The error-driven ranking model of the acquisition of phonotactics: some computational results

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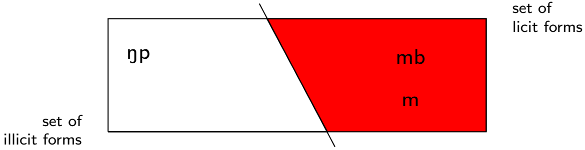
1.

Error-driven learning of phonotactics

Phonology = phonotactics + repairs

- Phonotactics: knowledge of the language specific acceptability of forms, construed as a categorical distinction: licit vs illicit [Gorman 2013]



- Repairs: knowledge of the language specific repairs of illicit forms, as revealed by alternations from URs to SRs: [Kenstowicz and Kisseberth 1977]

Indonesian:	*Quechua:*
/məŋ+pilih/ → [məmilih] 'to vote'	/kaŋ+pa/ → [kamba] 'yours'
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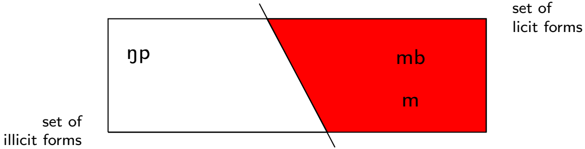

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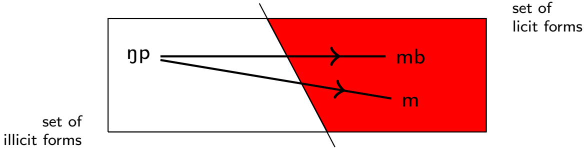

Learnability logics and acquisition phenomenology

- **Knowledge of repairs ⇒ knowledge of phonotactics:**
  - if you know alternations, you know what is licit and what is not
  - i.e. phonotactics is the *range* of a phonological grammar G

- **Learnability logics:** [Prince and Tesar 2008]
  - start by focusing on a smaller aspect of the problem of learning phonology, namely start with the problem of learning the phonotactics
  - then use knowledge of phonotactics to bootstrap into knowledge of the whole system of phonological alternations

- **Acquisition phenomenology:**
  - nine-month-olds react differently to illicit sounds [Jusczyk et al. 1993]
  - they thus display knowledge of the target phonotactics
  - at a stage when morphology is lagging behind [Kazazis 1969; Hayes 2004]
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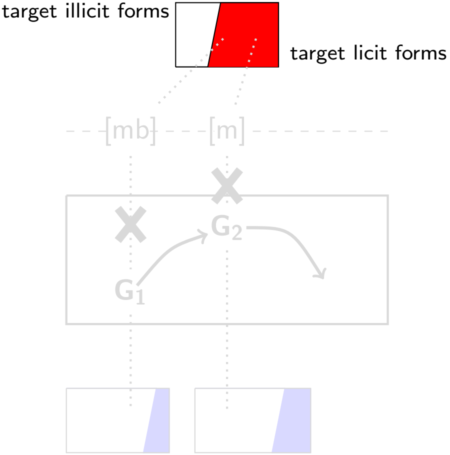

target adult phonotactics

stream of input data

space of all possible grammars

currently predicted phonotactics

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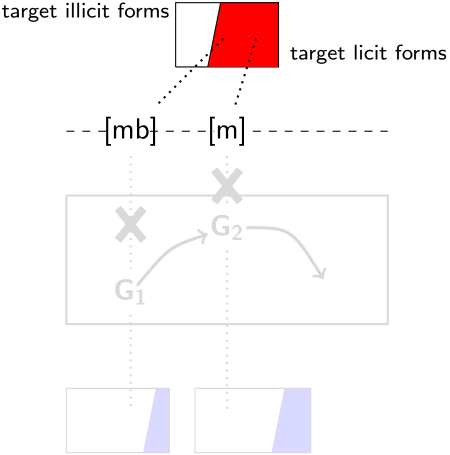

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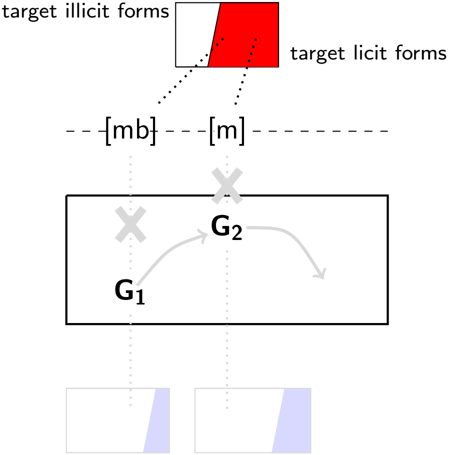

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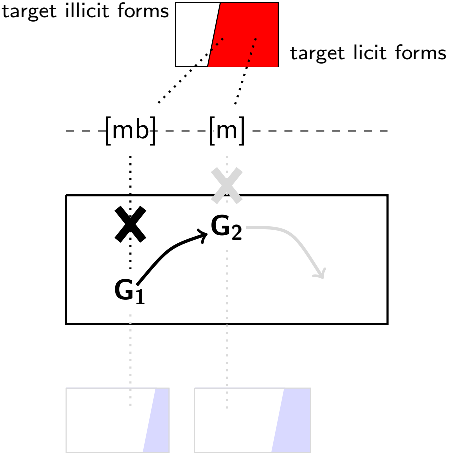

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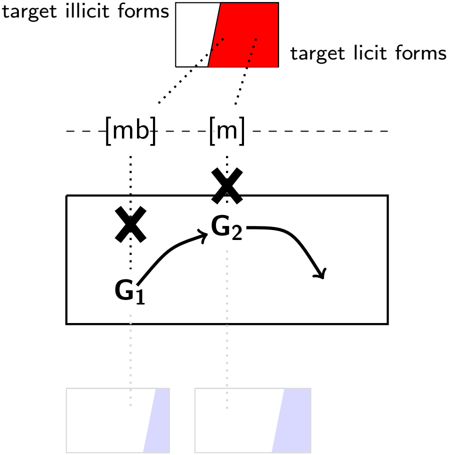

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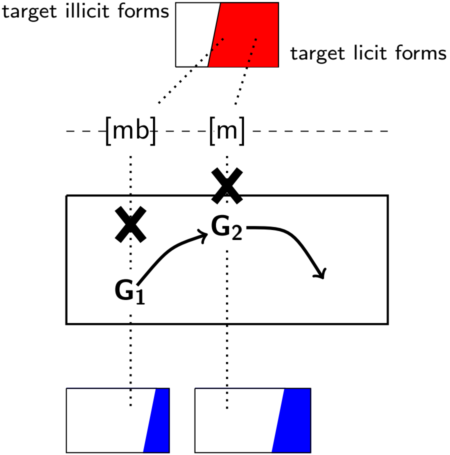

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Two modeling virtues

- **Memory-free**: [Gibson and Wexler 1994]
  - the error-driven learner doesn't keep track of previously seen data
  - doesn't need a lexicon of stored forms
  - it is therefore suitable to model early acquisition stages

- **Gradual**: [McLeod et al. 2001]
  - the error-driven learner describes a sequence of grammars
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- **Today**, I will focus [Cesa-Bianchi and Lugosi 2006; Tesar and Smolensky 1998; Boersma 1998]
  - on the early purely phonotactic learning stage
  - through error-driven learning
  - within the OT implementation of constraint-based phonology

⇒ error-driven ranking algorithms (EDRAs) for phonotactic learning

- **Does it work?** Are there guarantees that the learned phonotactics coincides with the target phonotactics?

- **The right initial question:** "a [learning model] that is powerful enough to account for the *fact* of language acquisition may be a more promising first approximation of an ultimately viable theory than one that is able to describe the *course* of language acquisition, which has been the traditional focus of developmental psycholinguistics" [Pinker 1979]

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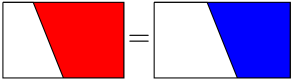

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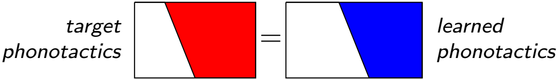

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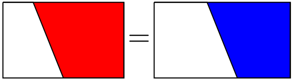

learned phonotactics

- **Two sub-questions:**
  - Consistency:

target

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  - Restrictiveness:

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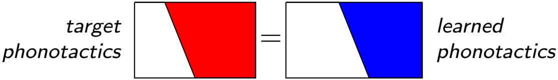

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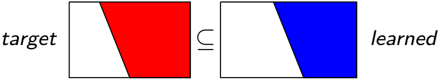

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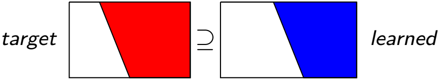

2.
OT implementation

Shortest overview of constraint-based phonology

- The core data unit is a comparison between two candidate mappings for the same underlying form: (/ŋp/, [m]) versus (/ŋp/, [mb])
- Languages differ in which candidate beats the other (stricken out):
  - (/ŋp/, [m]) beats (/ŋp/, ~~[mb]~~) according to Indonesian
  - (/ŋp/, [mb]) beats (/ŋp/, ~~[m]~~) according to Quechua
- A set of constraints map these phonological comparisons into $\mathbb{R}^n$

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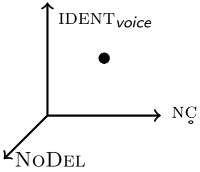

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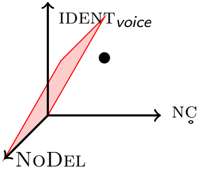

- A grammar corresponds to a hyperplane that leaves all its comparisons on one side
- A grammar is described by assigning constraint weights which define the corresponding hyperplane

[Prince and Smolensky 2004]

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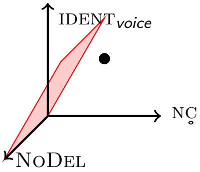

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Typological restrictions on the hyperplanes

- Are there restrictions on the hyperplanes that correspond to actual phonological grammars?

- **The OT empirical generalization:** [Prince and Smolensky 2004]
  natural language phonologies display no additive/gang-up effects

- Equivalently, natural language phonologies...
  - ...correspond to hyperplanes whose weights decay exponentially
  - ...correspond to very titled hyperplanes
  - ...can be re-parametrized with rankings
  - ...enforce strict domination

  This is a currently open typological debate [Pater 2007]

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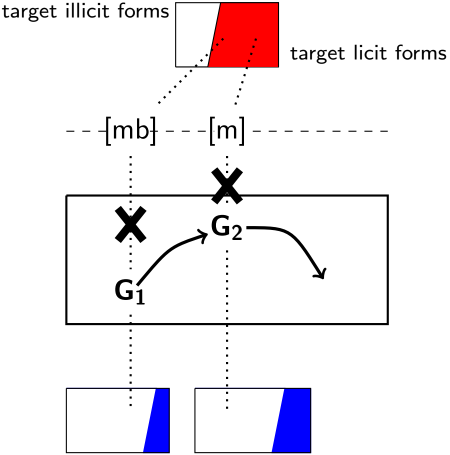

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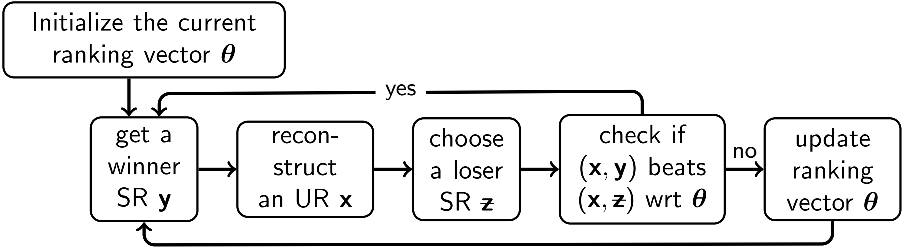
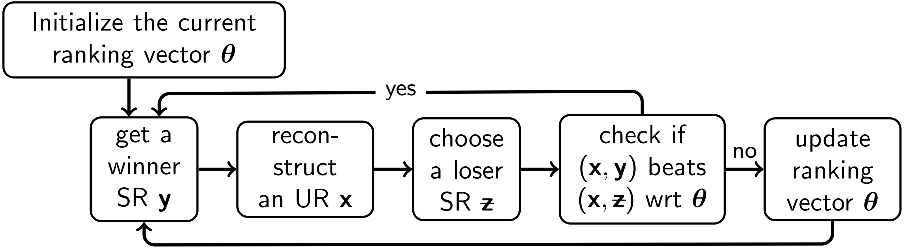
EDRA model


Initialize the current ranking vector $\boldsymbol{\theta}$
yes
get a winner SR $\mathbf{y}$
recon- struct an UR $\mathbf{x}$
choose a loser SR $\mathbf{z}$
check if $(\mathbf{x}, \mathbf{y})$ beats $(\mathbf{x}, \mathbf{z})$ wrt $\boldsymbol{\theta}$
no
update ranking vector $\boldsymbol{\theta}$

EDRA model: representation of the current grammar



- The algorithm maintains a current hypothesis of the target OT grammar, namely a current constraint ranking
- This current ranking is represented numerically: [Boersma 1998]
  - each constraint is assigned a ranking value
  - big ranking value $\leftrightarrow$ high ranked constraint
  - these ranking values are collected in a ranking vector (RV) θ
- Current RV is initialized and updated through a five step loop

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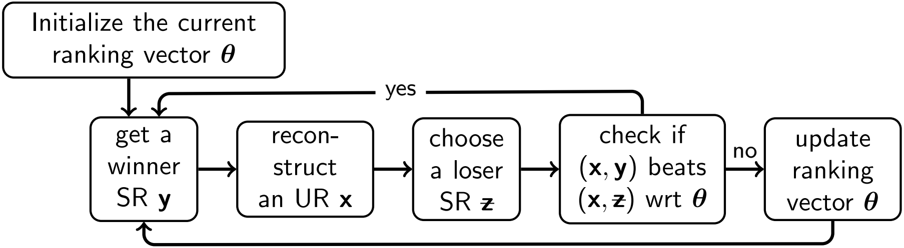

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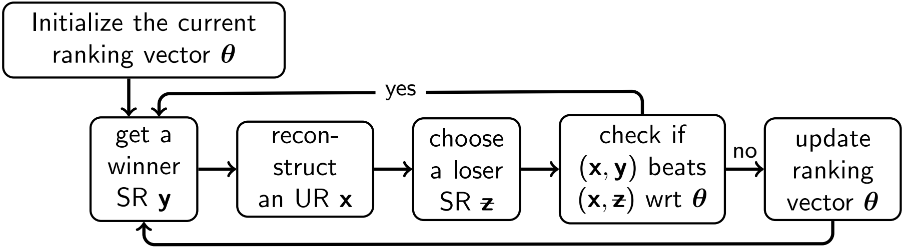

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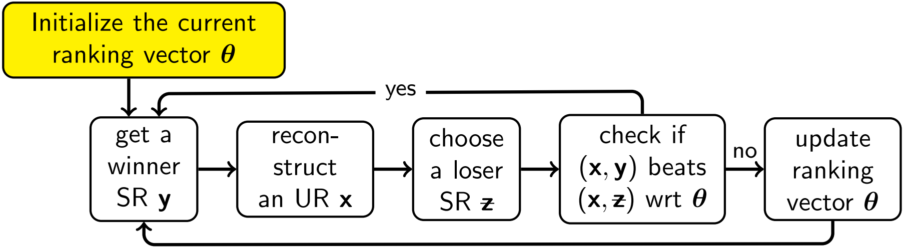
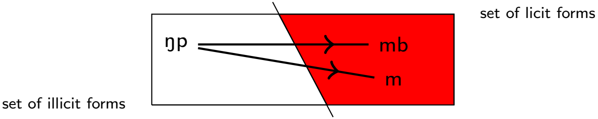


- Optimal repairs are subject to two desiderata:
  - cross the line and always end up with a licit form
  - but do not land too far away from the target



- Two types of OT constraints:
  - markedness: work towards neutralization of contrast (e.g. *NC̥)
  - faithfulness: work towards preservation of contrast (e.g. IDENT$_{\text{voi̥ce}}$)

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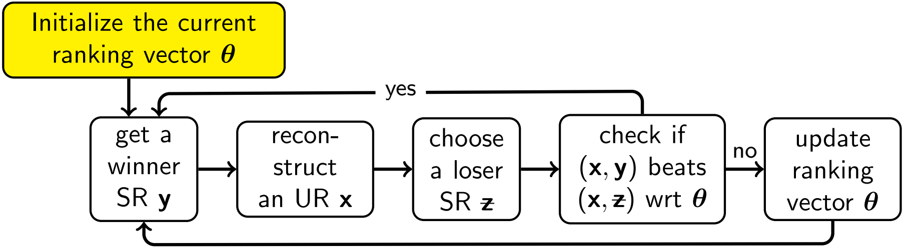

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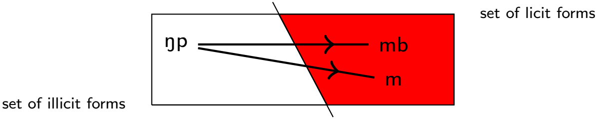

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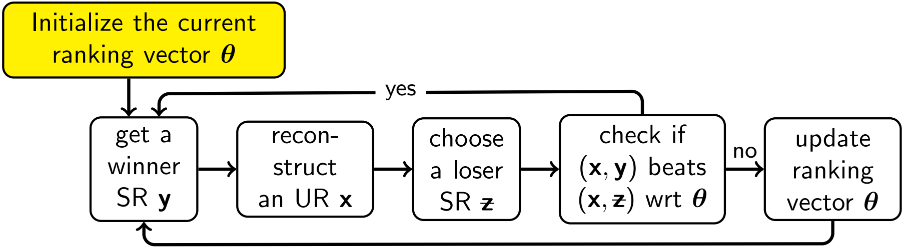
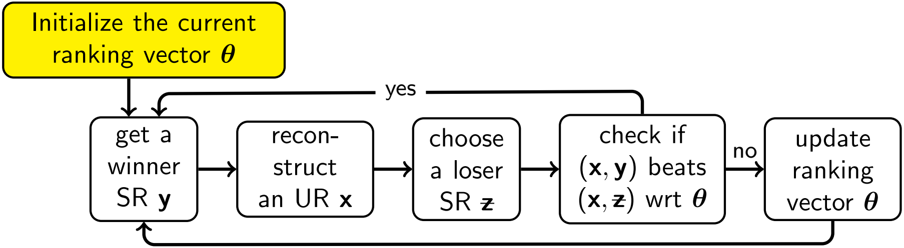


- Initialization of current RV: [Smolensky 1996b,a; Jusczyk, Smolensky, and Allocco 2002]
  - faithfulness constraints start with a small initial ranking value
  - markedness constraints start with a large initial ranking value

$\Rightarrow$ the initial $\mathcal{M} \gg \mathcal{F}$ predicts only unmarked forms to be licit

- The psycholinguistic literature seems to make the opposite assumption! [Davidson, Jusczyk, and Smolensky 2004; Mazuka, Cao, Dupoux, and Christophe 2011]

EDRA model: initialization



- Initialization of current RV: [Smolensky 1996b,a; Jusczyk, Smolensky, and Allocco 2002]
  - faithfulness constraints start with a small initial ranking value
  - markedness constraints start with a large initial ranking value

$\Rightarrow$ the initial $\mathcal{M} \gg \mathcal{F}$ predicts only unmarked forms to be licit

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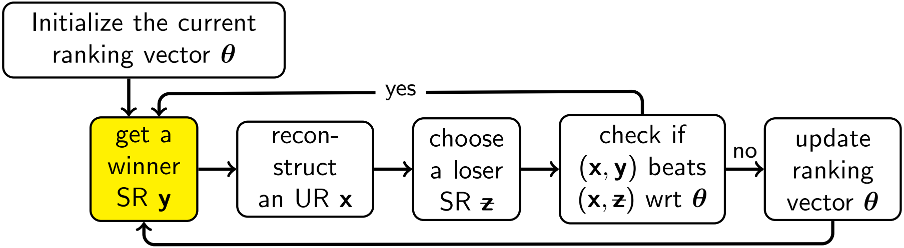
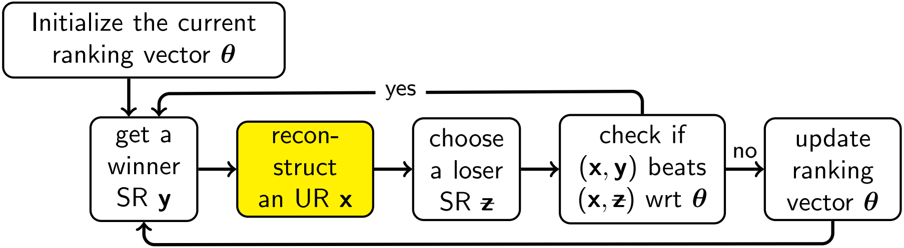
EDRA model: training SR



- At each iteration, the model is trained on a SR $\mathbf{y}$ assumed to be licit relative to the target phonotactics
- No assumptions whatsoever are made on this infinite sequence of training data [Cesa-Bianchi and Lugosi 2006]

EDRA model: reconstruction of the UR



- At each iteration, the model assumes a fully faithful UR $\mathbf{x}$ for the current training SR $\mathbf{y}$: $\boxed{\mathbf{x} = \mathbf{y}}$ [Prince and Tesar 2004]
- This assumption only makes sense if there are no representational differences between SRs and URs [Moreton 2004]
- This assumption is sound if the target grammar is idempotent: does not repair phonotactically licit forms [Magri 2016]
- Idempotency holds "by and large" but not always: chain shifts (a→e→i)

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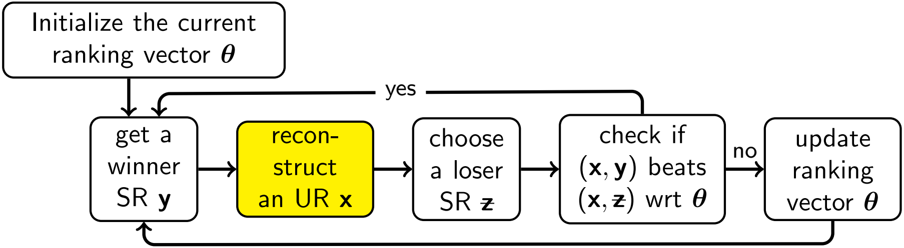

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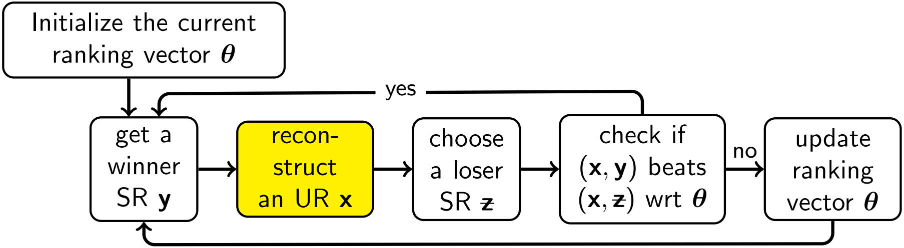

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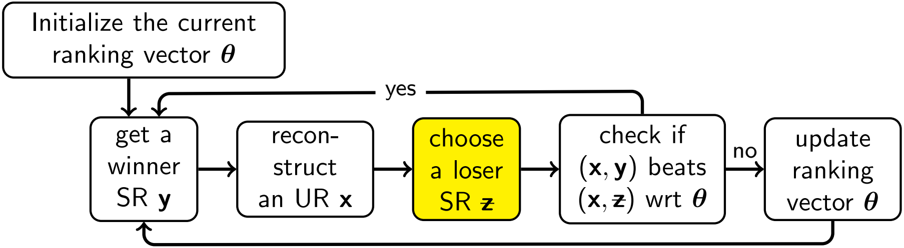

- At each iteration, the model compares the intended winner **y** with some properly chosen loser **z**
- **Rule I:** if there exists a loser SR **z** able to trigger an update, choose one such loser, so as not to waste data [Tesar and Smolensky 1998]
- **Rule II:** If there are multiple such losers, choose a loser SR **z** which is "as close as possible" to the intended winner SR **y** [Magri and Kager 2015]

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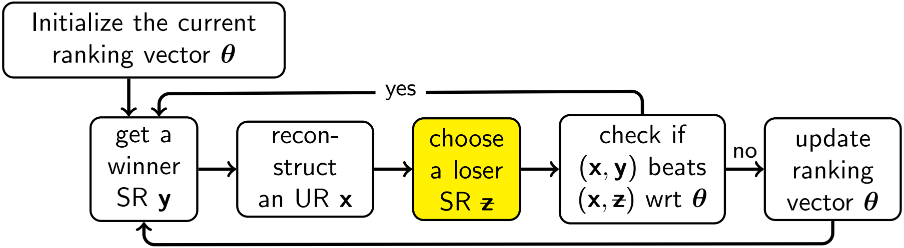

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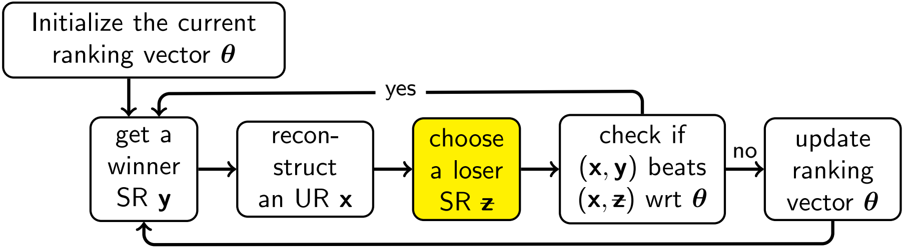

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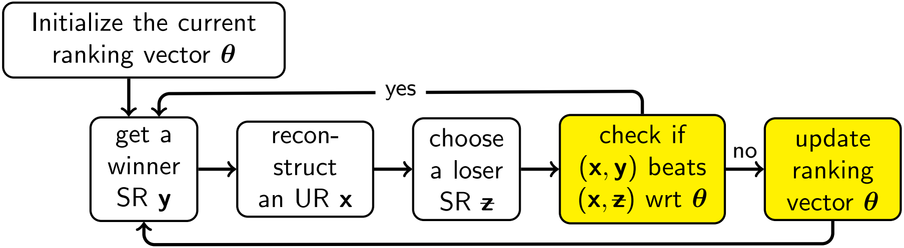
EDRA model: check and update

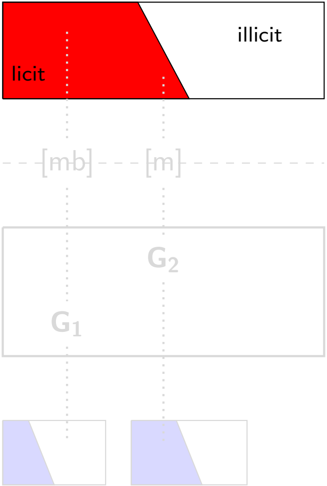


- At each iteration, model checks if the winner mapping $(\mathbf{x}, \mathbf{y})$ beats the loser mapping ($\mathbf{x}$, **ƶ**) according to the current RV
- If that is the case, the learner has nothing to learn from the current piece of data, loops back, and waits for more data
- Otherwise, the learner properly updates the current RV in response to its failure on the current comparison $(\mathbf{x}, \mathbf{y})$ versus ($\mathbf{x}$, **ƶ**)

3.
Convergence, efficiency, consistency

Consistency, convergence, efficiency

- **Learning scenario:**



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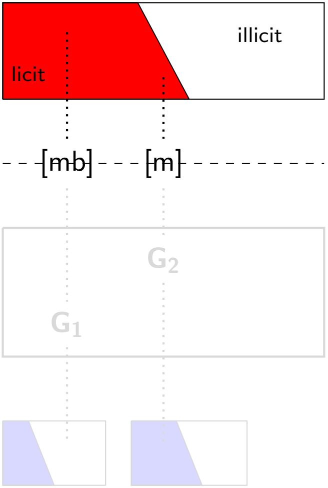

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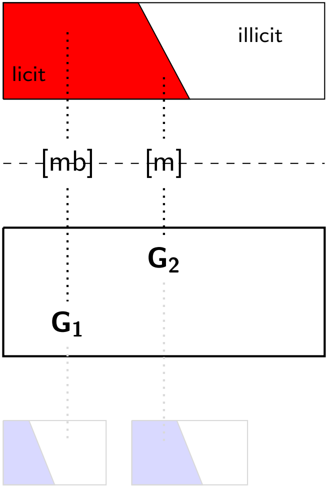

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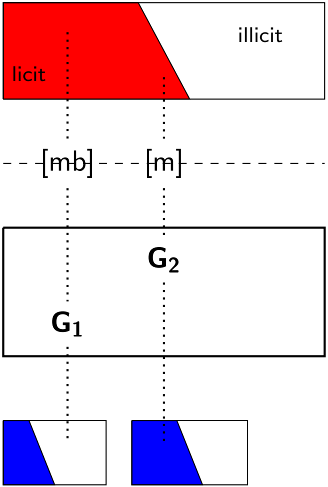

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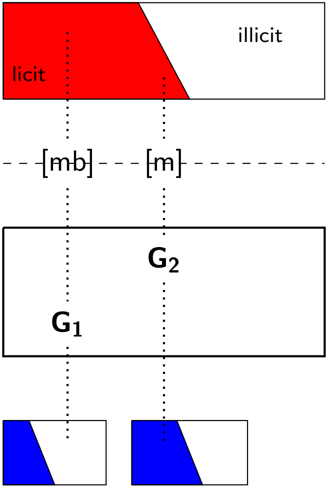

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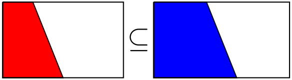

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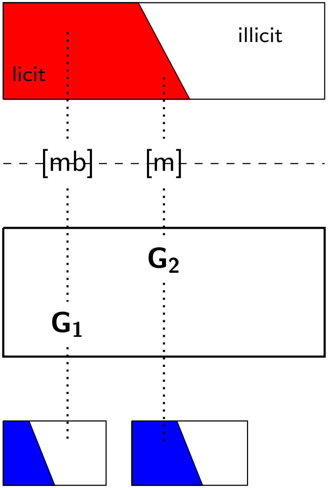

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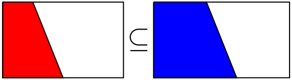

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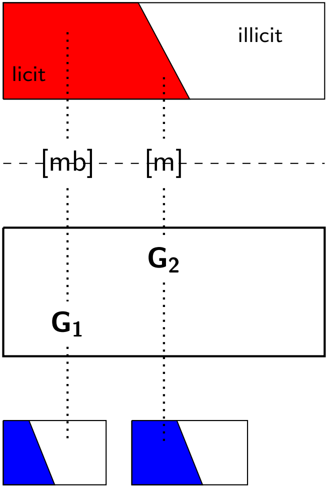

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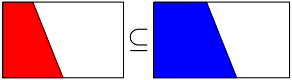

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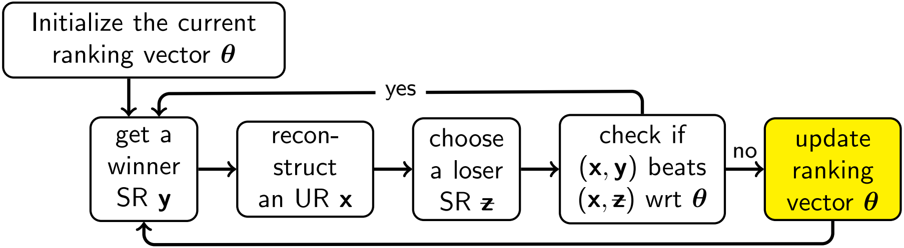
How to tune the promotion amount to get convergence



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- **Examples:**
  - $p = 0$: (gradual) EDCD [Tesar and Smolensky 1998]
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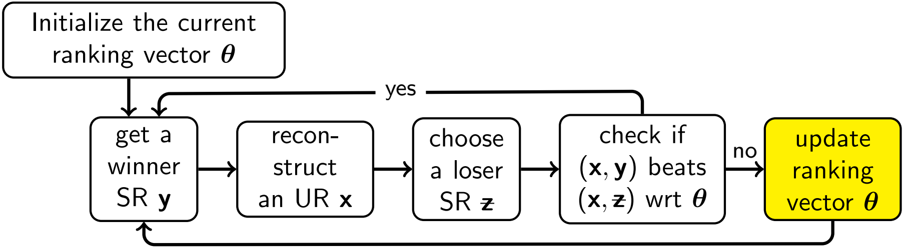

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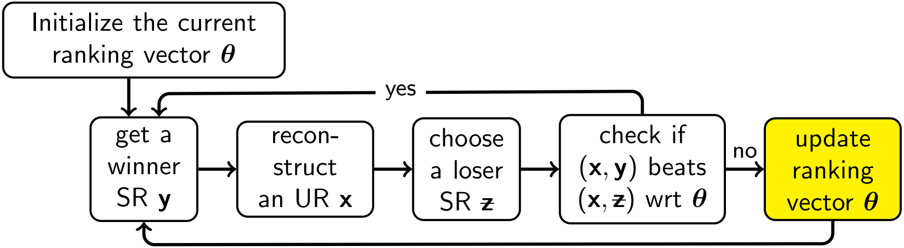

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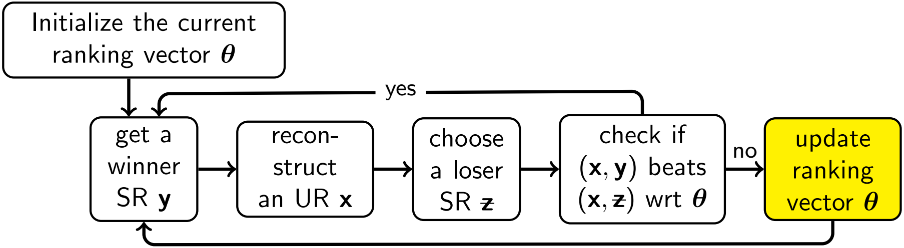

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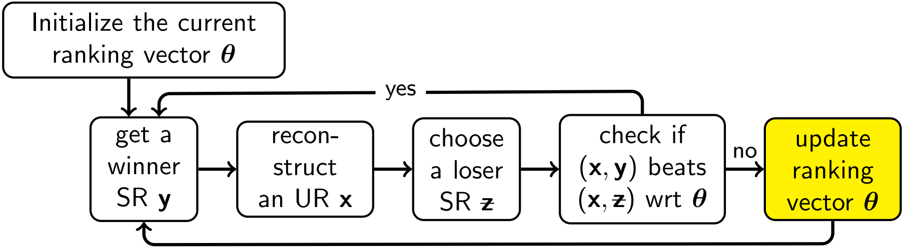

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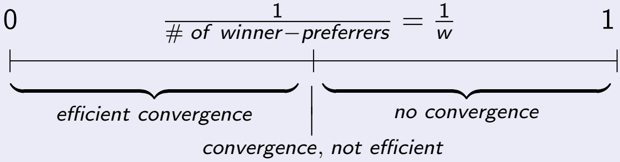
A complete theory of convergence

First result of this talk

Efficient convergence holds iff the promotion amount p is calibrated namely smaller than the inverse of the number of winner-preferrers:



[Tesar and Smolensky 1998; Pater 2008; Magri 2012]

- Some boasting:
  - complete theory: necessary and sufficient
  - no assumptions on initialization, URs, losers
  - extends to noisy setting [Magri to appearb]
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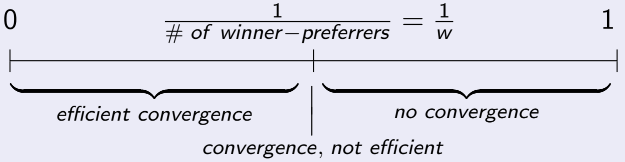

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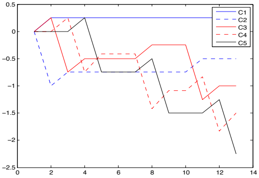
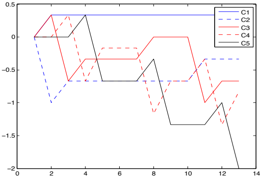
- $p = \frac{1}{w+2}$ 

$p = \frac{1}{w+1}$ 

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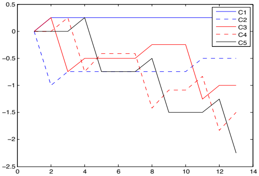
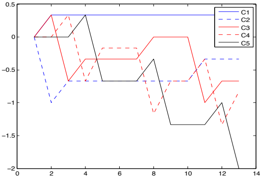
- $p = \frac{1}{w+2}$  $p = \frac{1}{w+1}$ 
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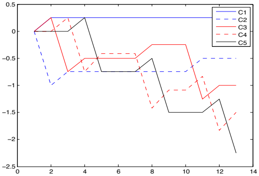

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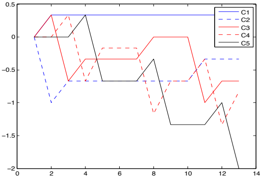

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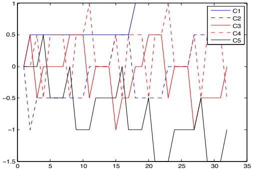

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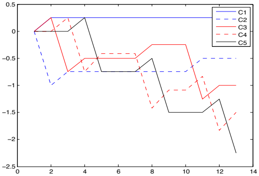
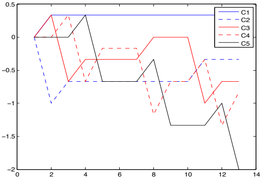
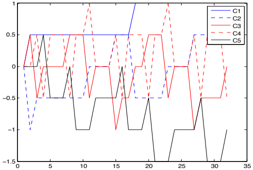
- $p = \frac{1}{w+2}$ 

  $p = \frac{1}{w+1}$ 

- $p = \frac{1}{w+0}$ 

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Discussion/II

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- "a [. . .] possibility is that demands of learnability provide a pressure for strict domination among constraints, [although] it remains an open problem to formally characterize exactly what is essential about strict domination to guarantee efficient learning" [Legendre et al. 2006]
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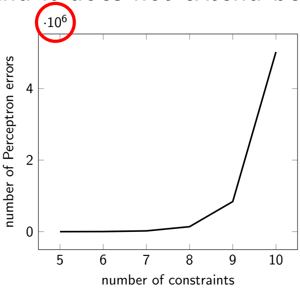

4.
Restrictiveness

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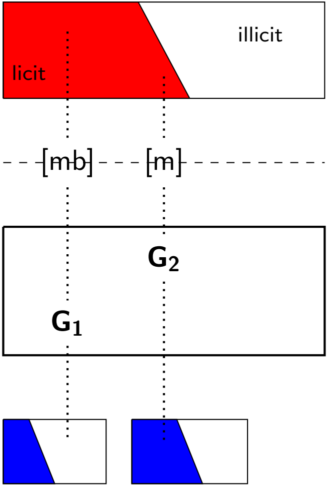

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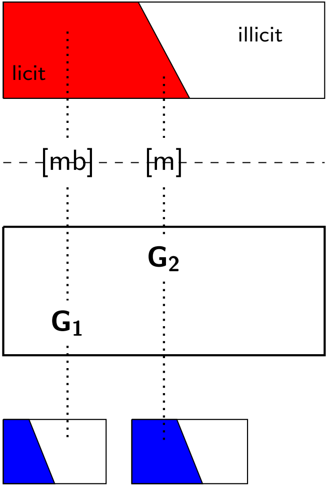

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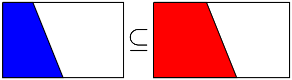

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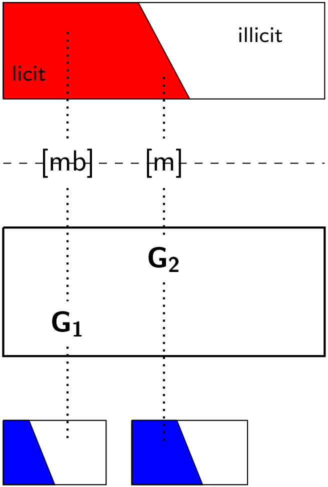

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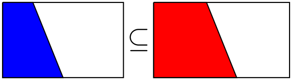

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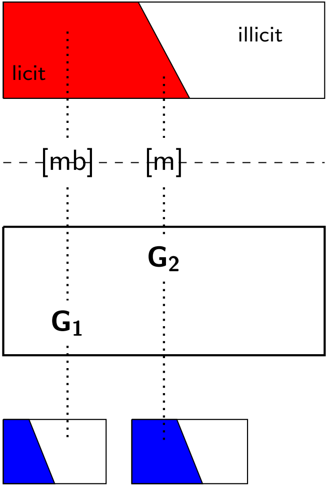

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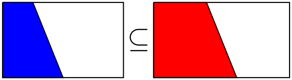

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  - a finite OT typology specified through candidates and constraints
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- **Size:** $\max\{\text{number of constraints}, \text{number of candidates}\}$

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This formulation of the Subset problem in OT is intractable [Magri 2013]

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Faithfulness constraints and phonotactics

- **Velar inventories:**
  - let's focus of segment inventories
  - out of the four velar obstruents: [g], [k], [ɣ], [x]
  - here are some representative examples: [g k ~~ɣ~~ ~~x~~] [~~g~~ k ~~ɣ~~ ~~x~~]

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An assumption on the target phonotactic pattern

- **Intuition:**
  - OT constraints come in two varieties: $\mathcal{F}$ and $\mathcal{M}$ constraints
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- **Formalization:**

A phonotactic pattern is $\mathcal{F}$-irrelevant (relative to a certain constraint set) provided it can be generated by

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Third result of this talk

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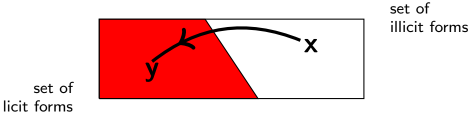

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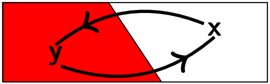

set of licit forms

- ☐ symmetry requires that vice versa **x** be a candidate of **y**
- ☐ Plausible if candidacy defined in terms of phonological operations

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- EDRA only demotes as much as needed: [Tesar and Smolensky 1998]
  - the top ranked constraint is never demoted
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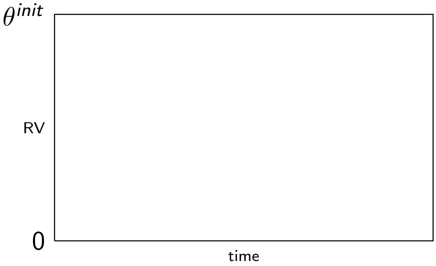

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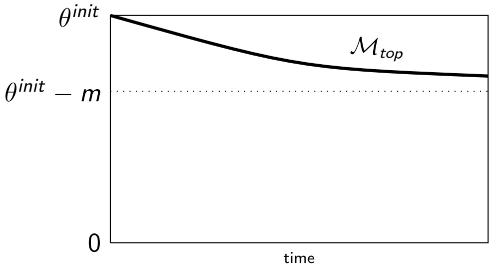

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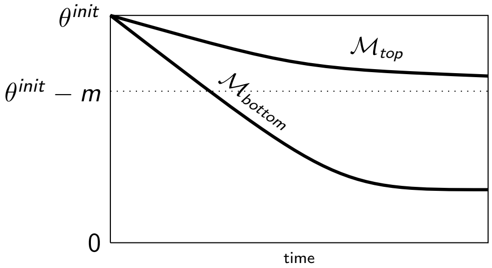

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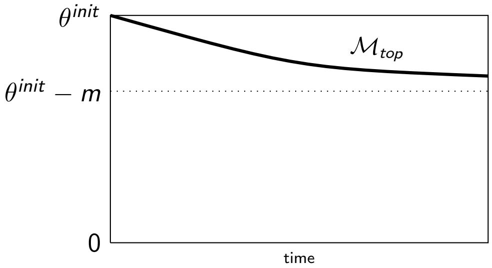

- $\mathcal{M}_{top}$ need to be ranked high and indeed stay high

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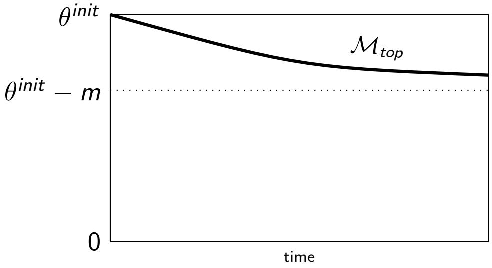

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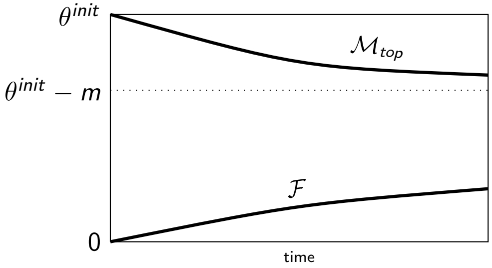


- If promotion amount p is null or small, $\mathcal{F}$ stays lower than $\theta^{init} - m$:

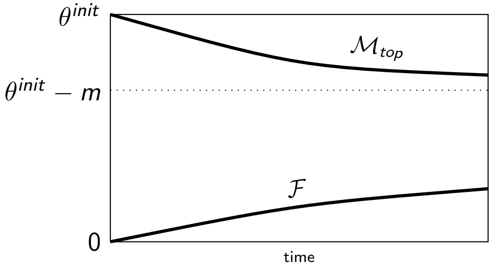
$$p = \frac{1}{w + \Delta} \qquad \Delta(m) \simeq \frac{m}{k \log m}$$

- As good as we might have hoped for:
  - $\Delta(m)$ needs to increase with m and cannot be constant
  - yet, the rate of increase (derivative) of $\Delta(m)$ decreases with m

- The target ranking condition $\mathcal{M}_{top} \gg \mathcal{F}$ is thus learned

Step 1: EDRAs learn $\mathcal{M}_{top} \gg \mathcal{F}$

- As we have seen, $\mathcal{M}_{top}$ stay above $\theta^{init} - m$:



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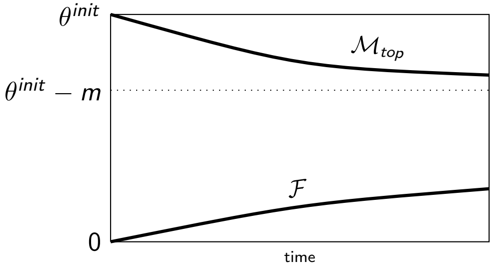

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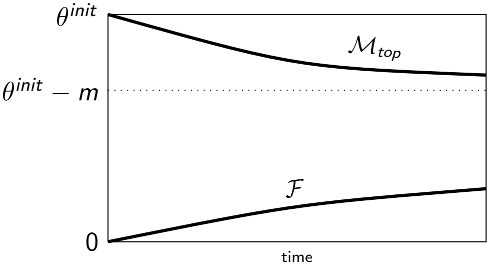

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Step 2: $\mathcal{M}_{top} \gg \mathcal{F}$ suffices for restrictiveness

Suppose **x** is illicit and thus neutralized to **y** by the target phonology

target ranking	**prefers** /**x**/→[**x**] vs	/**x**/→[**y**]
M		$\surd$
$M' M''\ldots$	$\surd$	
$F' F''\ldots$	$\surd$	

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target ranking	**learned ranking**	**prefers** /**x**/→[**x**]	vs /**x**/→[**y**]
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M	M		√
$M' M'' \ldots$		√	
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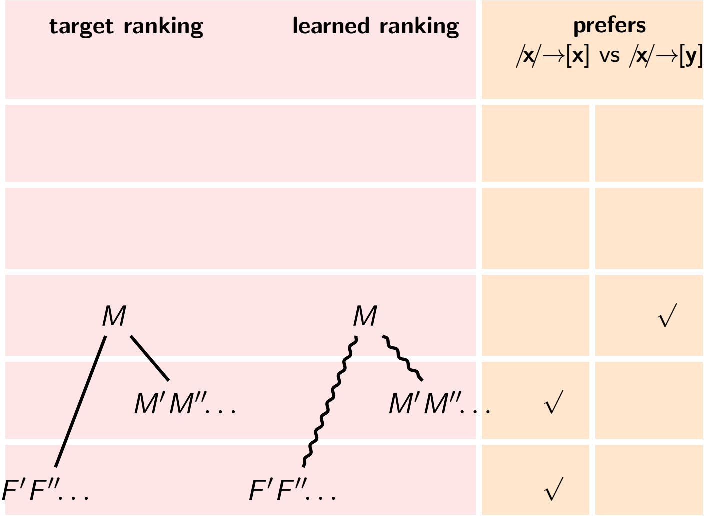

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	M'	√	
M	M		√
$M' M'' \ldots$	$M'' \ldots$	√	
$F' F'' \ldots$	$F' F'' \ldots$	√	

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target ranking	**learned ranking**	**prefers** /**x**/→[**x**] vs /**x**/→[**y**]		**prefers** /**y**/→[**x**] vs /**y**/→[**y**]	
	M'	$\surd$			
M	M		$\surd$		
$M'\,M''\ldots$	$M''\ldots$	$\surd$			
$F'\,F''\ldots$	$F'\,F''\ldots$	$\surd$			

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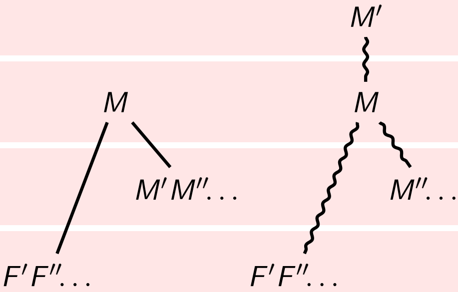

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	M'	√		√	
M	M		√		
$M' M'' \ldots$	$M'' \ldots$	√			
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	C				√
	M'	√		√	
M	M		√		
$M' M'' \ldots$	$M'' \ldots$	√			
$F' F'' \ldots$	$F' F'' \ldots$	√			

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	F				√
	M'	√		√	
M	M		√		
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	M'	√		√	
M	M		√		
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$F' F'' \dots$	$F' F'' \dots$	√			

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Suppose **x** is illicit and thus neutralized to **y** by the target phonology

target ranking	**learned ranking**	**prefers** /**x**/→[**x**] vs /**x**/→[**y**]		**prefers** /**y**/→[**x**] vs /**y**/→[**y**]	
	$\overline{M}$				√
	M'	√		√	
M	M		√		
$M' M'' \ldots$	$M'' \ldots$	√			
$F' F'' \ldots$	$F' F'' \ldots$	√			

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	$\overline{M}$		√		√
	M'	√		√	
M	M		√		
$M' M'' \ldots$	$M'' \ldots$	√			
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	$\overline{M}$		√
	M'	√	
M			√
$M' M'' \dots$	$M'' \dots$	√	
$F' F'' \dots$	$F' F'' \dots$	√	

5.
Conclusions

Concluding remark I

Second result of this talk

The OT problem of the acquisition of phonotactics (stated as a Subset problem) is intractable, even in the best conditions [Magri 2013]

- GL usually motivated through poverty of the stimulus arguments: child's linguistic input is ambiguous, incomplete, degenerate [Thomas 2002]
- Poverty of the stimulus arguments are difficult to make:
  - empirical side: they are about child input
  - theoretical side: what suffices for learnability [Clark and Lappin 2011]
- Results such as the one above show that learning is hard even when the input is rich and idealized:
  - pairs of underlying and surface representations
  - pristine and uncorrupted
  - no hidden structure
- From poverty of the stimulus to hardness of the task: intractability results provide further ammunition for GL

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Concluding remark II

First result of this talk

The EDRA model converges efficiently under the OT mode of constraint interaction ("tilted planes" assumption) [Tesar and Smolensky 1998; Pater 2008; Magri 2012]

Third result of this talk

The EDRA model is restrictive when the target/training phonotactic pattern is $\mathcal{F}$-irrelevant (under mild additional assumptions) [Magri in preparation]

- Current skepticism about phonological universals: [Evans and Levinson 2009]
  - "the study of universals is fraught with difficulties" [Hyman 2008]
  - "there appear to be so few absolute universals" [Maddieson 1984]
  - "the quest for universals has failed" [van Oostendorp 2013]
- This is because phonology has focused on the wrong type of universals: "every language has a coronal stop" [Hyman 2008; Blevins 2009]

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- This is because phonology has focused on the wrong universals: "every language has a coronal stop" [Hyman 2008; Blevins 2009]
- The right universals are motivated by computational considerations (learnability, algorithm for production and interpretation)

Thanks!

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