Keywords: logic, reasoning, truth-conditional semantics
14. Formal methods in semantics

1. Introduction.
2. First order logic and natural language.
3. Formal systems, proofs and decidability.
4. Semantic models, validity and completeness.
5. Formalizing linguistic methods.
7. Linguistic applications of semantic methods.
8. Conclusions

Abstract

Covering almost an entire century, this article reviews in general and non-technical terms how formal, logical methods have been applied to the meaning and interpretation of natural language. This paradigm of research in natural language semantics produced important new linguistic results and insights, but logic also profited from such innovative applications. Semantic explanation requires properly formalized concepts, but only provides genuine insight when it accounts for linguistic intuitions on meaning and interpretation or the results of empirical investigations in an insightful way. The creative tension between the linguistic demand for cognitively realistic models of human linguistic competence and the logicians demand for a proper and explicit account of all and only the valid reasoning patterns initially led to an interesting divergence of methods and associated research agenda’s. With the maturing of natural language semantics as a
 branch of cognitive science an increasing number of logicians trained in linguistics and linguists apt in using formal methods are developing more convergent empirical issues in interdisciplinary research programs.

1. Introduction

The scientific analysis of patterns of human reasoning properly belongs to the ancient discipline of logic, bridging more than twenty centuries from its earliest roots in the ancient Greek philosophical treatises of Plato and Aristotle on syllogisms to its contemporary developments in connecting dynamic reasoning in context to underlying neurobiological and cognitive processes. In reasoning with information from various sources available to us, we systematically exploit (i) the meaning of the words, (ii) the way they are put together in clauses, as well as (iii) the relations between these clauses and (iv) the circumstances in which we received the information in order to arrive at a conclusion (Frege 1892; Tarski 1956). The formal analysis of reasoning patterns not only offers an important window on the meaning and interpretation of logical languages, but also of ordinary, acquired, i.e. natural languages. It constitutes a core component of cognitive science, providing the proper scientific methods to model human information processing as constitutive structure and form.

Any explanatory scientific theory of the meaning and interpretation of natural language must at some level aim to characterize all and only those patterns of reasoning that guarantee to preserve in one way or another the assumed truth of the information on which the conclusions are based. In analyzing patterns of inferences, content and specific aspects of the interpretation can only be taken into consideration, if they can be expressed in syntactic form and constitutive structure or in the semantic meta-language. The
contemporary research program of natural language semantics has significantly expanded the expressions of natural language to be subjected to such formal methods of logical analysis to cover virtually all syntactic categories, as well as relations between sentences and larger sections of discourse or text, mapping syntactic, configurational structures to sophisticated notions of semantic content or information structure (Barwise & Perry 1983; Chierchia 1995; Cresswell 1985; Davidson & Harman 1972; Dowty, Wall & Peters 1981; Gallin 1975; Partee, ter Meulen & Wall 1990).

The classical division of labor between syntactic and semantic theories of reasoning is inherited from the mathematical logical theories developed in the early twentieth century, when logical, i.e. unnatural and purposefully designed languages were the primary subject of investigation. Syntactic theories of reasoning exploit as explanatory tools merely constitutive, configurational methods, structural, i.e. formal operations such as substitution and pure symbol manipulation of the associated proof theories. The semantic theories of reasoning require an interpretation of such constitutive, formal structure in models to characterize truth-conditions and validity of reasoning as systematic interaction between form and meaning (Boolos & Jeffrey 1980). The syntactic, proof theoretic strategy with its customary disregard for meaning as intangible, has originally been pursued most vigorously for natural language in the research paradigm of generative grammar. Its earliest mathematical foundational research in automata theory, formal grammars and their associated design languages regarded structural operations as the only acceptable formal methods. Reasoning or inference was as such not the target of their investigations, as grammars were principally limited to characterize sentence internal properties and their computational complexity (Chomsky 1957; Chomsky 1959; Chomsky & Miller 1958, 1963; Hopcroft & Ullman 1979; Gross & Lentin 1970). The
semantic, model-theoretic strategy of investigating human reasoning has been pursued most vigorously in natural language semantics, Lambek and Montague grammars and game theoretic semantics, and their 21st century successors, the various dynamic theories of meaning and interpretation founded on developments in intensional and epistemic logics of (sharing) belief and knowledge (Barwise 1989; van Benthem 1986; van Benthem & ter Meulen 1997; Cresswell 1985; Davidson & Harman 1972; Kamp 1981; Kamp & Reyle 1993; Lambek 1958; Lewis 1972, 1983; Montague 1974; Stalnaker 1999).

Formal methods deriving from logic are also applied in what has come to be known as formal pragmatics, where parameters other than worlds or situations, such as context or speaker/hearer, time of utterance or other situational elements may serve in the models to determine meaning and situated inference. This article will address formal methods in semantics as main topic, as formal pragmatics may be considered a further generalization of these methods to serve wider linguistic applications, but as such does not in any intrinsic way differ from the formal methods used in semantics.

In both logical and natural languages, valid forms of reasoning in their most general characteristics exploit the information available in the premises, assumed to be true in an arbitrary given model, to draw a conclusion, guaranteed to be also true in that model. Preserving this assumed truth of the premises is a complex process that may be modeled in various formal systems, but if their admitted inference rules are somehow violated in the process the conclusion of the inference is not guaranteed to be true. This common deductive approach accounts for validity in forms or patterns of reasoning based on the stable meaning of the logical vocabulary, regardless of the class of models under consideration. It has constituted the methodological corner stone of the research program
of natural language semantics, where ordinary language expressions are translated into logical expressions, their ‘logical form’, to determine their truth conditions in models as a function of their form and subsequently characterize their valid forms of reasoning (May 1985; Montague 1974; Dowty, Wall & Peters 1981). Some important formal methods of major schools in this research program are reviewed, mostly to present their conceptual foundations and discuss their impact on linguistic insights, while referring the reader for more technical expositions and formal details to the relevant current literature and articles 10 (Newen & Schröder) *Logic and semantics*, 11 (Kempton) *Formal semantics and representationalism*, 33 (Zimmermann) *Model-theoretic semantics*, 43 (Keenan) *Quantifiers*, 37 (Kamp & Reyle) *Discourse Representation Theory* and 38 (Dekker) *Dynamic semantics* (see also van Benthem & ter Meulen 1997; Gabbay & Guenthner 1983; Partee, ter Meulen & Wall 1990).

Excluded from consideration as *formal* methods in the sense intended here are other mathematical methods, such as statistical, *inductive* inference systems, where the assumptions and conclusions of inferences are considered more or less likely, or various quantitative approaches to meaning based on empirical studies, or optimality systems, which ordinarily do not account for inference patterns, but rank possible interpretations according to a given set of constraints of diverse kinds. In such systems form does not directly and functionally determine meaning and hence the role of inference patterns, if any, is quite different from the core role in logical, deductive systems of reasoning which constitute our topic. Of course, as any well defined domain of scientific investigation, such systems too may be further formalized and perhaps eventually even axiomatized as a logical, formal system. But in the current state of linguistics their methods are often informal, appealing to semantic notions as meaning only implicitly, resulting in
fragmented theories, which, however unripe for formalization, may still provide genuinely interesting and novel linguistic results.

In section 2 of this article the best known logical system of first order logic is discussed. It served as point of departure of natural language semantics, in spite of its apparent limitations and idealizations. In section 3 the classical definition of a formal system is specified with its associated notions of proof and theorem, and these are related to its early applications in linguistic grammars. The hierarchy of structural complexity generated by the various kinds of formal grammars is still seen to direct the current quest in natural language semantics for proper characterizations of cognitive complexity. Metalogical properties such as decidability of the set of theorems are introduced. In section 4 the general notion of a semantic model with its definition of truth conditions is presented, without formalization in set-theoretic notation, to serve primarily in conceptually distinguishing contingent truth (at a world) in a model from logical truth in regardless which model to represent valid forms of reasoning. Completeness is presented as a desirable, but perhaps not always feasible property of logical systems that have attained a perfect harmony between their syntactic and semantic sides. Section 5 discusses how the syntactic and semantic properties of pronouns first provided a strong impetus for using formal methods in separate linguistic research programs, that have later converged on a more integrated account of their behavior, currently still very much under investigation. In section 6 we review a variety of proof theoretic methods that have been developed in logic to understand how each of them had an impact on formal linguistic theories later. This is where constraints on derivational complexity and resource bounded generation of expressions is seen to have their proper place, issues that have only gained in importance in contemporary linguistic research. Section 7 presents an overview of the semantic
methods that have been developed in logic over the past century to see how they have influenced the development of natural language semantics as a flourishing branch of cognitive science, where formal methods provide an important contribution to their scientific methods. The final section 8 contains the conclusion of this article stating that the application of formal methods, deriving from logical theories, have greatly contributed to the development of linguistics as an independent academic discipline with an integrated, interdisciplinary research agenda. Seeking a continued convergence of syntactic and semantic methods in linguistic applications will serve to develop new insights in cognitive capacity underlying much of human information processing.

2. First order logic and natural language.

Many well known systems of first order logic (FOL), in which quantifiers may only range over individuals, i.e. not over properties or sets of individuals, have been designed to study inference patterns deriving from the meaning of the classical Boolean connectives of conjunction (…and…), disjunction (…or…), negation (not…) and conditionals (if…then…) and biconditionals (…if and only if…), besides the universal (every N) and existential (some N) quantifiers. Syntactically these FOL systems differed in the number of axioms, logical vocabulary or inference rules they admitted, some were optimal for simplicity of the proofs, others more congenial to the novel user, relying on the intuitive meaning of the logical vocabulary (Boolos & Jeffrey 1980; Keenan & Faltz 1985; Link 1991).

The strongly reformist attitudes of the early 20th century logicians Bertrand Russell, Gottlob Frege, and Alfred Tarski, today considered the great-grandfathers of modern logic, initially steered FOL developments away from natural language, as it was regarded
as too ambiguous, hopelessly vague, or content- and context-dependent. Natural language was even considered prone to paradox, since you can explicitly state that something is or is not true. This is what is meant when natural language is accused of “containing its own truth-predicate”, for the semantics of the predicate “is true” cannot be formulated in a non-circular manner for a perfectly grammatical, but self-referential sentence like *This statement is false*, which is clearly true just in case it is false. Similarly treacherous forms of self-referential acts with circular truth-conditions are found in simple statements such as *I am lying*, resembling the ancient Cretense Liar Paradox, and the syntactically overtly self-referential, but completely comprehensible *The claim that this sentence contains eleven words is false*, that lead later model theoretic logicians to develop innovative models of self-reference and logically sound forms of circularity, abandoning the need for rock bottom atomic elements of sets, as classical set theory had always required (Barwise & Etchemendy 1987).

Initially FOL was advocated as a ‘good housekeeping’ act in understanding elementary forms of reasoning in natural language, although little serious attention was paid on just how natural language expressions should be systematically translated into FOL, given their syntactic constituent structure. This raised objections of what linguists often called ‘miraculous translation’, appealing to the implicit intuitions on truth-functional meaning only trained logicians apparently had easy access to. Although this limited logical language of FOL was never intended to come even close to modeling the wealth of expressive power in natural languages, it was considered the basic Boolean algebraic core of the logical inference engine. The descriptive power of FOL systems was subsequently importantly enriched to facilitate the Fregean adagio of compositional translation by admitting lambda abstraction, representing the denotation of a predicate as a set $A$ by a
characteristic function $f$ that tells you for each element $d$ in the underlying domain $D$ whether or not $d$ is an element of that set $A$, i.e. $f_A(d)$ is true iff. $d$ is an element of the set $A$ (Partee, ter Meulen & Wall 1990). Higher order quantification with quantifiers ranging over sets of properties, or sets of those ad infinitum, required a type structure, derived from the abstract lambda calculus, a universal theory of functional structure. Typing formal languages resembled in some respects Bertrand Russell’s solution to avoid vicious circularity in logic (Barendregt 1984; Carpenter 1997; Montague 1974; Link 1991).

More complex connectives or other truth functional expressions and operators were added to FOL, besides situations or worlds to the models to analyze modal, temporal or epistemic concepts (van Benthem 1983; Carnap 1947; Kripke 1972; Lewis 1983; McCawley 1981). The formal methods of FOL semantics varied from classical Boolean full bi-valuation with total functions that ultimately take only true and false as values, to three or more valued models in which formulas may not always have a determined truth value, initially proposed to account for presupposition failure of definite descriptions that did not have a referent in the domain of the intended model. Weaker logical systems were also proposed, admitting fewer inference rules. The intuitionistic logics are perhaps the best known of these, rejecting for philosophical and perhaps conceptual reasons the classical law of double negation and the correlated rule of inference that allowed you to infer a conclusion, if its negation had been shown to lead to contradictions. In classical FOL the truth functional definition of the meaning of negation as set-theoretic complement made double negation logically equivalent to no negation at all, e.g. *It is not the case that every student did not hand in a paper* should at least truth-functionally mean the same as *Some student handed in a paper* (Partee, ter Meulen & Wall 1990). Admitting partial functions that allowed quantifiers to range over possible extensions of
their already fixed, given range, ventured into for logic also innovative higher order methods, driven by linguistic considerations of pronoun resolution in discourse and various sophisticated forms of quantification found in natural language, to which we return below (Chierchia 1995; Gallin 1975; Groenendijk & Stokhof 1991; Kamp & Reyle 1993).

3. Formal systems, proofs and decidability.

Whatever its exact language and forms of reasoning characterized as valid, any particular logical system must adhere to some very specific general requirements, if it is to count as a formal system. A formal system must consist of four components:

(i) a lexicon specifying the terminal expressions or words, and a set of non-terminal symbols or categories,

(ii) a set of production rules which determine how the well formed expressions of any category of the formal language may be generated,

(iii) a set of axioms or expressions of the lexicon that are considered primitive,

(iv) a set of inference rules, determining how expressions may be manipulated.

A formal system may be formulated purely abstractly without being intended as representation of anything, or it may be designed to serve as a description or simulation of some domain of real phenomena or, as intended in linguistics, modeling aspects of empirical, linguistic data.

A formal proof is the product of a formal system, consisting of (i) axioms, expressions of the language that serve as intuitively obvious or in any case unquestionable first principles, assumed to be true or taken for granted no matter what, and (ii) applications of the rules of inference that generate sequences of steps in the proof, resulting in its
conclusion, the final step, also called a \textit{theorem}. The \textit{grammar} of a language, whether logical or natural, is a system of rules that generates all and only all the grammatical or well-formed sentences of the language. But this does not mean that we can always get a definite answer to the general question whether an arbitrary string belongs to a particular language, something a child learning its first language may actually need. There is no general decision procedure determining for any arbitrary given expression whether it is or is not derivable in any particular formal system (Arbib 1969; Davis 1965; Savitch, Bach & Marsh 1987). However, this question is provably \textit{decidable} for sizable fragments of natural language, even if some form of higher order quantification is permitted (Nishihara 1990; Pratt 2003, 2006). Current research on generative complexity is focused on expanding fragments of natural language that are known to be decidable to capture realistic limitations on search complexity, in attempting to characterize formal counterparts to experimentally obtained results on human limitations of cognitive resources, such as memory or processing time. Automated theorem provers certainly assist people in detecting proofs for complex theorems or huge domains. People actually use smart, but still little understood heuristics in finding derivations, trimming down the search space of alternative variable assignments, for example, by marked prosody and intonational meaning. Motivating much of the contemporary research in applying formal methods to natural language is seeking to restrain the complexity or computational power of the formal systems to less powerful, learnable and decidable fragments. In such cognitively realistic systems the inference rules constrain the search space of valuation functions or exploit limited resources in linguistically interesting and insightful ways.

The general research program of determining the generative or computational complexity of natural languages first produced in the late 1950s the well known \textit{Chomsky Hierarchy}
of formal language theory, which classifies formal languages, their corresponding automata and the phrase-structure grammar that generate the languages as regular, context-free, context-sensitive or unrestricted rewrite systems (Arbib 1969; Gross & Lentin 1970; Hopcroft & Ullman 1979). Initially, Chomsky (1957, 1959) claimed that the rich structure of natural languages with its complex forms of agreement required the strength of unrestricted rewrite systems, or Turing machines. Quickly linguists realized that for grammars to be learnable and to claim to model in any cognitively realistic way our human linguistic competence, whether or not innate, their generative power should be substantially restricted (Peters & Ritchie 1973). Much of the contemporary research on resource-bounded categorial grammars and the economy of derivation in minimalist generative grammar, comparing formal complexity of derivations, is still seeking to distill universal principles of natural languages, considered cognitive constants of human information processing, linguistically expressed in structurally very diverse phenomena (Ades & Steedman 1982; Moortgat 1996; Morrill 1994, 1995; Pentus 2006; Stabler 1997).

4. Semantic models, validity and completeness.

On the semantic side of formal methods, the notion of a model $M$ plays a crucial role in the definition of truth-conditions for formulas generated by the syntax. The meaning of the Boolean connectives is considered not to vary from one model to a next one, as it is separated in the vocabulary as the closed class of expressions of the logical vocabulary. The conjunction (…and…) is true just in case each of the conjuncts is, the disjunction (…or…) is false only in the case neither disjunct is true. The conditional is only false in the case the antecedent (if… clause) is true, but the consequent (then… clause) is false.
The bi-conditional \( \ldots \text{if and only if}\ldots \) is true just in case the two parts have the same truth value. Negation \((\text{not}\ldots)\) simply reverses the truth value of the expression it applies to. For the interpretation of the quantifiers an additional tool is required, a variable assignment function \( f \), which assigns to each variable \( x \) in the vocabulary of variables a referent \( d \), an element of the domain \( D \) of the model \( M \). Proper names and the descriptive vocabulary containing all kinds of predicates, i.e. adjectives, nouns, and verbs, are interpreted by a function \( P \), given with model \( M \), that specifies who was the bearer of the name, or who had a certain property corresponding to a one place predicate, or stood in a certain relation for relational predicates in the given model \( M \).

Linguists were quick to point out that in natural language at least conjunctions and disjunctions connect not only full clauses, but also noun-phrases, which is not always reducible to a sentential connective, e.g. \( \text{John and Mary met} \neq \text{John met and Mary met} \). This kind of linguistic criticism quickly led to generalizations of Boolean operations in a logical system, where variable assignment functions are generalized and given the flexibility to assign referents of appropriate complex types in a richer higher order logic, but such complexities need not concern us here.

A formal semantic model \( M \) consists hence of: (i) a domain \( D \) of semantic objects, sometimes classified into types or given a particular internal structure, and (ii) a function \( P \) that assigns appropriate denotations to the descriptive vocabulary of the language interpreted. If the language also contains quantifiers, the model \( M \) comes equipped with a given variable assignment function \( g \), often considered arbitrary, to provide a referent, which is an element of \( D \), for all free variables. The given variable assignment function is sometimes considered to represent the current context in some contemporary systems that investigate context dependencies and indexicals. In FOL the universal quantifier \( \forall x \)}
$N(x)$ is interpreted as true in the model $M$, if all possible alternative assignment functions $g'$ to the variable $x$ that it binds provide a referent $d$ in the denotation of $N$. So not only the given variable assignment function $g$ provides a $d$ in the denotation of $N$, but all alternative assignments $g'$ that may provide another referent, but could equally well have been considered the given one, also do. For the existential quantifier $\textit{some } x \ [N(x)]$ only one such variable assignment, the given one or another alternative variable assignment function, suffices to interpret the quantifier as true in the model $M$. An easy way to understand the effect of the interpretation of quantifiers in clauses is to see that for a universal $NP$ the set denoted by $N$ should be a subset of the set denoted by the $VP$, e.g. $\textit{every student sings}$ requires that the singers include all the students. Similarly, the existential quantifier requires that the intersection of the denotation of the $N$ and the denotation of the $VP$ is not empty, e.g. $\textit{some student sings}$ means that among the singers there is at least one student.

Intensional models generalize this elementary model theory for extensional FOL to characterize truth in a model relative to a possible world, situation or some other index, representing, for instance, temporal or epistemic variability. Intensional operators typically require a clause in its scope to be true at all or at only some such indices, mirroring strong, universal and existential quantifiers respectively. The domain of intensional models must be enriched with the interpretation of the variables referring to such indices, if such meta-variables are included in the language to be interpreted. Otherwise they are considered to be external to the model, added as a set of parameters indexing the function interpreting the language. Domains may also be structured as (semi)lattices or other partial orders, which has proven useful for the semantics of plurals, mass terms and temporal reference to events, but such further variations on FOL must
remain outside the scope of the elementary exposition in this article.

To characterize logical truths, reflecting the valid reasoning patterns, one simply generalizes over all formulas true in all logically possible models, to obtain those formulas that must be true as a matter of necessity or form only, due to the meaning assigned to their logical vocabulary, irrespective of what is actually considered to be factually the case in the models. By writing out syntactic proofs with all their premises conjoined as antecedents of a conditional of which the conclusion is the consequent one may test in a semantic way the validity of the inference. If such a conditional cannot be falsified, the proof is valid and vice versa.

A semantic interpretation of a language, natural or otherwise, is considered formal only if it provides such precise logical models in which the language can be systematically interpreted. Accordingly, contingent truths, which depend for their truth on what happens to be the case in the given model, are properly distinguished from logical truths, which can never be false in any possible model, because the meaning of the logical vocabulary is fixed outside the class of models and hence remains invariable.

A logical system in which every proof of a theorem can be proven to correspond to a semantically valid inference pattern and vice versa is called a complete system, as FOL is. If a formal system contains statements that are true in every possible model but cannot be proven as theorems within that system by the admitted rules of inference and the given axiom base, the system is considered incomplete. Familiar systems of arithmetic have been proven to be incomplete, but that does not disqualify them from their sound use in practice and they definitely still serve as a valuable tool in applications.

5. Formalizing linguistic methods.
Given the fundamental distinction between syntactic, proof-theoretic and semantic, model-theoretic characterizations of reasoning in theories that purport to model meaning and interpretation, the general question arises what (dis)advantages these two methodologically distinct approaches respectively may have for linguistic applications. Although in its generality this question may not be answerable in a satisfactory way, it is clear that at least in the outset of linguistics as its own, independent scientific discipline, quite different and mostly disconnected, if not antagonizing research communities were associated with the two strategies. This separation of minds was only too familiar to logicians from the early days of modern logic, where proof theorists and model theoretic semanticists often drew blood in their disputes on the priority, conceptual or otherwise, of their respective methods. Currently seeking convergence of research issues in syntax and semantics is much more en vogue and an easy go-between in syntactic and semantic methods has already proven to pay off in obtaining the best linguistic explanations and new insights.

One of the best examples of how linguistic questions could fruitfully be addressed both by syntactic and semantic formal methods, at first separately, but later in tandem, is the thoroughly studied topic of binding pronouns and its associated concept of quantifier scope. Syntacticians focused primarily on the clear configurational differences between free (1a) and bound pronouns (1b), and reflexive pronouns (1c), which all depend in different ways on the subject noun-phrase that precedes and commands them in the same clause. Co-indexing was their primary method of indicating binding, though no interpretive procedure was specified with it, as meaning was considered elusive (Reinhart 1983a, 1983b).

b. [Every student], loves [[his], teacher], i, j.

c. [Every student], who knows [John/a professor], loves [himself], i, j.

This syntactic perspective on the binding behavior of pronouns within clauses had deep relations to constraints on movement as a transformation on a string, to which we return below. It limited its consideration of data to pronominal dependencies among clauses within sentences, disregarding the fact that singular universal quantifiers cannot bind pronouns across sentential boundaries (2a,b), but proper names, plurals and indefinite noun-phrases typically do (2c).

(2) a. [Every student], handed in a paper. [He], i, j passed the exam.

b. [Every student], who handed in a paper [t], passed the exam.

c. [John/ A student/All students], handed in a paper.

[He/they], i, j. passed the exam.

Semanticists had to understand how pronominal binding in natural language was in some respects similar, but in other respects quite different from the ordinary variable binding of FOL. From a semantic perspective the first puzzle was how ordinary proper names and other referring expressions, including freely referring pronouns, that were considered to have no logical scope and hence could not enter into scope ambiguities, could still force pronouns to corefer (3a), even in intensional contexts (3b).

(3) a. John/He loves his mother.

b. John believes that Peter loves his mother.

c. Every student believes that Peter loves his mother.

In (3a) the reference of the proper name John or the contextually referring free pronoun he fixes the reference of the possessive pronoun his. In (3b) the possessive pronoun his in the subordinate clause could be interpreted as dependent on Peter, but equally easily as
dependent upon John in the main clause. If FOL taught semanticists to identify bound variables with those variables that were syntactically within the scope of a existential or universal quantifier, proper names had to be reconsidered as quantifiers having scope, yet referring rigidly to the same individual, even across intensional contexts. By considering quantifying in as a primary semantic technique to bind variables simultaneously, first introduced in Montague (1974), semanticists fell into the logical trap of identifying binding with configurational notions of linear scope. This fundamental connection ultimately had to be abandoned, when the infamous Geach sentence (4)

(4) Every farmer who owns a donkey beats it.

made syntacticians as well as semanticists realize that existential noun-phrases in restrictive relative clauses of universal subject noun-phrases could inherit, as it were, their universal force, creating for the interpretation of (4) cases of farmers and donkeys over which the quantifier was supposed to range. This is called unselective binding, introduced first by David Lewis (Lewis 1972, 1983), but brought to the front of research in semantic circles in Kamp (1981). Generalizing quantifying in as a systematic procedure to account also for intersentential binding of pronouns, as in (2), was soon also realized to produce counterintuitive results. This motivated an entirely new development of dynamic semantics, where the interpretation of a given sentence would partially determine the interpretation of the next sentence in a text and pronominal binding was conceptually once and for all separated from the logical or configurational notion of scope (Groenendijk & Stokhof 1991; Kamp & Reyle 1993). The interested reader is referred to article 38 (Dekker) Dynamic semantics and article 40 (Büring) Pronouns for further discussion and details of the resulting account.

In logic itself quite a few different flavors of formal systems had been developed based on formal proof-theoretic (syntactic) or model-theoretic (semantic) methods. The best known on the proof-theoretic side are axiomatic proof theory (Jeffrey 1967), Gentzen sequent calculus (Gentzen 1934), combinatorial logic (Curry 1961), and natural deduction (Jeffrey 1967; Partee, ter Meulen & Wall 1990), each of which have made distinct and significant contributions in various developments in semantics. Of the more semantically flavored developments most familiar are Tarski’s classical notion of satisfaction in models (Tarski 1956), the Lambek and other categorial grammars (Ajduciewicz 1935; Bar Hillel 1964; van Benthen 1987, 1988; Buszkowski 1988; Buszkowski & Marciszewski 1988; Lambek 1958; Oehrle, Bach & Wheeler 1988), tightly connecting syntax and semantics via type theory (Barendregt 1984; van Benthen 1991; Carpenter 1997; Morrill 1994), Beth’s tableaux method (Beth 1970), game theoretic semantics (Hintikka & Kulas 1985), besides various intensional logical systems, enriched with indices representing possible worlds or other modal notions, which each also have led to distinctive semantic applications (Asher 1993; van Benthem 1983; Cresswell 1985; Montague 1974). The higher order enrichment of FOL with quantification over sets, properties of individuals or properties of properties in Montague Grammar (Barwise & Cooper 1981; van Benthem & ter Meulen 1985; Montague 1974) at least initially directed linguists’ attention away from the global linguistic research program of seeking to constrain the generative capacity and computational complexity of the formal methods in order to model realistically the cognitive capacities of human language users, as it focused everyone’s attention on compositionality, type theory and
type shifting principles, and adopted a fully generalized functional structure.

The next two sections selectively review some of the formal methods of these logical systems that have led to important semantic applications and innovative developments in linguistic theory.

An axiomatic characterization of a formal system is the best way to investigate its logic, as it matches its semantics to prove straightforwardly its soundness and completeness, i.e. demonstrating that every provable theorem is true in all models (*semantically valid*) and vice versa. For FOL a finite, in fact small number of logically true expressions suffice to derive all and only all valid expressions, i.e. FOL is provably complete. But in actually constructing new proofs an axiomatic characterization is much less useful, for it does not offer any reliable heuristics as guidance for an effective proof search. The main culprit is the rule of inference guaranteeing the transitivity of composition, i.e. to derive $A \rightarrow C$ from $A \rightarrow B$ and $B \rightarrow C$ requires finding an expression $B$ which has no trace in the conclusion $A \rightarrow C$. Since there are infinitely many possible such B$s, you cannot exhaustively search for it. The Gentzen sequent calculus (Gentzen 1934) is known to be equivalent to the axiomatic characterization of FOL, and it proved that any proof of a theorem using the transitivity of composition, or its equivalent, the so called Cut inference in Gentzen sequent calculus, may be transformed into a proof that avoids using this rule. Therefore, transitivity of composition is considered ‘logically harmless’, since the Gentzen sequent calculus effectively limits searching for a proof of any theorem to the expressions constituting the theorem you want to derive, called the *subformula* property. The inference rules in Gentzen sequent calculus are hence guaranteed to decompose the complexity of the expressions in a derivation, making the question
whether an expression is a theorem decidable. But from the point of view of linguistic applications the original Gentzen calculus harbored another drawback as generative system. It allowed structural inference rules that permuted the order of premises in a proof or rebracketed any triple (associativity), making it hard to capture linguistically core notions of dominance, governance or precedence between constituents of a sentence to be proven grammatical. To restore constitutive order to the premises, the premises had to be regarded to create an linearly ordered sequence or \( n \)-tuple, or a multiset (in mathematics, a multiset (or bag) is a generalization of a set. A member of a multiset can have more than one instances, while each member of a set has only one, unique instance.)

This is now customary within the current categorical grammars deriving from Gentzen’s system, reviewed below in the semantic methods that affected developments in natural language semantics (Moortgat 1997).

Perhaps the most familiar proof theoretic characterization of FOL is Natural Deduction, in which rules of inference systematically introduce or eliminate the connectives and quantifiers (Jeffrey 1967; Partee, ter Meulen & Wall 1990). This style of constructing proofs is often taught in introductory logic classes, as it does provide a certain intuitive heuristics in constructing proofs and closely follows the truth-conditional meaning given to the logical vocabulary, while systematically decomposing the conclusion and given assumptions until atomic conditions are obtained. Proving a theorem with natural deduction rules still requires a certain amount of ingenuity and insight, which may be trained by practice. But human beings will never be perfected to attain logical omniscience, i.e. the power to find each and every possible proof of a theorem. No actual success in finding a proof may mean either you have to work harder at finding it or that the expression is not a theorem, but you never know for sure which situation you are in
(Boolos & Jeffrey 1980; Stalnaker 1999).

The fundamental demand that grammars of natural languages must realistically model the human cognitive capacities to produce and understand language has led to a wealth of developments in searching how to cut down on the generative power of formal grammars and their corresponding automata. Early in the developments of generative grammar, the unrestricted deletion transformation was quickly considered the most dangerously powerful operation in an unrestricted rewrite system or Turing machine, as it permitted the deletion of any arbitrary expressions that were redundant in generating the required surface expression (Peters & Ritchie 1973). Although deletion as such has still not been eliminated altogether as possible effect of movement, it is now always constrained to leave a trace or some other formal expression, making deleted material recoverable. Hence no expressions may simply disappear in the context of a derivation. The Empty Category Principle (ECP) substantiated this requirement further, stating that all traces of moved noun-phrases and variables must be properly governed (Chomsky 1981; Haegeman 1991; van Riemsdijk & Williams 1986). This amounts to requiring them to be

\(c\)-commanded by the noun-phrase interpreted as binding them (c-command is a binary relation between nodes in a tree structure defined as follows: Node A c-commands node B iff A \(\neq\) B, A does not dominate B and B does not dominate A, and every node that dominates A also dominates B.). Extractions of an adjunct phrase out a \(wh\)-island as in

\(5\) * How, did Mary ask whether someone had fixed the car \(t_i\)?

or moving \(wh\)-expressions out of a \(that\)-clause as in

\(6\) * Who, does Mary believe that \(t_i\) will fix the car?
are clearly ungrammatical, because they violate this ECP condition, as the traces $t_i$ are co-indexed with and hence intended to be interpreted as bound by expressions outside their proper governance domain.

The classical logical notion of quantifier scope is much less restricted, as ordinarily quantifiers may bind variables in intensional context without raising any semantic problems of interpretation, as we saw above in (3c) (Dowty, Wall & Peters 1981; Partee, ter Meulen & Wall 1990). For instance, in intensional semantics the sentence (7)

(7) Mary believes that someone will fix the car.

has at least one so called ‘de re’ interpretation in which someone is ‘quantified in’ and assigned a referent in the actual world, of whom Mary believes that he will fix the car in a future world, where Mary’s beliefs have come true. Such wide scope interpretations of noun-phrases occurring inside a complementizer that-CP is considered in generative syntax a form of opaque or hidden movement at LF, regarded a matter of semantic interpretation, and hence external to grammar, i.e. not a question of derivation of syntactic surface word order (May 1985). Surface wide scope wh-quantifiers in such ‘de re’ constructions binding overt pronouns occurring within the intensional context i.e. within the that-clause are perfectly acceptable, as in (8).

(8) Of whom, does Mary believe that he will fix the car?

Anyone intending to convey that Mary’s belief regarded a particular person, rather than someone hypothetically assumed to exist, would be wise to use such an overt wide scope clause as in (8), according to a background pragmatic view that speakers should select the optimal syntactic form to express their thought and to avoid any possible misunderstanding.

This theoretical division of labor between tangible, syntactic movement to generate
proper surface word order and intangible movement to allow disambiguation of the semantic scope of quantifiers has perhaps been the core bone of contention over many years between on the one hand the generative formal methods, in which semantic ambiguity does not have to be syntactically derived, and on the other hand, categorial grammar and its later developments in Montague grammar, which required full compositionality, i.e. syntactic derivation must determine semantic interpretation, disambiguating quantifier scope by syntactic derivation. In generative syntax every derivational difference had to be meaningful, however implicit this core notion remained, but in categorial grammars certain derivational differences could be provably semantically equivalent and hence meaningless, often denigratingly called the *problem of spurious ambiguities*. To characterize the logical equivalence of syntactically distinguished derivations required an independent semantic characterization of their truth conditions, considered a suitable task of logic, falling outside the scope of linguistic grammar proper, according to most generativists. This problem of characterizing which expressions with different derivational histories would be true in exactly the same models, hence would be logically equivalent, simply does not arise in generative syntax, as it hides semantic ambiguities as LF movement not reflected in surface structure, avoiding syntactic disambiguation of semantic ambiguities.

A much stronger requirement on grammatical derivations is to demand methodologically that all and only the constitutive expressions of the derived expression must be used in a derivation, often called *surface compositionality* (Cresswell 1985; Partee 1979). This research program is aiming to eliminate from linguistic theory anything that is not absolutely necessary. Chomsky (1995) claimed that both deep structure, completely determined by lexical information, and surface structure, derived from it by
transformations, may be dispensed with. Given that a language consists of expressions which match sound structure to representations of their meaning, Universal Grammar should consist merely of a set of phonological, semantic, and syntactic features, together with an algorithm to assemble features into lexical expressions and a small set of operations, including *move* and *merge*, that constitute syntactic objects, the computational system of human languages. The central thesis of this minimalist framework is that the computational system is the optimal, most simple, solution to legibility conditions at the phonological and semantic interface. The goal is to explain all the observed properties of languages in terms of these legibility conditions, and properties of the computational system. Often advocated since the early 1990s is the *Lexicalist Hypothesis* requiring that syntactic transformations may operate only on syntactic constituents, and can only insert or delete designated elements, but cannot be used to insert, delete, permute, or substitute parts of words. This Lexicalist Hypothesis, which is certainly not unchallenged even among generativists, comes in two versions: (a) a weak one, prohibiting transformations to be used in derivational morphology, and (b) a strong version prohibiting use of transformations in inflection. It constitutes the most fundamentally challenging attempt from a syntactic perspective to approach surface compositionality as seen in the well-known theories of natural language semantics to which we now turn.

7. Linguistic applications of semantic methods.

The original insight of the Polish logician Alfred Tarski was that the truth conditional semantics of any language must be stated recursively in a distinct meta-language in terms of satisfaction of formulas consisting of predicates and free variables to avoid the paradoxical forms of self-reference alluded to above (Tarski 1956; Barwise &
By defining satisfaction directly, and deriving truth conditions from it, a proper recursive definition could be formulated for the semantics of any complex expression of the language. For instance, in FOL an assignment satisfies the complex sentence \( S \) and \( S' \) if and only if it satisfies \( S \) and it also satisfies \( S' \). For universal quantification it required an assignment \( f \) to satisfy the sentence ‘Every \( x \) sings’ if and only if for every individual that some other assignment \( f' \) assigns to the variable \( x \), while assigning the same things as \( f \) to the other variables, \( f' \) satisfies \( \text{sings}(x) \), i.e. the value of every such \( f'(x) \) is an element in the set of singers. Tarski’s definition of satisfaction is compositional, since for an assignment to satisfy a complex expression depends only on the syntactic composition of its constituents and their semantics, as Gottlob Frege had originally required (Frege 1892). Truth conditions can subsequently be stated relative to a model and an arbitrary given assignment, assigning all free variables their reference. Truth cannot be compositionally defined directly for ‘Every \( x \) sings’ in terms of the truth of \( \text{sings}(x) \), because \( \text{sings}(x) \) has a free variable \( x \), so its truth depends on which assignment happens to be the given one. The Tarskian truth-conditional semantics of FOL also provided the foundation for natural language semantics, limited to fragments that do not contain any truth- or falsity predicate, nor verbs like to lie, nor other expressions directly concerned with veridicality. The developments of File Change Semantics (Heim 1982), Discourse Representation Theory (Kamp 1981; Kamp & Reyle 1993), Situation Theory (Barwise & Perry 1983; Seligman & Moss 1997) and dynamic Montague Grammar (Chierchia 1995; Groenendijk & Stokhof 1991), that all allowed free variables or reference markers representing certain use of pronouns to be interpreted as if bound by a widest scope existential quantifier, even if they occurred in different sentences, fully exploit this fundamental Tarskian approach to compositional semantics.
by satisfaction.

Other formal semantics methods for FOL were subsequently developed in the second half of the 20th century as alternatives to Tarskian truth-conditional semantics. Beth (1970) designed a tableaux method in which a systematic search for counterexamples to the assumed validity of a reasoning pattern seeking to verify the premises, but falsify its conclusion leads in a finite number of decompositional steps either to such a counterexample, if one exists, or to closure, tantamount to the proof that no such counterexample exists (Beth 1970; Partee, ter Meulen & Wall 1990). This semantic tableaux method provided a procedure to enumerate the valid theorems of FOL, because it only required a finite number of substitutions in deriving a theorem: (i) the expression itself, (ii) all of its constituent expressions, and (iii) certain simple combinations of the constituents depending on the premises. Hence any tableau for a valid theorem eventually closes, and the method produces a positive answer. It does not however constitute a decision procedure for testing the validity of any derivation, since it does not enumerate the set of expressions that are not theorems of FOL.

Game-theoretic semantics characterizes the semantics of FOL and richer, intensional logics in terms of rules for playing a verification game between a truth-seeking player and falsification seeking, omniscient Nature (Hintikka & Kulas 1985; Hintikka & Sandu 1997; Hodges 1985). Its interactive and epistemic flavor made it especially suitable for the semantics of interrogatives in which requests for information are acts of inquiry resolved by the answerer, providing the solicited information (Hintikka 1976). Such information-theoretic methods are currently further explored in the generalized context of dynamic epistemic logic, where communicating agents each have access to partial, private and publicly shared information and seek to share or hide information they may
have depending on their communicative needs and intentions (van Ditmarsch, van der Hoek & Kooi 2007). Linguistic applications to the semantics of dialogue or multi-agent conversations in natural language already seem promising (Ginzburg & Sag 2000).

It was first shown in Skolem (1920) how second order methods could provide novel tools for logical analysis by rewriting any linear FOL formula with an existential quantifier in the scope of a universal quantifier into a formula with a quantification prefix consisting of existential quantifiers ranging over assignment functions, followed by only monadic (one place) universal quantifiers binding individual variables. The dependent first order existential quantifier is eliminated by allowing such quantification over second-order choice-functions that assign the value of the existentially quantified dependent variable as a function of the referent assigned to the monadic, universally quantified individual variable preceding it. Linguistic applications using such Skolem functions have been given in the semantics of questions (Engdahl 1986) and the resolution of functional pronouns (Winter 1997). The general strategy to liberate FOL from the linear dependencies of quantifiers by allowing higher order quantification or partially ordered, i.e. branching quantifier prefixes, was linguistically exploited in the semantic research on branching quantifiers (Hintikka & Sandu 1997; Barwise 1979). From a linguistic point of view the identification of linear quantifier scope with bound occurrences of variables in their bracketed ranges never really seemed justified, since informational dependencies such as coreference of pronouns bound by an indefinite noun-phrase readily cross sentential boundaries, as we saw in (2c). Furthermore, retaining perfect information on the referents already assigned to all preceding pronouns smells of unrealistic logical omniscience, where human memory limitations and contextual constraints are disregarded. It is obviously much too strong as epistemic requirement on ordinary people
sharing their necessarily always limited, partial information (Seligman & Moss 1997). Game-theoretic semantics rightly insisted that a proper understanding of the logic of information independence and hence of the lack of information was just as much needed for natural language applications, as the logic of binding and other informational dependencies. Such strategic reconsiderations of the limitations of foundational assumptions of logical systems have prompted innovative research in logical research programs, considerably expanding the formal methods available in natural language semantics (Muskens, van Benthem & Visser 1997).

By exploiting the full scale higher order quantification of the type-theoretic categorial grammars Montague Grammar first provided a fully compositional account of the translation of syntactically disambiguated natural language expressions to logical expressions by treating referential noun-phrases semantically on a par with quantificational ones as generalized quantifiers denoting properties of sets of individuals. This was accomplished obviously at the cost of generating spurious ambiguities ad libitum, giving up on the program of modeling linguistic competence realistically (van Benthem & ter Meulen 1985; Keenan & Westerståhl 1997; Link 1991; Montague 1974; Partee 1979). Its type theory, based only on two primitive types, e for individual denoting expressions and t for truth-value denoting expressions, forged a perfect fit between the syntactic categories and the function-argument structure of their semantics. For instance, all nouns are considered syntactic objects that require a determiner on their left side to produce a noun-phrase and semantically denote a set of entities, of type <e, t>, which is an element in the generalized quantifier of type <<e, t>, t> denoted by the entire noun-phrase. Proper names, freely referring pronouns, universal and existential NPs are hence treated semantically on a par as denoting a set of sets of individuals. This fruitful strategy
led to a significant expansion of the fragments of natural languages that were provided with a compositional model-theoretic semantics, including many kinds of adverbial phrases, degree and measurement expressions, unusual and complex quantifier phrases, presuppositions, questions, imperatives, causal and temporal expressions, but also lexical relations that affected reasoning patterns (Chierchia 1995; Krifka 1989). Logical properties of generalized quantifiers prove to be very useful in explaining, for instance, not only which noun-phrases are acceptable in pleonastic or existential contexts, but also why the processing time of noun-phrases may vary in a given experimental situation and how their semantic complexity may also constrain their learnability. In pressing on for a proper, linguistically adequate account of pronouns in discourse, and for a cognitively realistic logic of information sharing in changing contexts, new tools that allowed for non-linear structures to represent information content play an important conceptually clarifying role in separating quantifier scope from the occurrence of variables in the linear or partial order of formulas of a logical language, while retaining the core model-theoretic insights in modeling inference as concept based on Tarskian satisfaction conditions.

8. Conclusions.

The development of formal methods in logic has contributed essentially to the emancipation of linguistic research into an academic community where formal methods were given their proper place as explanatory tool in scientific theories of meaning and interpretation. Although logical languages are often designed with a particular purpose in mind, they reflect certain interesting computational or semantic properties also exhibited, though sometimes implicitly, in natural languages. The properties of natural languages
that lend themselves for analysis and explanation by formal methods have increased steadily over the past century, as the formal tools of logical systems were more finely chiseled to fit the purpose of linguistic explanation better. Even more properties will most likely become accessible for linguistic explanation by formal methods over the next century. The issues of cognitive complexity, characterized at many different levels from the neurobiological, molecular structure detected in neuro-imaging to interactive behavioral studies, and experimental investigations of processing time provide a new set of empirical considerations in the application of formal methods to natural language. They drive experimental innovations and require an interdisciplinary research agenda to integrate the various modes of explanation into a coherent model of human language use and communication of information.

The current developments in dynamic natural language semantics constitute major improvements in expanding linguistic application to a wider range of discourse phenomena. The forms of reasoning in which context dependent expressions may change their reference during the processing of the premises are now considered to be interesting aspects of natural languages, that logical systems are challenged to simulate, rather than avoid, as our great-grandfathers’ advice originally directed us to do. There is renewed attention to limit in a principled and empirically justified way the search space complexity to decidable fragments of FOL and to restrict the higher order methods in order to reduce the complexity to model cognitively realistic human processing power. Such developments in natural language will converge eventually with the syntactic research programs focusing on universals of language as constants of human linguistic competence.
770 7. References.


Alice G.B. ter Meulen, Geneva (Switzerland).