# Intensity interferometry

# R.Hanbury Brown and R.Q.Twiss (HB&T)

In 1956 hey prove with a lab experiment that

“time of arrival of photons in coherent beams of light is correlated”

The same year they measure the diameter of Sirius using this method using a 2.5 m baseline.

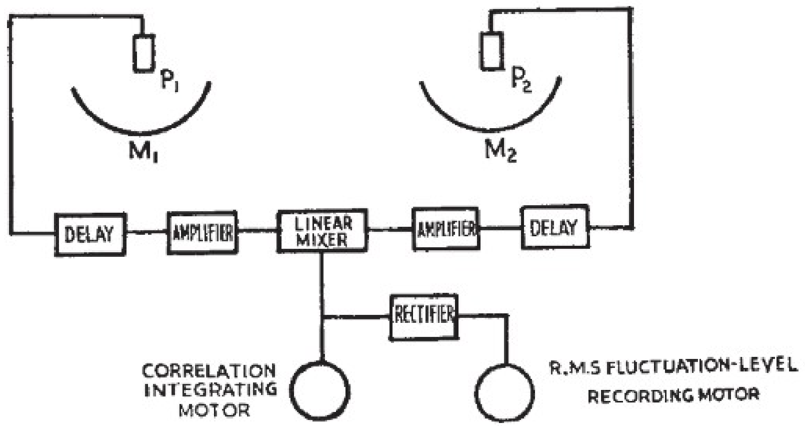
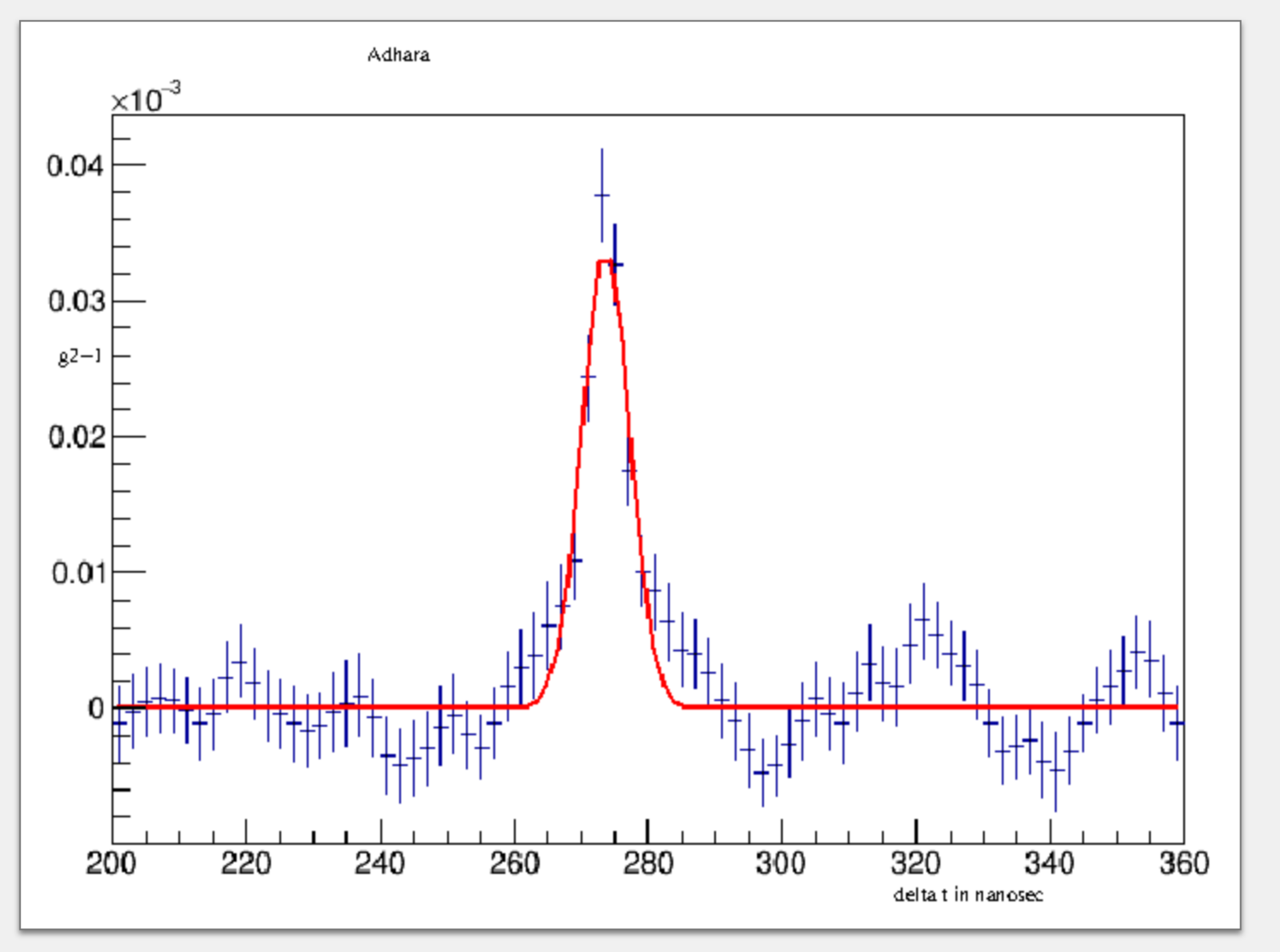



Fig. 1. Simplified diagram of the apparatus

# Count correlations




$$n = N - \langle N \rangle, \sigma = \sqrt{\langle n^2 \rangle}$$

$$\frac{\langle n_1 n_2 \rangle}{\sigma_1 \sigma_2}$$

# The problem

The standard reference is: Van Cittert-Zernike Theorem

I was trying to read papers published before the HB&T paper with questions like:

What is coherence?

Why it does not work with laser light?

How does it work?

I did not find useful answer to all those questions.

Causality

Until I found paper of Roy Glauber. But this was published after the publication of HB&T

He got a Nobel prize for those papers

He explains that people were not understanding this before

It is a purely quantum effect, semiclassical explanation are not convincing

HB&T experiment worked because their light was not coherent! (I mean it was coherent at order 1 but NOT coherent at order 2)

# Roy Glauber

2005 Nobel prize

Publish his serie of paper in 1963

He first define clearly what

Coherence means.

It show that there is a order of coherence. Light can be first order coherent but also second order coherent or nth order coherent. Maser were existing at this time ans suspect that they do possess nth order coherence. (first laser 1960)

# What is shown in paper

A monochromatic beam of light is first order coherent and it is what people means coherent up to his work.

So people where creating coherent light by using a thermal lamp and select a very fine bandwidth and call it coherent light.

Paper show that this light is first order coherent and has no coherence at order 2..n

First order coherent light is creating the biggest possible interference pattern.

Beam that have second order coherence show no HB&T effect.

This is precisely why HB&T light was not fully coherent that the experiment worked.

# So lets dive in paper and use Glauber own slides

https://www.nobelprize.org/prizes/physics/2005/glauber/lecture/

**Lets only use Electrical field (we can transform B into E with Lorentz transformation). Lets disregard polarization.**

$Energy = n h \nu, Energy \propto E^2 + B^2$ $\quad$ $So\ E^2 \propto n$

**The electric field can be described by its Fourier component.**

**Positive frequency of the Electrical field is the creation operator.**

**Negative frequency is the destruction operator.**

# Remember counting operator

**Quantum Field Theory** – for bosons

Field oscillation modes $\leftrightarrow$ harmonic oscillators

For harmonic oscillator:

$a$ lowers excitation $a|n\rangle = \sqrt{n}\,|n-1\rangle$

$a^\dagger$ raises excitation $a^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle$

$$aa^\dagger - a^\dagger a = 1$$

$$a^+ a|n\rangle = a^+\sqrt{(n)}|n-1\rangle = n|n\rangle$$

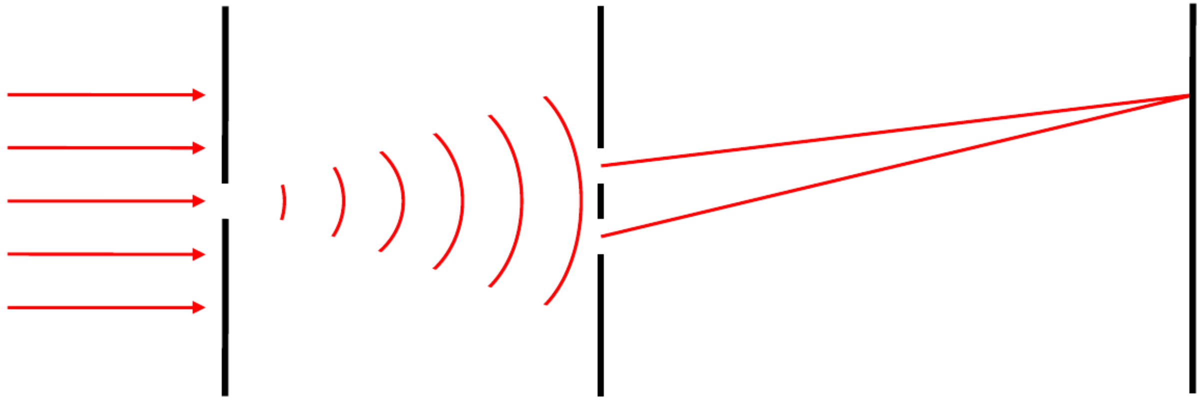
# Young pinhole (not very good analogy)

Define correlation function

$$G^{(1)}(r_1 t_1 r_2 t_2) = \langle E^{(-)}(r_1 t_1) E^{(+)}(r_2 t_2) \rangle$$

Young's 2-pinhole experiment measures:

$$G^{(1)}(\mathrm{r}_1\mathrm{t}_1\mathrm{r}_1\mathrm{t}_1) + G^{(1)}(\mathrm{r}_2\mathrm{t}_2\mathrm{r}_2\mathrm{t}_2) + G^{(1)}(\mathrm{r}_1\mathrm{t}_1\mathrm{r}_2\mathrm{t}_2) + G^{(1)}(\mathrm{r}_2\mathrm{t}_2\mathrm{r}_1\mathrm{t}_1)$$



*Coherence maximizes fringe contrast*

# Definition of coherance

*Let* $x = (r,t)$

Schwarz Inequality:

$$\left|G^{(1)}(x_1 x_2)\right|^2 \leq G^{(1)}(x_1 x_1) G^{(1)}(x_2 x_2)$$

Optical coherence:

$$\left|G^{(1)}(x_1 x_2)\right|^2 = G^{(1)}(x_1 x_1) G^{(1)}(x_2 x_2)$$

Sufficient condition: $G^{(1)}$ factorizes

*i.e.* $G^{(1)}(\mathrm{x}_1 \mathrm{x}_2) = \mathcal{E}^*(\mathrm{x}_1)\, \mathcal{E}(\mathrm{x}_2)$

~ also necessary:

Titulaer & G. *Phys. Rev.* 140 (1965), 145 (1966)



$\vec{a}\,\vec{b} = |\vec{a}||\vec{b}| \cos(angle(\vec{a}, \vec{b}))$

# Technicality

Initial states $\left|i\right\rangle$ random

Take ensemble average over $\left|i\right\rangle$

**Density Operator:** $\rho = \{|i\rangle\langle i|\}_{Average}$

~ Then **averaged counting probability** is

$$\{\langle i \mid E^{(-)}(rt)E^{(+)}(rt) \mid i\rangle\}_{Av.} = Trace\{\rho E^{(-)}(rt)E^{(+)}(rt)\}$$

# First order versus second order

Ordinary (Amplitude) interferometry measures

$$G^{(1)}(rtr't') \equiv \left\langle E^{(-)}(rt)E^{(+)}(r't') \right\rangle_{Ave.}$$

Intensity interferometry measures

$$G^{(2)}(rtr't'r't'rt) = \left\langle E^{(-)}(rt)E^{(-)}(r't')E^{(+)}(r't')E^{(+)}(rt) \right\rangle$$

**The two photon dilemma!**

# QUANTUM MECHANICS

BY

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*THIRD EDITION*

§3, P.9 . . .

Some time before the discovery of quantum mechanics people realized that the connexion between light waves and photons must be of a statistical character. What they did not clearly realize, however, was that the wave function gives information about the probability of *one* photon being in a particular place and not the probable number of photons in that place. The importance of the distinction can be made clear in the following way. Suppose we have a beam of light consisting of a large number of photons split up into two components of equal intensity. On the assumption that the intensity of a beam is connected with the probable number of photons in it, we should have half the total number of photons going into each component. If the two components are now made to interfere, we should require a photon in one component to be able to interfere with one in the other. Sometimes these two photons would have to annihilate one another and other times they would have to produce four photons. This would contradict the conservation of energy. The new theory, which connects the wave function with probabilities for one photon, gets over the difficulty by making each photon go partly into each of the two components. Each photon then interferes only with itself. Interference between two different photons never occurs.

The association of particles with waves discussed above is not restricted to the case of light, but is, according to modern theory, of universal applicability. All kinds of particles are associated with waves in this way and conversely all wave motion is associated with

# The famous factor 2

Two-fold joint count rate:

$$G^{(2)}(x_1 x_2 x_2 x_1) = G^{(1)}(x_1 x_1) G^{(1)}(x_2 x_2) + \underbrace{G^{(1)}(x_1 x_2) G^{(1)}(x_2 x_1)}_{\text{HB-T Effect}}$$

Note for $x_2 \to x_1$:

$$G^{(2)}(x_1\, x_1\, x_1\, x_1) = 2\,[G^{(1)}(x_1\, x_1)]^{\,2}$$



If 1st order coherent!!!

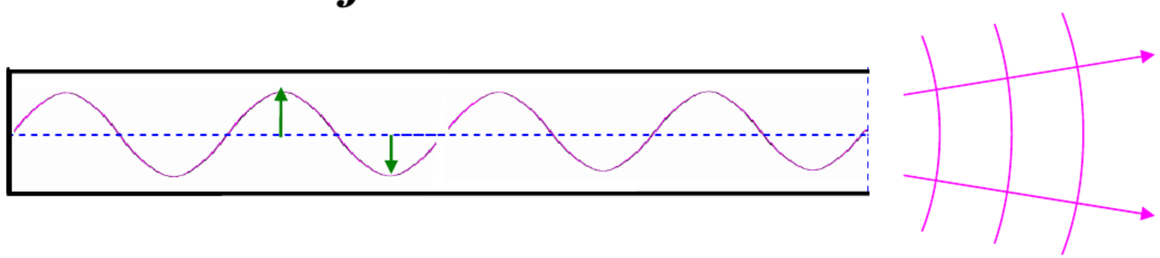
# Photon beams created by current are coherent (if no back reaction on current)

Any classical (i.e., predetermined) current $\boldsymbol{j}$ radiates coherent states

~ R.G. *Phys. Rev.* 84, '51

*What is current $\boldsymbol{j}$ for a laser?*

$\boldsymbol{j}$



Strong oscillating polarization current $\boldsymbol{j} = \dfrac{\partial \boldsymbol{P}}{\partial t}$

**Quantum Optics = Photon Statistics**

# Coherent field are eigenvector of destruction operator! (strange)

**Special states:** $a|\alpha\rangle = \alpha|\alpha\rangle$

$\alpha$ = any complex number

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

$$P(n) = \frac{|\alpha|^{2n}}{n!} e^{-|\alpha|^2}$$ , Poisson distribution

$$\langle n \rangle = |\alpha|^2$$

~ single mode coherent states



a is not event hermitian!

# Confusion due to semi classic developments

**If the density operator for a single mode can be written as:**

$$\rho = \int P(\alpha)|\alpha\rangle\langle\alpha|d^2\alpha$$

Then $\langle a^{\dagger n} a^m \rangle = Tr\,(\rho a^{\dagger} a^m) = \int P(\alpha)\alpha^{*n}\,\alpha^m d^2\,\alpha$

Operator averages become integrals

$P(\alpha)$ = quasi-probability density

Scheme works well for pseudo-classical fields, but is not applicable to some classes of fields *e.g.* "squeezed" fields, (no P-function exists).



$|\alpha\rangle\langle\alpha|$ Is not an orhtonormal base !!!

# Consequence of this paper

Coherence is well defined (Schwarz inequality, eigen vector of destruction operator).

There is coherence at order 1..n

Order 1 coherence mean maximal interference fringes

Order 2 coherence mean NO HB&T effect

Ideal laser have coherence 1..n (all photon created by a electrical current that does not react back)

Gaussian or thermal source have no coherence => they can show HB&T effect

You can create order 1 coherence in selecting very sharp bandwidth

You can create squeezed light with less noise then vacuum (LIGO) at some points.

HB&T is a pure quantum effect. Can be somewhat understood semi-classically.

HB&T had the correct intuition and built on it extremely big apparatus, but in a time where those things were not really understood.

# Glauber papers

**You can find almost all his papers in:**

**https://academictree.org/physics/publications.php?pid=49888**

**In particular read**

1963 Glauber RJ. The quantum theory of optical coherence Physical Review. 130: 2529-2539