Problem Set N. 1

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Instructions. You should hand in the solutions (typewriting or very clearly readable handwriting, please!) to the TA at the latest at the beginning of the class of 27 March 2019 at 10:15am, when the problems will be solved. You can also send your solutions via email to the TA before the class. No late submission will be accepted. Remember to keep a copy for yourselves as you will be marking your own work.

QUESTIONS

1. Consumption-leisure trade-off

   Francis's preferences are represented by the utility function

   $$U(C, L) = C\left(\frac{L}{24}\right)$$

   where L measures leisure hours and C is the numerary consumption good. Francis earns $R = 20$ CHF from a property that he rents to tenants. Total time available is $L_0 = 24$ hours.

   (a) (3 points) Compute her marginal rate of substitution (MRS).

   (b) (2 points) Which bundle can she choose if she is off the labor market?

   (c) (2 points) How high is her utility level when she chooses the bundle in item b?

   (d) (4 points) Compute her reservation wage and explain what the reservation wage measures.

   (e) (2 points) Suppose the wage rate is $W = 0.5$. Is she going to participate in the labor market? Explain.

   (f) (2 points) Suppose the wage rate is $W = 1$. Is she going to participate in the labor market? Explain.

   (g) (5 points) Find the utility level in item f.

2. Income and substitution effects with quasi-linear preferences The utility of a typical individual is $U = C - \frac{1}{2\gamma}H^{2\gamma}$, with $\gamma > 1$. The term C denotes consumption and the term H denotes hours worked. Total income (non-labour plus labour income) is denoted by $R + WH$.

(a) (2 points) Write down the problem of the individual and represent graphically the budget constraint and the map of indifference curves in the space C, H.

(b) (6 points) Derive the Hicksian and Marshallian labor supply elasticities.

(c) (2 points) Show that the reservation wage is equal to zero.

3. Labour supply with taxes

The basic labour supply model assumes that wage income is untaxed. Suppose instead that the marginal tax rate is such that the first $\overline{H}$ hours of labour were untaxed, the next $L_0 - \overline{H}$ hours of labour (where L_0 denotes total time endowment) were taxed at a rate equal to τ.

(a) (3 points) Show on a graph how this would affect the budget line. How might this alter work effort?

(b) (8 points) Assume that the utility function of the worker is given by $U = C^{1-\frac{\beta}{2}} L^{\frac{\beta}{2}}$ where C denotes consumption, L leisure and $\beta \in (0,1)$. Express analytically their labour supply as a function of w, L_0 and τ.

(c) (3 points) What is the value of the reservation wage? What is the elasticity of leisure with respect to wages?

(d) (6 points) At which value of β does the tax starts to have an impact on labour supply? And on consumption? How does the answer depend on the tax rate τ?

4. Intertemporal labor supply

Consider an intertemporal model of labour supply with the following per-period utility function $u_t = \ln c_t + \alpha ln(1-h_t)$ with $\alpha > 0$ and h_t denotes hours worked at time t. Assume that non-labour income is zero and that the interest rate is fixed over time at r. The individual discounts time at the rate $\beta < 1$. Assume also that there are only two periods, $t = 0$ and $t = 1$ and that total wealth at time 1 must equal zero. Consumption prices are normalised to 1 at all times.

(a) (8 points) Write down the maximisation problem, the Lagrangian for the individual problem and derive the full set of first order conditions

(b) (5 points) Combine the budget constraints at times $t = 0$ and $t = 1$ to obtain the intertemporal budget constraint that relates resources in all periods to consumption in all periods.

(c) (5 points) Derive the Frisch labour supply functions at time $t = 0$ and $t = 1$.

(d) (5 points) Derive the Frisch labour supply elasticity at time $t = 0$ and $t = 1$.

(e) (5 points) Find the level of β at which the individual smooths consumption perfectly over the two periods ($c_0 = c_1$).

(f) (6 points) Assume that $\beta = \frac{1}{1+r}$ and derive the optimal consumption level.

(g) (8 points) Derive the Marshallian elasticity of labour supply, i.e. allowing the wage change to also affect the marginal utility of wealth ν_0.

(h) (8 points) Derive the (Marshallian) effect of a change in the wage at time $t = 1$ on the labour supply at time $t = 0$.

5. Labor demand in the short run

In the short run, labor services L are the sole flexible input in the production of a firm's output Y. The firm's short-run production function is given by $Y = L^{\frac{3}{5}}$. The inverse demand for the firm's output is $P(Y) = Y^{-\frac{1}{3}}$.

(a) (6 points) Derive the short-run cost function of the firm, $\tilde{C}(W, Y)$, where W is the wage.

(b) (6 points) Assuming that the firm maximizes its profit, determine the optimal output level (as a function of wages W).

(c) (6 points) Determine the (unconditional) labor demand of the firm.

(d) (3 points) If the wage rate increases by 1%, by how much does the firm's labor demand change in the short run?

(e) (3 points) If the wage rate increases by 1%, by how much does the firm's output vary in the short run?

6. Labor demand in the long run

In the long run, the firm can adjust its capital stock and both capital and labor are flexible factors. Assume that the firm's long-run production function is

$$Y = \left(K^{\frac{1}{3}} L^{\frac{2}{3}}\right)^{\frac{6}{5}} \tag{1}$$

where K denotes capital services. The inverse demand for the firm's output is: $P(Y) = Y^{-\frac{1}{2}}$. R denotes the price of capital and W is the wage rate.

(a) (10 points) Write the Lagrangian of the cost minimization problem and give the full set of first-order conditions.

(b) (10 points) Derive the conditional demand functions for labor and capital services.

(c) (8 points) Show that the long-run cost function of the firm can be written in the form:

$$C(W, R, Y) = (\text{constant})\ W^{\alpha} R^{\beta} Y^{\gamma} \tag{2}$$

What are the values of the parameters α, β and γ?

(d) (8 points) Use Shephard's lemma to re-derive the conditional labor and capital demand functions. Check that the result is identical to what you obtained under point (b).

(e) (12 points) Give the elasticities of conditional labor demand with respect to the wage rate ($\bar{\eta}_W^L$) and to the price of capital services ($\bar{\eta}_R^L$). Could these results also be obtained using cost shares and the elasticity of substitution between capital and labor?

(f) (12 points) Recall that the profit function is defined as $\Pi(W,R) = \max_Y\{P(Y)Y - C(W,R,Y)\}$. Find the output level that maximizes profit. Show that the profit function can be written in the form:

$$\Pi(W,R) = (\text{constant})\ W^{\delta}R^{\eta}.$$

What are the values of δ and η?

(g) (8 points) Use Hotelling's lemma to derive the unconditional labor and capital demand functions (without attaching too much importance to the constant term).

(h) (8 points) Give the elasticities of unconditional labor demand with respect to the wage rate (η_W^L) and to the price of capital services (η_R^L). Are capital and labor gross substitutes or gross complements?