared the drawings produced by the adults and the children. The frequency of horizontal rectangles, which were prototypical of rectangle drawings in each age group, was not different between adults and children ($\chi^2 = 1.91$, ddl $= 1$, $p = .17$). A *t*-test for independent samples revealed that the average ratio between the longest and shortest sides of the rectangle was smaller in adults (mean $= 2.24$, $SD = 0.61$) than in 5-year-old children (mean $= 2.74$, $SD = 1.30$; $t = 2.96$, ddl $= 121$, $p < .01$). Nevertheless, most adults and children tended to produce rectangles with a ratio between sides around 2 as indicated by the modal interval of their productions, a value slightly superior to the golden ratio.

Similarly, the prevalence of horizontal triangles was equivalent between the two age groups ($\chi^2 = 0.04$, ddl $= 1$, $p = .83$). The proportion of acute, flattened, and equilateral triangles did not differ between the two groups ($\chi^2 = 0.72$, ddl $= 2$, $p = .69$). Moreover, the ratio of triangles produced by adults (mean $= 1.13$, $SD = 0.19$) and children (mean $= 1.08$, $SD = 0.21$) did not differ significantly ($t = 1.33$, ddl $= 116$, $p = .23$). Note that when the analysis of the ratio was restricted to acute triangles, the mean ratio reached the same value in both groups (1.2), indicating that the triangle proportions were still largely inferior to the golden ratio.

Discussion

Our findings show that adults and children tend to draw shapes centred on a prototype, for both rectangles and triangles. In both cases, the distributions of the measures of aspect ratio between shape longest and smallest sides were unimodal. In addition, drawings were reliably oriented along the horizontal edge of the table. Regarding the rectangle category, children and adults drew rectangular shapes with the longest sides parallel to the horizontal orientation, approximately 6-cm long, and at least twice as long as the shortest sides. For the triangle category, in both groups, the stereotypical triangle aggregates around the acute triangle, with both the longest sides, approximately 4.5-cm long, and at least 1.23 as long as the shortest side (approximately 3.7-cm long).

Yet the prototypical ratios of the drawings departed significantly from the value of the golden ratio (1.62). Even though the aspect ratio of the prototypical rectangle changed slightly between children and adults, in both populations the ratio values were significantly superior to the golden ratio for rectangles and inferior to the golden ratio for triangles.

Comparison between perception and production in children

The present findings only partially generalize previous findings from our labs in the perceptual domain (Kalénine, Pinet, & Gentaz, 2011; Pinet & Gentaz, 2007, 2008). Q12 Consistent with these previous studies, children's drawings of rectangles and triangles were organized around a prototype with specific orientation and ratio characteristics. While the favoured orientation tends to be constant across the perceptual and production tasks, the preferential ratio appears different. Rectangle ratio tends to be greater in production than in perception, while ratio of triangle drawing tends to be situated between the ratio of the two triangle shapes best recognized by 5-year olds: equilateral triangle (ratio = 1) and acute isosceles triangle with 1.5 ratio. Regarding the triangle category, one possibility could be that young children aimed at drawing an equilateral triangle but that their triangle would result in an isosceles triangle with a ratio slightly above one given the imprecision one may expect in drawings at age. If this hypothesis is correct, stereotypical triangles would be similar in perception and production and converge towards an equilateral triangle. A second possibility may be that geometrical shape prototypes do not fully overlap in perception and production. Although smaller than the ratio of the best-recognized isosceles triangle at 5 years of age, the mean ratio of children's triangle drawings was significantly different from 1, and similar to the mean ratio of adult triangle drawings. This suggests that the ratio deviation from the equilateral triangle in production tends to be systematic rather than related to 5-year olds' drawing imprecision and may reflect a certain dissociation between production and perception tasks, certainly for rectangles and likely for triangles.

Recent theoretical advancements suggest that knowledge in geometry does not rely on a unitary system (Spelke, Lee, & Izard, 2010). For example, length/distance perception for two-dimensional shape analysis and for navigation tends to show different developmental pathways. Length two-dimensional perception may also not tap into the same exact cognitive and neural mechanisms as length reproduction, which might explain why prototypical shape representations are not identical in the production domain. Supporting this view, Rival, Olivier, Ceyte, and Bard (2004) used the Dunker Q13 illusion in children from various age groups and showed a dissociation between distance perception and reproduction from 7 years of age that the authors related to the separation of the allocentric visual perception pathway from the egocentric action pathway. Our findings may suggest that in geometrical tasks, distance perception and reproduction might actually rely on distinct mechanisms from 5 years of age. In particular, motor requirements might constraint shape prototypes in production and may account for potential discrepancies between perception and production performance.

Overall, we considered that the results reported here support the idea of certain dissociation between length and distance concepts in perception and production. Moreover, they stress the importance of considering both perceptual and production performance when assessing geometrical abilities in children.

Origins of children and adults prototypes

The existence of similar prototypical drawings from 5 years of age raises the question of the potential origins of these prototypes. One first hypothesis would be to relate these prototypes to formal education in geometry received at school. However, since the 5-year-old children included in our study had not received any formal teaching in geometry and show prototypes just like adults, and in a similar orientation, it is unlikely that school plays a crucial role in shaping these prototypes. Moreover, relying on formal rules is at odds with a bias towards a preferred shape (e.g., according to formal rules all rectangles are equally good rectangles). The only result that might reflect the greater formal geometrical knowledge of adults is the slightly greater ratio in adult rectangle productions, compared to children. One can speculate that adult rectangle representation is more distinct from adult square representation (i.e., adults know better the rules that makes a square a rectangle, but with additional specific properties), which might have pushed adult rectangle drawing dimensions away from the square. Taken as a whole, the results reported here do not suggest that adult prototypical drawings are largely driven by their more advanced geometrical knowledge. Early education, social interactions, and feedback from adults on children's early productions might however have an important role in individuals' expectations of what is a 'good' shape and drive drawings towards certain characteristics. The role of social/parental expectations and feedback on performance would be an important factor to test in further studies assessing geometrical representations.

Alternatively, prototypical shapes may have deep origins in the functional properties of the visual system, present since early infancy. It is interesting to note that, at least for triangles, the prototypical shape is regular (isosceles, and very close to an equilateral triangle). From the Gestaltists, the preference for regular forms in perception has been widely observed in the literature. In experiments of slant perception, Slater and Morison (1985) found that newborn infants showed a preference for squares that could not be extinguished by previous familiarization with other shapes. In the completion of occluded figures, global symmetry of the completed figure has been shown to override local properties such as good continuation (Sekuler, Palmer, & Flynn, 1994). Wagemans, Van Gool, Swinnen, and Van Horebeek (1993) observed that human regularity detection is at least global enough to operate on patterns of four dots (i.e., virtual quadrilaterals). The origins of this bias towards regular forms are, however, still largely debated.

Another possibility is that both adults and children draw the shape that they encounter most frequently in their perceptual environment. Although this hypothesis would nicely explain why children and adults' drawings show the same characteristics, in the case of rectangles at least, it does not appear consistent with the discrepancy observed between children's prototypical shapes in perception and production tasks. This, however, does not mean that early visual experience does not influence productions. From an educational perspective, further training studies testing the role of the shape exemplars used during learning (different orientations and side-length ratios) on prototypical shape formation would be very informative. It would help determining whether prototypical geometrical shapes with particular orientations and side-length ratio characteristics can emerge from perceptual exemplar regularities during training.

In conclusion, the present findings demonstrate that (a) 5-year-old children and adults' productions of geometrical shapes converge around a prototype that show similar characteristics in both populations, (b) for children, the production prototype is similar in orientation to the perception prototype, however, perception and production prototypes differ somewhat in length ratio, (c) specifically, prototype dimensions do not respect the golden ratio in production for both adults and at 5-year olds. Further studies will be needed to identify the origin(s) of these shape prototypes.

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References

Burger, W. F., & Shaughnessy, J. M. (1986). Characterizing the van Hiele levels of development in geometry. *Journal for Research in Mathematics Education*, *17*, 31–48.

Clements, D., & Battista, M. (1992). Geometry and spatial reasoning. *Handbook of research in mathematics teaching and learning*. NY: Macmillan.

Clements, D., Swaminathan, S., Hannibal, M., & Sarama, J. (1999). Young children's concepts of shape. *Journal for Research in Mathematics Education*, *30*, 192–212.

Cleyet-Michaud, M. (1978). *Le nombre d'or*. Paris: PUF.

Fechner, G. (1876). *Vorschule der aesthetik* [*Preschool of aesthetics*]. Leipzig: Druck und Verlag von Breitkopf & Hartel.

Feldman, J. (2000). Bias toward regular form in mental shape spaces. *Journal of Experimental Psychology: Human Perception & Performance*, *26*, 152–165.

Green, C. (1996). All that glitters: A review of psychological research on the aesthetics of the golden section. *Perception*, *24*, 937–968.

Hartigan & Hartigan. (1985). The dip test of unimodality. *Annals of Statistics*, *13*, 70–84. Q15

Kalénine, S., Pinet, L., & Gentaz, E. (2010). The visual and visuo-haptic exploration of geometrical shapes increases their recognition in preschoolers. *International Journal of Behavioral Development*, *35*, 18–26.

Karmiloff-Smith, A. (1990). Constraints on representational change: Evidence from children's drawing. *Cognition*, *34*, 57–83.

Kouba, V., Brown, C., Carpenter, T., Linsquist, M., Silver, E., & Swafford, J. (1998). Results of the fourth NAEP assessment of mathematics: Measurement, geometry, data interpretation, attitudes, and other topics. *Arithmetic Teacher*, *33*, 10–16. Q16

Lange-Kuttner, C., & Vinter, A. (Eds.). (2007). *The development of drawing and non verbal intelligence.* Cambridge: Cambridge University Press.

Livio, M. (2002). *The golden ratio: The story of phi, the world's most astonishing number*. New York: Broadway Books.

Medin, D., & Schaffer, M. (1978). Context model of classification learning. *Psychological Review*, *85*, 207–238.

Nosofsky, R. (1988). Exemplar-based accounts of relations between classifications, recognition, and typically. *Journal of Experimental Psychology: Learning, Memory & Cognition*, *14*, 700–708.

Neisser, U. (1976). *Cognition and reality*. San Francisco, CA: Freeman.

Piaget, J., & Inhelder, B. (1947). *La représentation de l'espace chez l'enfant*. Paris: PUF. Q17

Picard, D., & Durand, K. (2005). Are young children's drawing canonically biased? *Journal of Experimental Child Psychology*, *90*, 48–64.

Pinet, L., & Gentaz, É. (2008). Évaluation d'entraînements multisensoriels de préparation à la reconnaissance de figures géométriques planes chez les enfants de 5 ans : Etude de la contribution du système haptique manuel. *Revue Française de Pédagogie*, *162*, 1–20.

Pinet, L., & Gentaz, E. (2007). La reconnaissance des figures géométriques planes par les enfants de 5 ans. *Grand N*, *80*, 17–24. Q18

Rival, C., Olivier, I., Ceyte, H., & Bard, C. (2004). Age-related differences in the visual processes implied in perception and action: Distance and location parameters. *Journal of Experimental Child Psychology*, *87*, 107–124.

Rosch, E. (1975). Cognitive reference points. *Cognitive Psychology*, *7*, 532–547.

Rosch, E., & Mervis, C. (1975). Family resemblance: Studies in the internal structures categories. *Cognitive Psychology*, *7*, 573–605.

Satlow, E., & Newcombe, N. (1998). When is a triangle not a triangle? Young children's developing concepts of geometric shape. *Cognitive Development*, *13*, 547–559.

Sekuler, A., Palmer, S., & Flynn, C. (1994). Local and global processes in visual completion. *Psychological Science*, *5*, 260–267.

Slater, A., & Morison, V. (1985). Shape constancy and slant perception at birth. *Perception*, *14*, 337–344.

Spelke, E., Lee, S. A., & Izard, V. (2010). Beyond core knowledge: Natural geometry. *Cognitive Science*, *34*, 863–884.

Van Sommers, P. (1984). *Drawing and cognition: Descriptive and experimental studies of graphic production processes*. Cambridge: Cambridge University Press.

Wagemans, J., Van Gool, L., Swinnen, V., & Van Horebeek, J. (1993). Higher-order structure in regularity detection. *Vision Research*, *33*, 1067–1088.

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