

# Radio- $\gamma$ response in blazars as a signature of adiabatic blob expansion: a self-consistent approach

Tramacere+ 2022 <https://arxiv.org/pdf/2112.03941.pdf> accepted A&A



# JetSeT

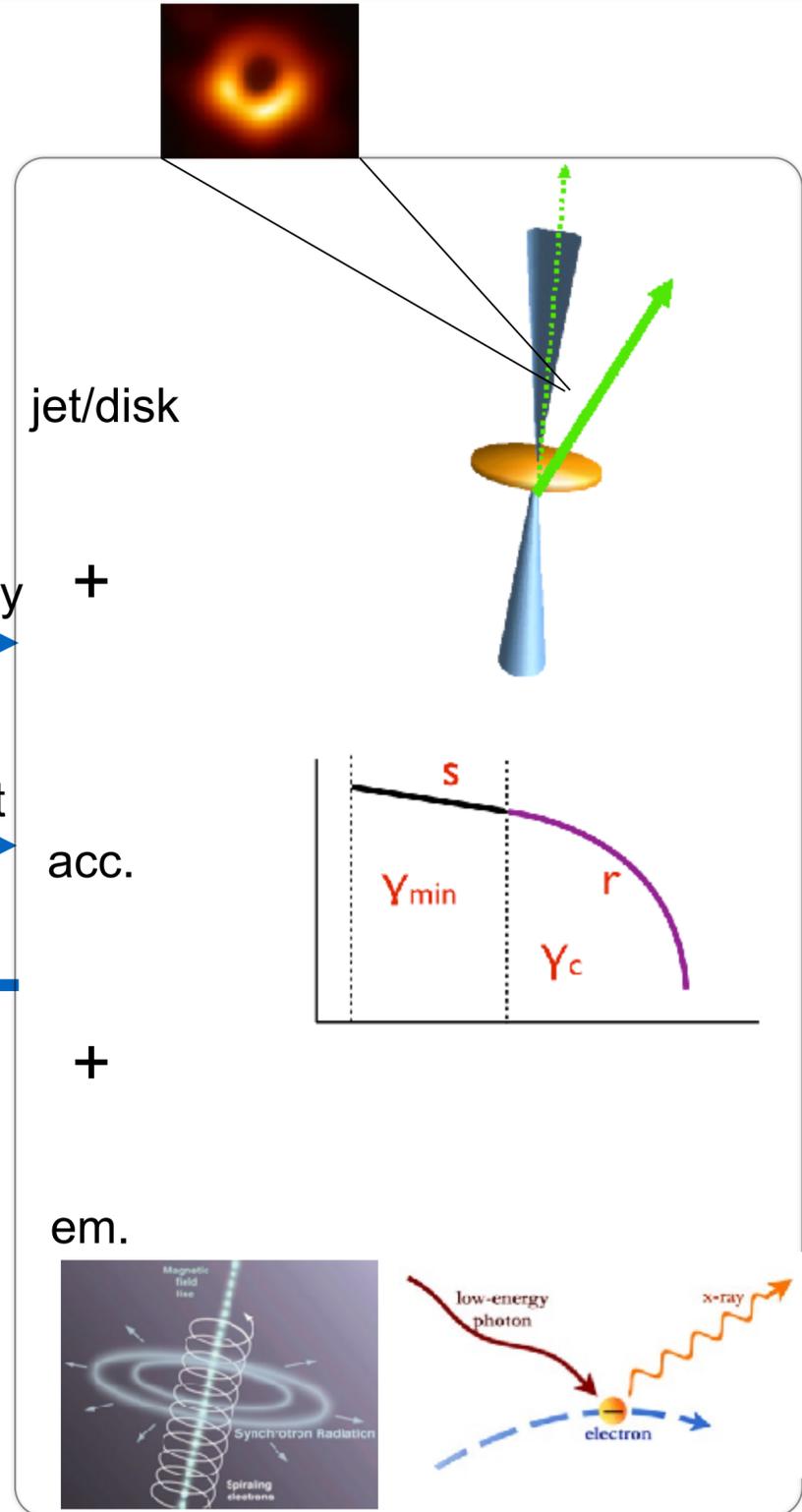
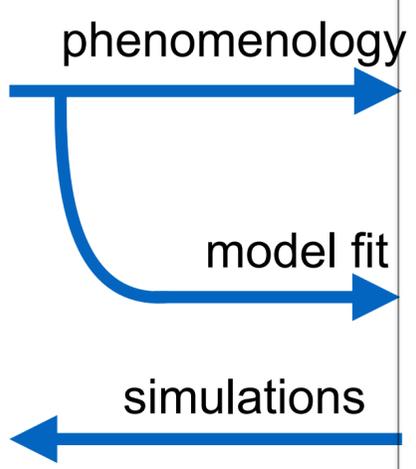
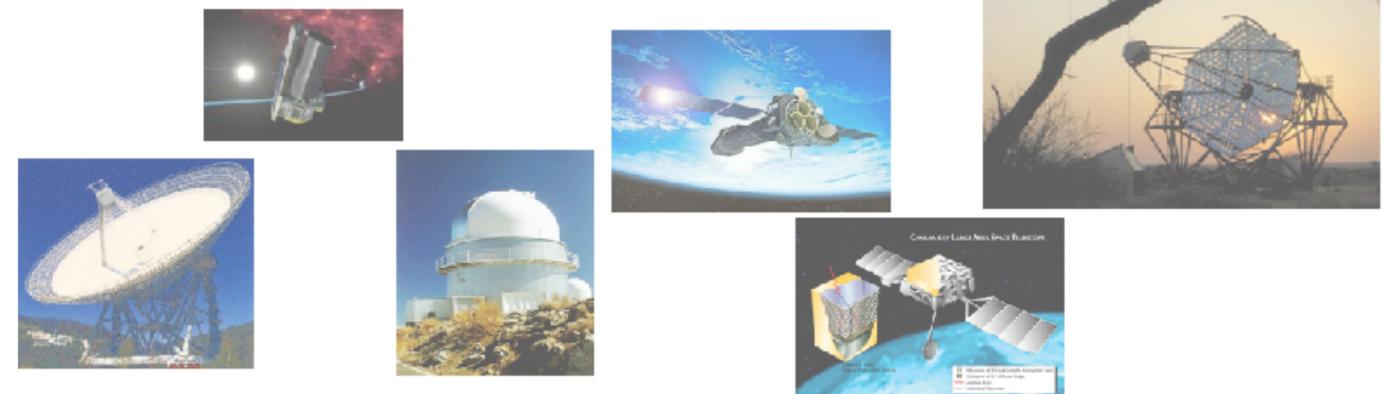
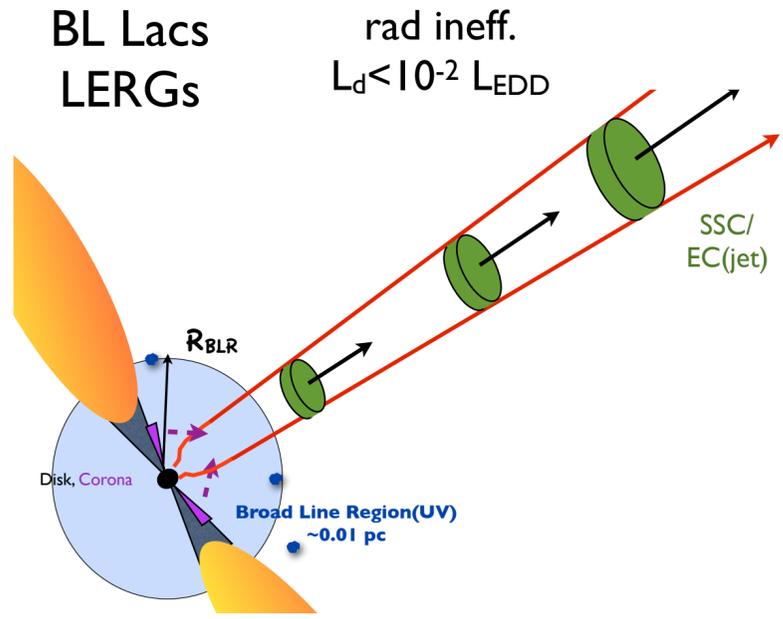
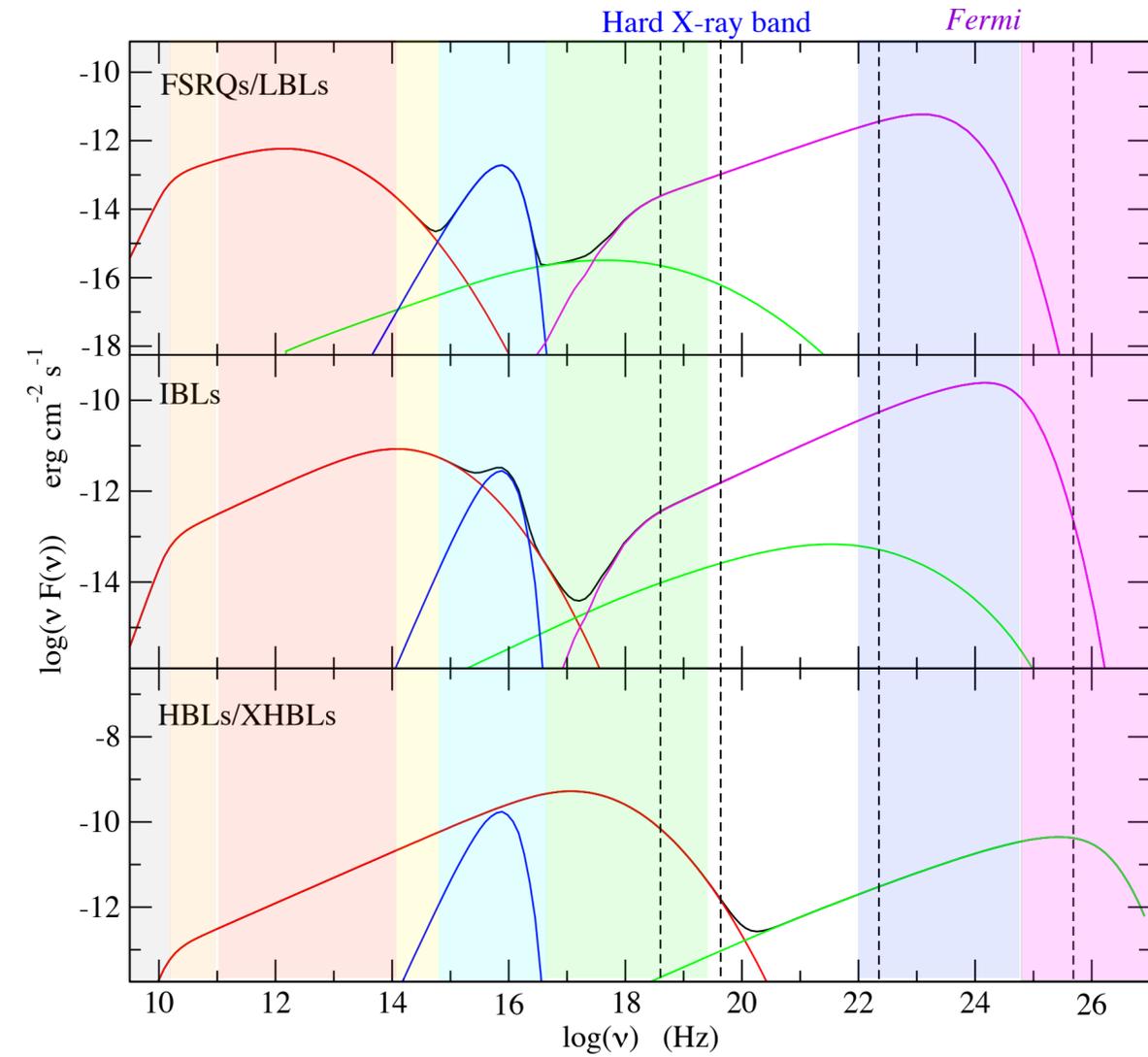
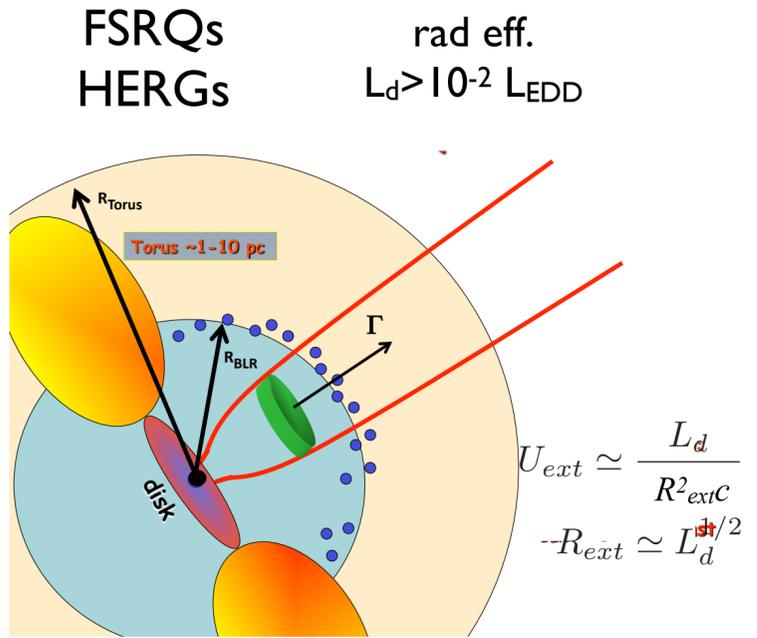
Jets SED modeler and fitting Tool  
**Andrea Tramacere**

in collaboration with  
V. Sliusar, R. Walter, J. Jurysek, M. Balbo

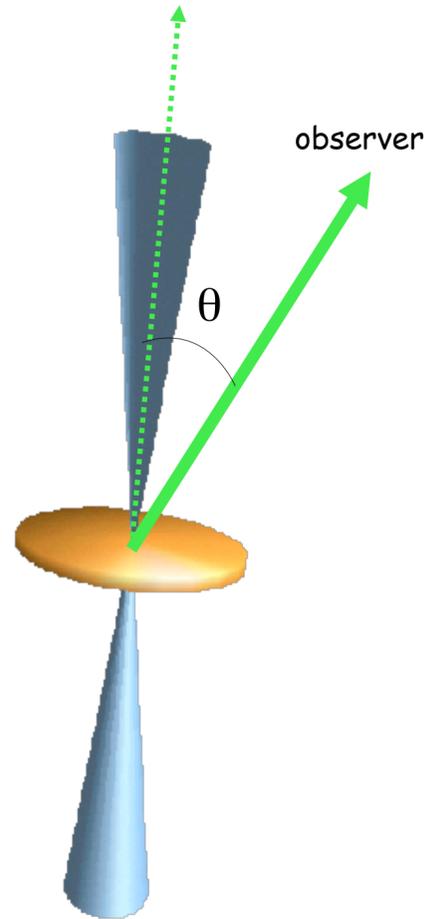
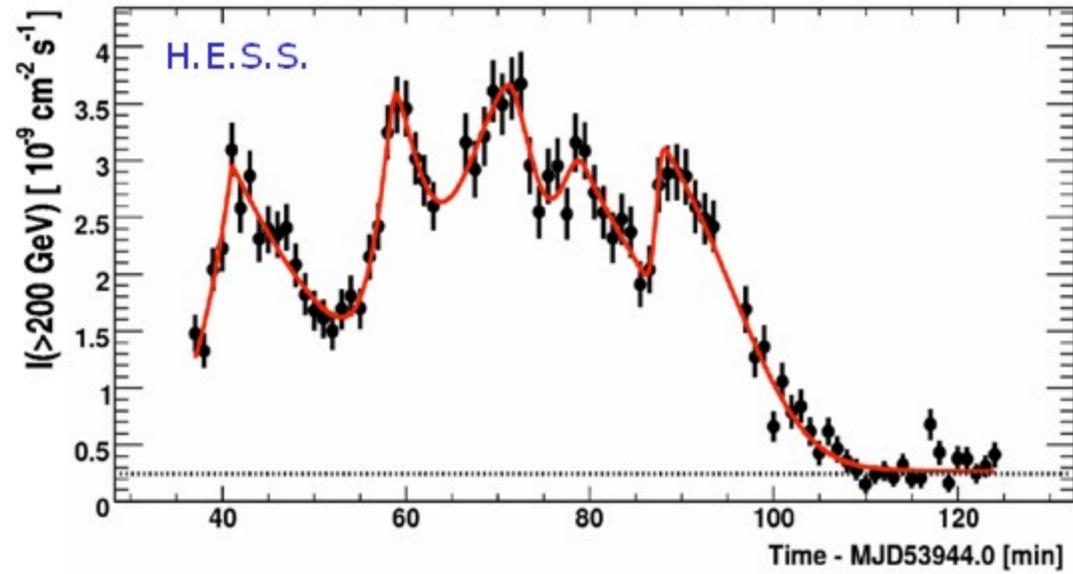
<https://jetset.readthedocs.io/en/latest/>

<https://github.com/andreatramacere/jetset>

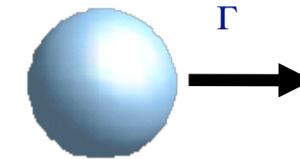
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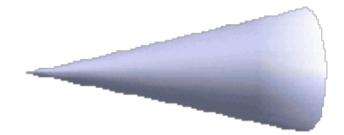
# Beamed Emission (achromatic var.)



rest frame :  
isotropic emission

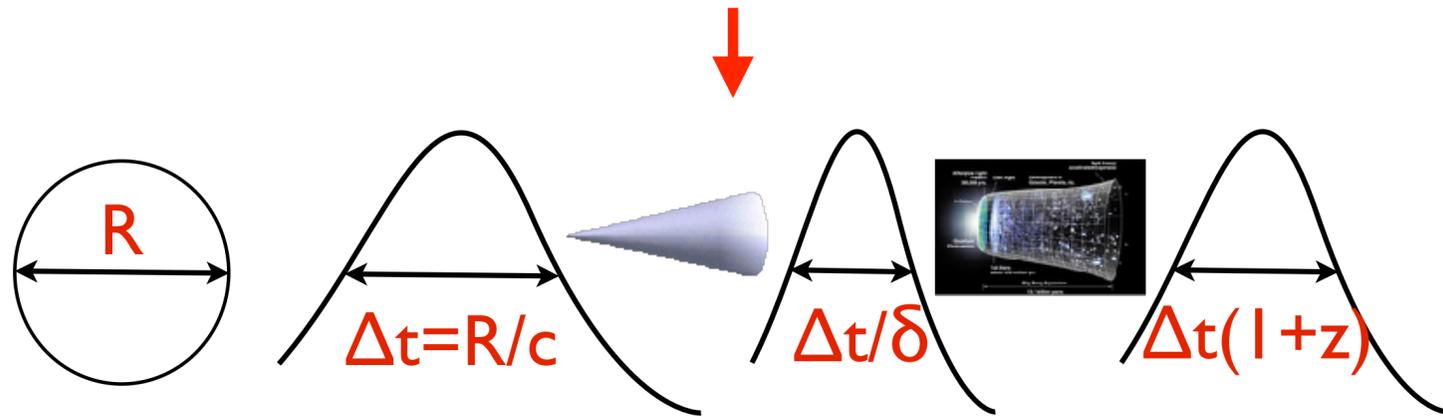


Observer frame: beamed

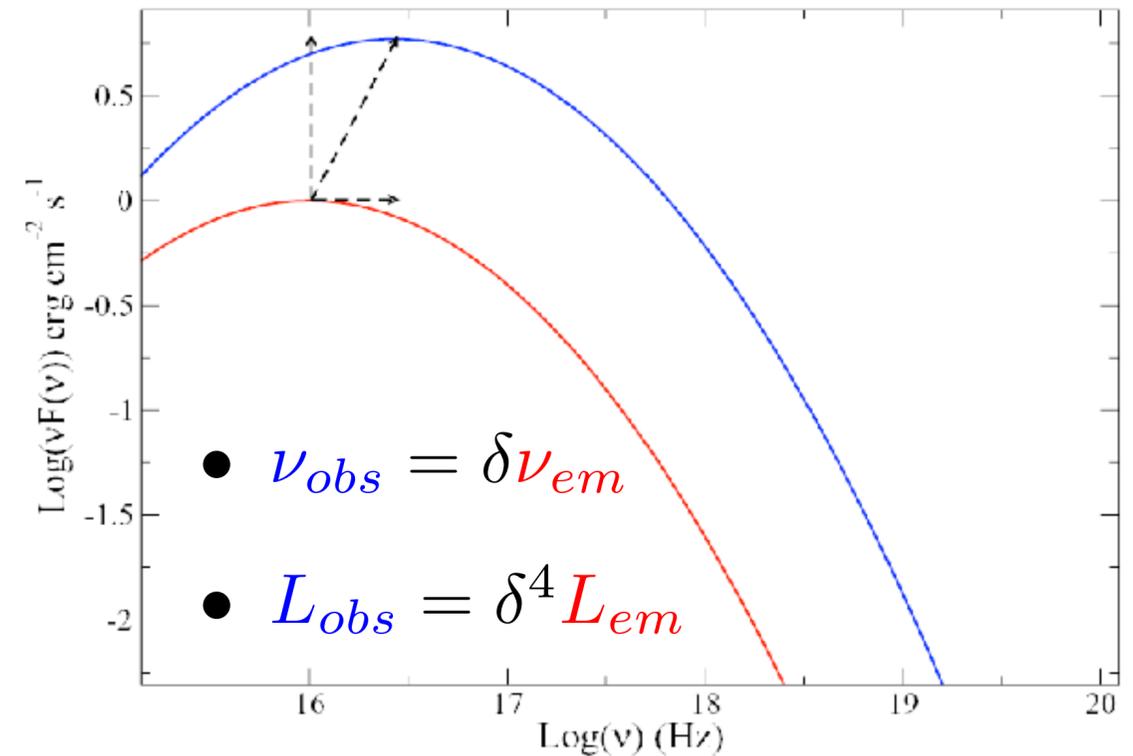


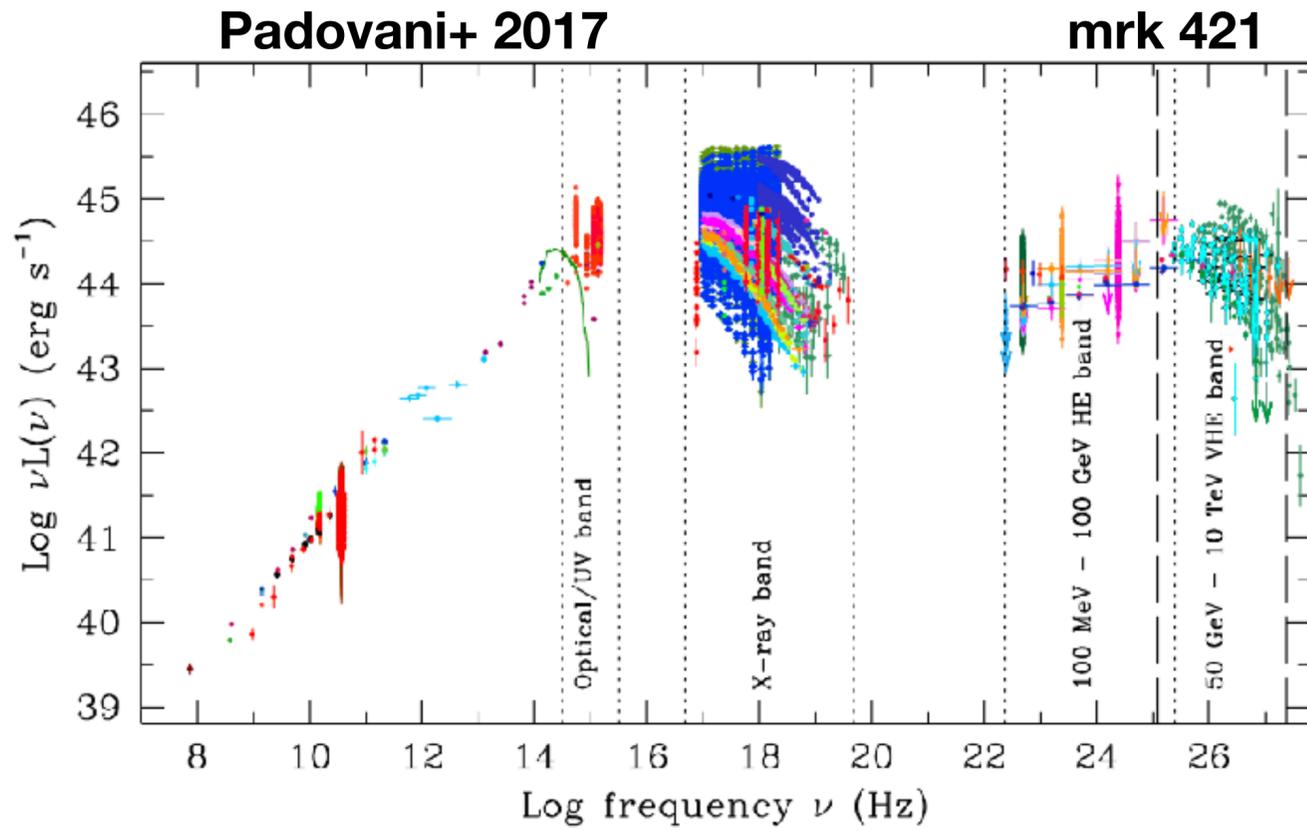
Beaming factor:

- $\delta = \frac{1}{\Gamma(1-\beta \cos(\theta))}$
- $\theta = 1/\Gamma$



$$R \leq c \Delta t \delta / (1+z)$$

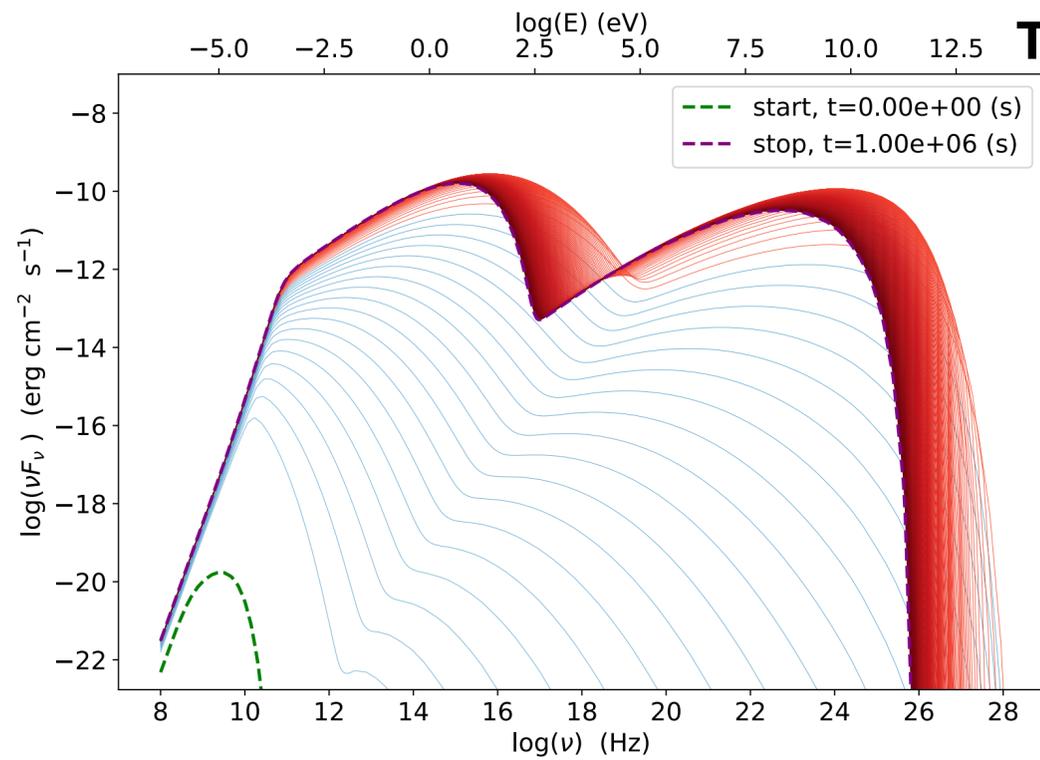




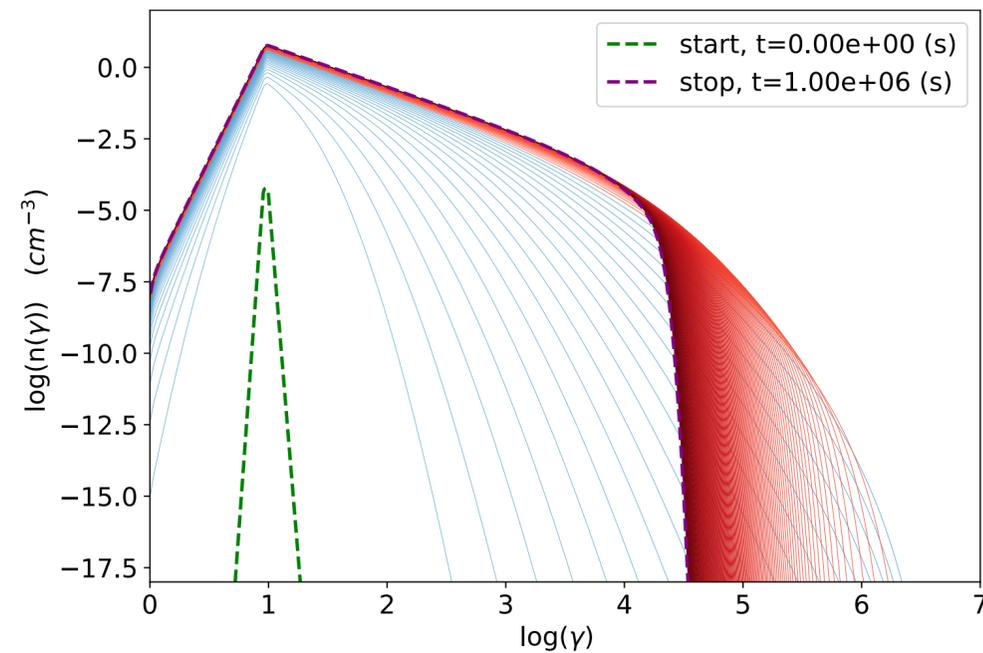
$t_{var}$  depends on  $t_{acc,cool.}$  and properties of radiative process plus geometry

Acc+cooling

Tramacere+ 2022,2011



$\ln(n(\gamma)) (cm^{-3})$



no crossing time

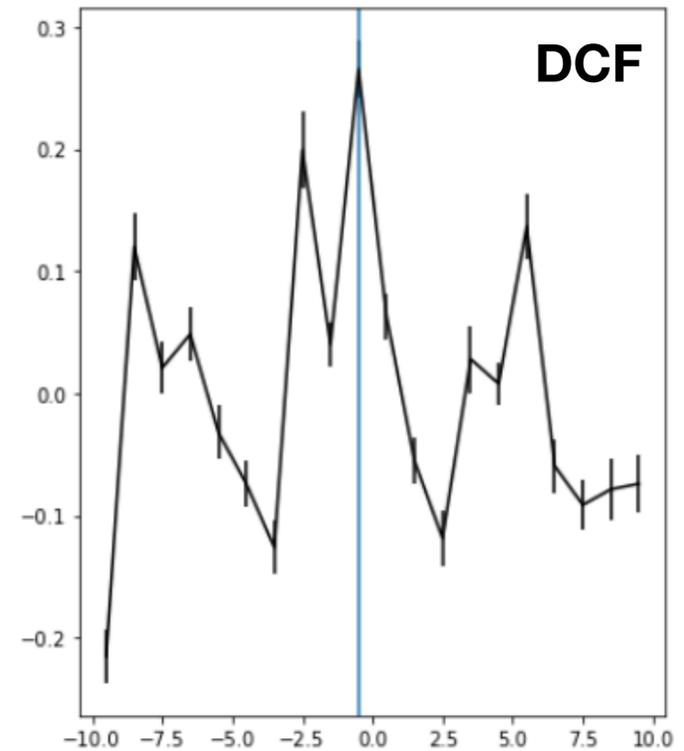
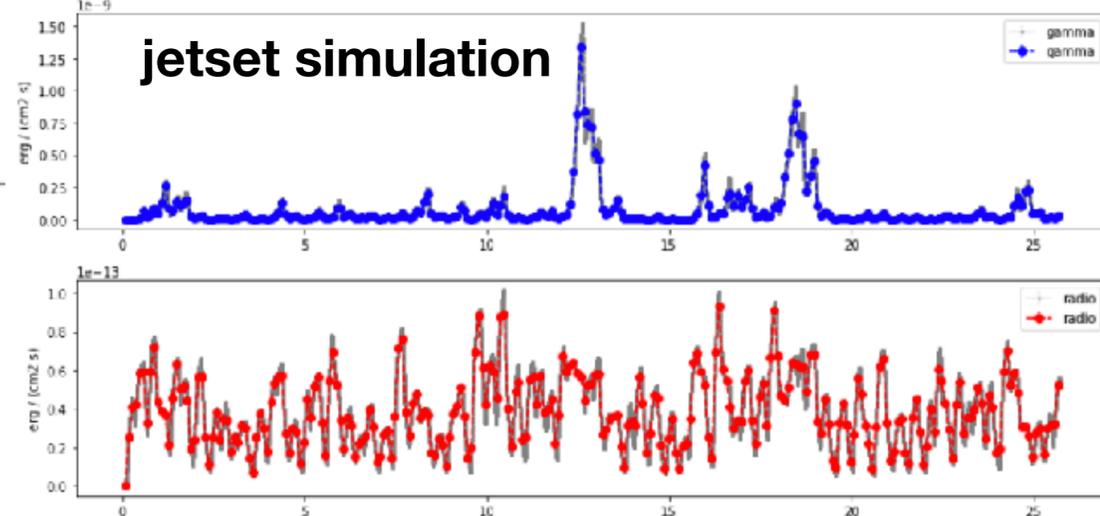
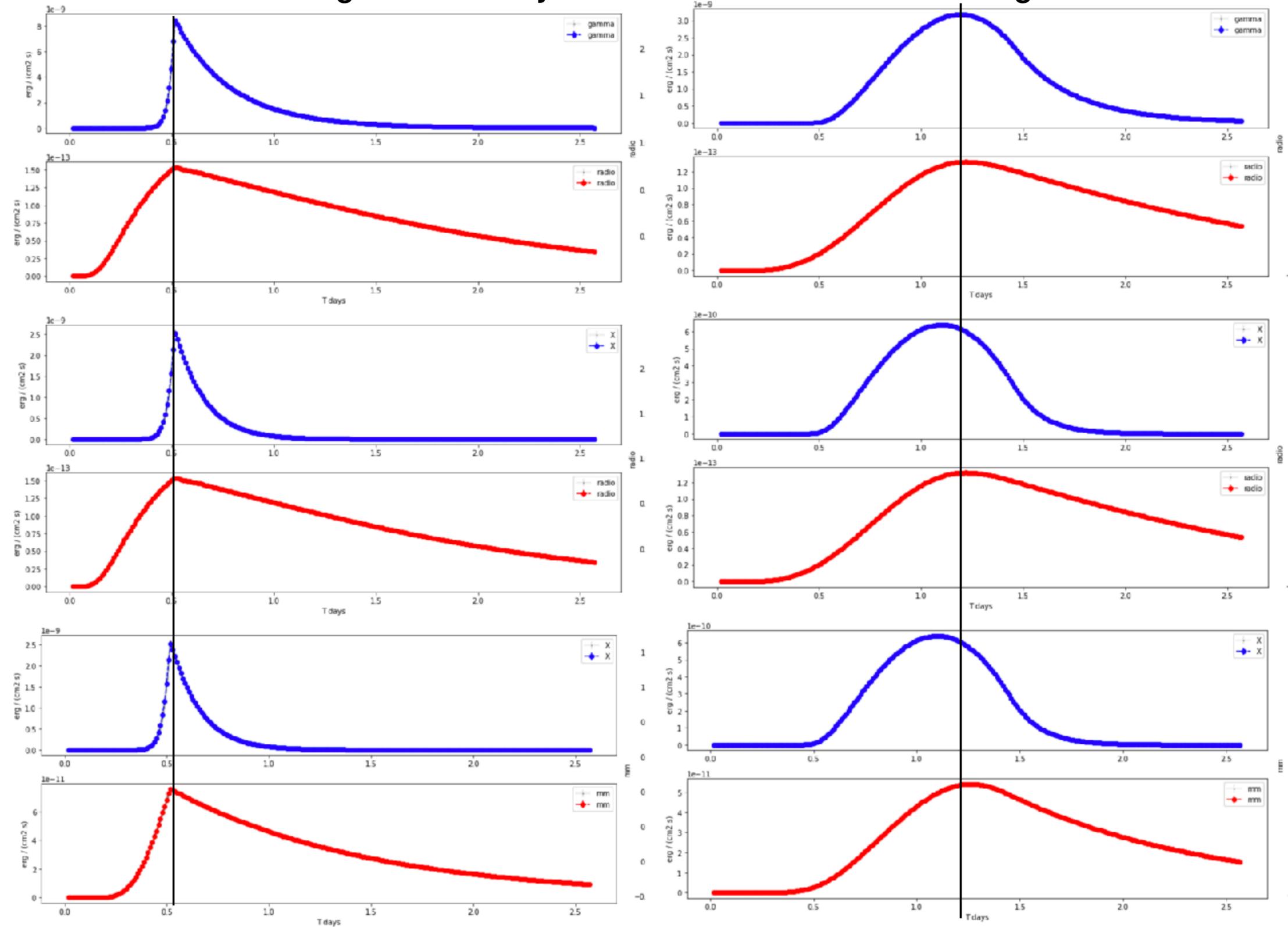
jetset simulation

with crossing time

hours to days

$$\Delta_T \sim R/c, t_{cool}$$

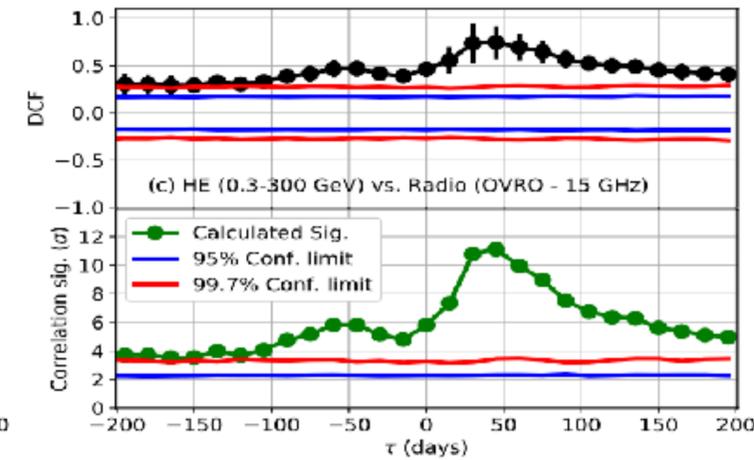
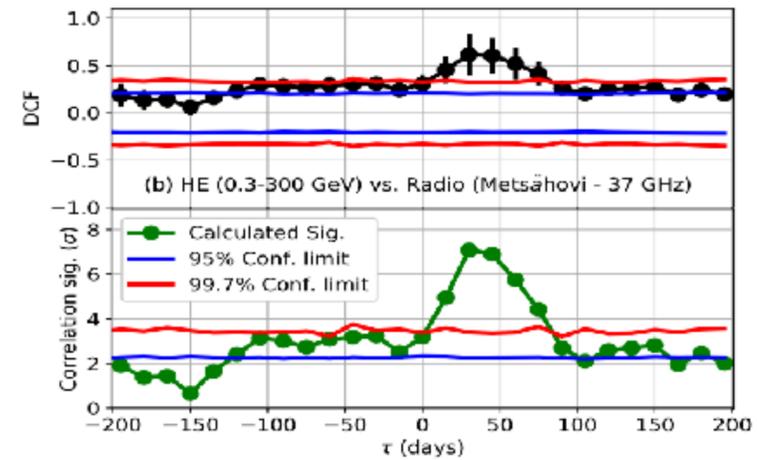
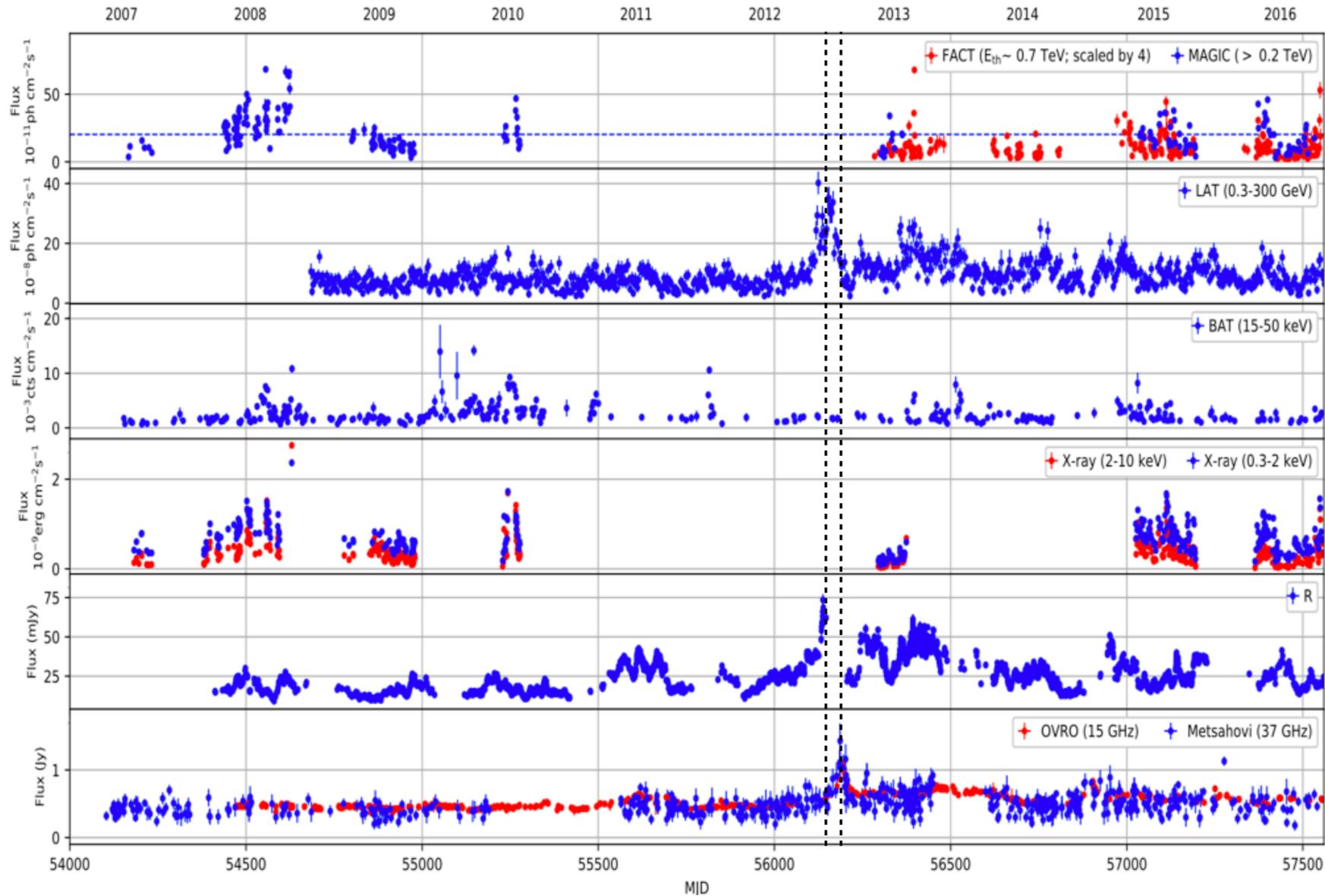
dispersion on  $\Delta_T \sim R/c, t_{cool}$



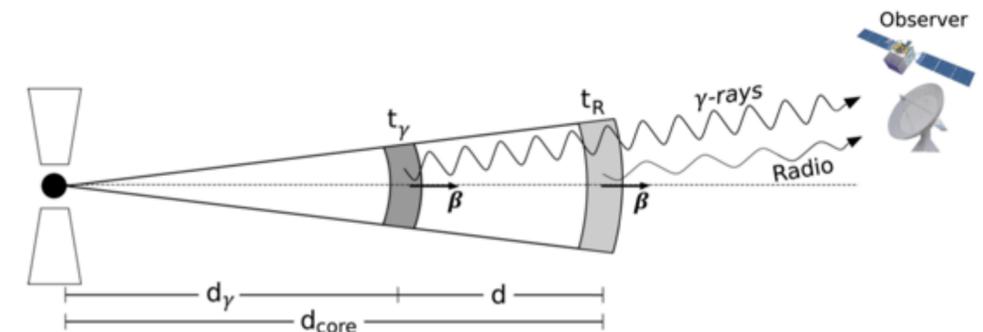
MW variability and correlation studies of Mrk 421 during historically low X-ray and  $\gamma$ -ray activity in 2015-2016

## Radio- $\gamma$ delay in Mrk 421 (months)

Magic coll. 2020



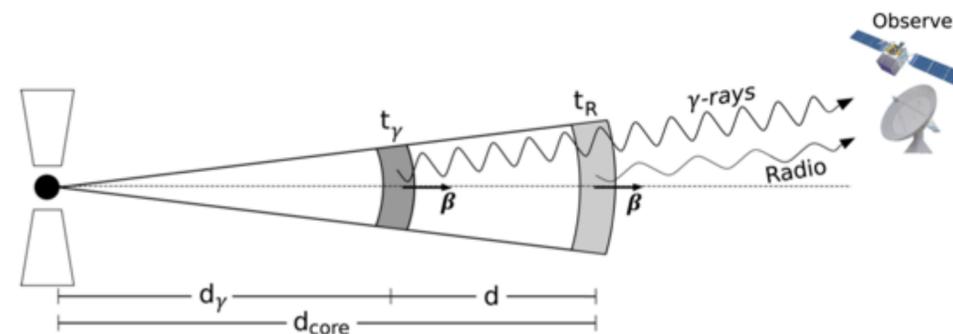
Radio- $\gamma$  delay  $\sim 1/\nu$



- W. Max-Moerbeck+ 2014
- B. Pushkarev+ 2010
- Ghisellini+1985
- McCray,R. 1968

- Observed lags are not compatible with cooling, acc., crossing (unless strong fine tuning)
- Explanations based on reacceleration, would be challenging due to MW observations

**We want to test if it is possible to reproduce a radio- $\gamma$  due ( $T \gg d$ ) due to blob expansion**



**W. Max-Moerbeck+ 2014**

**B. Pushkarev+ 2010**

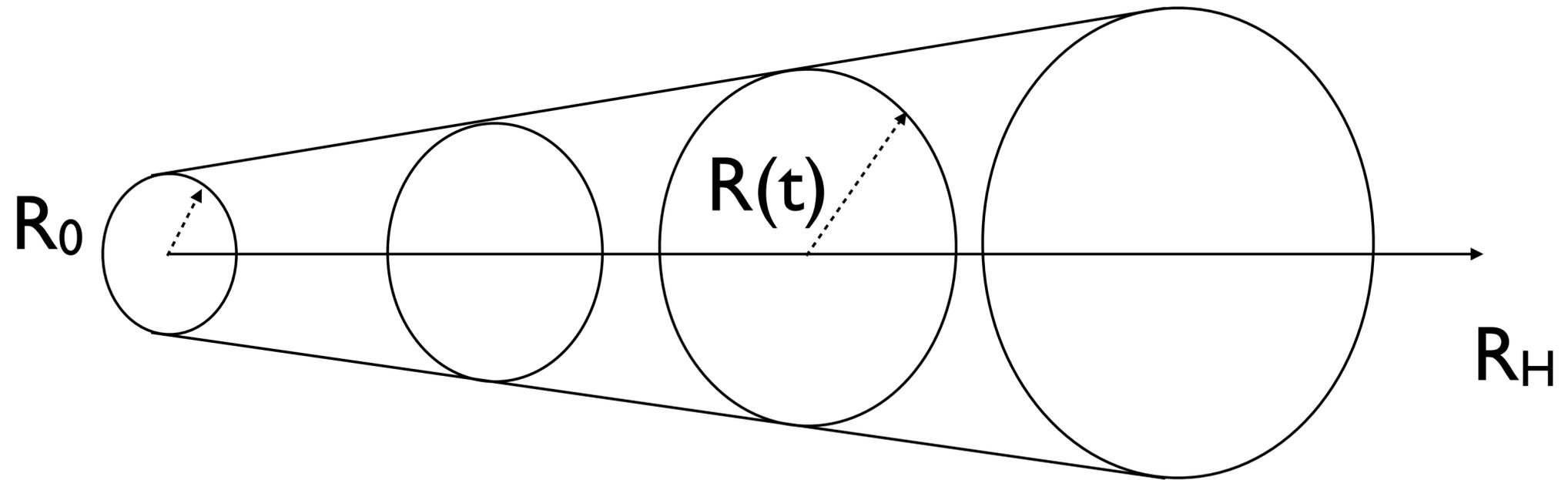
**Ghisellini+1985**

**McCray, R. 1968**

$$\beta_{\text{exp}} = v_{\text{exp}}/c$$

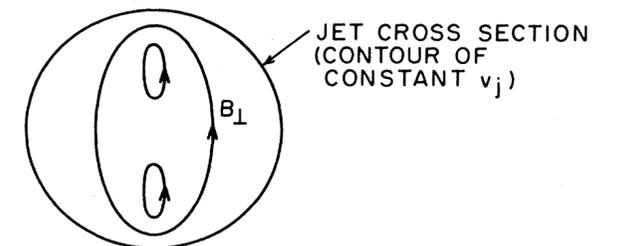
$$R(t) = R_0 + \beta_{\text{exp}}c(t - t_{\text{exp}})H(t - t_{\text{exp}})$$

$$B(t) = B_0 \left(\frac{R_0}{R(t)}\right)^{m_B} \quad m_B \in [1, 2]$$



**Magnetic field** (flux freezing and conservation): (Begelman, Blandford, and Rees 1984)

- $B_{||} \propto R^{-2}$  (poloidal)  $m_B=2$
- $B_{\perp} \propto R^{-1}$  (toroidal)  $m_B=1$
- for initial mixed configuration, **and no velocity gradient**,  $B_{\perp}$  will dominate with  $m_B \sim m_R$



$$R(t) = R_0 + \beta_{\text{exp}} c(t - t_{\text{exp}}) H(t - t_{\text{exp}})$$

$$B(t) = B_0 \left( \frac{R_0}{R(t)} \right)^{m_B} \quad m_B \in [1, 2]$$

**Geom.**

$$|\dot{\gamma}_{\text{synch}}(t)| = \frac{4\sigma_{TC}}{3m_e c^2} \gamma^2 U_B(t) = C_0 \gamma^2 U_B(t)$$

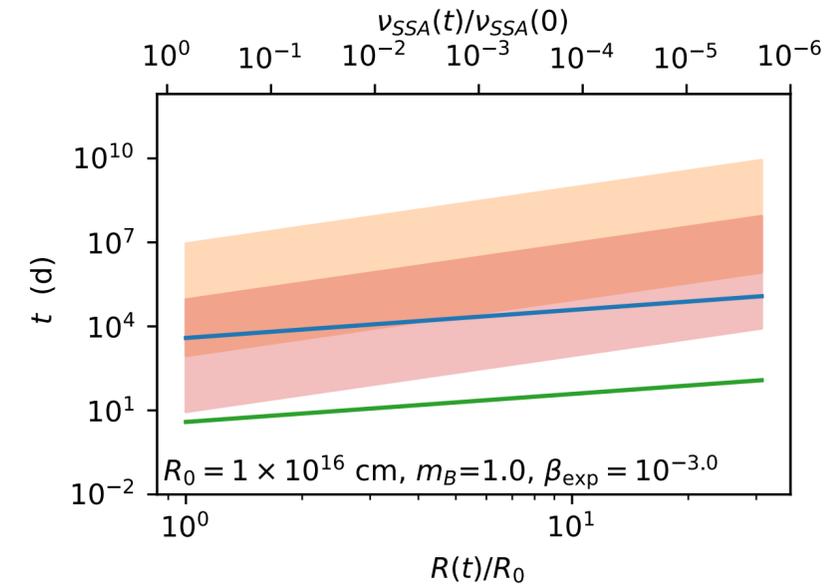
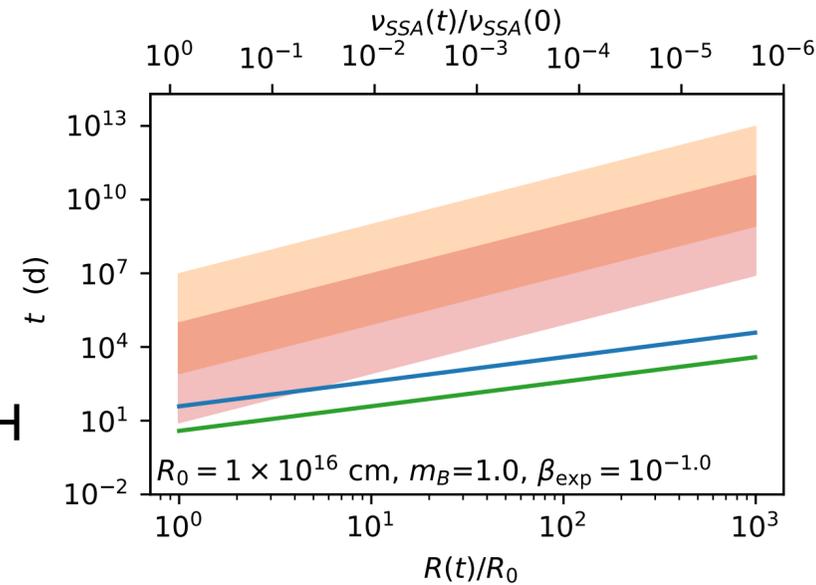
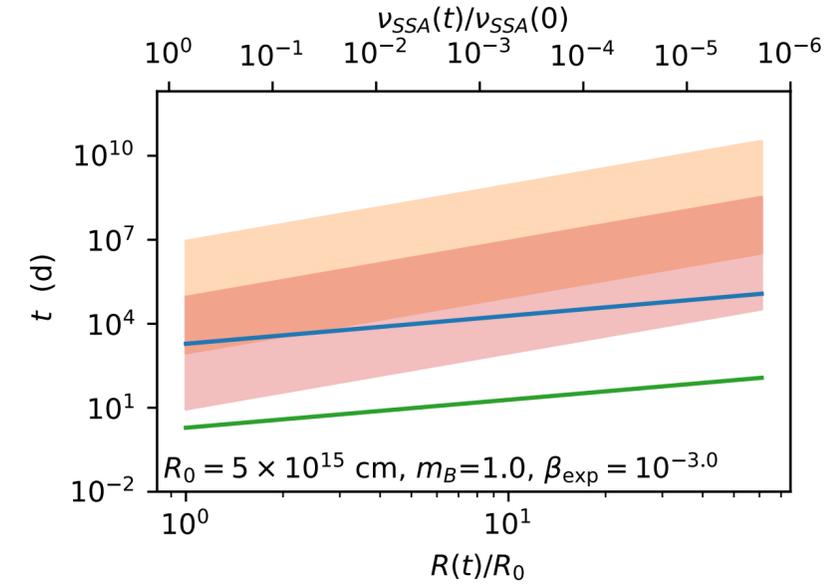
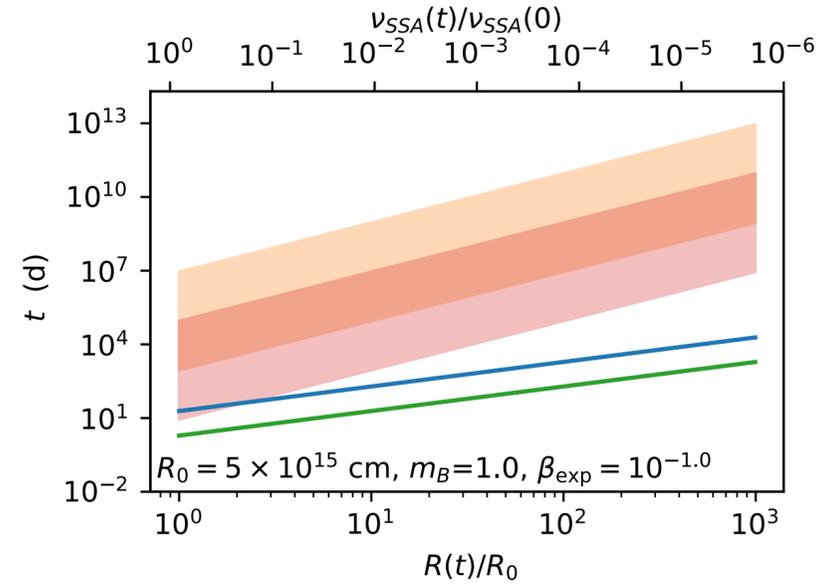
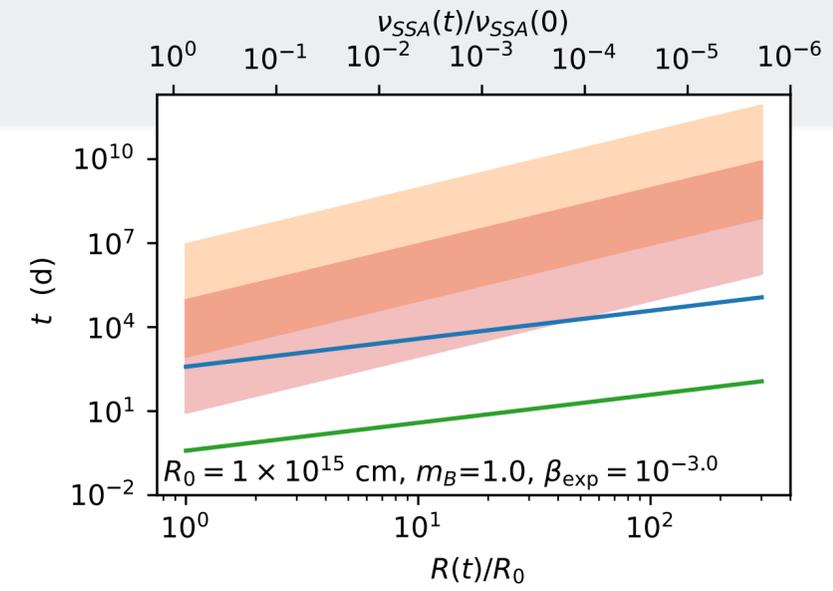
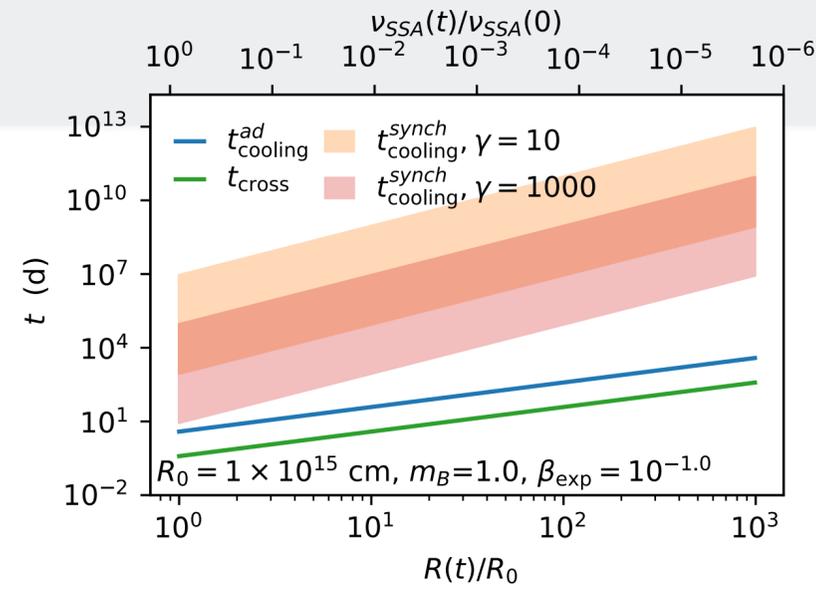
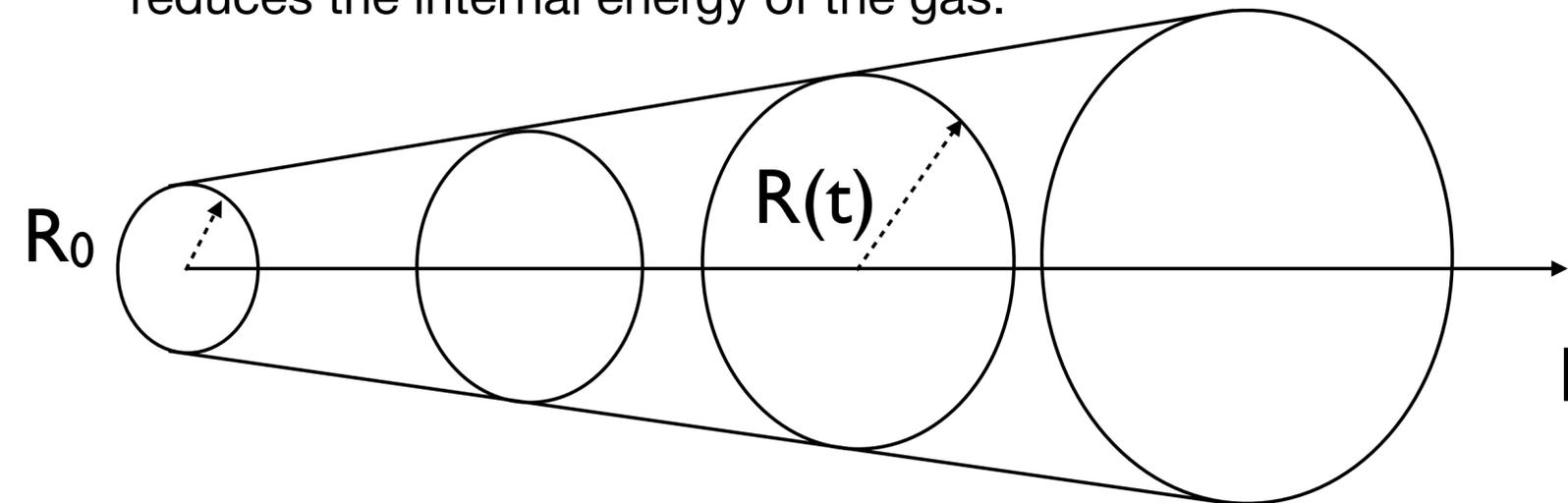
$$|\dot{\gamma}_{IC}(t)| = \frac{4\sigma_{TC}}{3m_e c^2} \gamma^2 \int f_{KN}(4\gamma\epsilon_0) \epsilon_0 n_{ph}(\epsilon_0, t) d\epsilon_0$$

$$= C_0 \gamma^2 F_{KN}(\gamma, t)$$

$$|\dot{\gamma}_{ad}(t)| = \frac{1}{3} \frac{\dot{V}}{V} \gamma = \frac{\dot{R}(t)}{R(t)} \gamma = \frac{\beta_{\text{exp}} c}{R(t)} \gamma$$

**Cooling**

If the electrons are confined within an expanding volume, they are subject to adiabatic losses as they do work which reduces the internal energy of the gas.



# self-consistent approach

here we add adiabatic cooling  
(t=time elapsed from the expansion)

$$|\dot{\gamma}_{ad}| = \frac{1}{3} \frac{\dot{V}}{V} \gamma = \frac{\dot{R}(t)}{R(t)} \gamma = \frac{\beta_{exp} c}{R(t)} \gamma$$

Tramacere+2022, Tramacere+2011

## injection term

$$L_{inj} = V_{acc} \int \gamma m_e c^2 Q(\gamma, t) d\gamma \quad (erg/s)$$

## systematic term

$$S(\gamma, t) = -C(\gamma, t) + A(\gamma, t)$$

### cooling term

$$C(\gamma) = |\dot{\gamma}_{synch}| + |\dot{\gamma}_{IC}| + |\dot{\gamma}_{ad}|$$

### sys. acc. term

$$A(\gamma) = A_{p0} \gamma, \quad t_A = \frac{1}{A_0}$$

$$\frac{\partial n(\gamma, t)}{\partial t} = \frac{\partial}{\partial \gamma} \left\{ - [S(\gamma, t) + D_A(\gamma, t)] n(\gamma, t) \right\} + \frac{\partial}{\partial \gamma} \left\{ D_p(\gamma, t) \frac{\partial n(\gamma, t)}{\partial \gamma} \right\} - \frac{n(\gamma, t)}{T_{esc}(\gamma)} - \frac{n(\gamma, t)}{T_{ad}} + Q(\gamma, t)$$

$$T_{ad} = \frac{1}{3} \frac{R(t)}{\beta_{exp} c} \quad (\text{Gould 1975})$$

## Turbulent magnetic field

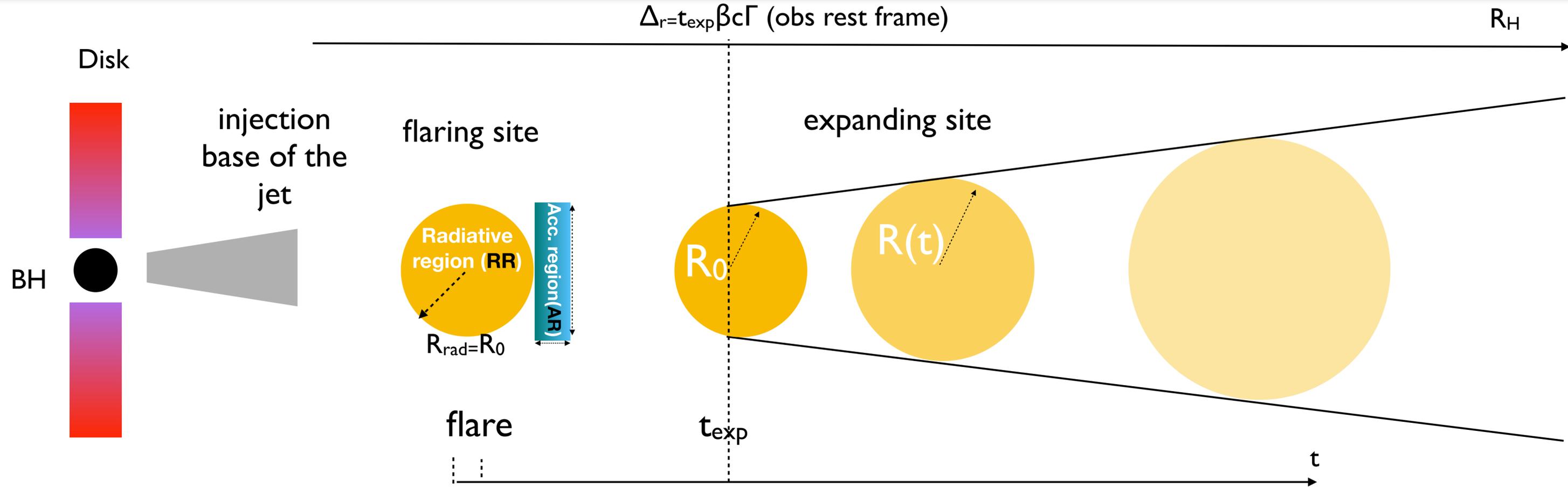
$$W(k) = \frac{\delta B(k_0^2)}{8\pi} \left( \frac{k}{k_0} \right)^{-q}$$



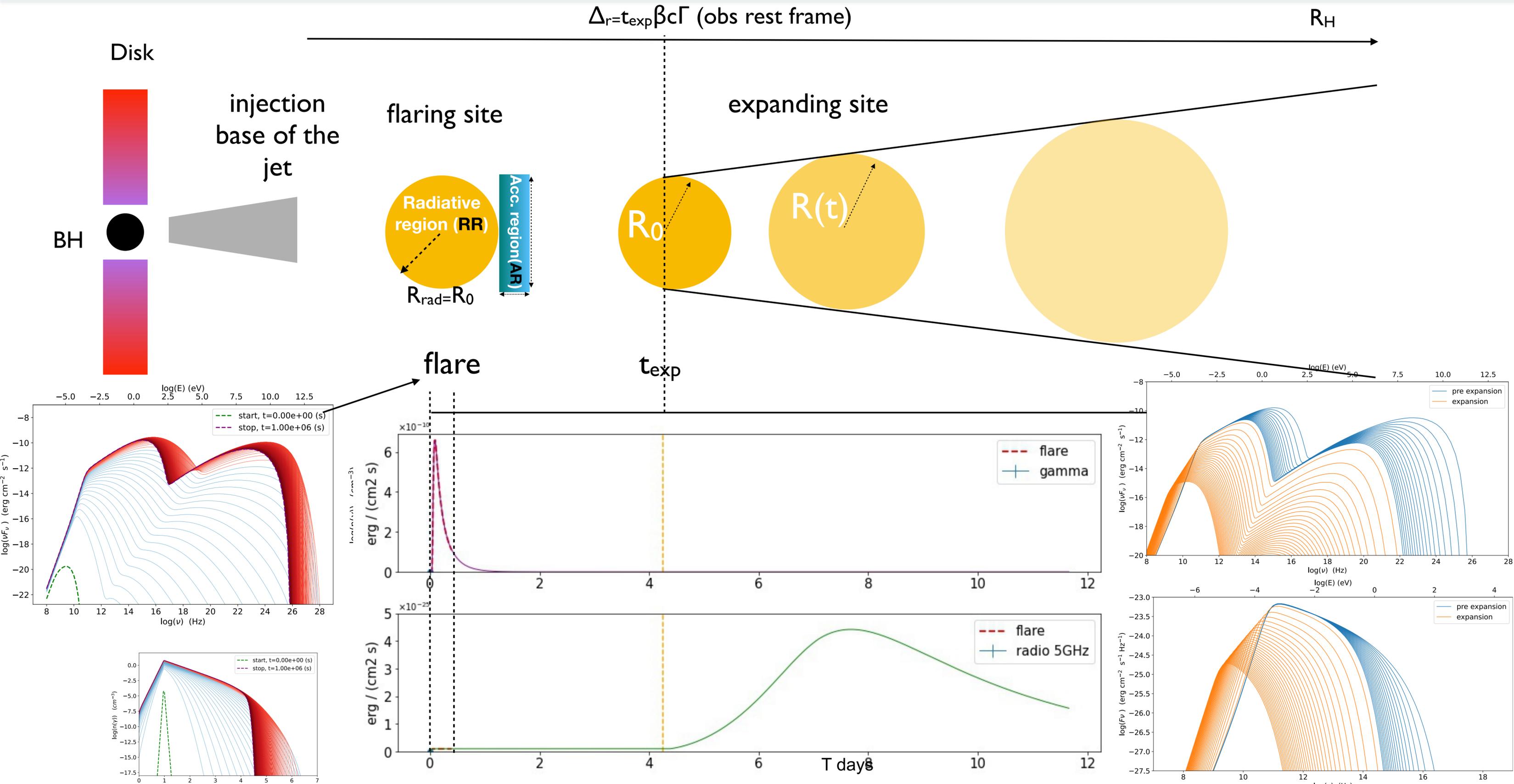
## momentum diffusion term

$$D_p \approx \beta_A^2 \left( \frac{\delta B}{B_0} \right)^2 \left( \frac{\rho_g}{\lambda_{max}} \right)^{q-1} \frac{p^2 c^2}{\rho_g c}$$

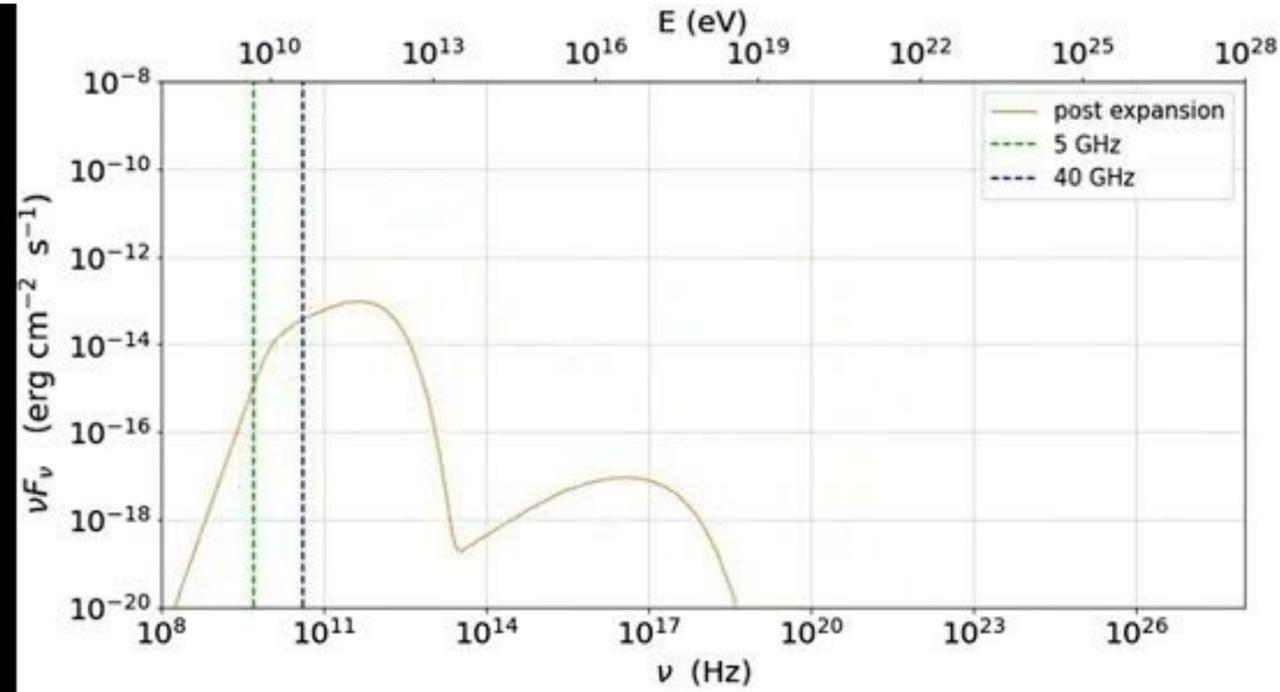
# self-consistent approach



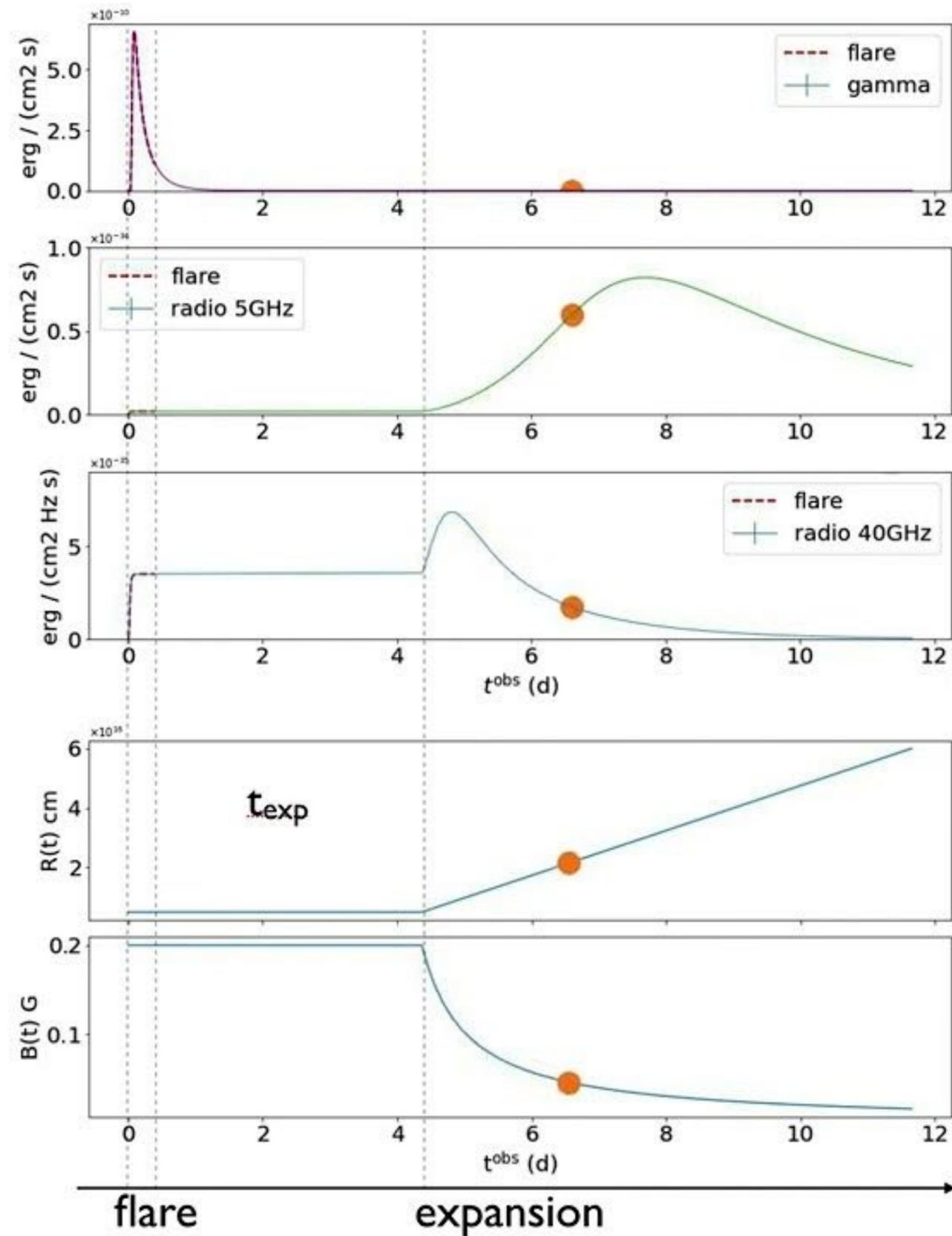
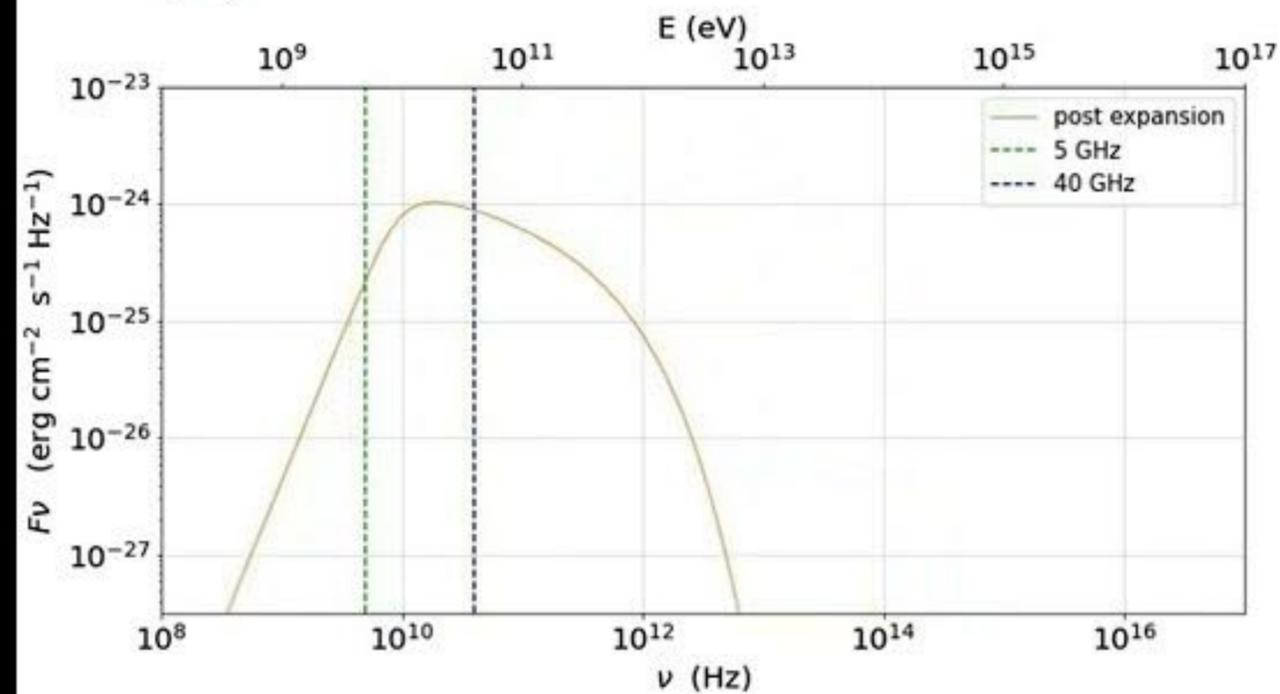
# self-consistent approach

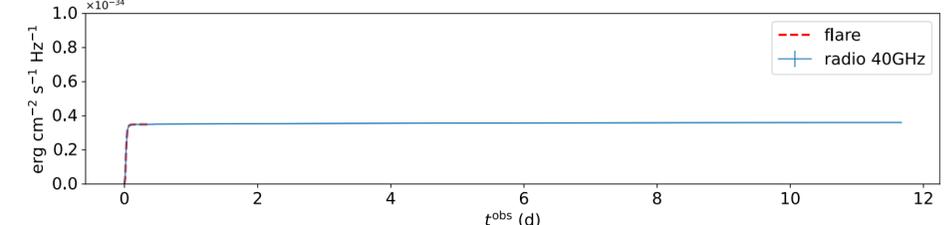
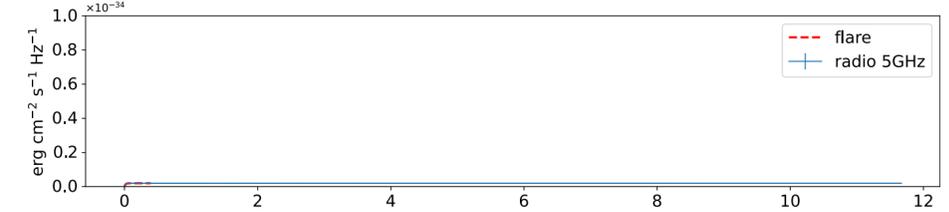
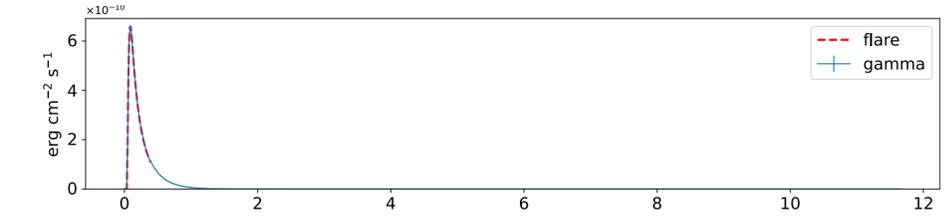
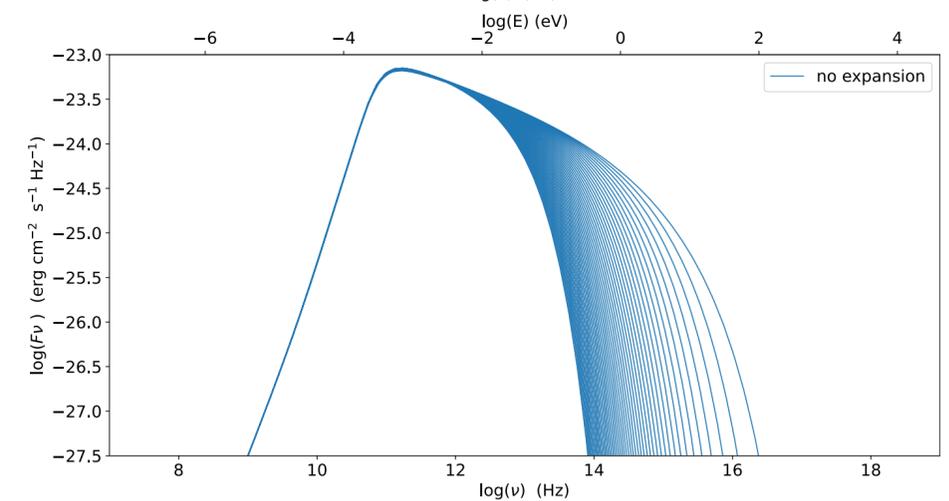
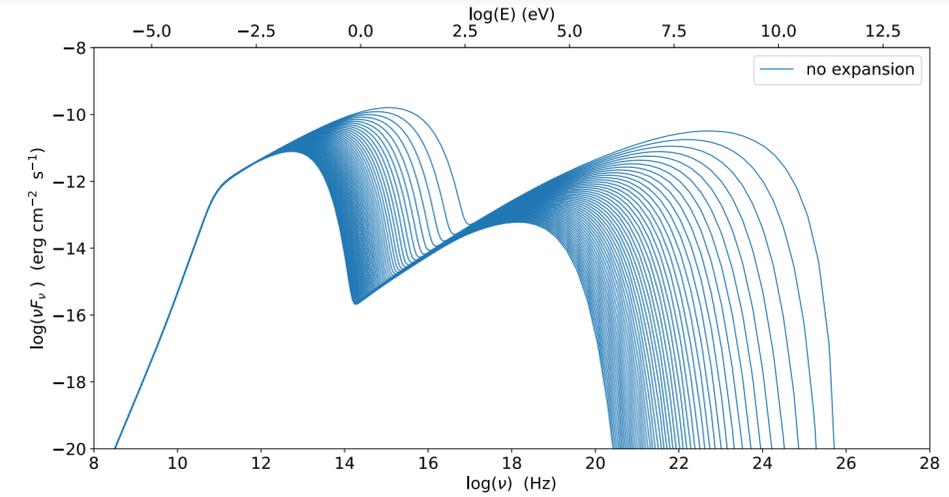
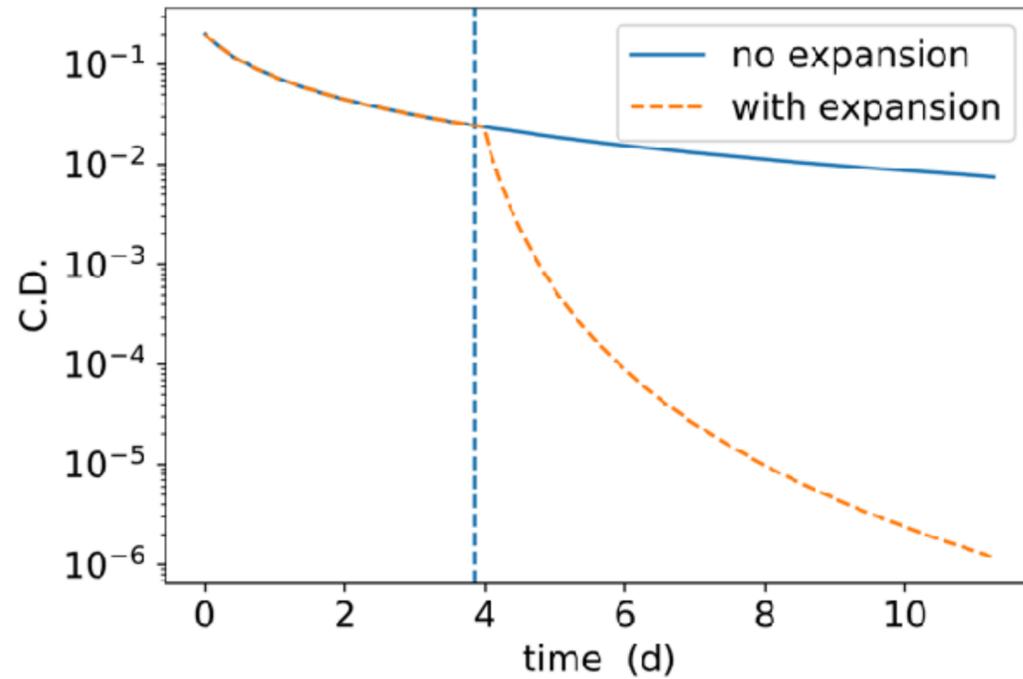
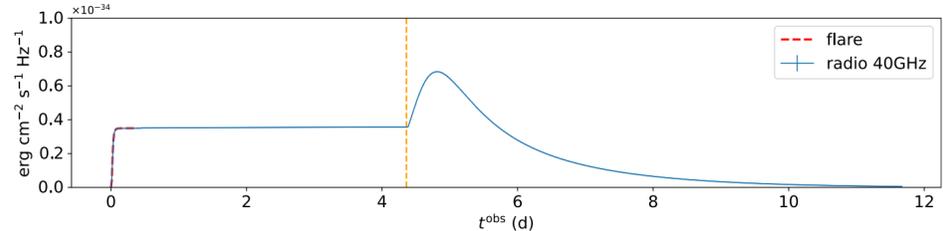
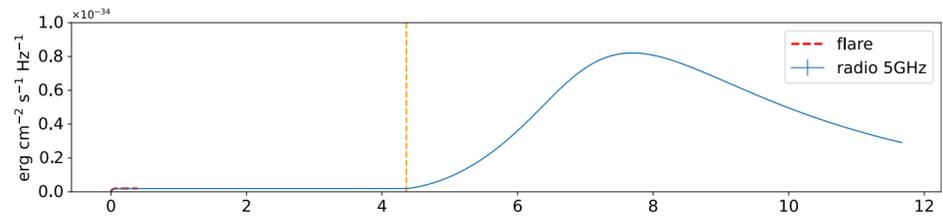
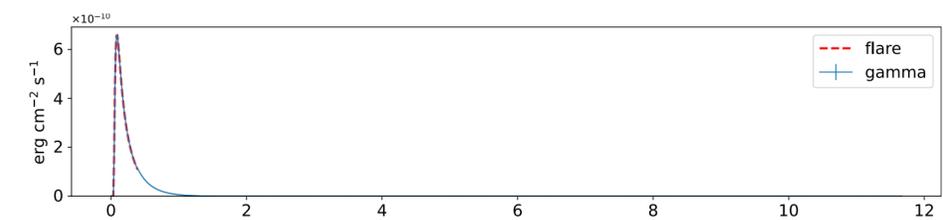
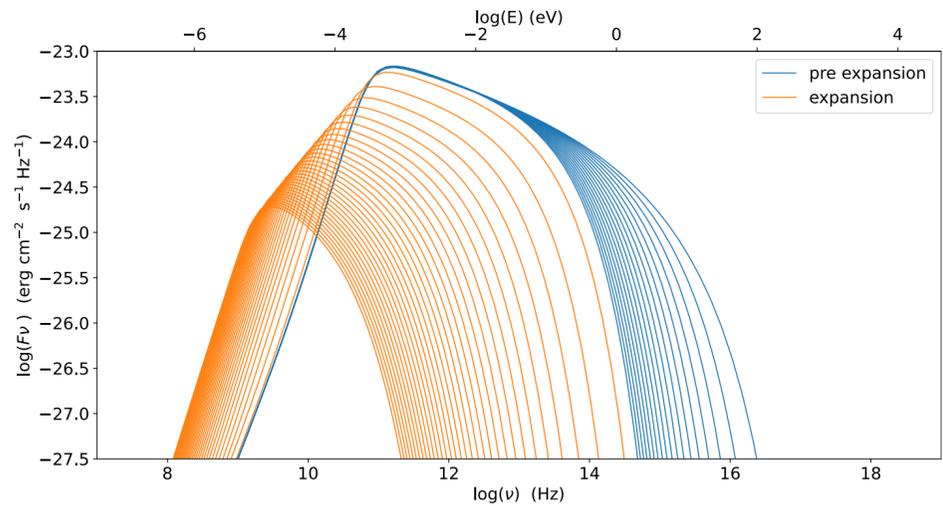
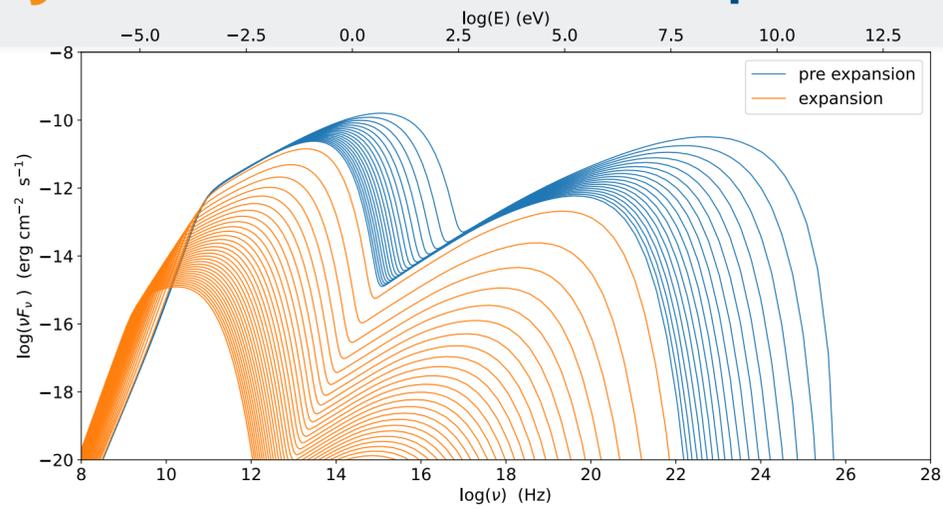


Click on the image to reproduce the animation on youtube



- duration  $\sim 3 \times 10^7$  s (blob frame)  $\sim 11$  d obs
- $t_{\text{exp}} = 1 \times 10^7$  s
- $\beta_{\text{exp}} = 0.1c$

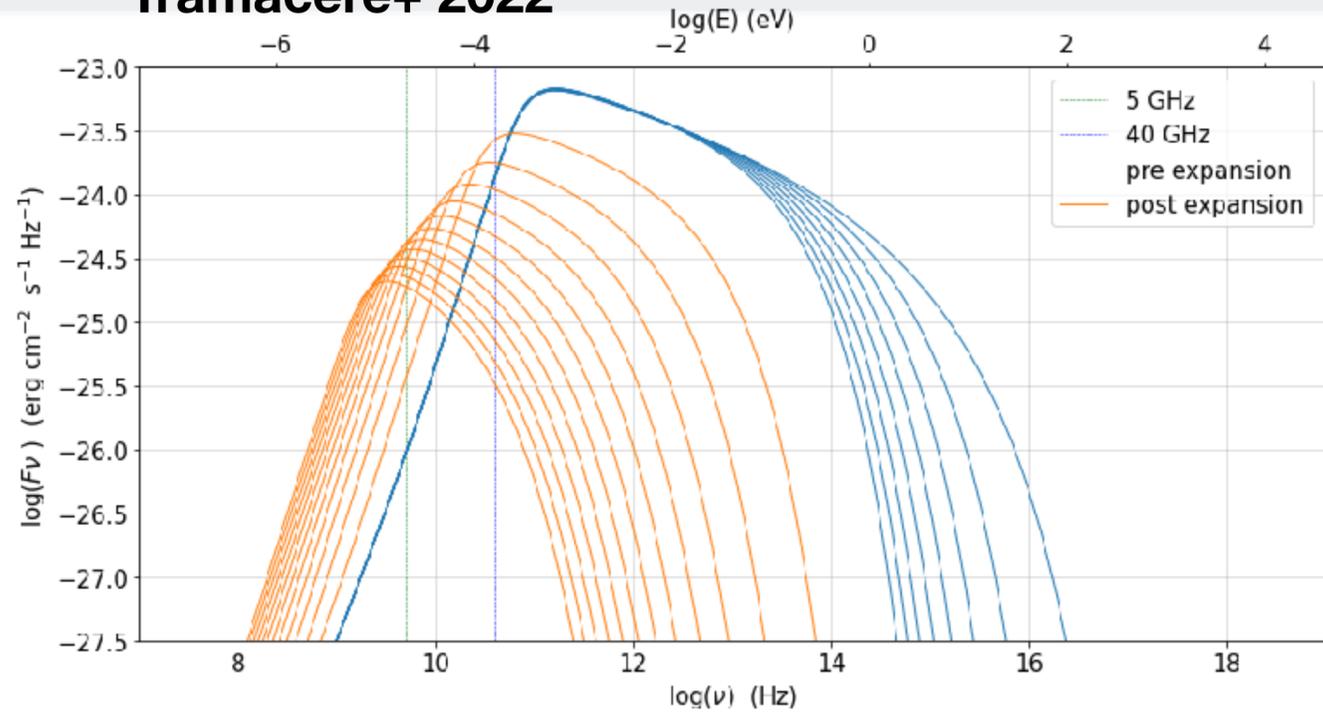




**The impact of the expansion on the Compton Dominance is significant and might be tested if the expansion begins already at the flaring stage**

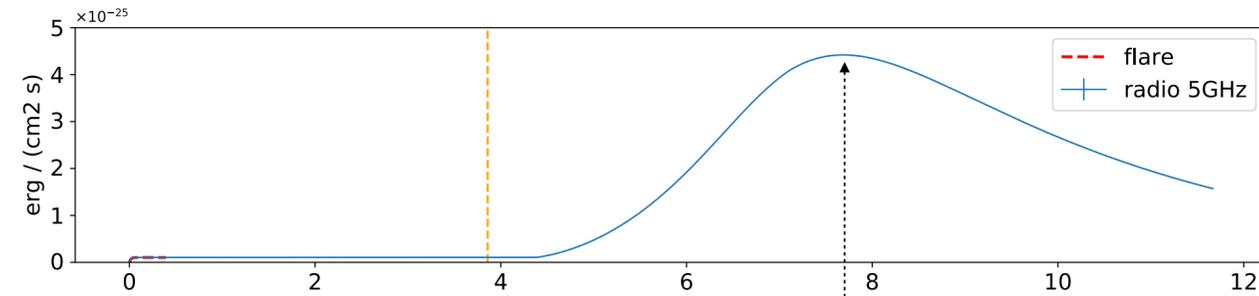
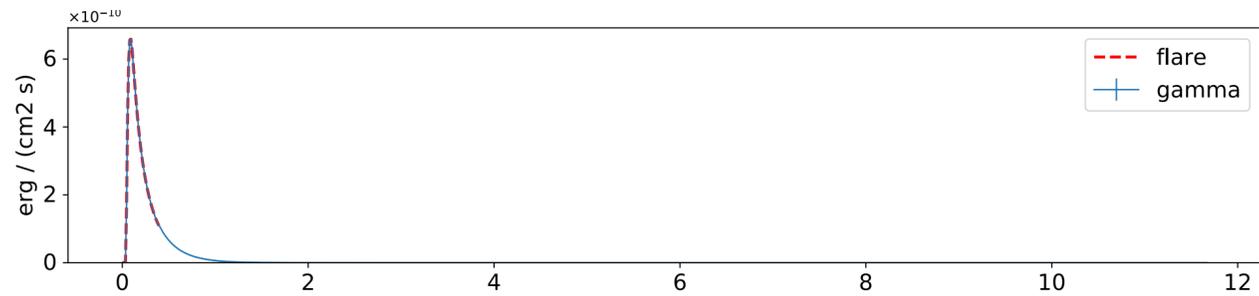
**In agreement with MAGIC 2021 paper on Mrk 421 2017 obs**





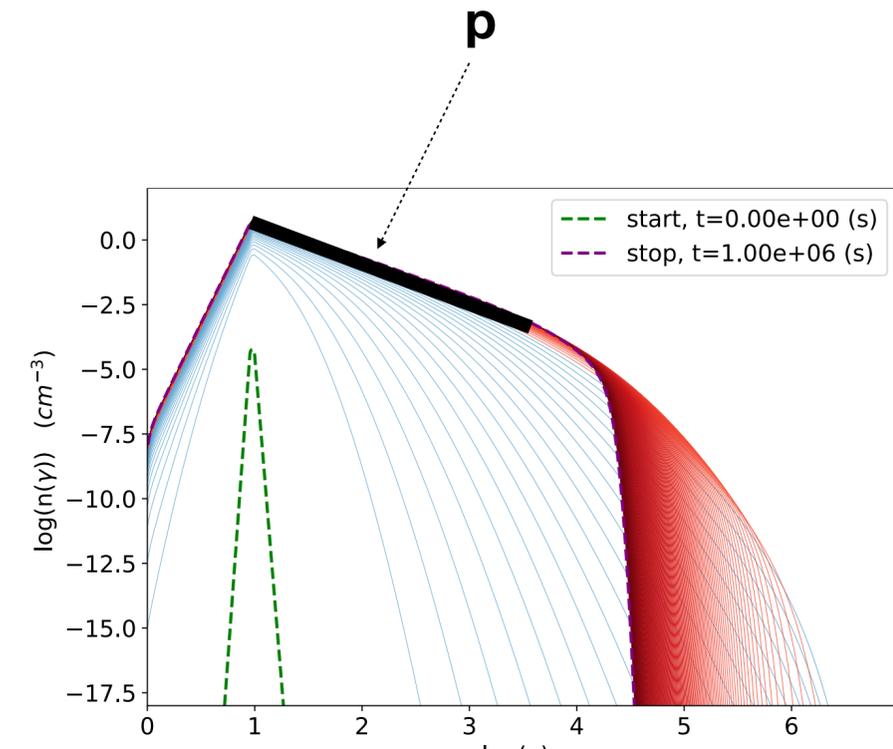
Rybicki&Lightman (1985) standard synchrotron theory

$$\nu_{SSA}(t) = \nu_L(t) \left[ \frac{\pi \sqrt{\pi} e R(t) N(t)}{4 B(t)} f_k(p) \right]^{\frac{2}{p+4}}$$

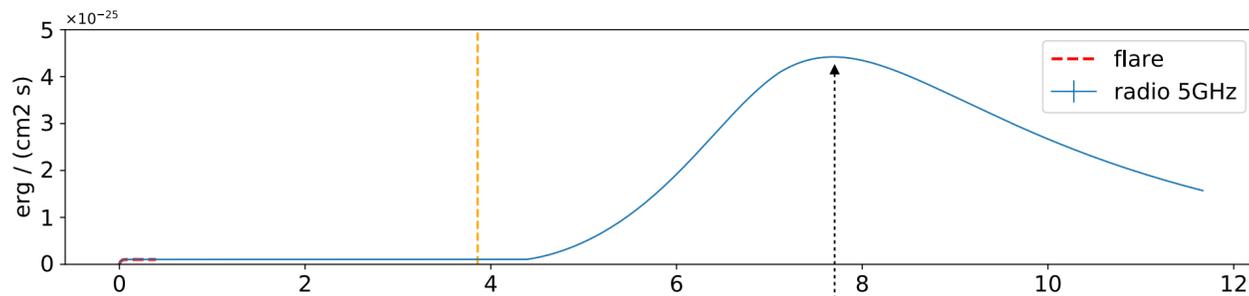
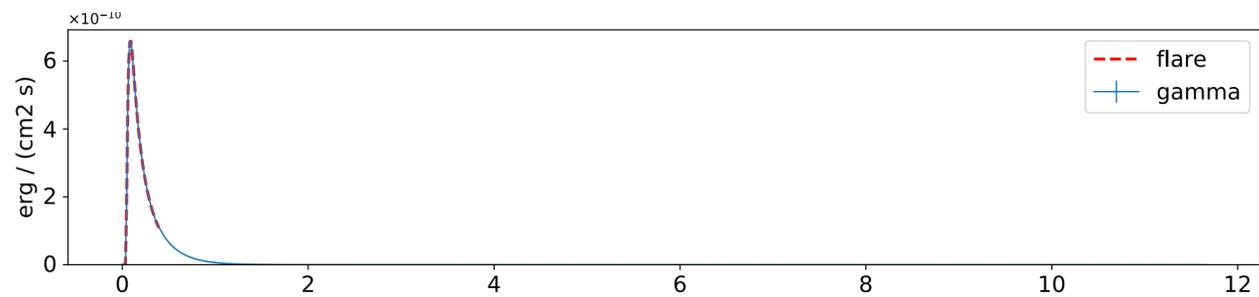
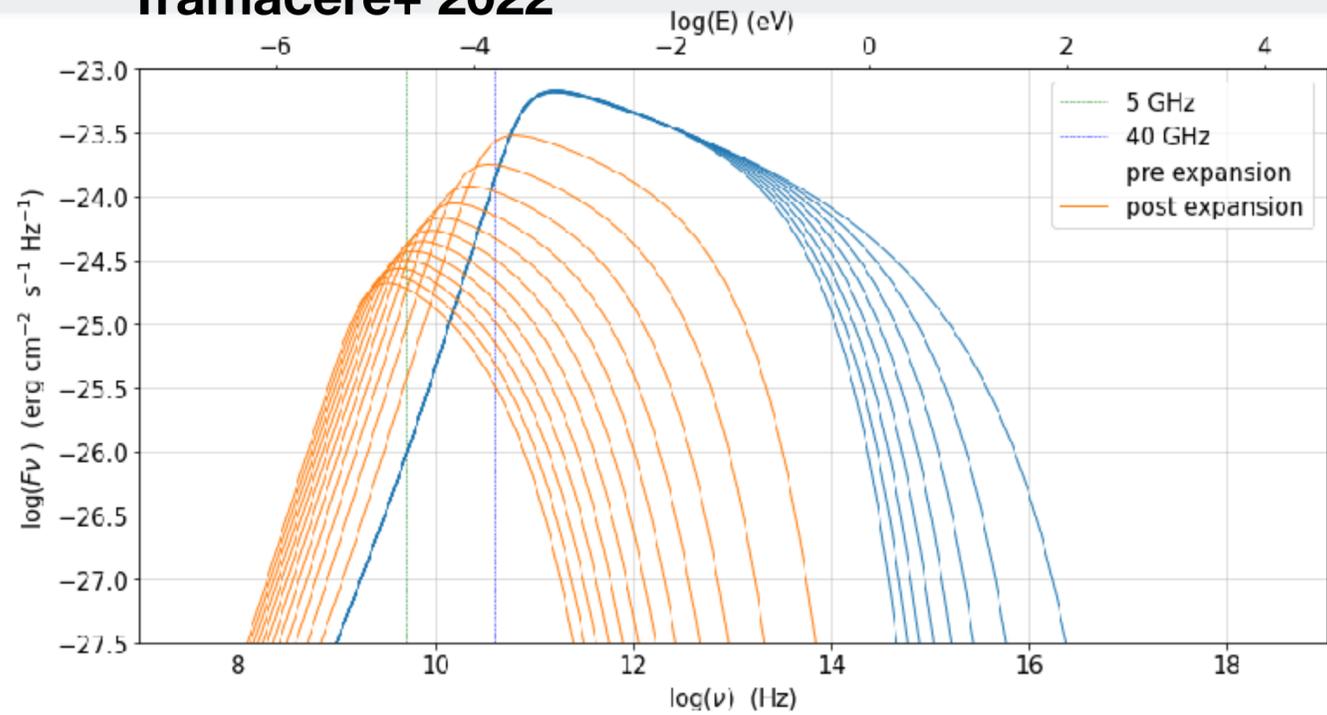


$t_{exp}$        $t_{peak}$        $t_{decay}$

src transp  
at  $\nu_{SSA}^*$   
 $R=R^*$



Tramacere+ 2022

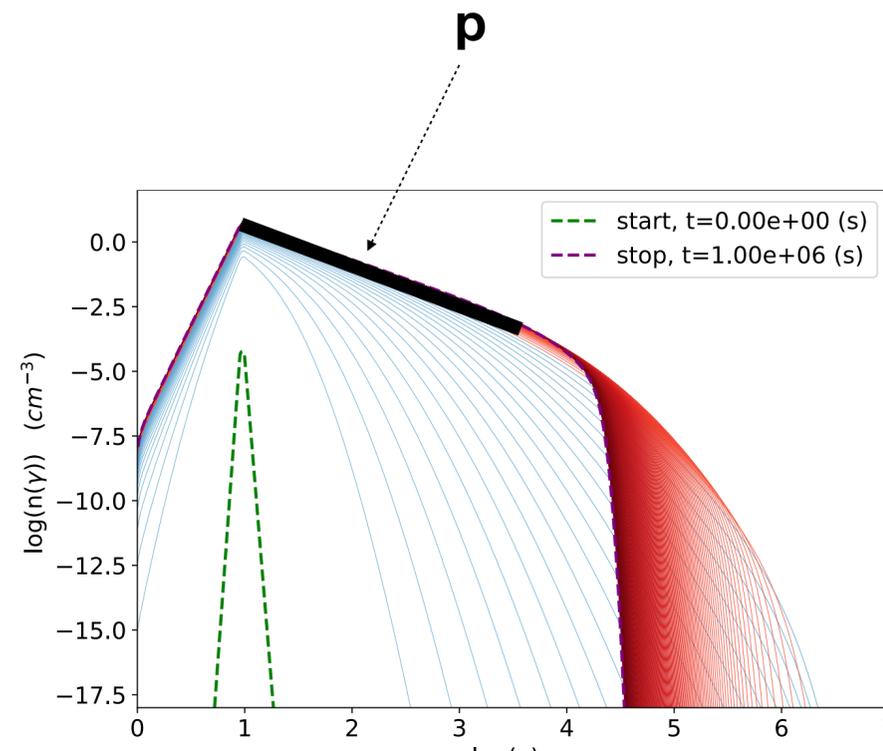


src transp  
at  $\nu_{\text{SSA}}^*$   
 $R=R^*$

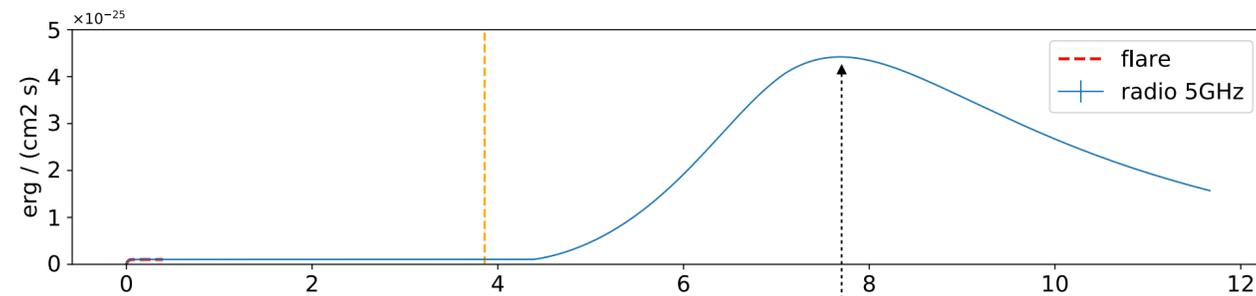
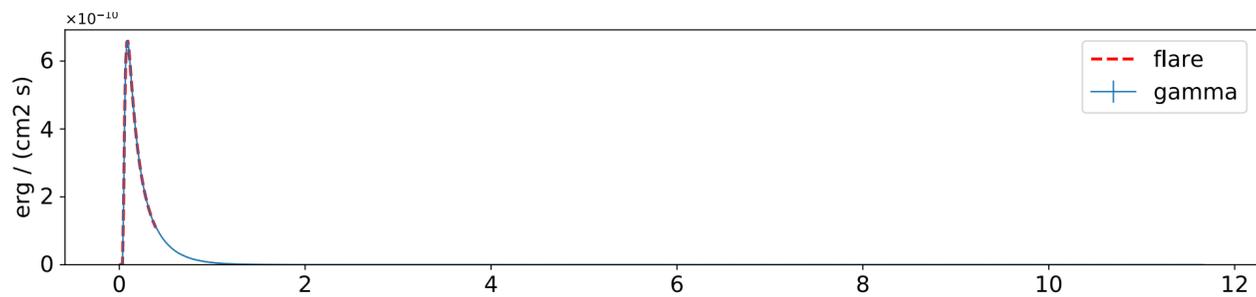
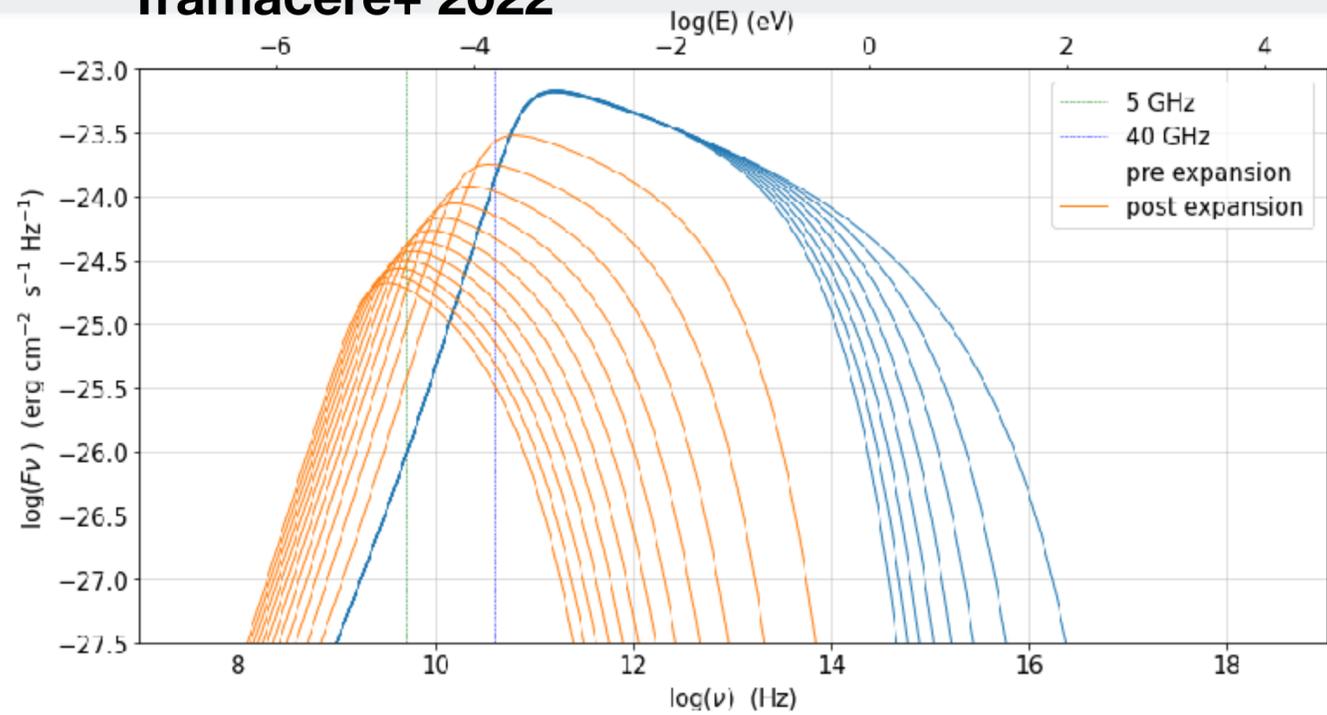
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$$R(t) = R_0 + \beta_{\text{exp}} c(t - t_{\text{exp}}) H(t - t_{\text{exp}})$$

$$B(t) = B_0 \left( \frac{R_0}{R(t)} \right)^{m_B}$$



Tramacere+ 2022



$t_{\text{exp}}$        $t_{\text{peak}}$        $t_{\text{decay}}$

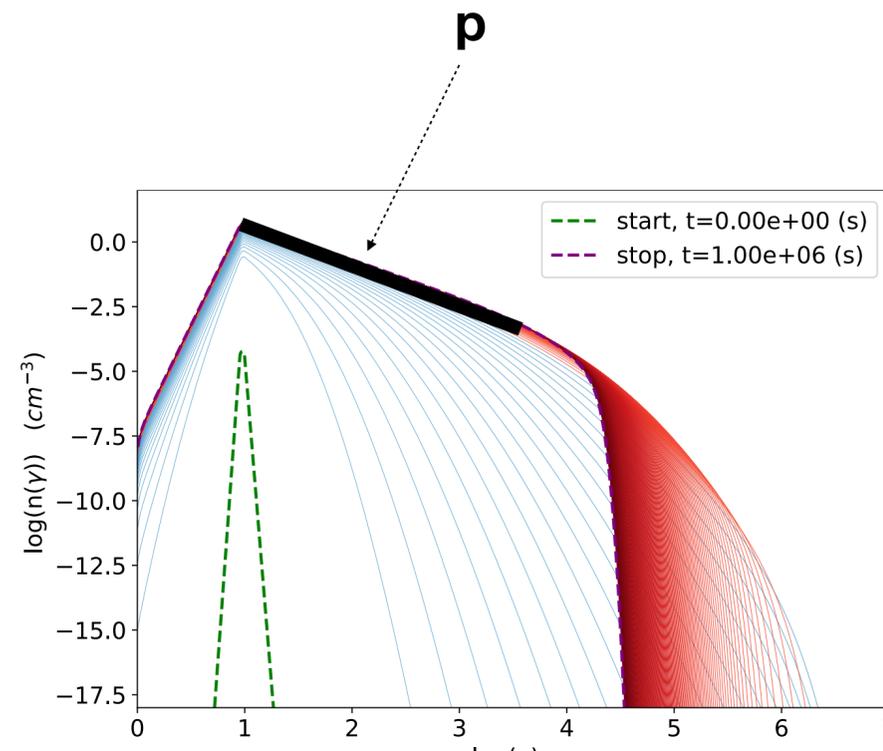
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at  $\nu_{\text{SSA}}^*$   
 $R=R^*$

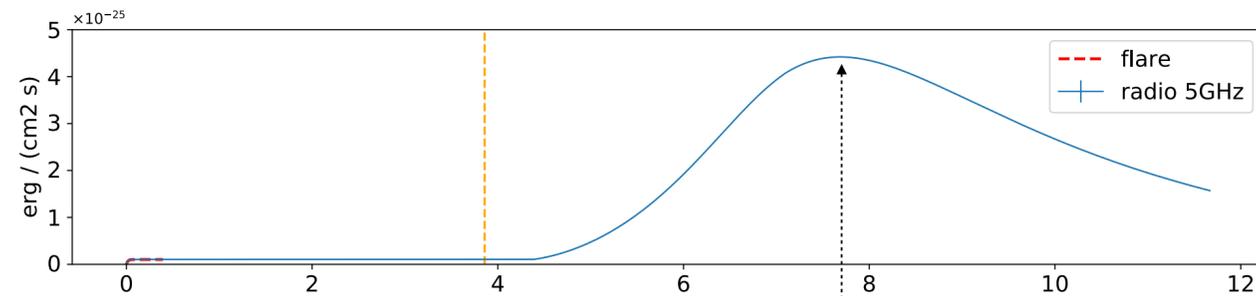
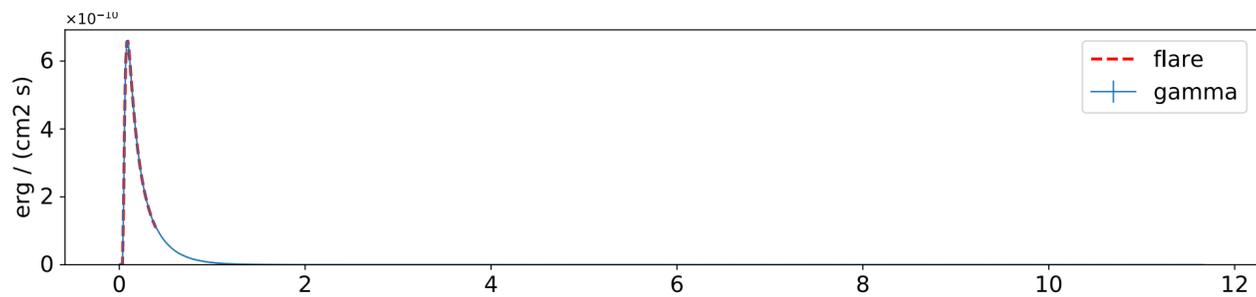
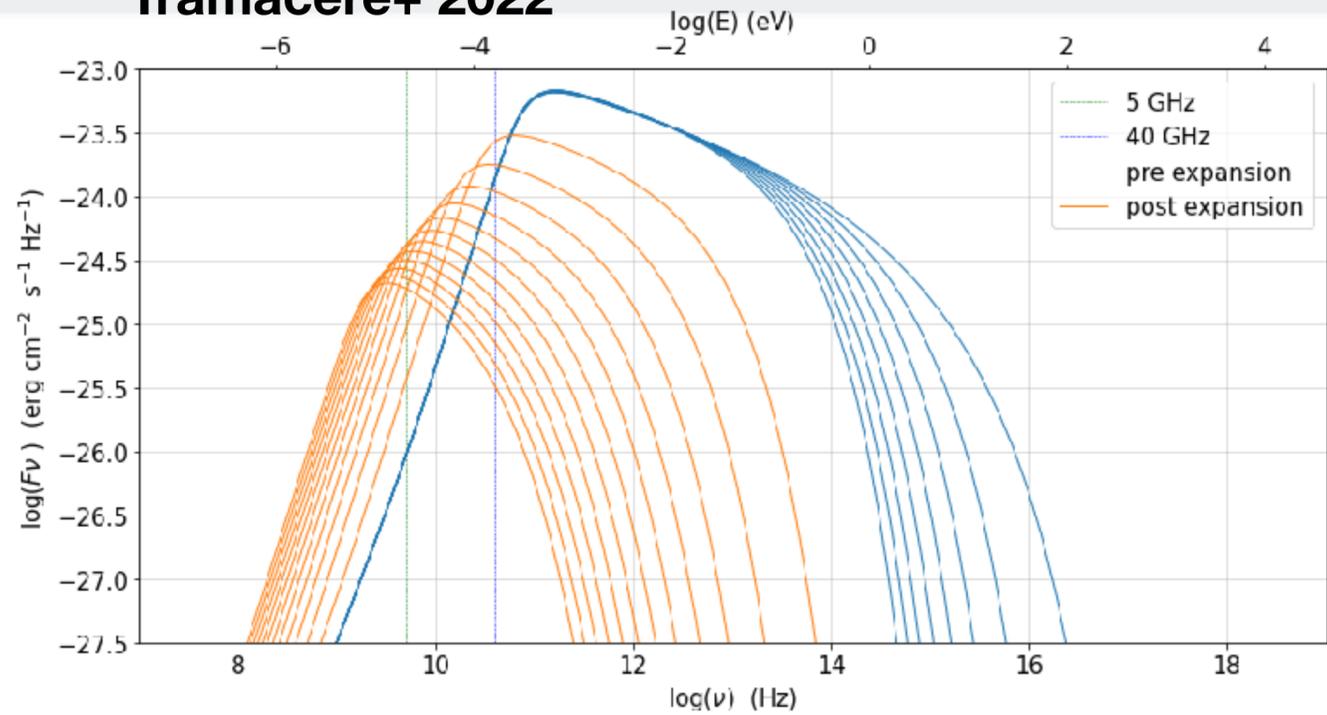
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$$B(t) = B_0 \left( \frac{R_0}{R(t)} \right)^{m_B}$$

$$\frac{\nu_{\text{SSA}}^*}{\nu_{\text{SSA}}^0} = \left[ \left( \frac{B^*}{B_0} \right)^{\frac{p+2}{2}} \left( \frac{R_0}{R^*} \right)^2 \right]^{\frac{2}{p+4}} = \left[ \frac{R_0}{R^*} \right]^{\frac{m_B(p+2)+4}{p+4}}$$





src transp  
at  $\nu_{\text{SSA}}^*$   
 $R=R^*$

$$R(t) = R_0 + \beta_{\text{exp}} c(t - t_{\text{exp}}) H(t - t_{\text{exp}})$$

$$\nu_{\text{SSA}}(t) = \nu_L(t) \left[ \frac{\pi \sqrt{\pi} e R(t) N(t)}{4 B(t)} f_k(p) \right]^{\frac{2}{p+4}}$$

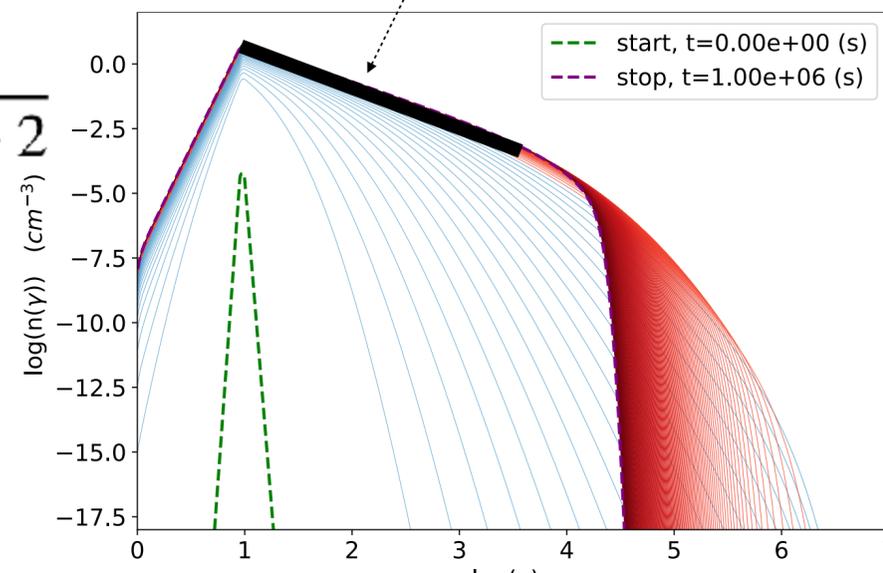
$$B(t) = B_0 \left( \frac{R_0}{R(t)} \right)^{m_B}$$

$$\frac{\nu_{\text{SSA}}^*}{\nu_{\text{SSA}}^0} = \left[ \left( \frac{B^*}{B_0} \right)^{\frac{p+2}{2}} \left( \frac{R_0}{R^*} \right)^2 \right]^{\frac{2}{p+4}} = \left[ \frac{R_0}{R^*} \right]^{\frac{m_B(p+2)+4}{p+4}}$$

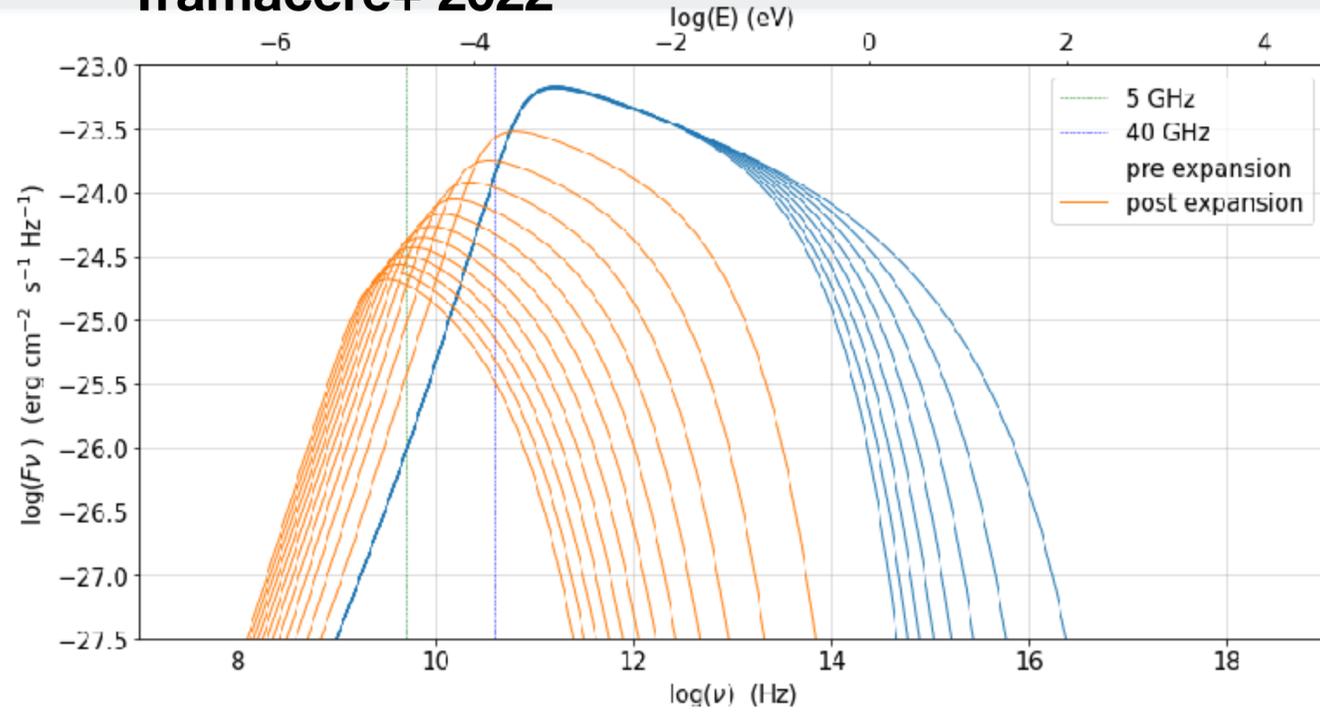
invert

$$R^* = R_0 \left( \frac{\nu_{\text{SSA}}^0}{\nu_{\text{SSA}}^*} \right)^\psi$$

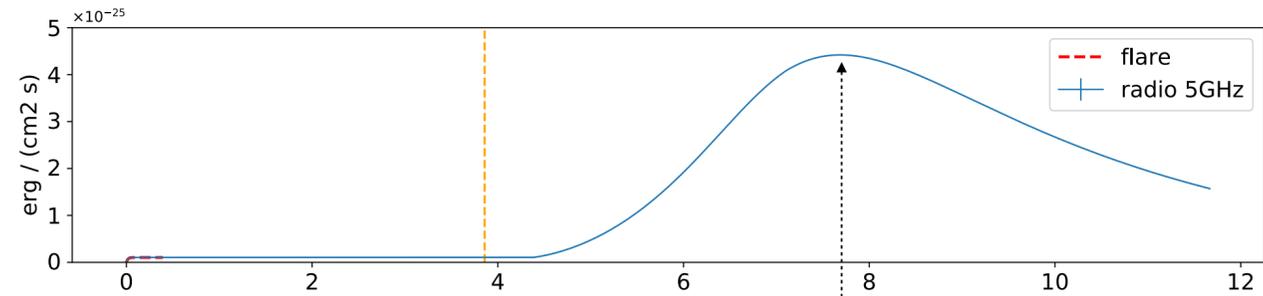
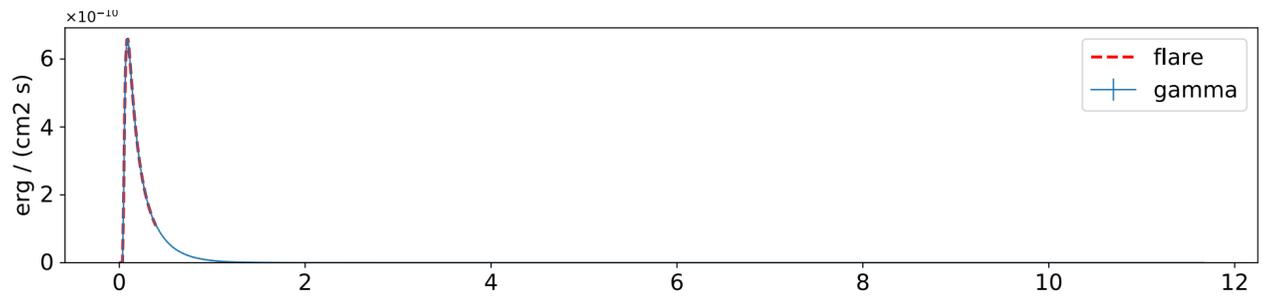
$$\psi = \frac{p+4}{m_B(p+2)-2}$$



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$$\Delta t_{\nu_{SSA}^0 \rightarrow \nu_{SSA}^*} = t_{\text{exp}} + t_{\text{peak}}$$

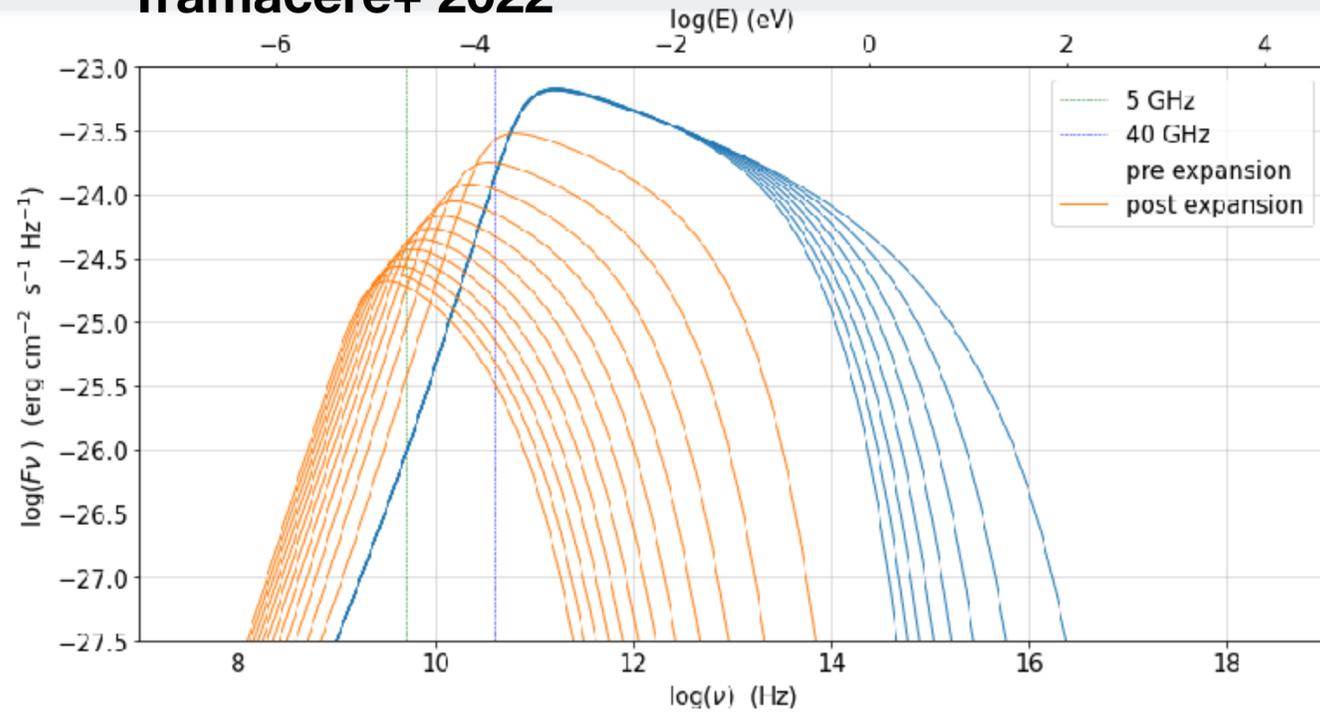


src transp  
at  $\nu_{SSA}^*$   
 $R=R^*$

$$R^* = R_0 \left( \frac{\nu_{SSA}^0}{\nu_{SSA}^*} \right)^\psi$$

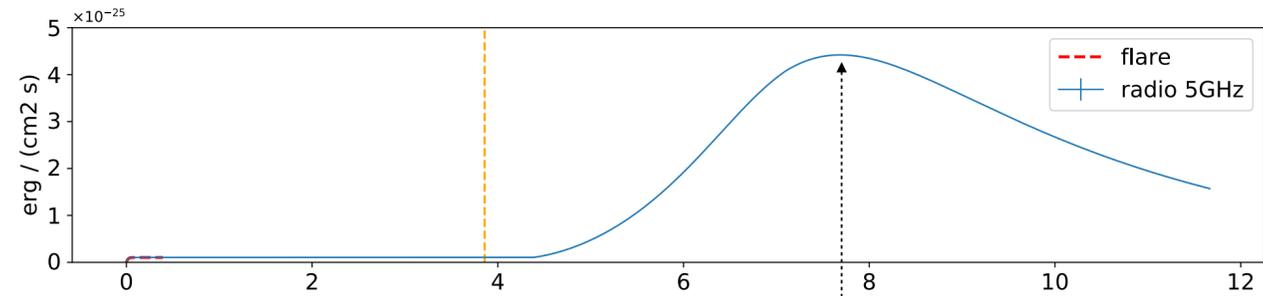
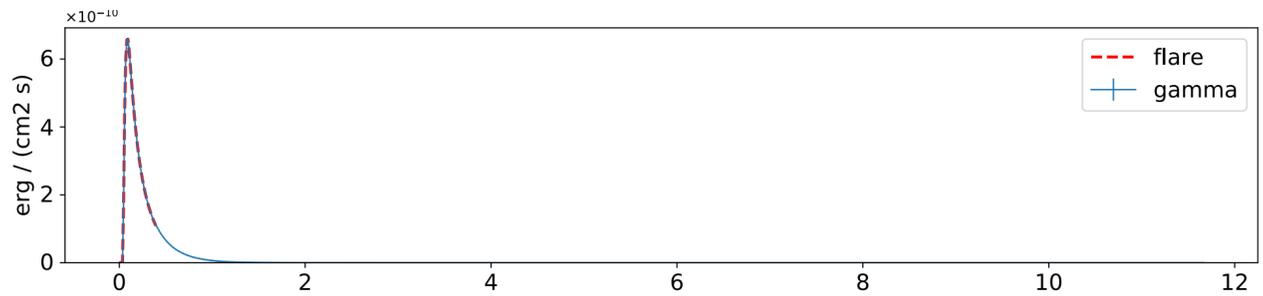
$$\psi = \frac{p + 4}{m_B(p + 2) - 2}$$

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$$\Delta t_{\nu_{SSA}^0 \rightarrow \nu_{SSA}^*} = t_{\text{exp}} + t_{\text{peak}}$$

$$t_{\text{peak}} = \Delta t_{R_0 \rightarrow R^*} = \frac{R^* - R_0}{\beta_{\text{exp}} c} = \frac{R_0}{\beta_{\text{exp}} c} \left[ \left( \frac{\nu_{SSA}^0}{\nu_{SSA}^*} \right)^\psi - 1 \right]$$

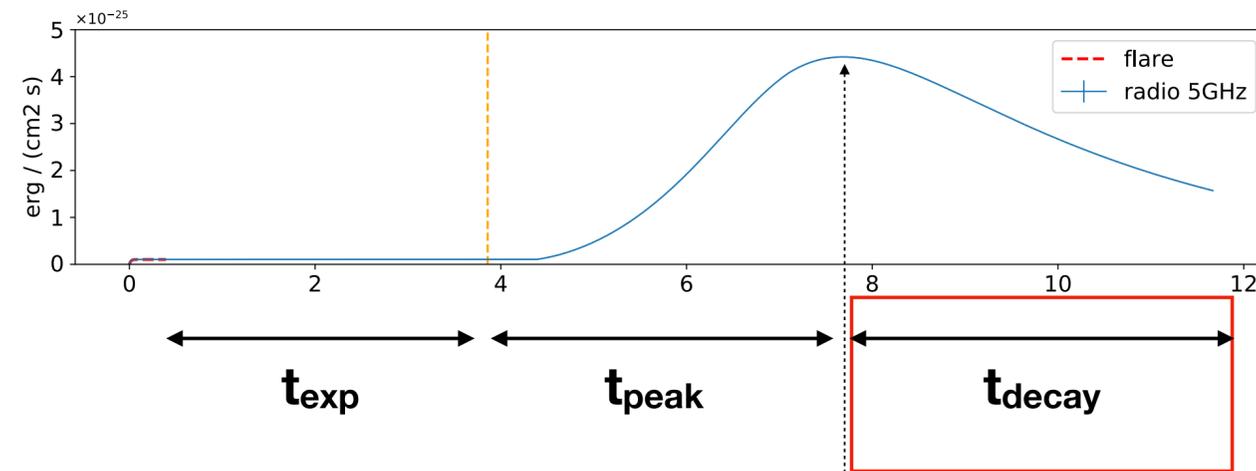
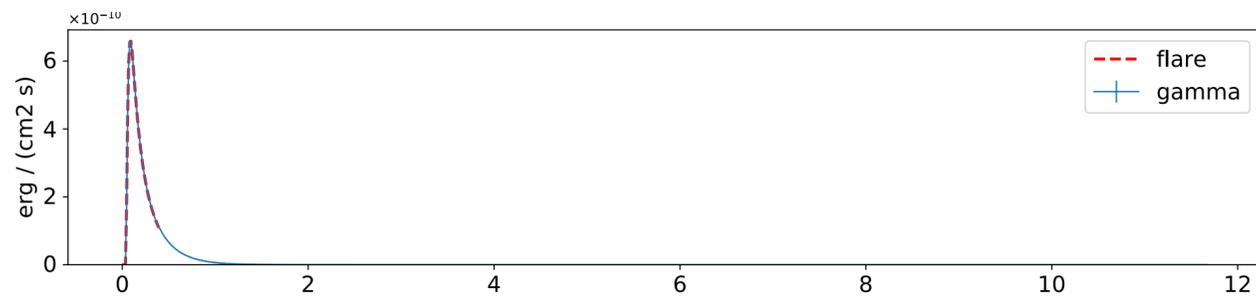
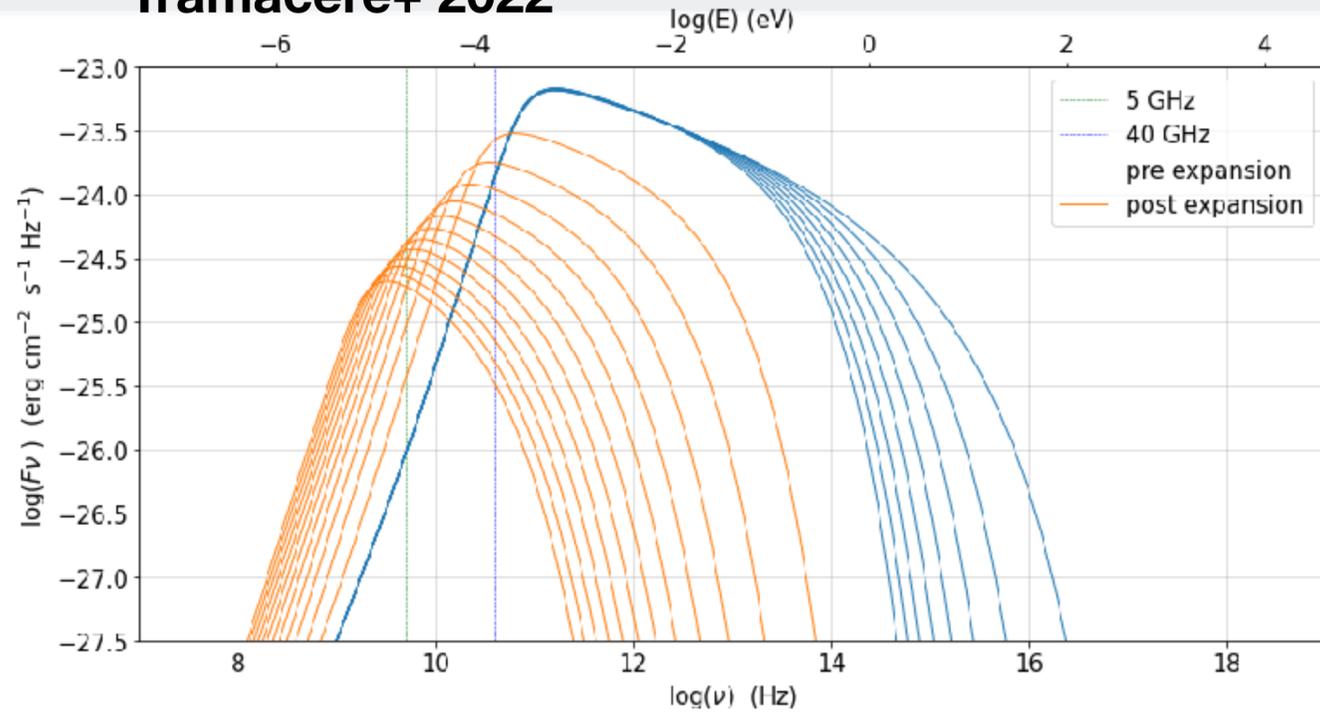


src transp  
at  $\nu_{SSA}^*$   
 $R=R^*$

$$R^* = R_0 \left( \frac{\nu_{SSA}^0}{\nu_{SSA}^*} \right)^\psi$$

$$\psi = \frac{p + 4}{m_B(p + 2) - 2}$$

Tramacere+ 2022



src transp  
at  $\nu_{SSA}^*$   
 $R=R^*$

$$\Delta t_{\nu_{SSA}^0 \rightarrow \nu_{SSA}^*} = t_{exp} + t_{peak}$$

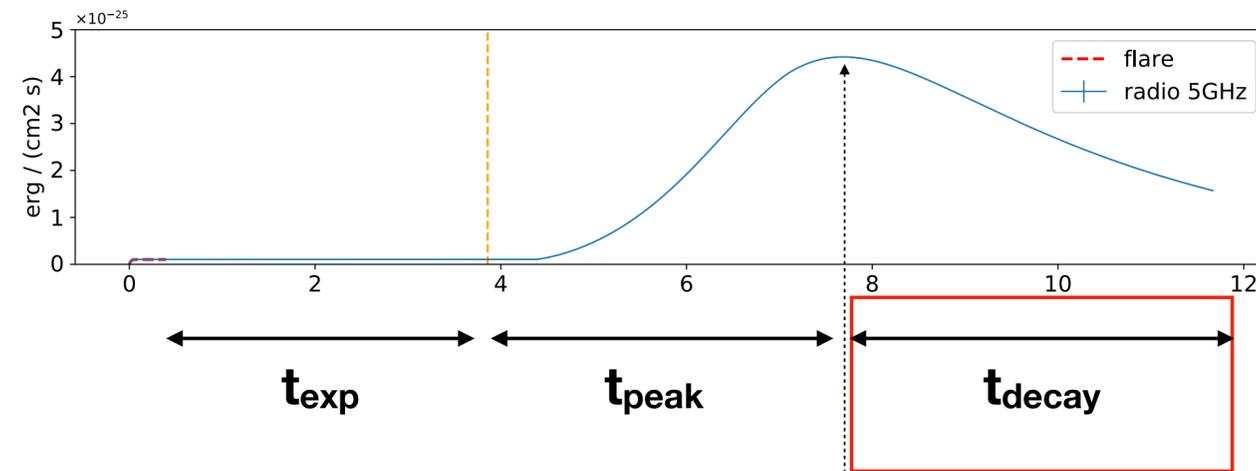
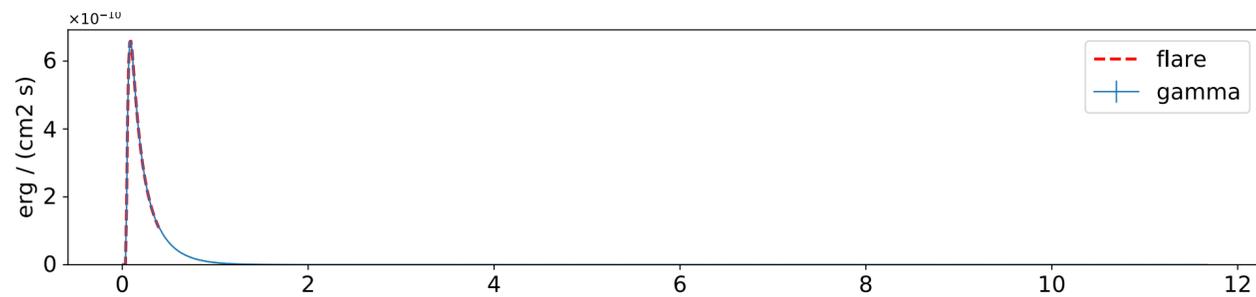
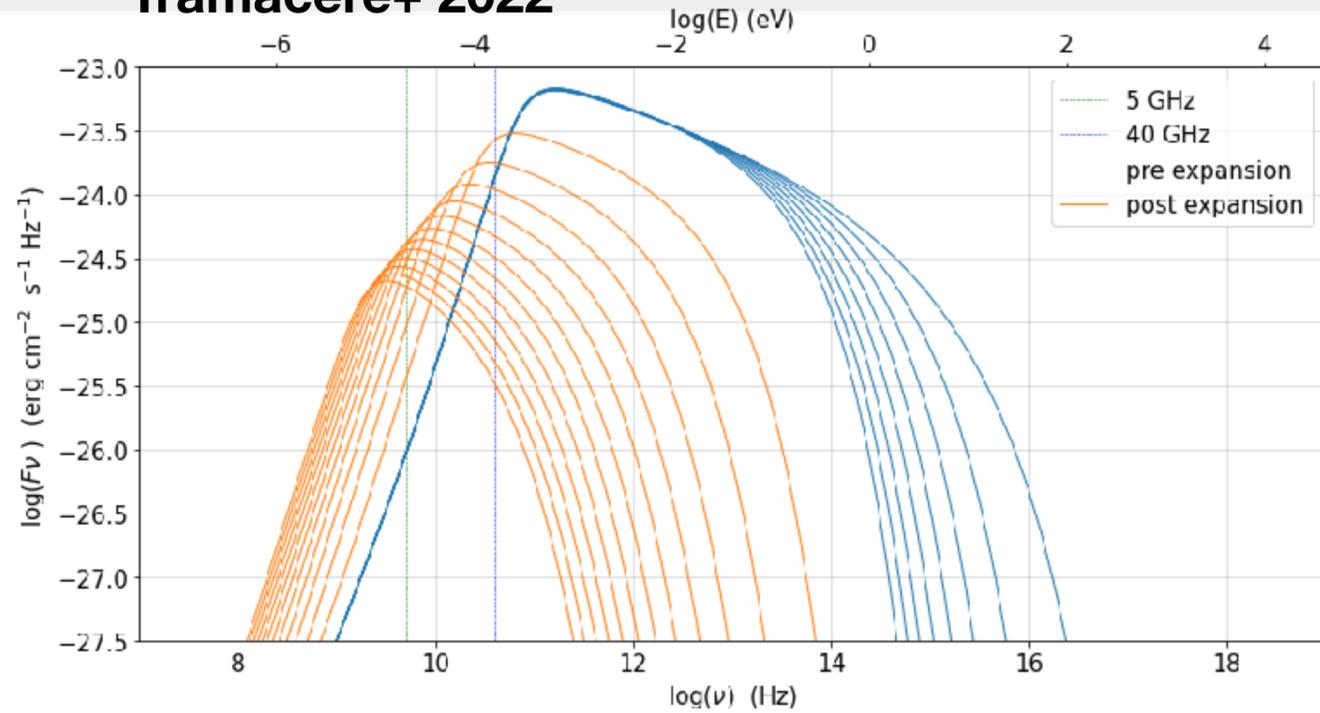
$$t_{peak} = \Delta t_{R_0 \rightarrow R^*} = \frac{R^* - R_0}{\beta_{exp} c} = \frac{R_0}{\beta_{exp} c} \left[ \left( \frac{\nu_{SSA}^0}{\nu_{SSA}^*} \right)^\psi - 1 \right]$$

$$t_{decay} \rightarrow t_{decay}^{ad}(t^*) \propto \frac{R^*}{\beta_{exp} c} = \frac{R_0}{\beta_{exp} c} \left( \frac{\nu_{SSA}^0}{\nu_{SSA}^*} \right)^\psi$$

$$R^* = R_0 \left( \frac{\nu_{SSA}^0}{\nu_{SSA}^*} \right)^\psi$$

$$\psi = \frac{p + 4}{m_B(p + 2) - 2}$$

Tramacere+ 2022



src transp  
at  $\nu^*_{\text{SSA}}$   
 $R=R^*$

$$\Delta t_{\nu_{\text{SSA}}^0 \rightarrow \nu_{\text{SSA}}^*} = t_{\text{exp}} + t_{\text{peak}}$$

$$t_{\text{peak}} = \Delta t_{R_0 \rightarrow R^*} = \frac{R^* - R_0}{\beta_{\text{exp}} c} = \frac{R_0}{\beta_{\text{exp}} c} \left[ \left( \frac{\nu_{\text{SSA}}^0}{\nu_{\text{SSA}}^*} \right)^\psi - 1 \right]$$

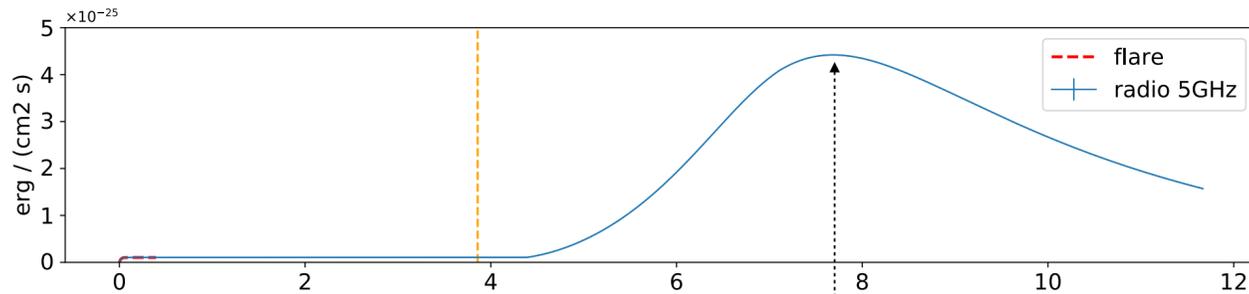
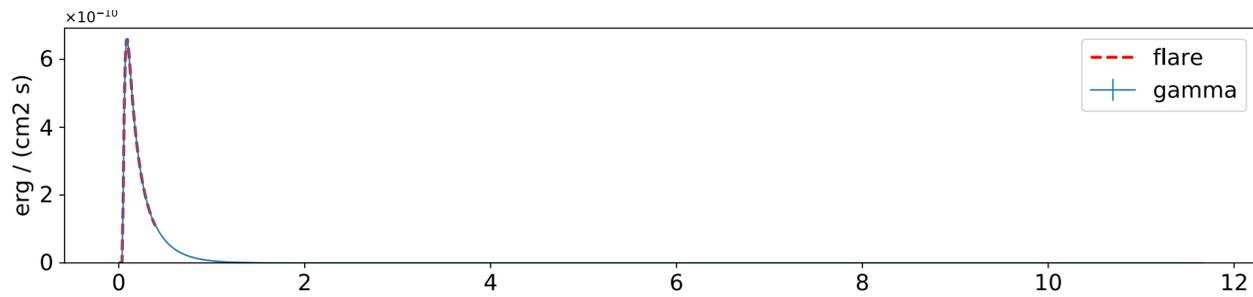
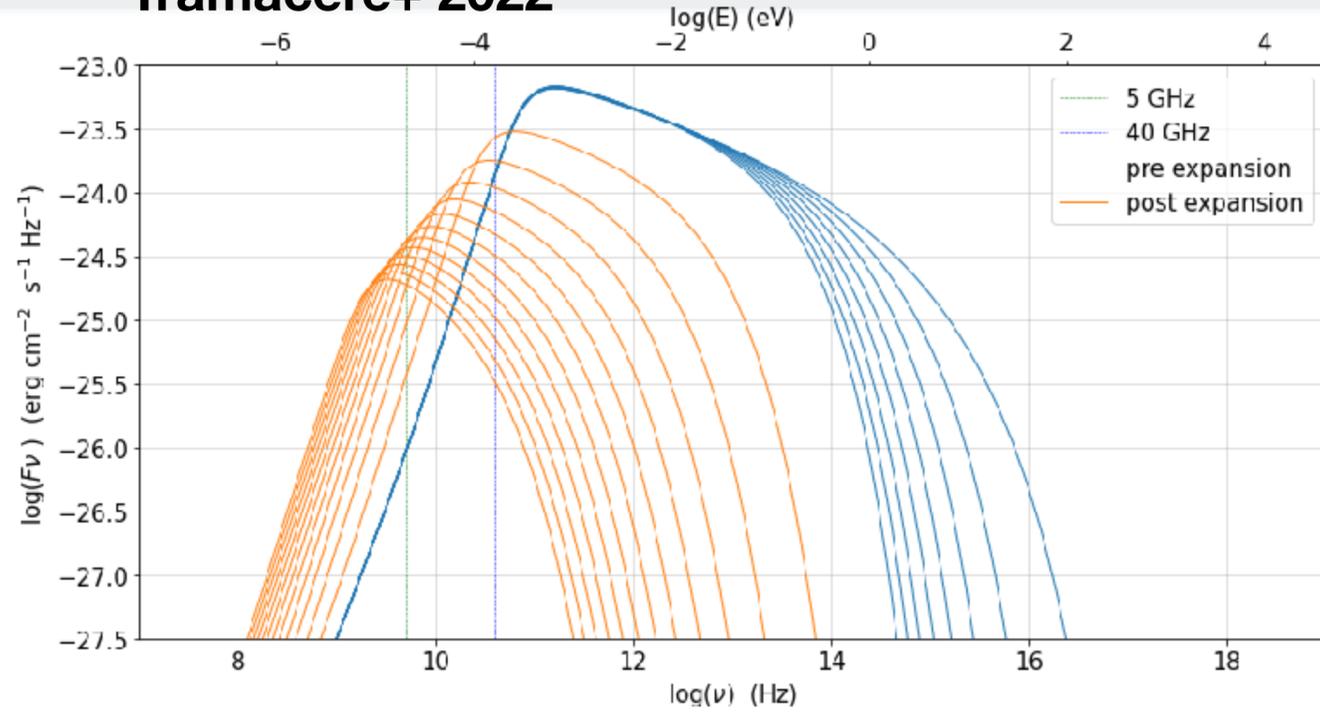
$t_{\text{decay}}$  branches into:

- radiative**:  $t_{\text{decay}}^{\text{ad}}(t^*) \propto \frac{R^*}{\beta_{\text{exp}} c} = \frac{R_0}{\beta_{\text{exp}} c} \left( \frac{\nu_{\text{SSA}}^0}{\nu_{\text{SSA}}^*} \right)^\psi$
- geom**:  $t_{\text{decay}}^{\text{geom}}(t^*) \propto \frac{F_{\nu_{\text{SSA}}}(t^*)}{\dot{F}_{\nu_{\text{SSA}}}(t^*)} \propto \frac{R^*}{m_B \beta_{\text{exp}} c} = \frac{t_{\text{decay}}^{\text{ad}}(t^*)}{m_B}$

$$R^* = R_0 \left( \frac{\nu_{\text{SSA}}^0}{\nu_{\text{SSA}}^*} \right)^\psi$$

$$\psi = \frac{p + 4}{m_B(p + 2) - 2}$$

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src transp  
at  $\nu_{\text{SSA}}^*$   
 $R=R^*$

Tramacere+ 2022

$$\Delta t_{\nu_{\text{SSA}}^0 \rightarrow \nu_{\text{SSA}}^*} = t_{\text{exp}} + t_{\text{peak}} = t_{\text{exp}}^{\text{obs}} + \frac{t_{\text{var}}^{\text{obs}}}{\beta_{\text{exp}}} \left[ \left( \frac{\nu_{\text{SSA}}^0}{\nu_{\text{SSA}}^*} \right)^\psi - 1 \right]$$

$$t_{\text{peak}}^{\text{obs}} = \frac{t_{\text{var}}^{\text{obs}}}{\beta_{\text{exp}}} \left[ \left( \frac{\nu_{\text{SSA}}^0}{\nu_{\text{SSA}}^*} \right)^\psi - 1 \right]$$

$$t_{\text{decay}}^{\text{obs}} = \frac{t_{\text{var}}^{\text{obs}}}{m_B \beta_{\text{exp}}} \left( \frac{\nu_{\text{SSA}}^0}{\nu_{\text{SSA}}^*} \right)^\psi,$$

$$\delta = \frac{1}{\Gamma(1 - \beta_\Gamma \cos(\theta))}$$

$$\nu^{\text{obs}} = \nu \frac{\delta}{z+1}$$

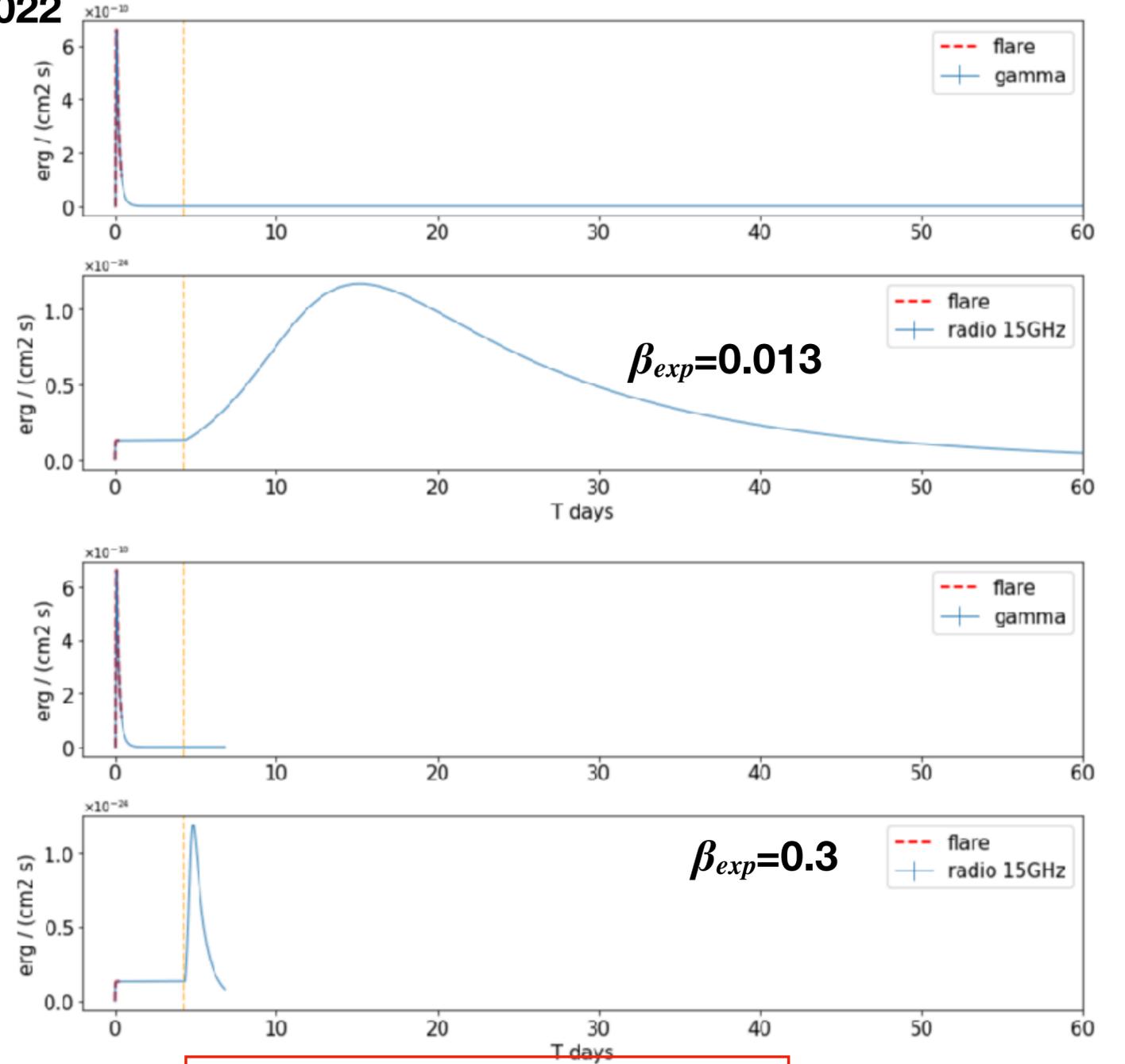
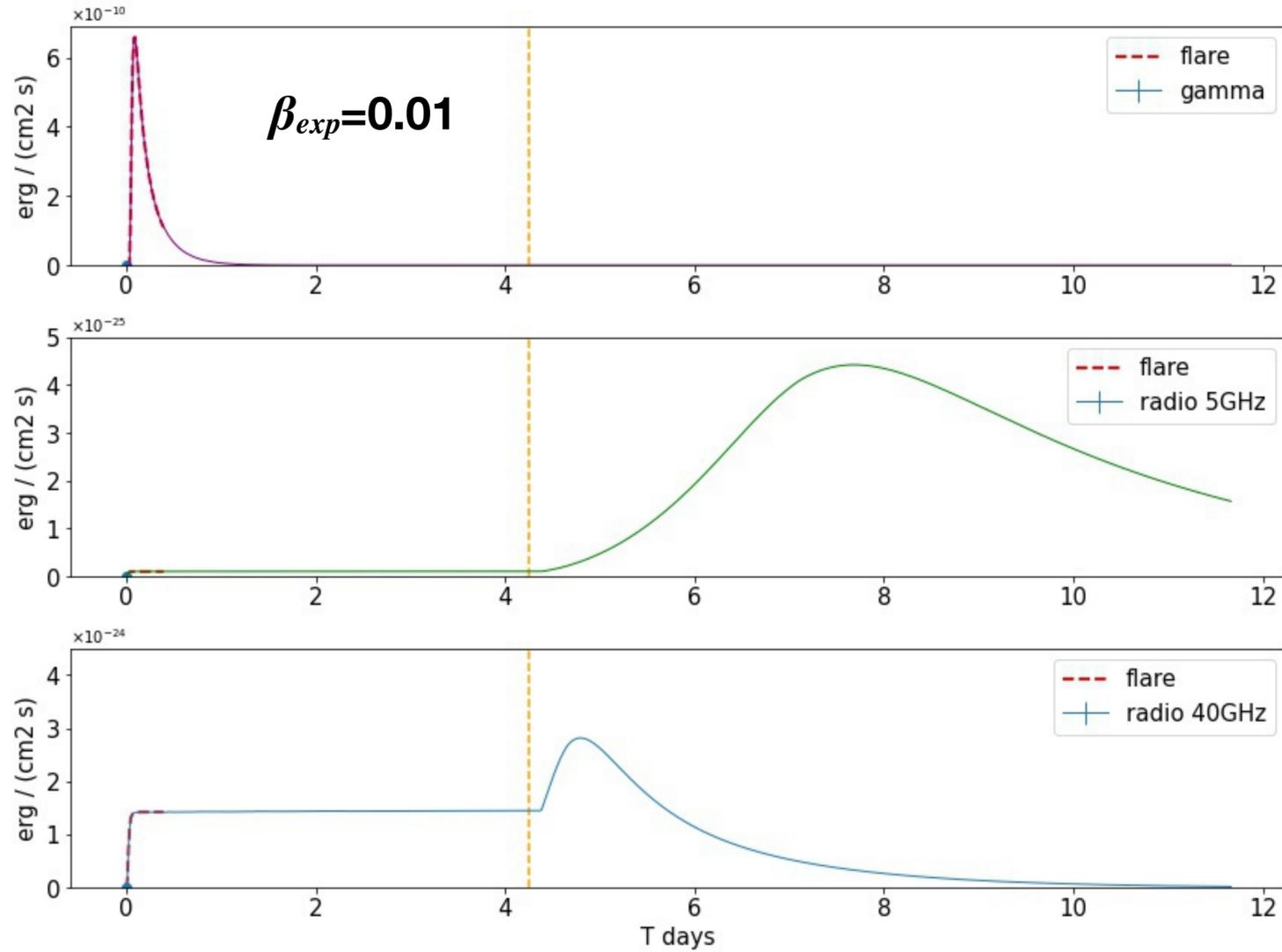
$$t_{\text{var}}^{\text{obs}} = \frac{(1+z)R_0}{\delta c}$$

$$R_{\text{obs}}^0 = R_0 \frac{1+z}{\delta}$$

$$R^* = R_0 \left( \frac{\nu_{\text{SSA}}^0}{\nu_{\text{SSA}}^*} \right)^\psi$$

$$\psi = \frac{p+4}{m_B(p+2) - 2}$$

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$$t_r \sim \nu^0_{SSA} / \nu^*_{SSA}$$

$$t_d \sim \nu^0_{SSA} / \nu^*_{SSA}$$

$$\Delta t \sim t_{exp} + t_r$$

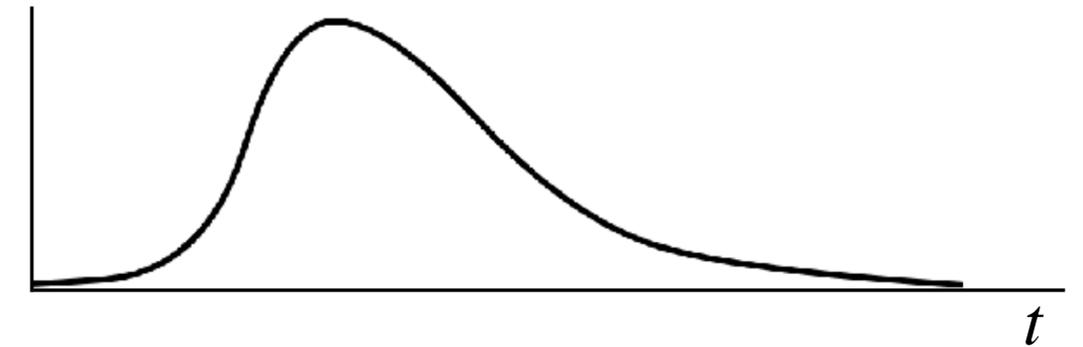
$$t_r \sim 1 / \beta_{exp}$$

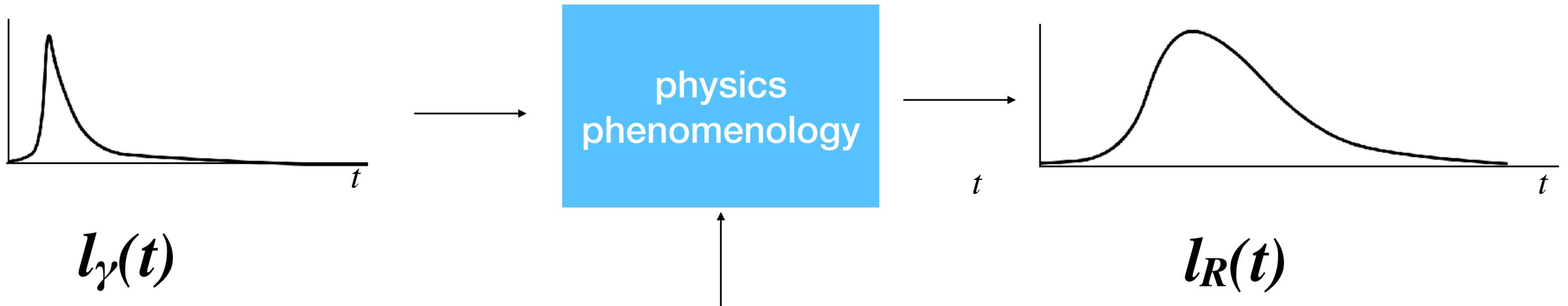
$$t_d \sim 1 / \beta_{exp}$$

$$\Delta t \sim t_{exp} + t_r$$

 $l_\gamma(t)$ 

physics  
phenomenology

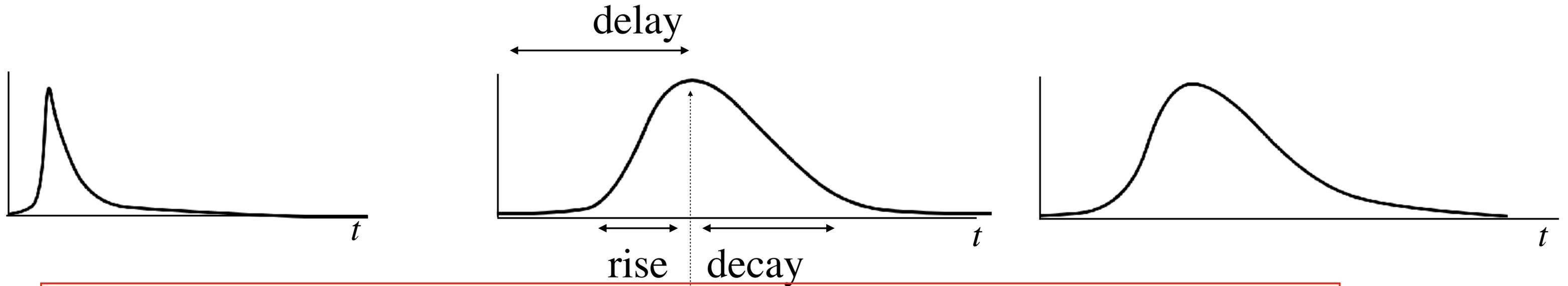
 $l_R(t)$



$$t_{\text{decay}}^{\text{obs}} = \frac{R_0^{\text{obs}}}{m_B \beta_{\text{exp}} c} \left( \frac{\nu_{\text{SSA}}^{0,\text{obs}}}{\nu_{\text{SSA}}^{*,\text{obs}}} \right)^\phi$$

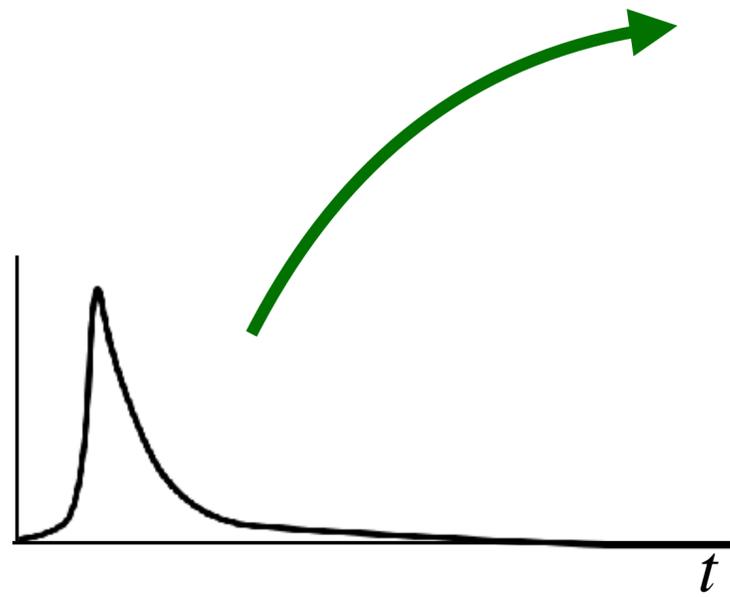
$$t_{\text{rise}}^{\text{obs}} = \frac{1}{2} t_{\text{peak}}^{\text{obs}} = \begin{cases} \frac{1}{2} \frac{R_0^{\text{obs}}}{\beta_{\text{exp}} c} \left[ \left( \frac{\nu_{\text{SSA}}^{0,\text{obs}}}{\nu_{\text{SSA}}^{*,\text{obs}}} \right)^\phi - 1 \right] & \text{if } \nu_{\text{SSA}}^{0,\text{obs}} > \nu_{\text{SSA}}^{*,\text{obs}} \\ 0 & \text{otherwise} \end{cases}$$

$$\Delta t^{\text{obs}} = t_{\text{exp}}^{\text{obs}} + t_{\text{peak}}^{\text{obs}} = t_{\text{exp}}^{\text{obs}} + \frac{R_0^{\text{obs}}}{\beta_{\text{exp}} c} \left[ \left( \frac{\nu_{\text{SSA}}^{0,\text{obs}}}{\nu_{\text{SSA}}^{*,\text{obs}}} \right)^\phi - 1 \right].$$

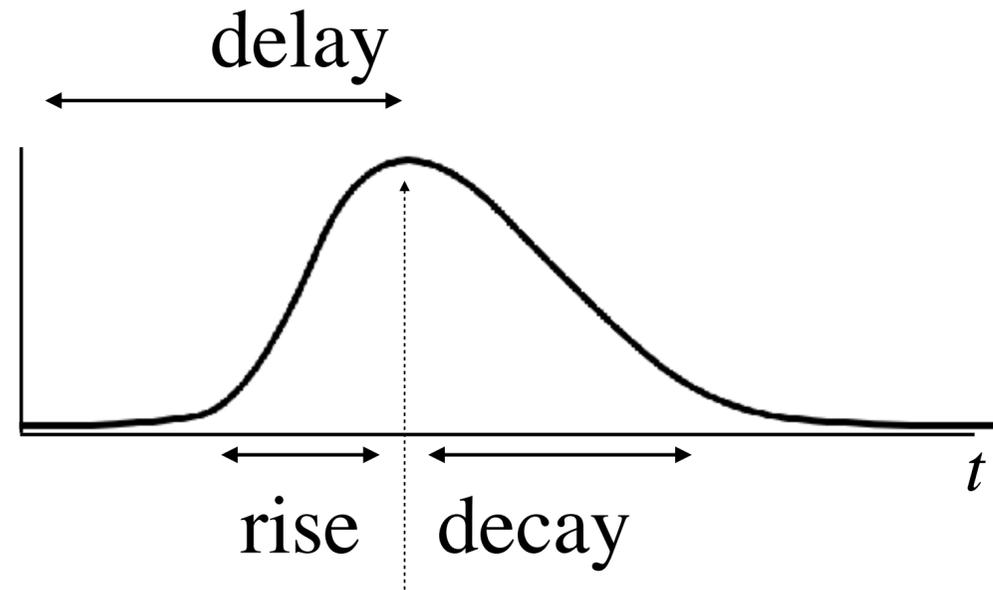


$$l_{\gamma}(t) * S(t, t_{\text{rise}}, t_{\text{decay}}, t_{\text{delay}}) = l_R(t)$$

optimise  $S$  to match  $l_R$



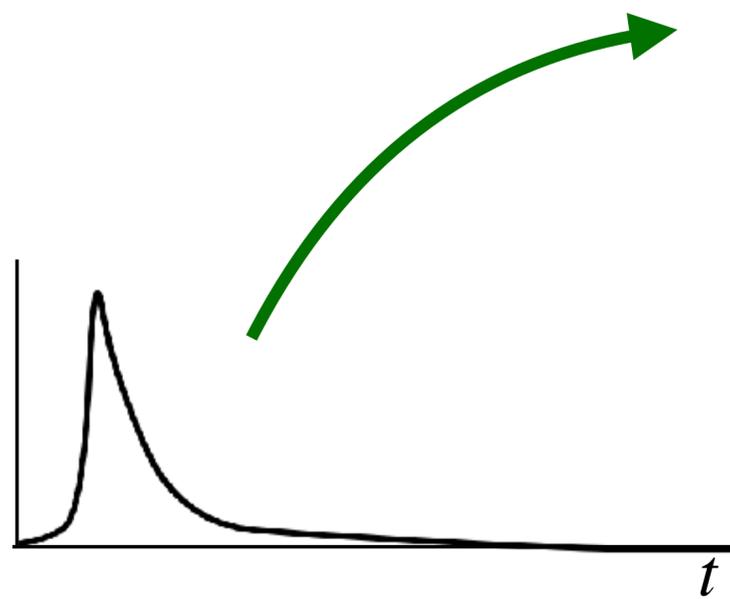
$l_\gamma(t)$



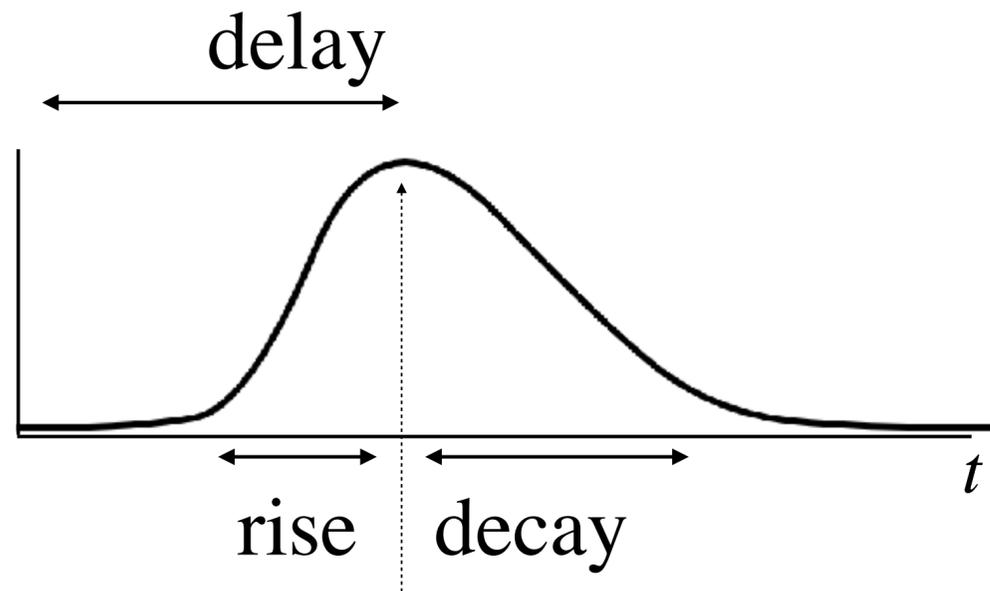
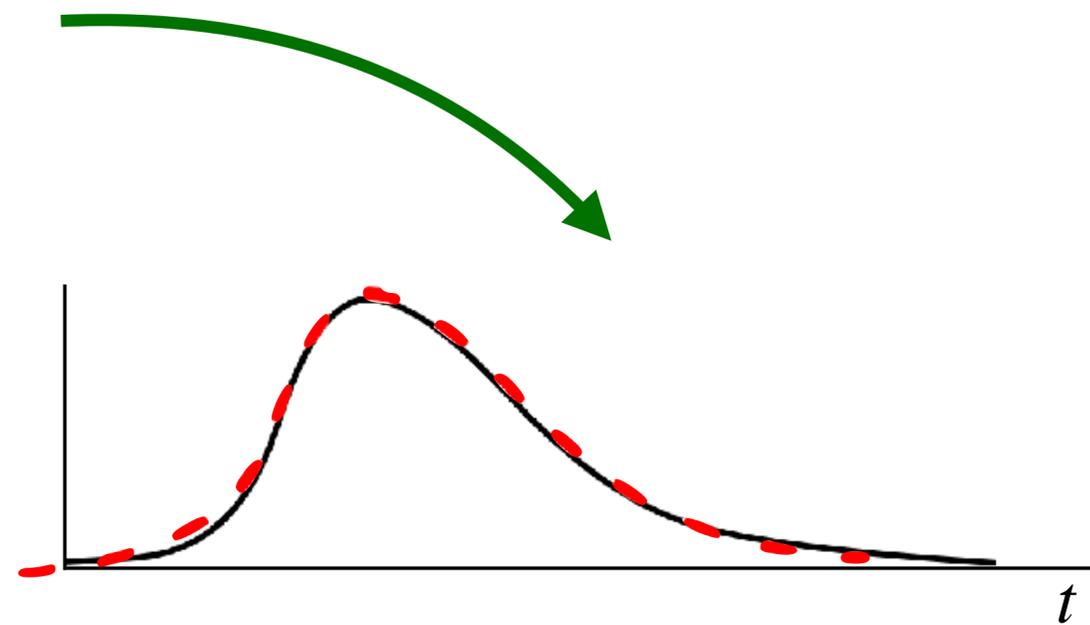
$$* S(t, t_{\text{rise}}, t_{\text{decay}}, t_{\text{delay}}) = l_R(t)$$



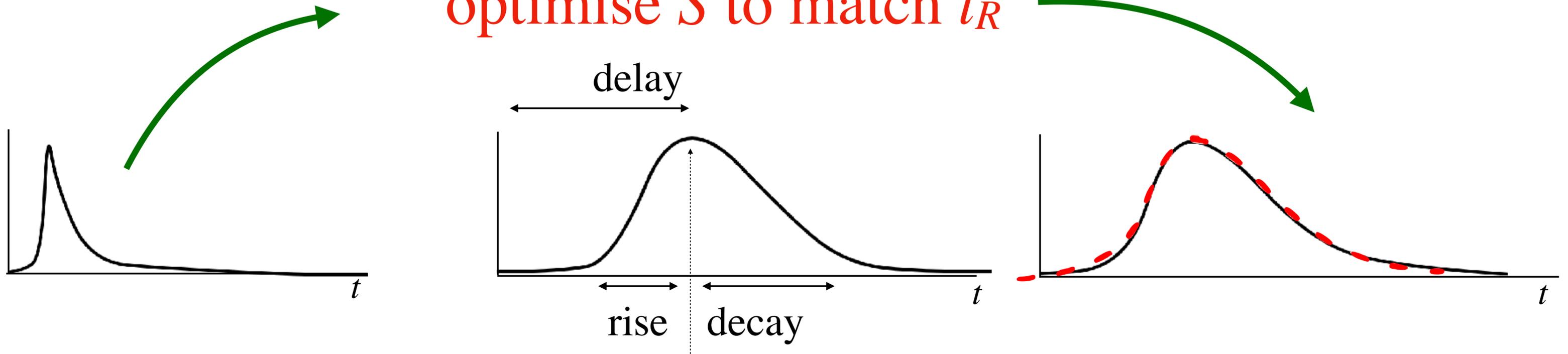
$l_R(t)$


 $l_{\gamma}(t)$ 

optimise  $S$  to match  $l_R$


 $*$ 
 $S(t, t_{\text{rise}}, t_{\text{decay}}, t_{\text{delay}})$ 
 $=$ 
 $l_R(t)$ 


optimise  $S$  to match  $l_R$



$l_\gamma(t)$

$$* S(t, t_{rise}, t_{decay}, t_{delay}) = l_R(t)$$

phenomenology-physics

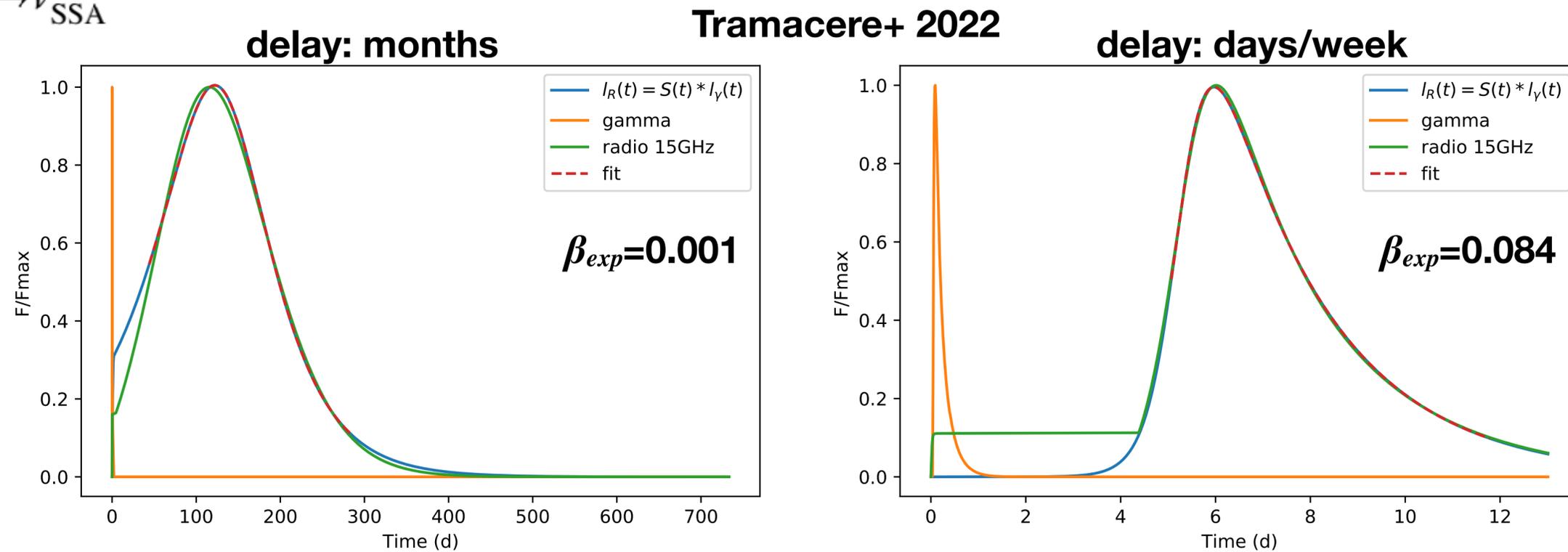
$t_{[rise,decay,delay]} (m_B, \beta_{exp}, B, R, v_{SSA})$

exp-logistic

$$S(t) = A \frac{\exp^{-(t-\Delta_t)/t_{\text{decay}}^*}}{1 + \exp^{-(t-\Delta_t)/t_{\text{rise}}}}$$

$t_{\text{decay}}^{\text{obs}}$   
 $t_{\text{rise}}^{\text{obs}}$   
 $\Delta t^{\text{obs}}$   
 $\nu_{\text{SSA}}^{0,\text{obs}} \rightarrow \nu_{\text{SSA}}^{*,\text{obs}}$

The proposal is a simplified and physically motivated (within the context of expansion) version of Turler+ 1999, Sliusar+ 2019



Best fit for the radio- $\gamma$  response, at 15 GHz, for  $\beta_{\text{exp}} = 0.001$  (left panel), and at 15 GHz for  $\beta_{\text{exp}} = 0.084$  (right panel), and  $t_{\text{exp}} = 1 \times 10^7$  s. The lightcurves are in the observer frame. The red dashed line represents the actual fit interval, the orange line the simulate  $\gamma$  - ray lightcurve, the green one the simulated radio lightcurve, and the blue one is the best-fit of the radio lightcurve obtained from the convolution of the  $\gamma$  - ray lightcurve with the best fit response

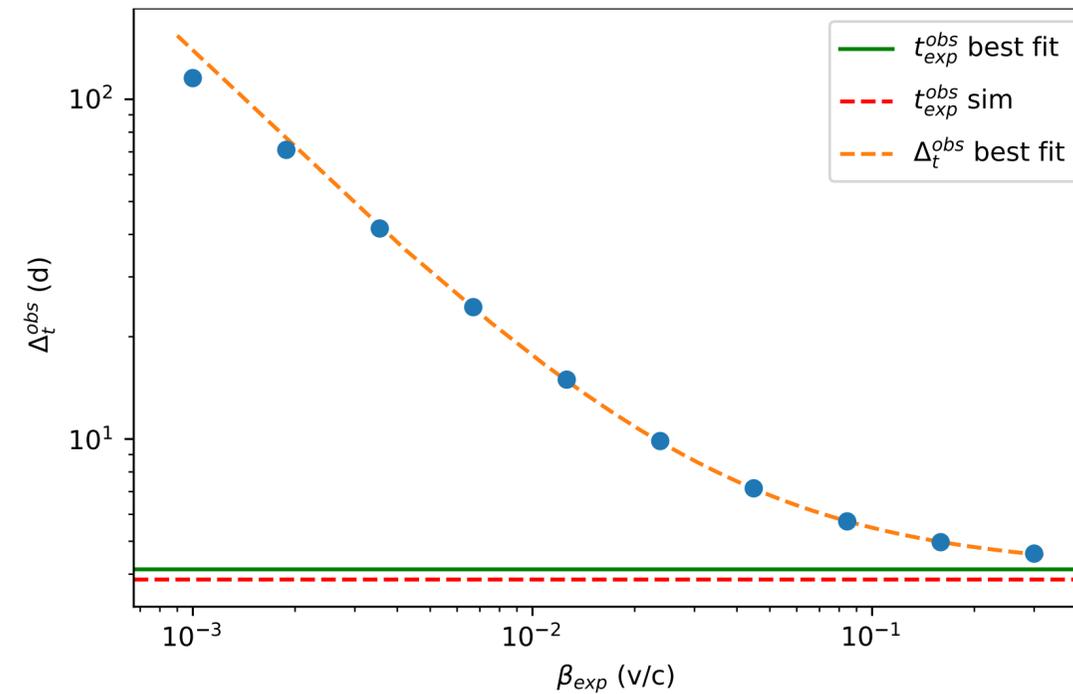
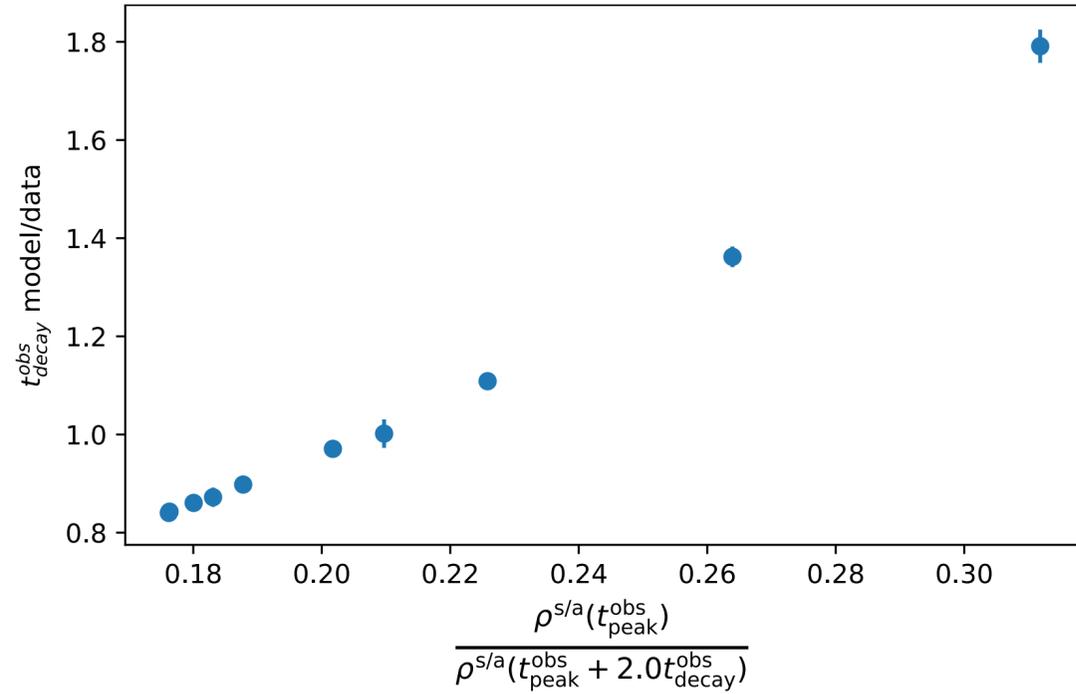
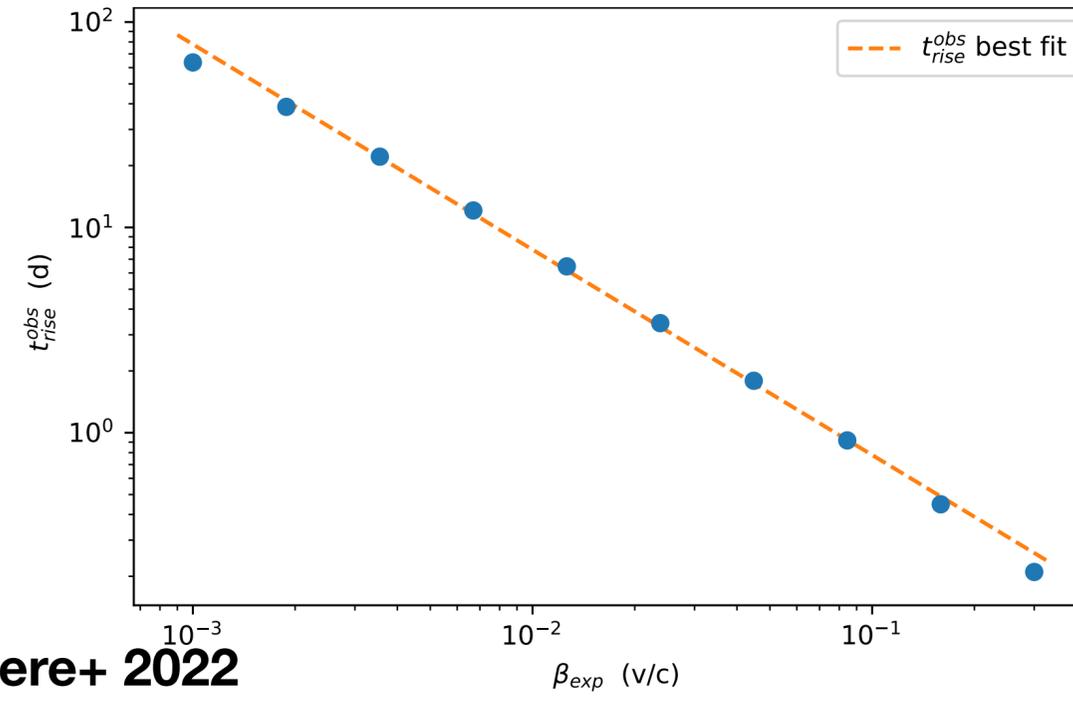
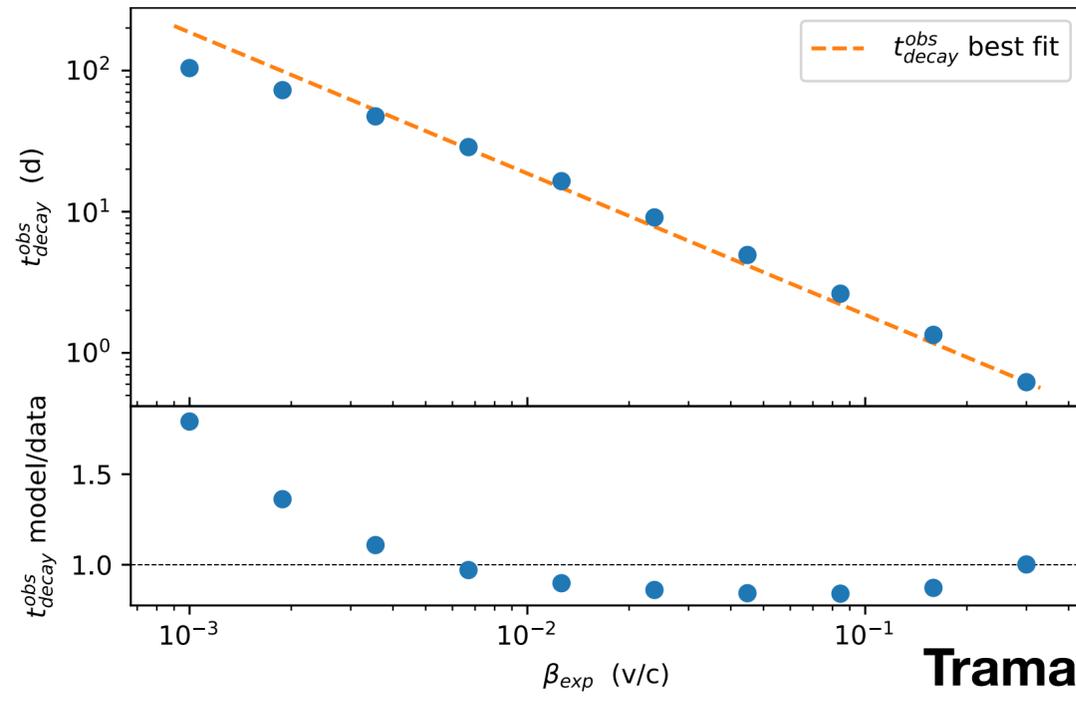
# trends vs $\beta_{exp}$

$\beta_{exp}=[0.001-0.3]$

$$t_{decay}^{obs} = \frac{R_0^{obs}}{m_B \beta_{exp} c} \left( \frac{v_{SSA}^{0,obs}}{v_{SSA}^{*,obs}} \right)^\phi$$

$$t_{rise}^{obs} = \frac{1}{2} t_{peak}^{obs} = \begin{cases} \frac{1}{2} \frac{R_0^{obs}}{\beta_{exp} c} \left[ \left( \frac{v_{SSA}^{0,obs}}{v_{SSA}^{*,obs}} \right)^\phi - 1 \right] & \text{if } v_{SSA}^{0,obs} > v_{SSA}^{*,obs} \\ 0 & \text{otherwise} \end{cases}$$

$$\Delta t^{obs} = t_{exp}^{obs} + t_{peak}^{obs} = t_{exp}^{obs} + \frac{R_0^{obs}}{\beta_{exp} c} \left[ \left( \frac{v_{SSA}^{0,obs}}{v_{SSA}^{*,obs}} \right)^\phi - 1 \right].$$



		actual values		values from $\beta$ trend best fit		
		blob	obs	$t_{rise}^{obs}$	$t_{decay}^{obs}$	$\Delta t^{obs}$
$R_0$	cm	$5 \times 10^{15}$	$1.66 \times 10^{14}$	$(1.9 \pm 0.5) \times 10^{14}$	$(1.7 \pm 0.1) \times 10^{14}$	$(1.8 \pm 0.1) \times 10^{14}$
$v_{SSA}^0$	GHz	3	90	$110 \pm 40$	$100 \pm 10$	$100 \pm 5$
$t_{exp}$	s	$1 \times 10^7$	$3.3 \times 10^5$			$(3.57 \pm 0.01) \times 10^5$
$m_B$		1			$0.96 \pm 0.06$	
$\phi$				$0.6 \pm 0.1$	$0.52 \pm 0.04$	$0.54 \pm 0.02$
$p$		1.46		$1.6 \pm 0.3$	$1.5 \pm 0.01$	$1.57 \pm 0.05$

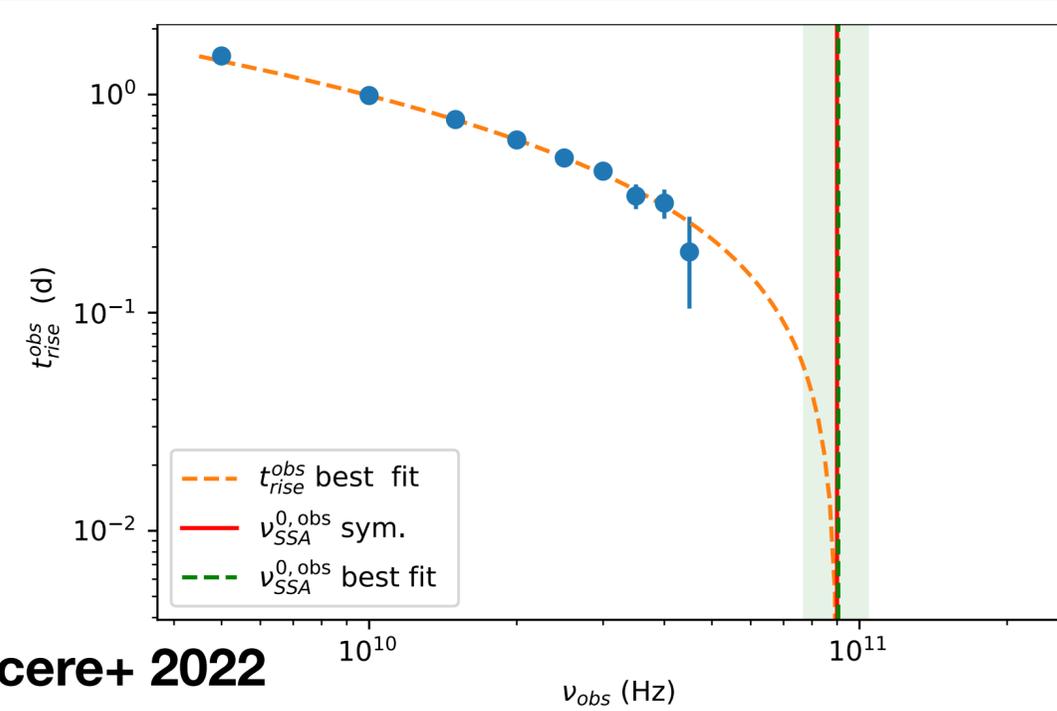
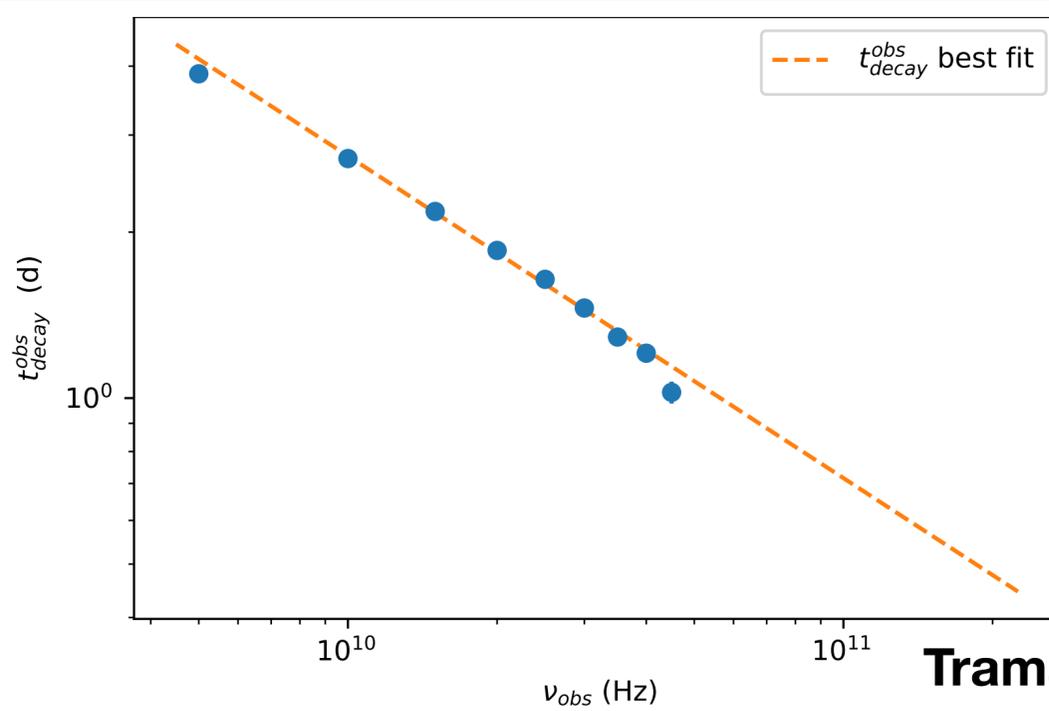
# trends vs $\nu_{obs}$

$$\beta_{exp} = [0.1]$$

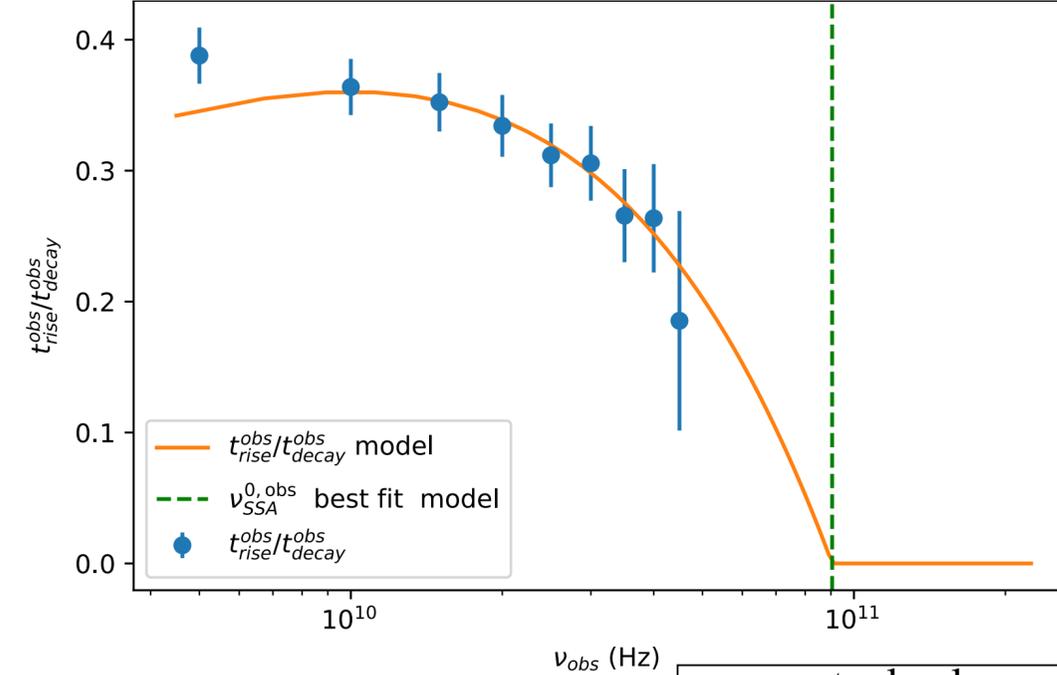
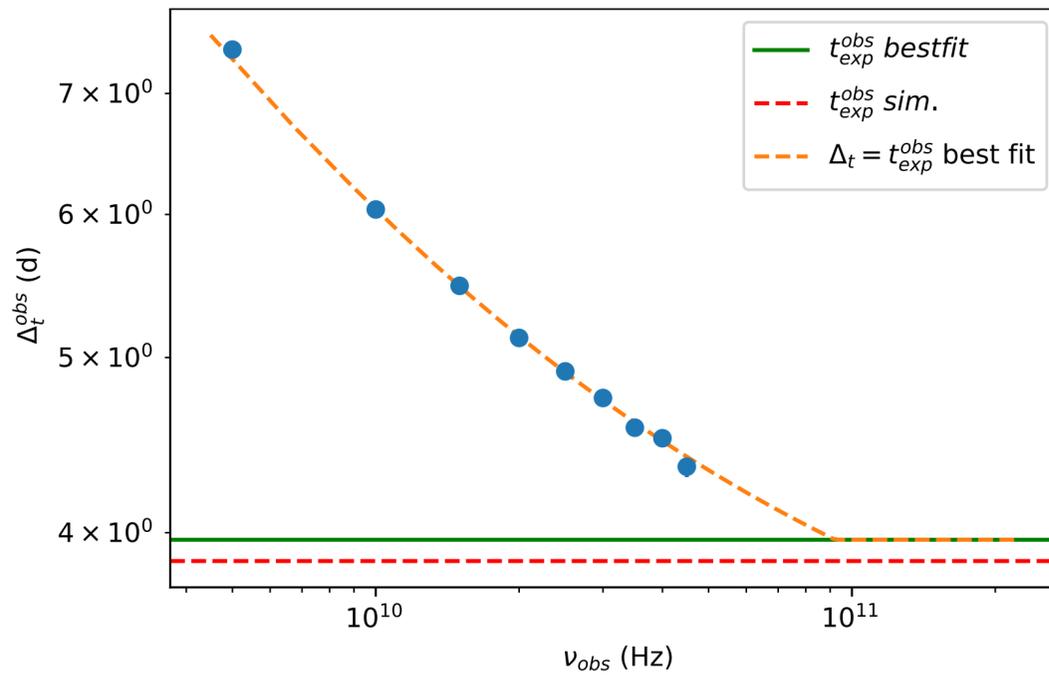
$$t_{decay}^{obs} = \frac{R_0^{obs}}{m_B \beta_{exp} c} \left( \frac{\nu_{SSA}^{0,obs}}{\nu_{SSA}^{*,obs}} \right)^\phi$$

$$t_{rise}^{obs} = \frac{1}{2} t_{peak}^{obs} = \begin{cases} \frac{1}{2} \frac{R_0^{obs}}{\beta_{exp} c} \left[ \left( \frac{\nu_{SSA}^{0,obs}}{\nu_{SSA}^{*,obs}} \right)^\phi - 1 \right] & \text{if } \nu_{SSA}^{0,obs} > \nu_{SSA}^{*,obs} \\ 0 & \text{otherwise} \end{cases}$$

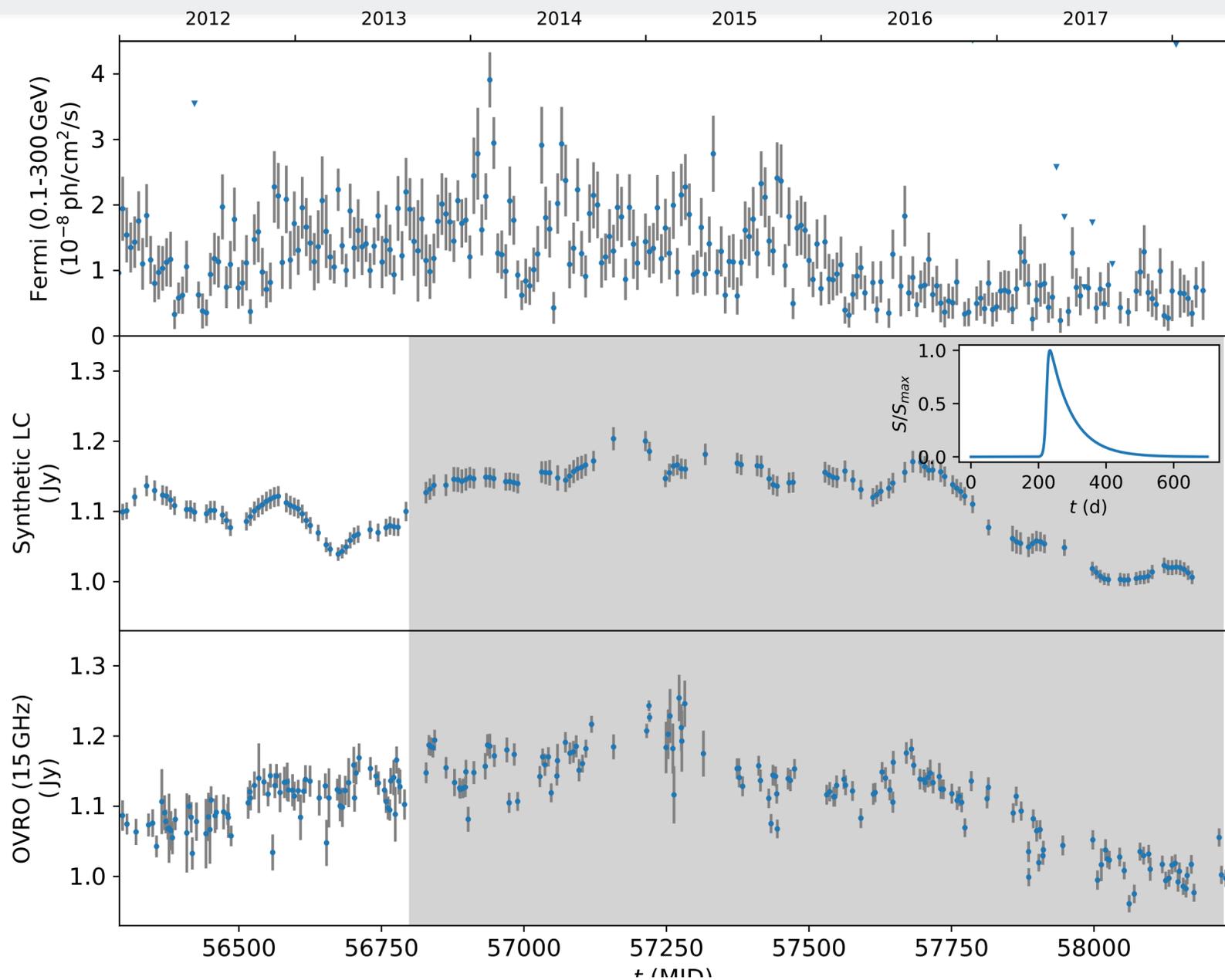
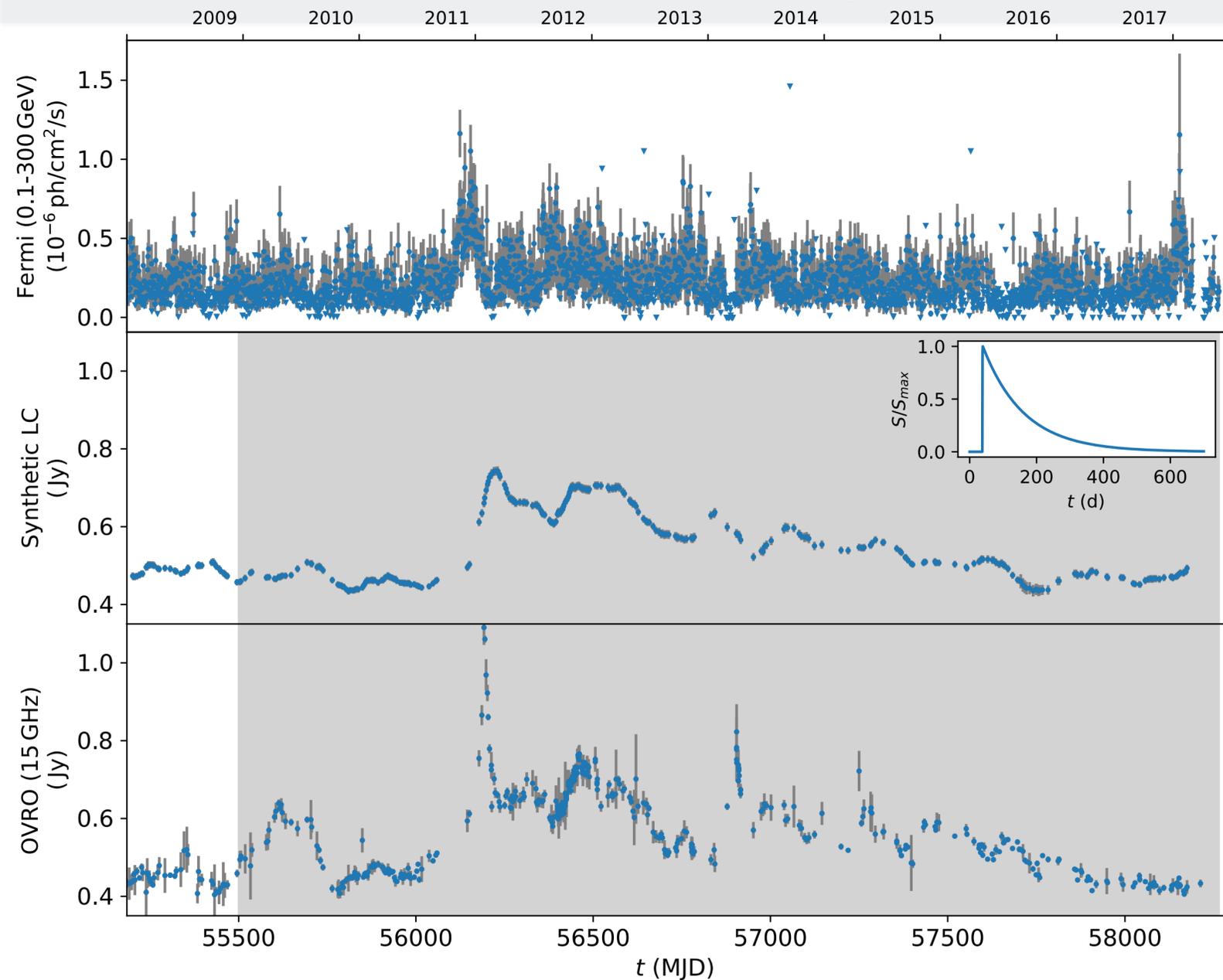
$$\Delta t^{obs} = t_{exp}^{obs} + t_{peak}^{obs} = t_{exp}^{obs} + \frac{R_0^{obs}}{\beta_{exp} c} \left[ \left( \frac{\nu_{SSA}^{0,obs}}{\nu_{SSA}^{*,obs}} \right)^\phi - 1 \right].$$



Tramacere+ 2022



		actual values		values from $\nu$ trend best fit		
		blob	obs	$t_{rise}^{obs}$	$t_{decay}^{obs}$	$\Delta t^{obs}$
$R_0$	cm	$5 \times 10^{15}$	$1.66 \times 10^{14}$	$(2.4 \pm 1.0) \times 10^{14}$	$(1.7 \pm 0.2) \times 10^{14}$	$(1.6 \pm 0.1) \times 10^{14}$
$\nu_{SSA}^0$	GHz	3	90	$90 \pm 10$	$100 \pm 20$	$90 \pm 10$
$t_{exp}$	s	$1 \times 10^7$	$3.3 \times 10^5$			$(3.4 \pm 0.1) \times 10^5$
$m_B$		1			$1.0 \pm 0.1$	
$\beta_{exp}$	c	0.1		$0.03 \pm 0.01$	$0.09 \pm 0.01$	$0.06 \pm 0.01$
$\phi$				$0.24 \pm 0.07$	$0.58 \pm 0.02$	$0.50 \pm 0.02$
$p$		1.46		$0.6 \pm 0.2$	$1.7 \pm 0.1$	$1.4 \pm 0.1$



## Tramacere+ 2022

Parameter	Value
$A$	$12.5^{+0.5}_{-0.013} \times 10^3 \text{ Jy cm}^2 \text{ s/ph}$
$t_{\text{rise}}$	$\lesssim 1 \text{ day}$
$t_{\text{decay}}$	$126.5^{+1.3}_{-1.3} \text{ days}$
$\Delta t$	$37.58^{+0.13}_{-0.13} \text{ days}$
$F_{\text{background}}$	$0.18^{+0.008}_{-0.0004} \text{ Jy}$

$$S(t) = A \frac{\exp^{-(t-\Delta t)/t_{\text{decay}}^*}}{1 + \exp^{-(t-\Delta t)/t_{\text{rise}}}}$$

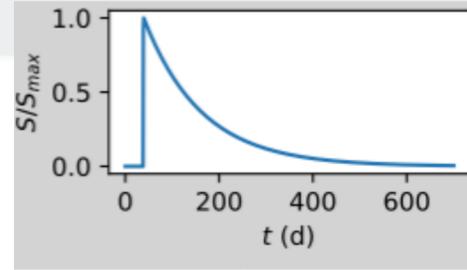
Parameter	Value
$A$	$166^{+5}_{-3} \times 10^4 \text{ Jy cm}^2 \text{ s/ph}$
$t_{\text{rise}}$	$12^{+4}_{-4} \text{ days}$
$t_{\text{decay}}$	$73^{+3.6}_{-3.6} \text{ days}$
$\Delta t$	$234^{+10}_{-10} \text{ days}$
$F_{\text{background}}$	$0.915^{+0.004}_{-0.004} \text{ Jy}$

$$\mathcal{L} = \mathcal{L}_{\text{rise}} + \mathcal{L}_{\text{decay}} + \mathcal{L}_{\text{delay}}$$

Tramacere+ 2022

Model

$$\mathcal{L} \propto \sum_{i=[1,2,3]} -\frac{1}{2} \frac{(x_i - \mu_i)^2}{2\sigma_i^2} - \frac{1}{2} \ln(\sigma_i^2)$$



conv analysis best fit values

$$\Delta t, t_{\text{rise}}^{\text{obs}}, t_{\text{decay}}^{\text{obs}}$$

$$t_{\text{decay}}^{\text{obs}} = \frac{R_0^{\text{obs}}}{m_B \beta_{\text{exp}} c} \left( \frac{\nu_{\text{SSA}}^{0,\text{obs}}}{\nu_{\text{SSA}}^{*,\text{obs}}} \right)^\phi$$

$$t_{\text{rise}}^{\text{obs}} = \frac{1}{2} t_{\text{peak}}^{\text{obs}} = \begin{cases} \frac{1}{2} \frac{R_0^{\text{obs}}}{\beta_{\text{exp}} c} \left[ \left( \frac{\nu_{\text{SSA}}^{0,\text{obs}}}{\nu_{\text{SSA}}^{*,\text{obs}}} \right)^\phi - 1 \right] & \text{if } \nu_{\text{SSA}}^{0,\text{obs}} > \nu_{\text{SSA}}^{*,\text{obs}} \\ 0 & \text{otherwise} \end{cases}$$

$$\Delta t^{\text{obs}} = t_{\text{exp}}^{\text{obs}} + t_{\text{peak}}^{\text{obs}} = t_{\text{exp}}^{\text{obs}} + \frac{R_0^{\text{obs}}}{\beta_{\text{exp}} c} \left[ \left( \frac{\nu_{\text{SSA}}^{0,\text{obs}}}{\nu_{\text{SSA}}^{*,\text{obs}}} \right)^\phi - 1 \right].$$

flat priors

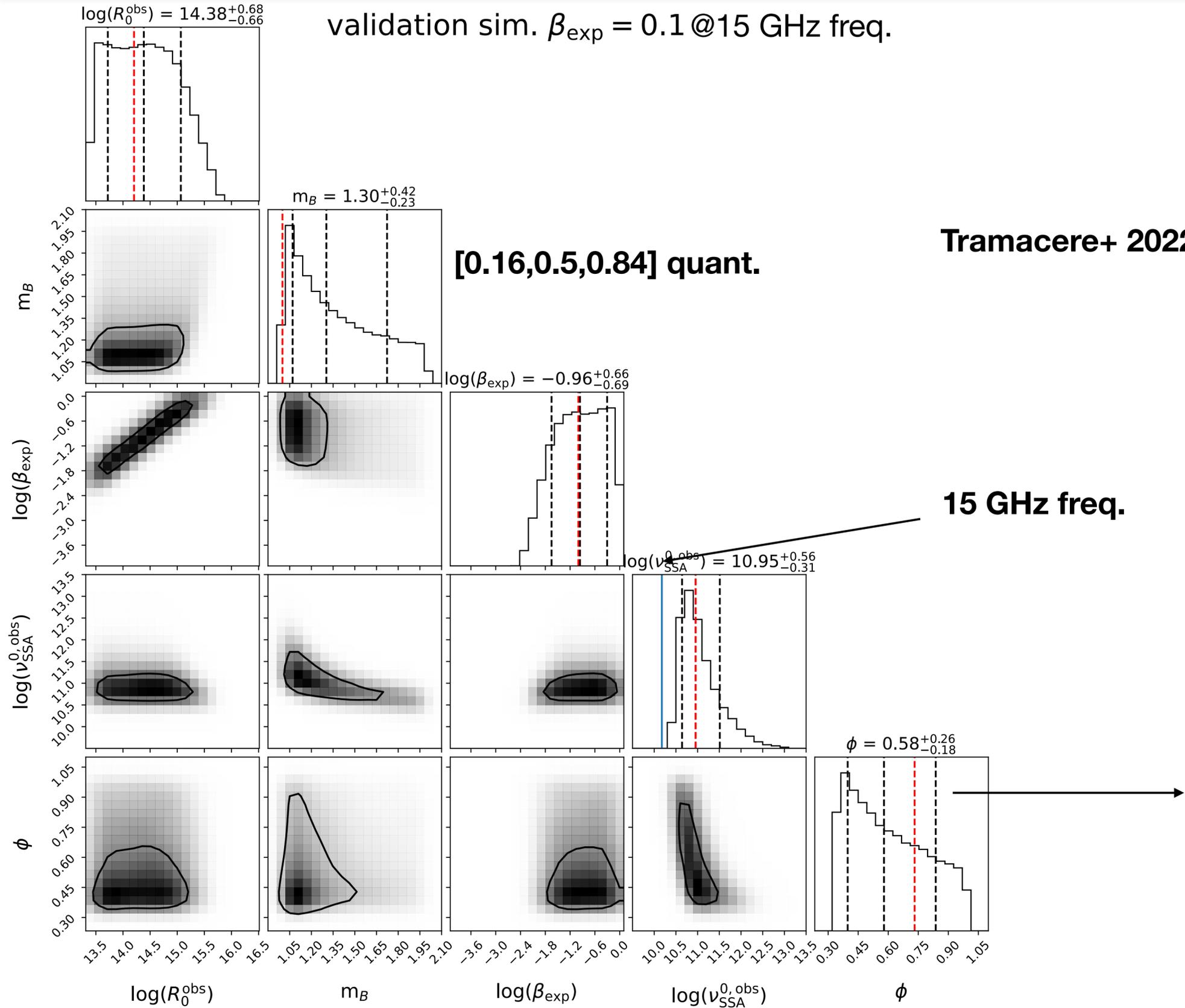
$$m_B \in [1, 2]$$

$$\nu_{\text{SSA}}^{0,\text{obs}} \in [10, 10^4] \text{ GHz}$$

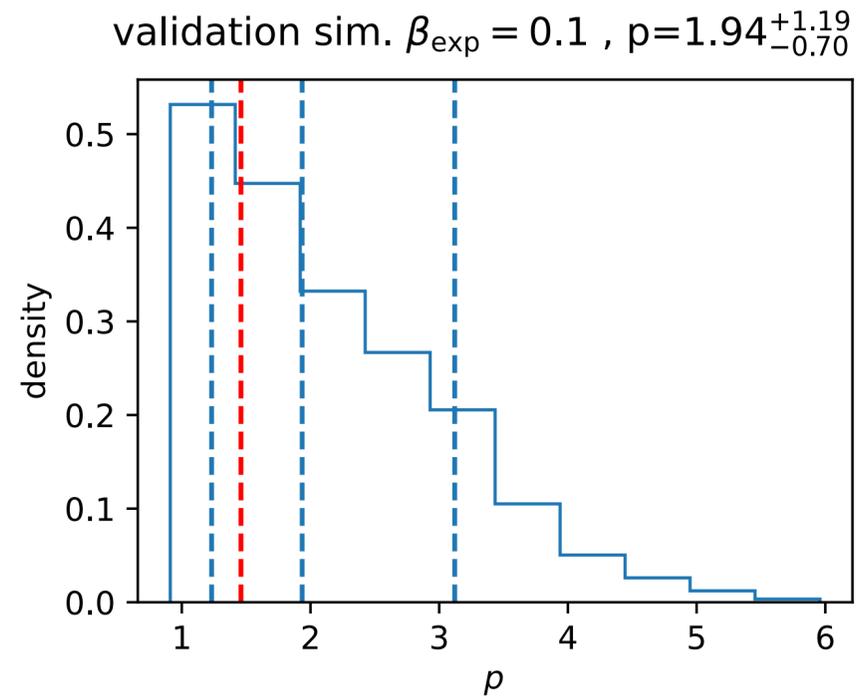
$$\phi \in [1/3, 1]$$

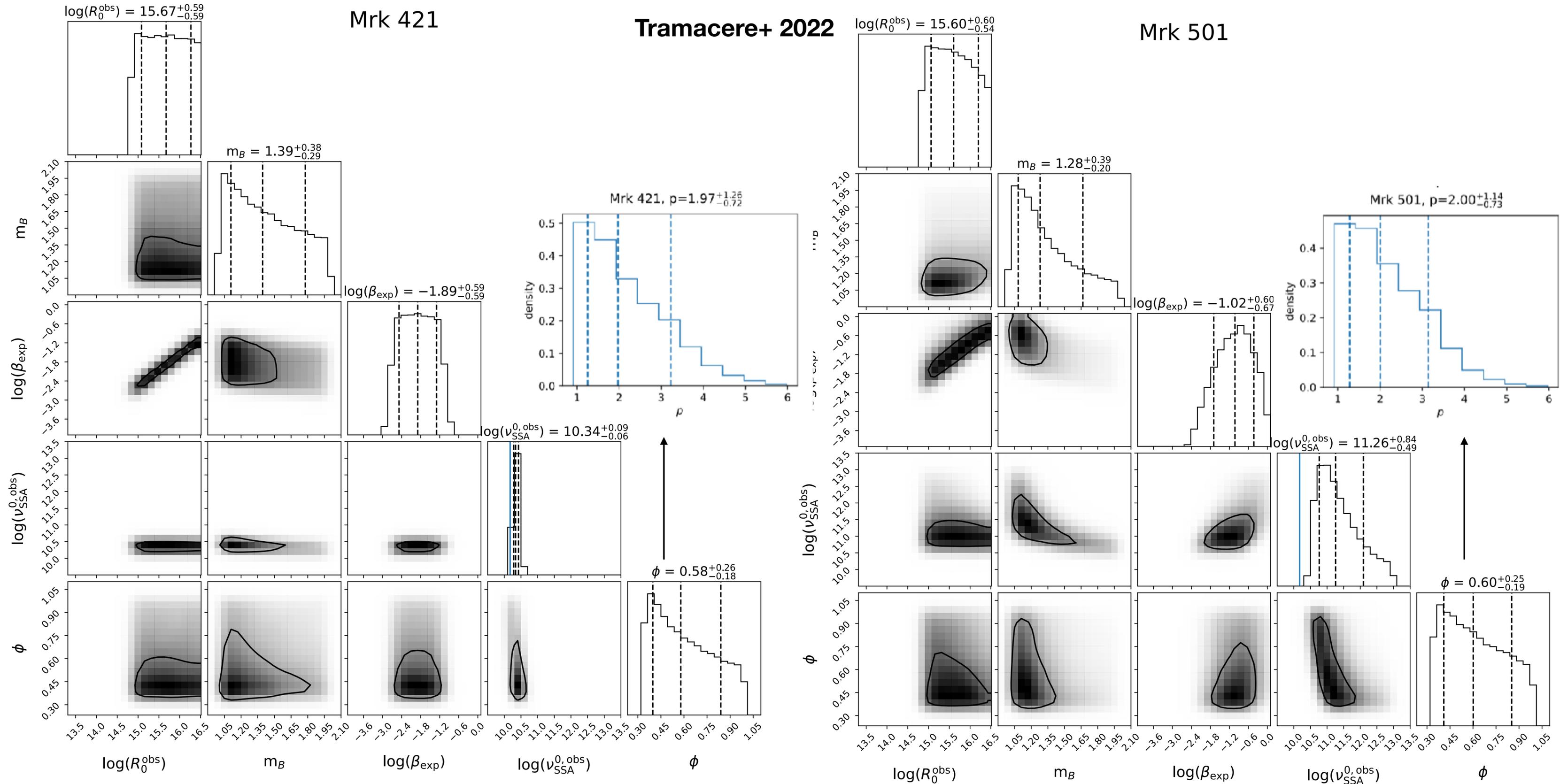
$$t_\gamma^{\text{var}} \in [0.25, 14] \text{ days}$$

$$R_{\text{obs}}^0 = R_0 \frac{1+z}{\delta} \quad R_0^{\text{obs}} \in [6.5 \times 10^{13}, 3.6 \times 10^{17}] \text{ cm}$$



red dashed line simulation values





- Radio-gamma delays on weeks to year timescales can be self consistently reproduced by adiabatic blob expansion
- We derived phenomenological relations, validated via accurate numerical simulations, and plugged to a response function, providing a direct link between the radio delay timescales and physics of the jet
- Implication on structure of magnetic fields, jet expansion, and MW connection open an interesting path to a deeper understanding of the how the engine of the jets work, and how jets evolve on larger scales, providing connection between micro and macro physics in relativistic jets
- **Analysis fully reproducible with JetSeT and convolution tool:**

[https://github.com/andreatramacere/adiabatic\\_exp\\_radio\\_gamma\\_delay](https://github.com/andreatramacere/adiabatic_exp_radio_gamma_delay)





$$S(t) = A \frac{\exp \frac{-(t-\Delta)}{t_f}}{1 + \exp \frac{-(t-\Delta)}{t_u}} \longrightarrow \Delta t = \Delta - t_u \ln \left( \frac{t_u}{t_f - t_u} \right)$$

**Analytical**

**Numerical**

$$S(t) = \frac{A}{2} \longrightarrow t_{\text{rise}} = t_u \left( 0.54 + 1.34 \left( \frac{t_f}{t_u} \right)^{1/4} \right)$$

$$S(t) = \frac{A}{e} \longrightarrow t_{\text{decay}} = t_f \left( 1.00 + 1.33 \left( \frac{t_f}{t_u} \right)^{-1.11} \right)$$

# adb./rad. cooling competition

