Idempotency and the triangle inequality

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1.

Idempotent phonological grammars

Phonology/I: phonotactics

- Phonological forms ($\simeq$ strings of segments) differ in acceptability in a language specific way: [Halle 1978]
  - [blɪk], *[bnɪk] in English
  - [pat], *[pad] in Dutch
- A large current debate on the nature of this acceptability judgements: categorical versus gradient [Gorman 2013; Hayes and Wilson 2008]
- This talk assumes a categorical distinction: the set of phonological forms is split into licit versus illicit ones
- Knowledge of this distinction is productive and thus grammatical, as shown by nonce words [Ohala and Ohala 1986]
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- Phonotactically illicit forms are repaired in a language specific way:
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- These mappings are revealed by alternations:

/pad/ → [pat]	/pad+ən/ → [padən]
/lad/ → [lat]	/lad+ən/ → [ladən]

- Knowledge of these mappings is productive and thus grammatical, as shown by wug-testing [Berko 1958]
- Classical terminology: [Kenstowicz and Kisseberth 1977; Burzio 1996]
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Formalism/I: candidates

- Intuition: for each possible UR, we consider a list of possible mappings that different grammars can pick from
- Candidates
  - are pairs $(\mathbf{a}, \mathbf{b})$ of an UR $\mathbf{a}$ and a SR $\mathbf{b}$
  - they are collected together in a candidate set denoted $\mathfrak{Can}$
- Example:

$$\mathfrak{Can} = \boxed{\begin{array}{cc} \ldots & (\mathbf{a}, \mathbf{b}) = (/\text{pad}/, [\text{pat}]) \\ (\mathbf{a}, \mathbf{b}) = (/\text{pad}/, [\text{pa}]) & \ldots \end{array}}$$

- Remarks:
  - $\mathfrak{Can}$ can be infinite
  - $\mathfrak{Can}$ contains multiple candidates which share the same UR
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- A stragihtforward formalization of phonological knowledge:
  - phonotactics: the set of licit forms = $\mathit{range}(G)$
  - repairs: a UR $\mathbf{a}$ is repaired to $\mathbf{b}$ if $G(\mathbf{a}) = \mathbf{b} \neq \mathbf{a}$

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Chain shifts

- "Theorem" of phonology:

  phonotactically illicit $\Longrightarrow$ repaired

- What about the reverse implication?

  repaired $\stackrel{?}{\Longrightarrow}$ phonotactically illicit

  informally: the good stuff should not be repaired

- This reverse implication holds by and large. . .
- . . . but not always:

/a/ → [e]	gáta 'cat-FEM'	gétu 'cat-MAS'
	sánta 'saint-FEM'	séntu 'saint-MAS'
/e/ → [i]	bnéna 'child-FEM'	nínu 'child-MAS'
	séks 'dry-FEM'	síku 'dry-MAS'

[Lena dialect of Spanish: Hualde 1989; Gnanadesikan 1997]

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G is idempotent provided it satisfies this implication [Prince and Tesar 2004]

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for every candidate $(\mathbf{a}, \mathbf{b})$ in the candidate set $\mathfrak{Can}$.

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The formal theory of idempotency

- The definition of idempotency is framework independent: it looks at grammars at an extensional, functional level
- I will now zoom in on a specific phonological framework: constraint-based phonology and its HG and OT implementations
- Question 1: which conditions on the constraints guarantee that OT or HG grammars are idempotent?
- Question 2: what do these constraint conditions mean? do they admit an intuitive interpretation?
- To preview, idempotency holds in constraint-based phonology when phonological distance between URs and SRs is measured in compliance with the triangle inequality [Magri 2016a,b]
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- Constraint conditions for idempotency are interesting for phonology:
  - want to model chain shifts in constraint-based phonology
  - just look up a constraint from the list of those which fail the conditions

- Constraint conditions for idempotency are interesting for learnability:
  - want to avoid chain shifts for the learner to soundly assume faithful URs for phonotactically licit training SR [Hayes 2004; Prince and Tesar 2004]
  - just make sure all constraints in your simulations belong to the list of constraints which satisfy the conditions for idempotency

- Can phonology and learnability be reconciled? Future development:
  - the learner is fine with the typology containing a chain shift **a** $\rightarrow$ **e** $\rightarrow$ **i**
  - provided the typology contains another grammar which is idempotent and phonotactically equivalent (**a** illicit; **e**, **i** licit)
  - can we use the constraint conditions for idempotency to show that attested chain shifts have this property? [Moreton and Smolensky 2002]

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2.

Constraint-based phonology

Optimality

- Intuition: map a UR to *the* SR which yields *the* optimal candidate among those candidates which share that UR
- Formalization:
  - order the candidates in $\mathfrak{Can}$ which share a certain UR $\mathbf{a}$:

$$(\mathbf{a}, \mathbf{b}) \text{ is better than } (\mathbf{a}, \mathbf{c})$$
$$(\mathbf{a}, \mathbf{c}) \text{ is better than } (\mathbf{a}, \mathbf{d})$$
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- Remarks:
  - if "better than" is a linear order among candidates, G is a function
  - otherwise, multiple optima: a rudimentary account of variation
  - might use the entire ordering for gradience acceptability [Coetzee 2004, 2008]

Optimality

- Intuition: map a UR to *the* SR which yields *the* optimal candidate among those candidates which share that UR
- Formalization:
  - order the candidates in $\mathfrak{Can}$ which share a certain UR $\mathbf{a}$:

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- Intuition: define "better than" by measuring the badness of each candidate from a variety of conflicting phonological perspectives
- Formalization:
  - constraint C maps a candidate $(\mathbf{a}, \mathbf{b})$ to number of violations $C(\mathbf{a}, \mathbf{b})$
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  - constraints are collected together in a constraint set $\mathfrak{Con}$
- Examples:
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HG and OT

- Intuition: a candidate is better if it violates less the constraints
- Formalization/I: f there is a unique constraint C:
  - candidate $(\mathbf{a}, \mathbf{b})$ is better than $(\mathbf{a}, \mathbf{c})$ iff $C(\mathbf{a}, \mathbf{b}) < C(\mathbf{a}, \mathbf{c})$
  - *the* optimal candidate minimizes constraint violations
  - the typology consists of a single grammar
- Formalization/II: Harmonic Grammar: [Legendre, Sorace, and Smolensky 2006]
  - a grammar assigns to each constraint C a (non-negative) weight θ_C
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- Typology of phonological grammars defined through a pair $(\mathfrak{Can}, \mathfrak{Con})$
- Assumptions on the candidate set $\mathfrak{Can}$:
  - reflexive: if $(\mathbf{a}, \mathbf{b}) \in \mathfrak{Can}$, then also $(\mathbf{a}, \mathbf{a}), (\mathbf{b}, \mathbf{b}) \in \mathfrak{Can}$
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3.

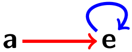
The OT faithfulness idempotency condition (OT-FIC)

Intuition

a ⟶ **e**

- Reasoning by contradiction:
  - suppose some UR is mapped to **[e]**, say **/a/**

Intuition



- Reasoning by contradiction:
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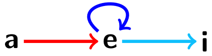

□ Reasoning by contradiction:

- suppose some UR is mapped to **[e]**, say **/a/**
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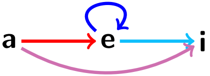

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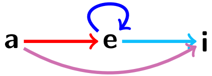

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- To get the contradiction, it is intuitively sufficient that each constraint *C* satisfies the following implication:

  if *C* prefers (**/e/**, **[i]**) to (**/e/**, **[e]**) or doesn't care
  then *C* prefers (**/a/**, **[i]**) to (**/a/**, **[e]**) or doesn't care

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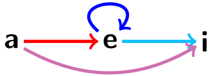

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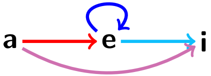
Intuition



- Reasoning by contradiction:
  - suppose some UR is mapped to **[e]**, say **/a/**
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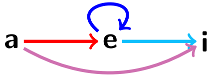

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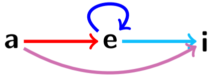

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OT faithfulness idempotency condition (OT-FIC)

$$\begin{array}{rl} \textbf{if:} & F(\mathbf{b}, \mathbf{c}) = 0 \\ \textbf{then:} & F(\mathbf{a}, \mathbf{c}) \leq F(\mathbf{a}, \mathbf{b}) \end{array}$$

Idempotency in OT

First result of this talk

Assume that:

— the candidate set $\mathfrak{Can}$ is reflexive and transitive

— the constraint set $\mathfrak{Con}$ only consists of faithfulness and markedness

If every faithfulness constraint F satisfies the OT-FIC, every grammar in the OT typology corresponding to $(\mathfrak{Can}, \mathfrak{Con})$ is idempotent [Magri 2016b; see also Moreton and Smolensky 2002; Tesar 2013; Buccola 2013]

- Take-home message:
  - the OT-FIC is a sufficient condition for idempotency in OT
  - at this level of generality, it is also necessary
- Big open question:
  - most phonological patterns are idempotent (chain shifts are "rare")
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  - how come? what does the OT-FIC really mean?

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[1] Suppose idempotency fails for the OT grammar $G_{\gg}$

[2a] $G_{\gg}(\mathbf{a}) = \mathbf{b}$
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4.
Extension to HG

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OT faithfulness idempotency condition (OT-FIC)

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OT faithfulness idempotency condition (HG-FIC)

if: $F(\mathbf{b},\mathbf{c}) = \xi$

then: $F(\mathbf{a},\mathbf{c}) \leq F(\mathbf{a},\mathbf{b}) + \xi$

for every threshold $\xi \geq 0$

Second result of this talk

Assume that $(\mathfrak{Can}, \mathfrak{Con})$ satisfy the usual assumptions. If every faithfulness constraint F satisfies the HG-FIC, every grammar in the HG typology corresponding to $(\mathfrak{Can}, \mathfrak{Con})$ is idempotent [Magri 2016b]

- Sanity check:
  - given $(\mathfrak{Can}, \mathfrak{Con})$, HG typology is larger than OT typology
  - a stronger condition is needed to discipline all HG grammars to comply
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[2a] $G_\theta(\mathbf{a}) = \mathbf{b}$
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[6] The latter is equivalent to the negation of the HG-FIC

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5.

A metric interpretation of idempotency: the case of HG

Faithfulness triangle inequality

- Faithfulness constraints:
  - $\textsc{Max}(\mathbf{a}, \mathbf{b})$ = number of segments of **a** deleted in **b**

    $\textsc{Max}(/\text{pad}/, [\text{pat}]) = 0$, $\textsc{Max}(/\text{pad}/, [\text{pa}]) = 1$
  - $\textsc{Ident}_{[voice]}(\mathbf{a}, \mathbf{b})$ = number of segments of **a** differing in voicing in **b**

    $\textsc{Ident}_{[voice]}(/\text{pad}/, [\text{pat}]) = 1$, $\textsc{Ident}_{[voice]}(/\text{pad}/, [\text{pa}]) = 0$
- Intuitively measure the phonological distance between URs and SRs
- Do faithfulness constraints satisfy the various conditions which pertain to the axiomatic definition of distance or metric? [Rudin 1953]
- One crucial metrical axiom is the triangle inequality:
  - the side of any triangle is shorter than the sum of the other two sides
  - $dist(\mathbf{a}, \mathbf{c}) \leq dist(\mathbf{a}, \mathbf{b}) + dist(\mathbf{b}, \mathbf{c})$

Faithfulness triangle inequality (FTI)

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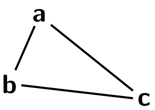

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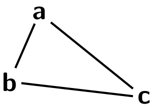

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Metric interpretation of the HG-FIC

Third result of this talk

For an arbitrary faithfulness constraint F:

HG-FIC		FTI
if: $F(\mathbf{b},\mathbf{c}) \leq \xi$ **then:** $F(\mathbf{a},\mathbf{c}) \leq F(\mathbf{a},\mathbf{b}) + \xi$	$\Longleftrightarrow$	$F(\mathbf{a},\mathbf{c}) \leq F(\mathbf{a},\mathbf{b}) + F(\mathbf{b},\mathbf{c})$

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- This equivalence holds because:
  - assume that $\xi = F(\mathbf{b}, \mathbf{c})$
  - then the FTI is analogous to the consequent of the HG-FIC
- This equivalence means that:
  - the HG-FIC simply requires a faithfulness constraint to measure phonological distance in compliance with the triangle inequality
  - HG idempotency follows from the assumption that the faithfulness constraints have good metrical properties

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6.

A metric interpretation of idempotency: categoricity and the case of OT

Towards a metric interpretation of the OT-FIC

Fourth result of this talk: preliminary formulation

For every binary faithfulness constraint F (which take values 0 or 1):

OT-FIC		**FTI**
if: $F(\mathbf{b}, \mathbf{c}) = 0$ **then:** $F(\mathbf{a}, \mathbf{c}) \leq F(\mathbf{a}, \mathbf{b})$	$\Longleftrightarrow$	$F(\mathbf{a}, \mathbf{c}) \leq F(\mathbf{a}, \mathbf{b}) + F(\mathbf{b}, \mathbf{c})$

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- Why the OT-FIC entails the FTI for binary constraints:
  - If the antecedent of the OT-FIC holds:
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  - If the antecedent of the OT-FIC fails:
    $\Longrightarrow$ that makes the right-hand side of the FTI large enough
- The FTI entails the OT-FIC independently of binarity
  but the equivalence fails for non-binary faithfulness constraints
- This makes sense: FTI = HG-FIC > OT-FIC
- In conclusion, FTI is unrelated to OT idempotency in the general case

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$\iff$

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Categoricity: the idea

McCarthy's categoricity conjecture [McCarthy 2003b]

Each faithfulness constraint F useful in phonology is categorical

- Intuitively, $\textsc{Ident}_{[\textsc{nasal}]}$ is categorical because:
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- In general, categoricity means that a phonological candidate can be broken up into "sub-candidates" in such a way that:
  - $F(\text{cand}) = \sum_{\text{sub-cand}} F(\text{sub-cand})$ — the violations assigned by F to the candidate is the sum of the violations it assigns to the "sub-candidates"
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Monotonicity: the idea

- Categoricity is intimately related to monotonicity
- Intuitively, $\text{IDENT}_{[\text{NASAL}]}$ is monotone because violations increase when candidates increase through additional correspondence relations:

$$\begin{pmatrix} \text{n t k} \\ | \\ \text{t ŋ} \end{pmatrix} \leq \begin{pmatrix} \text{n t k} \\ |\,/\,/ \\ \text{t ŋ} \end{pmatrix} \Longrightarrow \text{IDENT}\begin{pmatrix} \text{n t k} \\ | \\ \text{t ŋ} \end{pmatrix} \leq \text{IDENT}\begin{pmatrix} \text{n t k} \\ |\,/\,/ \\ \text{t ŋ} \end{pmatrix}$$

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Categoricity and monotonicity: more details

- Candidate = UR + SR + correspondence [McCarthy and Prince 1995]
- A candidate can be split into sub-candidates along any of these three dimensions, yielding three notions of categoricity and monotonicity
- Faithfulness categoricity:
  - C-categoricity: sub-candidates have one (few) corresponding pair (IDENT)
  - I-categoricity: sub-candidates have one (few) underlying segment (MAX)
  - O-categoricity: sub-candidates have one (few) surface segment (DEP)
- Faithfulness monotonicity:
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Categoricity and monotonicity: more details

- Candidate = UR + SR + correspondence [McCarthy and Prince 1995]
- A candidate can be split into sub-candidates along any of these three dimensions, yielding three notions of categoricity and monotonicity
- Faithfulness categoricity:
  - C-categoricity: sub-candidates have one (few) corresponding pair (IDENT)
  - I-categoricity: sub-candidates have one (few) underlying segment (MAX)
  - O-categoricity: sub-candidates have one (few) surface segment (DEP)
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Categoricity+monotonicity in natural language phonology

Extended categoricity conjecture

Any faithfulness constraint F relevant for Natural Language is

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Metric interpretation of the OT-FIC

Fourth result of this talk

For every F which satisfies the extended categoricity conjecture:

OT-FIC **if:** $F(\mathbf{b}, \mathbf{c}) = 0$ **then:** $F(\mathbf{a}, \mathbf{c}) \leq F(\mathbf{a}, \mathbf{b})$	$\Leftrightarrow$	**FTI** $F(\mathbf{a}, \mathbf{c}) \leq F(\mathbf{a}, \mathbf{b}) + F(\mathbf{b}, \mathbf{c})$

- The proof is not straightforward. Intuitively:
  - the equivalence holds for binary constraints (as we have seen)
  - and thus extends to categorical ones = sum of binary constraints
  - monotonicity is a technical assumption to grease the proof
- This equivalence for categorical + monotone constraints means that:
  - the OT-FIC simply requires a faithfulness constraint to measure phonological distance in compliance with the triangle inequality
  - OT idempotency follows from the assumption that the faithfulness constraints have good metrical properties

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7.

Establishing the OT-FIC

Faithfulness constraints which satisfy the OT-FIC

Second result of this talk

The following faithfulness constraints satisfy the OT-FIC:

- MAX, $\text{MAX}_{[\pm\varphi]}$ (Casali 1998), INTEGRITY
  - under no additional assumptions
- DEP, $\text{DEP}_{[\pm\varphi]}$, IDENT_{φ} (when φ is total), Pater's (1999) value-restricted variants thereof, UNIFORMITY, LINEARITY, MAX/DEPLINEARITY (Heinz 2005), ADJACENCY (Carpenter 2002)
  - provided correspondence relations cannot break underlying segments
- The disjunction of two $\text{MAX}_{[\pm\varphi]}$'s, two $\text{DEP}_{[\pm\varphi]}$'s, two IDENT_{φ}'s,
  - under the same conditions for the corresponding individual constraints

Take home message:

- constraints which only count over underlying segments (like MAX):
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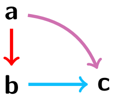
How to establish the OT-FIC: the case of $\textsc{Ident}_{\text{nas}}$

- We want to establish the OT-FIC for $\textsc{Ident}_{\text{nas}}$:

If $\textsc{Ident}_{\text{nas}}(\mathbf{b}, \mathbf{c}, \rho) = 0$
then $\textsc{Ident}_{\text{nas}}(\mathbf{a}, \mathbf{c}, \rho) \leq \textsc{Ident}_{\text{nas}}(\mathbf{a}, \mathbf{b}, \rho)$
provided there is no breaking of under. segments



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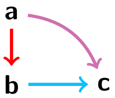

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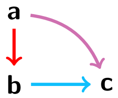

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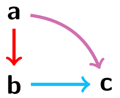

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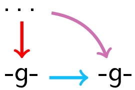

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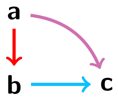

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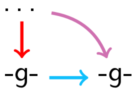

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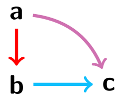

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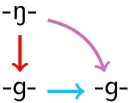

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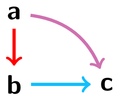

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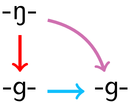

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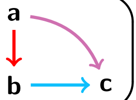

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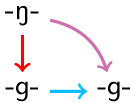

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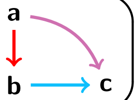

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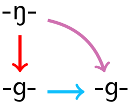

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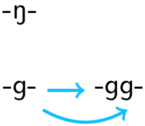

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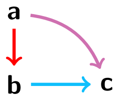

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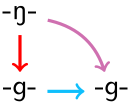

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  - $\textsc{Ident}_{\text{nas}}(\mathbf{a}, \mathbf{c}, \rho) = 2$
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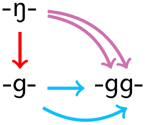

Faithfulness constraints which fail the OT-FIC

Third result of this talk

The following faithfulness constraints do not satisfy the OT-FIC:

- ANCHOR and CONTIGUITY
- Restricted versions of MAX and DEP (e.g.: DEP-V and MAX-C)
  - if correspondence relations can "cross" the restriction
- IDENT_φ (but not $\text{MAX}_{[\pm\varphi]}$ and $\text{DEP}_{[\pm\varphi]}$)
  - if φ is *partial* (e.g.: [strident], when defined only for coronals)
- All constraints which count not only underlying segments
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- The conjunction of two $\text{MAX}_{[\pm F]}$'s, two $\text{DEP}_{[\pm F]}$'s, two IDENT_F's,
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- Although the OT-FIC is only sufficient, idempotency can be shown to fail in each of these cases ⇒ the OT-FIC is a tight sufficient condition
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Third result of this talk

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Faithfulness constraints which fail the OT-FIC

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The following faithfulness constraints do not satisfy the OT-FIC:

- Anchor and Contiguity
- Restricted versions of Max and Dep (e.g.: Dep-V and Max-C)
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- Ident_φ (but not $\text{Max}_{[\pm\varphi]}$ and $\text{Dep}_{[\pm\varphi]}$)
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- All constraints which count not only underlying segments
  - if correspondence relations can break underlying segments
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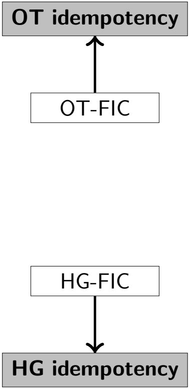
8.
Conclusions

Summary

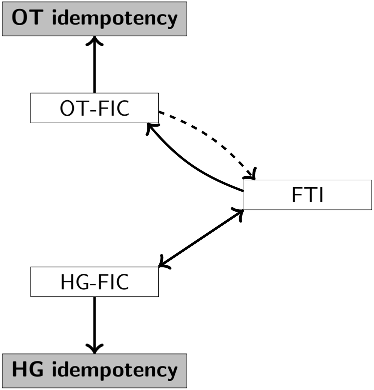
OT idempotency

HG idempotency

Summary



Summary



- Idempotency is related to the metrical nature of faithfulness:
  - HG: the relation holds unrestricted
  - OT: it requires categoricity
- A non-trivial implication of McCarthy's categoricity conjecture

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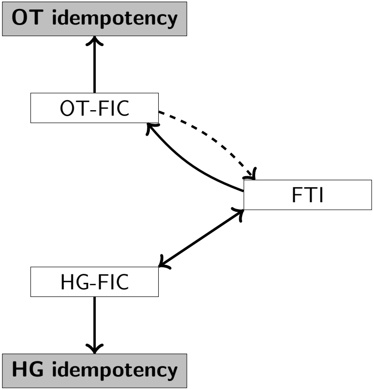

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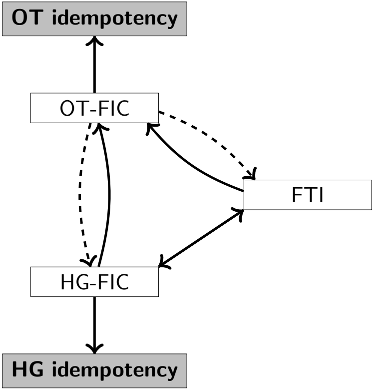

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- Given categoricity, idempotency in HG does not require additional constraint conditions than in OT
- The constraints which satisfy the OT-FIC, also satisfy the HG-FIC (and the FTI)

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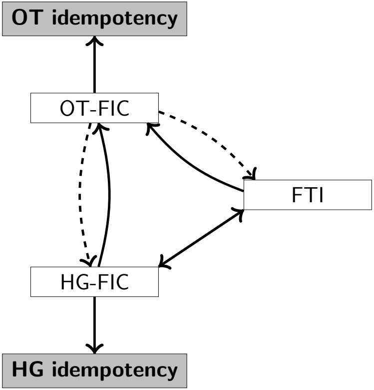

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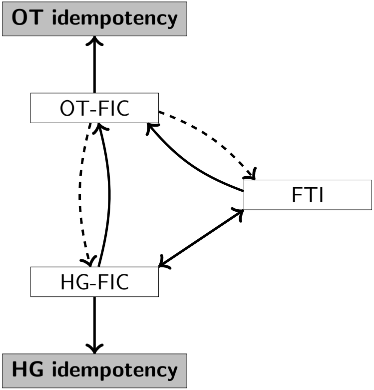
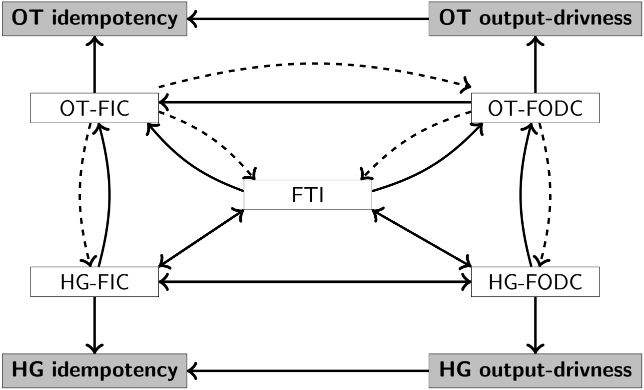


Extensions



OT idempotency
OT output-drivness
OT-FIC
OT-FODC
FTI
HG-FIC
HG-FODC
HG idempotency
HG output-drivness

Thank you!

[Slides available on my website, together with the two papers that this talk is based on: Magri (2016a) and Magri (2016b)]

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