B

Efficiency, LRT_ρ, RAIC and RC_p with biweight ρ-function

To develop the efficiency (3.20) and other quantities for the LRT_ρ, $RAIC$ and the RC_p with the biweight estimator with ρ-function (3.15), we will make use of $E\left[r^k\right] = \frac{(k)!}{2^{k/2}(k/2)!}$ to compute (even) the moments of a $\mathcal{N}(0,1)$, and of

$$\int_{-\infty}^{c} r^k d\Phi(r) = L_k = -c^{k-1}\varphi(c) + (k-1)\Phi(c)L_{k-2},$$

with $L_0 = \Phi(c)$ and $L_1 = -\varphi(c)$. We will need (even) moments up to the order 14, so that we can compute

$$\begin{aligned}
L_2 &= -c\varphi(c) + \Phi(c)^2 \\
L_4 &= -(c^3 + 3c\Phi(c))\varphi(c) + 3\Phi(c)^3 \\
L_6 &= -(c^5 + 5c^3\Phi(c) + 15c\Phi(c)^2)\varphi(c) + 15\Phi(c)^4 \\
L_8 &= -(c^7 + 7c^5\Phi(c) + 35c^3\Phi(c)^2 + 105c\Phi(c)^3)\varphi(c) + 105\Phi(c)^5 \\
L_{10} &= -(c^9 + 9c^7\Phi(c) + 63c^5\Phi(c)^2 + 315c^3\Phi(c)^3 + 945c\Phi(c)^4)\varphi(c) \\
&\quad +945\Phi(c)^6 \\
L_{12} &= -(c^{11} + 11c^9\Phi(c) + 99c^7\Phi(c)^2 + 693c^5\Phi(c)^3 + 3465c^3\Phi(c)^4 \\
&\quad +10395c\Phi(c)^5)\varphi(c) + 10395\Phi(c)^7 \\
L_{14} &= -(c^{13} + 13c^{11}\Phi(c) + 143c^9\Phi(c)^2 + 1287c^7\Phi(c)^3 + 9009c^5\Phi(c)^4 \\
&\quad +45045c^3\Phi(c)^5 + 135135c\Phi(c)^6)\varphi(c) + 135135\Phi(c)^8
\end{aligned}$$

and therefore

$$\int_{-c}^{c} d\Phi(r) = 1 - 2\Phi(-c)$$

$$\int_{-c}^{c} r^2 d\Phi(r) = \int r^2 d\Phi(r) - 2\int_{-\infty}^{-c} r^2 d\Phi(r) = 1 - 2\varphi(c)c - 2\Phi(-c)^2$$

$$\int_{-c}^{c} r^4 d\Phi(r) = 3 - 2\varphi(c)(c^3 + 3c\Phi(-c)) - 6\Phi(-c)^3$$

$$\int_{-c}^{c} r^6 d\Phi(r) = 15 - 2\varphi(c)(c^5 + 5c^3\Phi(-c) + 15c\Phi(-c)^2) - 30\Phi(-c)^4$$

$$\int_{-c}^{c} r^8 d\Phi(r) = 105 - 2\varphi(c)(c^7 + 7c^5\Phi(-c) + 35c^3\Phi(-c)^2 + 105c\Phi(-c)^3) - 210\Phi(-c)^5$$

$$\int_{-c}^{c} r^{10} d\Phi(r) = 945 - 2\varphi(c)(c^9 + 9c^7\Phi(-c) + 63c^5\Phi(-c)^2 + 315c^3\Phi(-c)^3 + 945c\Phi(-c)^4) - 1890\Phi(-c)^6$$

$$\int_{-c}^{c} r^{12} d\Phi(r) = 10395 - 2\varphi(c)(c^{11} + 11c^9\Phi(-c) + 99c^7\Phi(-c)^2 + 693c^5\Phi(-c)^3 + 3465c^3\Phi(-c)^4 + 10395c\Phi(-c)^5) - 20790\Phi(-c)^7$$

$$\int_{-c}^{c} r^{14} d\Phi(r) = 135135 - 2\varphi(c)(c^{13} + 13c^{11}\Phi(-c) + 143c^9\Phi(-c)^2 + 1287c^7\Phi(-c)^3 + 9009c^5\Phi(-c)^4 + 45045c^3\Phi(-c)^5 + 135135c\Phi(-c)^6) - 270270\Phi(-c)^8$$

For the efficiency (3.20), we have

$$e_c = \left[\frac{5}{c^4}\int_{-c}^{c} r^4 d\Phi(r) - \frac{6}{c^2}\int_{-c}^{c} r^2 d\Phi(r) + \int_{-c}^{c} d\Phi(r)\right]^2 \Bigg/ \left(\frac{1}{c^8}\int_{-c}^{c} r^{10} d\Phi(r) - \frac{4}{c^6}\int_{-c}^{c} r^8 d\Phi(r) + \frac{6}{c^4}\int_{-c}^{c} r^6 d\Phi(r) - \frac{4}{c^2}\int_{-c}^{c} r^4 d\Phi(r) + \int_{-c}^{c} r^2 d\Phi(r)\right)$$

For the LRT_ρ, and using the ρ-function given in (3.15), we have that (3.26) reduces to

$$\left(\frac{5}{c^4}\int_{-c}^{c} r^4 d\Phi(r) - \frac{6}{c^2}\int_{-c}^{c} r^2 d\Phi(r) + \int_{-c}^{c} d\Phi(r)\right) \Bigg/ \left(\frac{1}{c^8}\int_{-c}^{c} r^{10} d\Phi(r) - \frac{4}{c^6}\int_{-c}^{c} r^8 d\Phi(r) + \frac{6}{c^4}\int_{-c}^{c} r^6 d\Phi(r) - \frac{4}{c^2}\int_{-c}^{c} r^4 d\Phi(r) + \int_{-c}^{c} r^2 d\Phi(r)\right)$$

For the $RAIC$ given in (3.31), we have

$$a = \left(\frac{1}{c^8}\int_{-c}^{c} r^{10} d\Phi(r) - \frac{4}{c^6}\int_{-c}^{c} r^8 d\Phi(r) + \frac{6}{c^4}\int_{-c}^{c} r^6 d\Phi(r) - \frac{4}{c^2}\int_{-c}^{c} r^4 d\Phi(r) + \int_{-c}^{c} r^2 d\Phi(r)\right)$$

$$b = \left(\frac{5}{c^4}\int_{-c}^{c} r^4 d\Phi(r) - \frac{6}{c^2}\int_{-c}^{c} r^2 d\Phi(r) + \int_{-c}^{c} d\Phi(r)\right)$$

For the RC_p, **?** show that

$$\begin{aligned} U_p - V_p = n &\int \left(\frac{\partial}{\partial r}\rho(r)\right)^2 d\Phi(r) \\ &-2p \int \left(\frac{\partial}{\partial r}\rho(r)\right)^2 \frac{\partial}{\partial r \partial r}\rho(r) d\Phi(r) \Big/ \int \frac{\partial}{\partial r \partial r}\rho(r) d\Phi(r) \\ &+p\left(\int \left(\frac{\partial}{\partial r \partial r}\rho(r)\right)^2 d\Phi(r) + 2\int \frac{1}{r}\frac{\partial}{\partial r}\rho(r)\frac{\partial}{\partial r \partial r}\rho(r) d\Phi(r)\right. \\ &\left. -3\int \frac{1}{r^2}\left(\frac{\partial}{\partial r}\rho(r)\right)^2 d\Phi(r)\right) \int \left(\frac{\partial}{\partial r}\rho(r)\right)^2 d\Phi(r) \Big/ \\ &\left[\int \frac{\partial}{\partial r \partial r}\rho(r) d\Phi(r)\right]^2 \end{aligned}$$

and

$$V_P = p\int \frac{1}{r^2}\left(\frac{\partial}{\partial r}\rho(r)\right)^2 d\Phi(r) \int \left(\frac{\partial}{\partial r}\rho(r)\right)^2 d\Phi(r) \Big/ \left[\int \frac{\partial}{\partial r \partial r}\rho(r) d\Phi(r)\right]^2$$

For the biweight ρ-function (3.15), we have

$$\begin{aligned} (U_p - V_p) = n &\left(\frac{1}{c^8}\int_{-c}^{c} r^{10} d\Phi(r) - \frac{4}{c^6}\int_{-c}^{c} r^8 d\Phi(r) + \frac{6}{c^4}\int_{-c}^{c} r^6 d\Phi(r)\right) \\ &-n\left(\frac{4}{c^2}\int_{-c}^{c} r^4 d\Phi(r) - \int_{-c}^{c} r^2 d\Phi(r)\right) \\ &-2p\left(\frac{5}{c^{12}}\int_{-c}^{c} r^{14} d\Phi(r) - \frac{26}{c^{10}}\int_{-c}^{c} r^{12} d\Phi(r)\right. \\ &+\frac{55}{c^8}\int_{-c}^{c} r^{10} d\Phi(r) - \frac{60}{c^6}\int_{-c}^{c} r^8 d\Phi(r) + \frac{35}{c^4}\int_{-c}^{c} r^6 d\Phi(r) \\ &\left.-\frac{10}{c^2}\int_{-c}^{c} r^4 d\Phi(r) + \int_{-c}^{c} r^2 d\Phi(r)\right) \Big/ \\ &\left[\frac{5}{c^4}\int_{-c}^{c} r^4 d\Phi(r) - \frac{6}{c^2}\int_{-c}^{c} r^2 d\Phi(r) + \int_{-c}^{c} d\Phi(r)\right] \\ &+p\left(\frac{32}{c^8}\int_{-c}^{c} r^8 d\Phi(r) - \frac{80}{c^6}\int_{-c}^{c} r^6 d\Phi(r)\right. \\ &\left.+\frac{64}{c^4}\int_{-c}^{c} r^4 d\Phi(r) - \frac{16}{c^2}\int_{-c}^{c} r^2 d\Phi(r)\right) \end{aligned}$$

$$\left(\frac{1}{c^8}\int_{-c}^{c} r^{10} d\Phi(r) - \frac{4}{c^6}\int_{-c}^{c} r^{8} d\Phi(r) + \frac{6}{c^4}\int_{-c}^{c} r^{6} d\Phi(r) \right.$$
$$\left. -\frac{4}{c^2}\int_{-c}^{c} r^{4} d\Phi(r) + \int_{-c}^{c} r^{2} d\Phi(r)\right) \Bigg/$$
$$\left[\frac{5}{c^4}\int_{-c}^{c} r^{4} d\Phi(r) - \frac{6}{c^2}\int_{-c}^{c} r^{2} d\Phi(r) + \int_{-c}^{c} d\Phi(r)\right]^2$$

and

$$V_P = p\left(\frac{1}{c^8}\int_{-c}^{c} r^{8} d\Phi(r) - \frac{4}{c^6}\int_{-c}^{c} r^{6} d\Phi(r) + \frac{6}{c^4}\int_{-c}^{c} r^{4} d\Phi(r) \right.$$
$$\left. -\frac{4}{c^2}\int_{-c}^{c} r^{2} d\Phi(r) + \int_{-c}^{c} d\Phi(r)\right)\left(\frac{1}{c^8}\int_{-c}^{c} r^{10} d\Phi(r) - \frac{4}{c^6}\int_{-c}^{c} r^{8} d\Phi(r) \right.$$
$$\left. +\frac{6}{c^4}\int_{-c}^{c} r^{6} d\Phi(r) - \frac{4}{c^2}\int_{-c}^{c} r^{4} d\Phi(r) + \int_{-c}^{c} r^{2} d\Phi(r)\right) \Bigg/$$
$$\left[\frac{5}{c^4}\int_{-c}^{c} r^{4} d\Phi(r) - \frac{6}{c^2}\int_{-c}^{c} r^{2} d\Phi(r) + \int_{-c}^{c} d\Phi(r)\right]^2$$