Framework for Formal Ontology

Barry Smith and Kevin Mulligan

ABSTRACT. The discussions which follow rest on a distinction, first expounded by Husserl, between formal logic and formal ontology. The former concerns itself with (formal) meaning-structures; the latter with formal structures amongst objects and their parts. The paper attempts to show how, when formal ontological considerations are brought into play, contemporary extensionalist theories of part and whole, and above all the mereology of Leśniewski, can be generalised to embrace not only relations between concrete objects and object-*pieces*, but also relations between what we shall call dependent parts or *moments*. A two-dimensional formal language is canvassed for the resultant ontological theory, a language which owes more to the tradition of Euler, Boole and Venn than to the quantifier-centred languages which have predominated amongst analytic philosophers since the time of Frege and Russell. Analytic philosophical arguments against moments, and against the entire project of a formal ontology, are considered and rejected. The paper concludes with a brief account of some applications of the theory presented.

1. Form

We can distinguish, in relation to every object or region of objects, both formal and material truths. Material truths are, for example, the truths of the natural sciences. As examples of formal truths we might consider:

> *if r is part of s and s is part of t, then r is part of t,*

or:

> *if s is part of but not identical with t, then there is a part of t which is not part of s.*

Formal truths of this kind correspond, we shall argue, to formal structures or relations in the underlying region of objects, material truths to underlying material structures or relations. Form and matter can therefore be distinguished on two distinct levels: on the level of *truths*, and on the level of *things*.[1] Formal ontology consists in the investigation of formal structures or relations on this second level; it concerns itself only indirectly with formal truths on the first level. It is therefore to be distinguished from another, somewhat more familiar formal discipline, the discipline of logic. Formal logic does not concern itself with objects or object-relations in the world; nor does it concern itself specifically with sentences about such objects. It deals, rather, with *sentences in general* (including, for example, the sentences of mathematics),[2] and where it is applied to sentences about objects it can take no account of any formal or material object-structures which may be exhibited amongst the objects pictured. Its attentions are directed, rather, to the relations which obtain between sentences purely in virtue of what we can call their logical complexity (for example the deducibility-relations which obtain between any sentence of the form $A \,\&\, B$ and sentences of the forms A and B).

The distinction between formal logic and formal ontology reflects a corresponding distinction between logical and ontological complexity. The first kind of complexity is, as a result of the work of logicians since Frege, comparatively well-understood. It is that kind of complexity which is captured by the propositional and predicate calculi.[3] Complexity of the second kind is unfortunately less well-understood (though because of the ontological centrality of the relations of part and whole, we can expect to learn much that is of relevance to formal ontology from those philosophers, from Boole and Husserl to Leśniewski and Goodman, who have seriously investigated such relations[4]).

Our comparative lack of understanding of formal ontological complexity has its roots in a confusion, characteristic of analytic philosophy since its inception, which consists in the running together of the *formal* and the *formal logical*.[5] That this is not an arbitrary confusion will become clear when we investigate the sense in which formal logic and formal ontology can justifiably be regarded as sharing in common the trait that they are both formal disciplines. Both consist in the investigation of certain formal, as opposed to material, structures and relations manifested by their respective subject-matters; but we cannot yet say by what criterion the specifically formal structures in any given subject-matter are to be delineated. What do the formal relations between sentences investigated by logic have in common with the formal object-relations investigated by ontology? How do we establish, for ex-

Topoi **2** (1983) 73–85. 0167-7411/83/0021-0073$01.95.

ample, that *function, argument, conjunction, implication, truth, falsehood,* on the one hand, and *object, property, part, whole, overlapping, discreteness*, on the other hand, are formal concepts? Formal concepts and formal structures and relations amongst sentences or things do not come ready labelled as such. And since we have no independent definition of 'material', we shall make no headway, either, if we attempt a negative definition of the formal as 'that which is independent of, or such as to apply equally to, all matter'.

Our contention is that it is the concept of *operation* which is the key to the understanding of form. Formal concepts, whether logical or ontological, are most readily delineated by the fact that the meanings of the corresponding signs are determined exhaustively by the directives or operational rules which govern their use.[6] To paraphrase the *Tractatus* only slightly (cf. 6.126): one can calculate whether a logical or ontological constant is formal, by calculating the formal properties of the corresponding symbol. Without bothering about sense or meaning we construct the formal constants out of others using only *rules that deal with signs*, i.e. by successively applying certain operations that always generate further formal constants out of the initial ones. Thus the meaning of the formal logical constant '&' is determined exhaustively by the operational rules which govern its use. But this is true also, as we shall see, of the meanings of the constants ('is a whole', 'is a part', etc.) of formal ontology.

This operational understanding of the formal was first propounded by Husserl in his early writings on the philosophy of logic and mathematics.[7] It can be extracted cleanly from the *Tractatus*, but only by substituting the word 'formal' for relevant occurrences of the word 'logical' in Wittgenstein's text – which throws clear light on the nature of the analytic philosophical confusion.

That this confusion is not entirely without justification can now be shown as follows. If the most serviceable criterion of the formal is rooted in the operational behaviour of corresponding *symbols*, then because it is logic, and not ontology, which deals most directly with structures and relations exhibited in language, the assumption lies ready to hand that all that need by said concerning *form* can be said within the confines of the province of logic. The mainstream of analytic philosophy has accordingly acquired a tendency to ignore the formal structures and relations exhibited amongst objects in the world. Hence we see why the theory of part and whole, and related formal ontological disciplines, have so conspicuously failed to establish themselves within it.

It is Frege, more than anyone else, who is responsible for this development. Frege's attempts, in the *Begriffsschrift*, to understand the logical form of scientific sentences led to his discovery of the theory of quantification, a theory which made possible the first formally adequate representation of multiple generality.[8] Frege's success in the application of this theory to the problem of representing the forms of mathematical sentences, a success which is unparalleled in the history of logic, laid the foundations of a methodological tradition within which quantificational logic came to occupy a central and impregnable position – to the extent that the formal machinery at the disposal of the analytic philosopher has consisted almost entirely of the (logical) machinery of the prediate calculus.[9]

Logicians before Frege had taken for granted the existence of some kind of parallelism between the surface (subject-predicate) form of assertoric sentences and the nexus of substance and quality, or substance and accident, in the world. Whilst not directly calling into question the existence of such a mirroring relation between language and world, Frege denied its *logical* relevance, asserting that the logical structure-building principles of language – which he saw as consisting in the application of function to argument – lay at a level distinct from that of surface grammar. This recognition of the *sui generis* nature of logical structure had a tremendous positive impact on subsequent logical inquiries. But because one of the most important lessons of Frege's early thought, that there is a radical heterogeneity between logical and ontological structure, was never taken to heart by his successors, it has had equal and opposite negative consequences for the discipline of ontology. Where traditional philosophers had been too ready to read into language the structures of the world, post-Fregean philosophers have shown themselves too ready to read into the world the structures of (one particular formal logical) language. They have presumed, in effect, that the structures of the world can be adequately understood purely in terms of those structural moments isolated within the predicate calculus (or within its ontological step-mother, the theory of sets). We wish to suggest in what follows that it is possible once more to prise apart the disciplines of formal logic and formal ontology, by bringing into focus the heterogeneous nature of the two sorts of structure-building principles which form their subject-matters.

2. Propositional pictures I

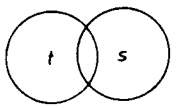
Formal ontology seeks to develop a formal language whose

syntax will mirror directly all and only those (formal) structure-building principles which are to be encountered in the world.[10] If, for example, there are no negative entities in the world (for example no negative states of affairs), then the directly depicting language of formal ontology will contain no negative propositions.

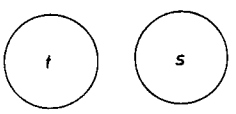
This ideal is by no means achieved by Leśniewski's mereology, whose symbolism embodies a misture of syntactic devices constructed for the purposes of representing *ontological* relations with, on the other hand, alien *logical* machinery (for example the machinery of quantification). The ideal is more nearly approximated to by an earlier and nowadays too often neglected attempt to formalise the relations of part and whole, the calculus of Venn (or Euler diagrams). Venn's original ideas will not, of course, pass muster as the basis of a formally rigorous notation, and the various attempts that were made in the 19th century to exploit these ideas in the direction of greater formal serviceability failed equally to achieve the required standards of rigour – principally, we may say, because their authors did not have the benefit of Frege's work, nor of the work of subsequent logicians.[11] Further, their use often involved precisely that running together of logical and ontological considerations criticised above.[12] Yet we believe that there is no fundamental incompatibility between Venn's ideas and Fregean and post-Fregean notions of rigour. We shall argue, more precisely, that it is possible, taking as our starting point certain ideas of Venn and his contemporaries, to develop a rigorous formal ontological language having precisely the requisite property that its constituent propositions or propositional pictures mirror formal structures and relations in the world. The Tractarian allusions here are not incidental: it will turn out that it is impossible to express, *within* the formal ontological language, those truths of formal ontology which it has been devised to elucidate. Such truths will, rather, show themselves in the *application* of the language.

We shall have space here to present only the briefest outlines of a Venn-like formal notation, providing only preliminary hints as to how it might be possible to state formation rules of sufficient generality that they can accommodate propositional pictures of arbitrary complexity.[13] We shall rather content ourselves, here and in Section 5, with a specification of the basic component-formulae and of their equivalents in a rough-and-ready mereological English.

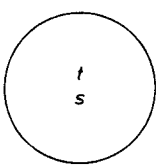
Italic letters enclosed within circular (eventually also non-circular) frames are *names* of objects in the world. The propositional pictures of our language consist entirely of names, compounded together in ways which reflect formal ontological relations between the objects named.[14] Because they consist entirely of names they are – in the terminology of the *Tractatus* – logically elementary, and therefore also logically (though not ontologically) independent.

(1) 

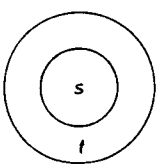
shall signify that two objects *t* and *s* exist, and that they overlap materially. Here the term 'object' is used in the widest possible sense. It embraces not only the perceptually discriminable furniture of the material universe, but also, for example, material qualities and relations, actions, events and processes, spatial and temporal stretches, mental acts, states and their contents, and more or less arbitrarily discriminated parts and aggregates of all of these.[15] Thus our circular frames carry no presupposition to the effect that their underlying matters are in any sense intrinsically unified. They may, for example, signify merely the demarcation of some area of interest.[16] (1), for example, might express the fact that *what Smith believes is true* overlaps, partially, with *what Mulligan belives is true.* The formal representation of unified wholes will occupy us in §5 below.

(2) 

will signify that *t* and *s* exist and are discrete from each other.[16] That two wholes are identical is represented by:

(3) 

(Here the letters '*t*' and '*s*' may be assumed to be superimposed, one upon the other; it is merely for reasons of legibility that they are printed separately.)

(4) 

might signify that the whole *s* is a proper part of a second whole *t*. 'Ⓣ' signifies simply that *t* exists; 'O' that a whole

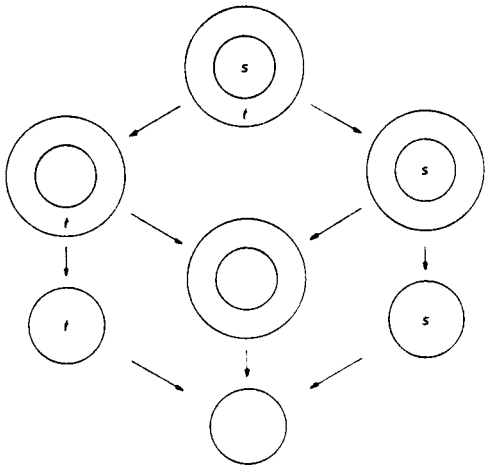
(something) exists. It is a simple matter to devise more complex ideographs after the manner of Euler and Venn (but incorporating, be it noted, only the purely positive fragment of their notational calculi).

Any pictorial complex of the sort indicated above can be read or understood in two distinct ways: either as a propositional picture, asserting that its underlying matter, as thus and thus constituted, exists; or as a complex name of this matter.[17] The reader may ask what relation such a language – which, according to the favoured reading of its constituent formulae, consists entirely either of names, or of existence propositions – might bear to any actually existing language with which he is familiar. Whilst no natural language consists exclusively either of names or of existence propositions, the work they perform is indispensable to any language. It is through names, in contexts in which they are bound up with serious suppositions of existence, that language achieves its relation to the world.[18] The formal ontological language, therefore, involves no constructions not already to be encountered within natural language as normally conceived.[19] But the formal language enables us to bring into light a property of existence propositions as these occur in ordinary discourse which has been too often overlooked. For excessive concentration on the problem of whether 'exists' is a genuine predicate has distracted attention away from a peculiarity of such propositions, namely that from the truth of any proposition of the form '*a* exists', we can infer ontologically the truth of an existence proposition about any part of *a* (see e.g. (5) below).

It is by reference to the vertical relation between language and the world that the vertically interrelated structures of names and nameables treated of by formal ontology are distinguished from the horizontal relations between sentences and their parts treated of by logic. The formal ontological language, then, is not a language in any usual sense. It has been devised for the purposes of laying bare the formal ontological complexity of names and nameables, abandoning entirely the task of representing the logical structures of the sentences in which names normally occur. A sentence (propositional picture) such as (4) is *logically* structureless; it contains no logical connectives, no quantifiers or predicate expressions, nor any analogues of these.[20] And it leaves no scope for the application of the Fregean analytic device of distinguishing function and argument: all its constituent (well-formed) expressions are complete. Hence we should exhibit no surprise at the fact that, from the logical point of view, the expressive powers of the language are so limited. Even the formal propositions that it has been devised to elucidate cannot be *said* within it, but are capable only of being *shown* – by the networks of propositional pictures derivable operationally from its constituent complexes.

(4), for example, generates the network:[21]

(5)

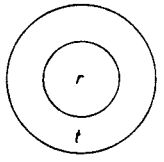


(5) represents a system of *ontological inferences*; it is in this sense that the propositional pictures of the formal ontological language are logically, but not ontologically independent. The inferences from:

(6)

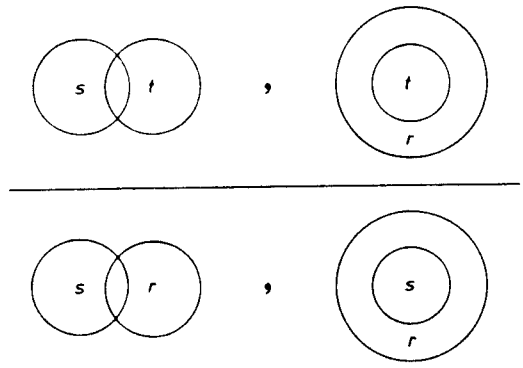
to

(6′)



may be said to represent a species of ontological syllogism, capturing some of those ontological insights which lay – intermingled with insights of a purely logical nature – at the root of Aristotle's original syllogistic theory.[22] Other figures of the ontological syllogism may be felicitously represented as inferences yielding multiple conclusions (cf. Shoesmith and Smiley, 1978), where

(7)



for example, is to be interpreted as signifying that if all propositional pictures above the line are true, then precisely one of the propositional pictures below the line is also true.

Formation and deduction rules for the above may be specified by regarding each propositional picture as a conjunction of sentences expressing Boolean relations between its constituent terms taken in pairs. Inspection reveals that the following is an exhaustive list of such relations:

$$tR_1s := t \cap s \neq 0 \wedge t \cap \bar{s} \neq 0 \wedge \bar{t} \cap s \neq 0$$
$$tR_2s := t \cap s = 0 \wedge t \cap \bar{s} \neq 0 \wedge \bar{t} \cap s \neq 0$$
$$tR_3s := t \cap s \neq 0 \wedge t \cap \bar{s} = 0 \wedge \bar{t} \cap s = 0$$
$$tR_4s := t \cap s \neq 0 \wedge t \cap \bar{s} \neq 0 \wedge \bar{t} \cap s = 0$$
$$tR_5s := t \cap s = 0 \wedge t \cap \bar{s} = 0 \wedge \bar{t} \cap s \neq 0$$

– corresponding, respectively, to (1), (2), (3), (4), and the converse of (4) above.

We shall use '$\mathbb{S}(m, n)$' to designate the manifold of ordered pairs $\langle i, j\rangle$, $m \leqslant i \leqslant j \leqslant n$, and '$\mathbb{IF}(m, n)$' to designate the manifold of functions from $\mathbb{S}(m, n)$ into $\{1, 2, 3, 4, 5\}$. The admissible propositional pictures may now be identified as those conjunctions of the form

$$\bigwedge_{\mathbb{S}(m,n)} a_i R_{f(i,j)} a_j, \quad 1 \leqslant m \leqslant n, \quad f \in \mathbb{IF}(m, n)$$

which are consistent with the axioms of Boolean algebra.[23] $a_iR_{f(i,i)}a_i$ is admissible if and only if $f(i, i) = 3$.

k consistent conjunctions in a_{i_p}, $m_p \leqslant i_p \leqslant n_p$, $1 \leqslant p \leqslant k$, determined, respectively, by the functions f_p selected from $\mathbb{IF}(m_p, n_p)$ are consistent *with each other* if and only if there is some function $g \in \mathbb{IF}(\min(m_p), \max(n_p))$, such that

$$g\Big|_{\mathbb{S}(m_p, n_p)} = f_p$$

and

$$\bigwedge_{\mathbb{S}(\min(m_p), \max(n_p))} a_i R_{g(i,j)} a_j$$

is consistent. The propositional pictures corresponding to the given k conjunctions can be combined into a *single* propositional picture only if g as thus defined is unique; otherwise the given manifold generates a family of mutually exclusive propositional pictures. Suppose $g_1, \ldots, g_l$ are the functions corresponding to the respective members of this family, then we can state the following general deduction rule:

$$\frac{\bigwedge_{0 < p \leqslant k \; \mathbb{S}(m_p, n_p)} a_i R_{f_p(i,j)} a_j}{\bigwedge_{\mathbb{S}_1} a_i R_{g_1(i,j)} a_j, \ldots, \bigwedge_{\mathbb{S}_l} a_i R_{g_l(i,j)} a_j}$$

whenever $\mathbb{S}_1, \ldots, \mathbb{S}_l \subseteq \mathbb{S}(\min(m_p), \max(n_p))$.

3. Pieces and moments

Let us introduce the term 'extensive' to signify those objects entering into the Boolean or mereological relations of discreteness, overlapping, etc., distinguished above. Extensive objects, for example lumps of metal or of cake, are objects which admit of piecing into constituent bits, mutually independent of each other.[24] A lump of cake can, within certain limits, be *sliced*, either actually or in our imagination, in such a way that of each of the resulting slices it is true that its continued existence does not depend upon the continued existence of the remaining slices.

We might initially be tempted to suppose that all extensive objects in the real material world are spatio-temporal concreta.[25] Leśniewski himself lends some support to this position in his conception of mereology as standing in need of completion by the theories of chronology (the theory of time) and stereology (the theory of space). Consider, however, an object such as a sheet of glass which is uniformly red in colour. Here the glass itself is an object entering into the mereological relations of discreteness, overlapping and so on: it is capable of being pieced. But consider now the specific *area of colour* which pervades it. The latter is, surely, conceptually distinguishable from the former, even though existentially inseparable from it.[26] And it is also, we shall argue, an object entering into mereological relations: slicing the sheet of glass in half, for example, whether actually or in thought, results in an exactly corresponding bisection of the colour of the glass. Yet this individual *colour* is not a materially extended concrete object.

There are, then, extensive objects which are not concreta. Objects of this kind form the basis of Goodman's *logische Aufbau* in *The Structure of Appearance*. We have

no means of knowing whether Leśniewski himself admitted the possibility of such objects. We do however know that he admitted cases of non-concrete objects – such as sense data and dreams – which were, in his eyes, without spatial location.[27] It is unfortunately impossible to determine what view he might have taken as to the relations between objects of this kind and the concreta (for example human beings) with which we should normally assume they are associated. He could not, of course, have held that they are *parts* of an individual in the mereological sense of 'part'. Yet is not clear, either, that they can properly be conceived as mereologically discrete from their bearers. The consideration of such objects suggests, in fact, that it is necessary to evolve a generalised concept of *part*, comprehending not only detachable *pieces*, but also non-concrete parts, parts of objects which can be discriminated within those objects only in thought.[28]

We shall introduce the term 'moment' to designate such abstractly discriminable but not materially detachable parts of a material thing.[29] Moments may, like objects in general, be either *extensive* or *non-extensive*.[30] Extensive moments are, like the colour of the sheet of glass, capable of being pieced; and the resultant pieces are capable of entering into mereological relations with each other. Non-extensive moments (examples of which will be provided below) are not capable of being pieced. The concept of moment is, as we shall see, a formal concept: the meanings of corresponding expressions can be fixed exhaustively by reference to purely formal operations governing their use.

Let us suppose that John is suffering from influenza and from a headache. The headache and the influenza are moments of John, standing in certain relations to each other: the former may be, for example, a symptom of the latter. We can compare John's headache with the (perhaps qualitatively indistinguishable) headache of Jane, and establish, say, that they temporally succeed each other. In making such comparisons we are dealing, surely, with entities no less distinct and separate than their respective bearers. Similarly, we would argue, the redness of the sheet of coloured glass on the table is distinct and separate from the redness of the sheet of coloured glass on the chair, even where the two sheets are (as we would normally say) *identical in colour*. The paradox is removed, of course, by distinguishing *numerical* from *qualitative* identity. Where analytic philosophers have normally supposed that the distinction between these two kinds of entity can be drawn only in relation to ordinary material bodies falling under sortal terms and to the pieces of such bodies, the proponent of moments is claiming, in effect, that the distinction applies also to entities of a third category, distinct from both of these.

4. Against moments

Within the Anglo-Saxon philosophical literature it is Stout who is most commonly associated with the claim that moments or particularised qualities and relations exist and that the acknowledgment of their existence is a precondition of an adequate ontological theory of material reality.[31] Moore, on the other hand, is most commonly associated with the denial of this claim. The defenders of moments have included also Husserl,[32] Meinong,[33] Ingarden,[34] Kenny,[35] Williams,[36] and – with some reservations – Strawson.[37] Their critics have included Brentano and Marty,[38] Gustav Bergmann and his pupils (for example Grossmann[39]), Armstrong,[40] and most notably – especially with regard to those moments which are mental acts and states – the later Wittgenstein.

In his famous dispute with Stout on this issue, Moore claimed that he could make nothing at all of what Stout was saying. And it is difficult to see what arguments could be advanced which would convince him that he should make the effort to understand. For there is no worldly *fact* which cannot be expressed in a vocabulary which eschews reference to moments. That there is a moment of influenza inhering in John, for example, can be expressed by asserting that John has influenza; that there is a moment of redness inhering in this sheet of coloured glass can be expressed simply as: the sheet of glass is red.[41] Here both 'influenza' and 'red', as they occur in the reformulated sentences, are general terms; their use involves ontological commitment not to individual moments but (at most) to general concepts or to universals.

It is not contingent facts which fall out of our purview when moments are denied, but rather certain *materially necessary* relations between objects in the world (relations reflected by propositions which are, in the jargon of philosophy, synthetic and *a priori*)[42]. Consider, for example, the thesis that a particular individual redness *r* cannot, *as a matter of necessity*, exist, except in co-existence with some specific extended object; or the thesis that the individual moments of hue, saturation and brightness which constitute a particular redness cannot, *as a matter of necessity*, exist, except in co-existence with each other.[43] These theses are not capable of being expressed except in a language which embraces ontological commitment to moments. But for analytic philosophers already suspicious

of (*de re*) necessity they will provide little reason to abandon suspicion of the new and dubious category of moments.

The most powerful motivating force underlying the resistance of the analytic philosopher to the acceptance of an ontology of moments is his tendency to run together ontological questions with questions of logic or 'grammar'. Confronted with sentences involving apparent reference to moments, the analytic philosopher will attempt to understand the *logical* form of such sentences, i.e. he will attempt to translate them into the canonical notation of the predicate calculus.[44] But this notation, as standardly understood, allows reference only to objects of non-dubious kinds. (This is a result of the fact that the employment of the predicate calculus as a tool of analytic philosophy has rested on a reading of its individual constant and variable terms as standing in for the proper names of ordinary language. The latter contains no proper names for moments, and therefore also no means of signalling, where moments are concerned, the distinction between numerical and qualitative identity.) The ontological question, whether non-dubious objects exhaust the furniture of the universe, is thereby replaced by the logical question: can *talk* in terms of objects (or in terms of objects and concepts) do the work of talk in terms of moments? Our own view, as will by now be clear, is that the new – or not so new[45] – category must indeed be recognised. The catalogue of moments would include not only individual colours, tones and other secondary qualities, diseases and their symptoms, mental acts and states, but also shapes, motions, velocities, electric fields and charges, and almost all the other phenomena of physics.[46]

It is not only our picture of the ontology of the world which gets mucked up when moments are excluded however. Our picture of mental experience, too, becomes distorted. For there are large and important classes of mental acts which have moments as their objects; and as we shall see, the analytic philosopher's attempts to come to grips with the structures of such acts without recourse to moments leads him to conclusions that are wholly counterintuitive.

It is possible to divide mental acts into two broad categories: acts directed to objects (in our wide sense): for example an act of seeing, noticing or remembering *the door*; and acts directed to states of affairs, called by analytic philosophers 'propositionally articulated': for example a seeing, noticing or remembering *that the door is closed*.

Now it would seem that there are many acts quite clearly falling within the former category which are directed not to individual objects falling under sortal terms but to moments. We can see, smell, feel, notice or remember, for example, the colour of Kissinger's hair, the anger of Monostatos, the light of Venice, the smell of Lake Nyassa, the gait of Groucho Marx, the pace of Manhattan, and so on. In virtue of their denial of moments, analytic philosophers are constrained to regard such acts as propositionally articulated (Pameno's seeing the anger of Monostatos, for example, is replaced by Pameno's seeing that Monostatos is angry; a thinking about John's influenza is replaced by a thinking about John combined together with a thinking that John has influenza). The analytic philosopher can give credence to this account only because his methodology allows him to come into contact with mental experience itself only indirectly: his immediate attentions are directed to the sentences normally used to express or signal mental acts. To understand mental experience is for him merely to understand the logical form of the sentences in which the relevant mental verbs figure. But whilst it may (perhaps) be true that 'Pameno sees the anger of Monostatos' behaves, from the logical point of view, indistinguishably from 'Pameno sees that Monostatos is angry', the two sentences picture mental acts whose ontological structures are – like the structures of their respective objects – crucially distinct.

5. Propositional pictures II

Given any arbitrary material aggregate we can introduce arbitrary piecings or partitions of the aggregate into constituent material parts. In an aggregate of cabbages and kings, for example, the partition Z_1 might discriminate the sub-aggregate consisting of all cabbages from the sub-aggregate consisting of kings; Z_2 might discriminate kingly legs from the sub-aggregate of cabbages aggregated with kingly residues; Z_3 might discriminate green from non-green members of the original totality.

Z_1, Z_2 and Z_3 are material partitions, not merely in the sense that they discriminate material parts, but also in the sense that the concepts in terms of which they articulate the initial totality are material, rather than formal concepts (the meanings of the corresponding expressions – 'green', 'leg', 'king', etc. – are not capable of being grasped completely, 'without bothering about sense or meaning', by reference to the formal operations which govern their use).[47] Can we assume that all partitions are material in this second sense? Consider, for example, that partition of the totality of kings and cabbages which discriminates its

constituent *individual objects*. The concept of an individual object, of an integral or unified whole, is, surely, a formal concept (and we shall see that the formal ontological language can be extended in such a way as to establish that this is indeed the case). But problems arise in virtue of the fact that individual objects can be discriminated at different levels within the initial totality. There is, first of all, the natural partition of this totality into individual cabbages and kings. But within each cabbage and king we can go on to discriminate individual cells, individual molecules, and so on.

The existence of this hierarchy of levels within the world of material things implies also that the mereological concept of piecing discussed in Section 2 above is more involved than may at first appear to be the case. That process of slicing a piece of cake which produces further sliceable pieces of cake (further pieces on the same level as the original whole) must somewhere – perhaps with the appearance of molecules of starch – come to an end. Spatial or temporal *continua*, in contrast, are homogeneously pieceable. This suggests a distinction between homgeneous and non-homogeneous extensivity: a non-homogeneously extensive whole contains within itself – corresponding to the successive levels of integrity to be disclosed within it – certain material contours which must be respected in any actually executed process of piecing.

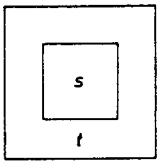
How, then, are we to extend the formal ontological calculus in such a way as to exhibit the formal character of the concept *individual object?* It is the notion of existential dependence which will form the basis of this extension. A colour moment is, we have said, *dependent* on its bearer: whilst the former cannot exist without the latter, the latter may well exist without the former (though not, we can assume, without *some* colour moment). A husband, similarly, is dependent on a wife. Here, however, we have a case of two-sided or mutual dependence, since the wife, equally, cannot exist (in her capacity as wife) without her husband.

Following Husserl we can say that an item t is *founded upon* a second item s if and only if t is such that, of its nature, it cannot exist except as bound up (in some more inclusive whole) with s.[48] If t is founded on r and there is no s such that t is founded on s and s on r, then we say that t is *immediately* founded upon r; otherwise t is *mediately* founded. An individual object – integral or unified whole – can now be defined as a whole all of whose parts are mediately or immediately founded on each other, and none of whose parts are founded on any item discrete from the whole itself. The existence of different levels of integrity is recognised, within the terms of this definition, in virtue of the fact that the foundation or dependence of one object upon another is itself a function of the ontological level or region from whose standpoint the given objects are considered. (Hans in his capacity as a husband is founded on Erna his wife; in his capacity as a human being, however, Hans can perfectly well exist independently of his wife.)[49]

Integral wholes are symbolised by polygonal frames.

'$\boxed{t}$' signifies that the integral whole t exists. That the integral whole s is part of the integral whole t is signified, in analogy with the notation introduced above, by:

(8)



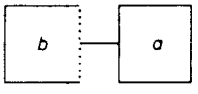
'$\square$' signifies 'an integral whole exists'.

Moments, that is to say, those items founded on other items, we symbolise by means of broken frames such as:

The meaning of the above can perhaps best be made clear by means of examples:

One-sided dependence

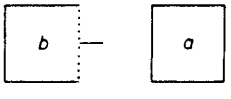
(9)



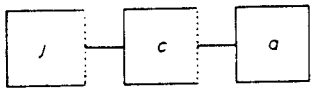
signifies that, say, the specific bruise, blueness or baronetcy b is founded or dependent on its bearer, Alfredo.

(A connected frame-complex like (9) should not be held to carry any presupposition to the effect that moment b and object a are mereologically discrete, that *b lies outside* its bearer. Whilst it is natural to conceive certain kinds of moments as discrete in this sense, for other kinds of moments it is equally natural to conceive moment and object as overlapping or, indeed, to conceive a moment as a proper part of its object. Presuppositions of mereological discreteness are however preserved in disconnected frame-complexes like:

(10)

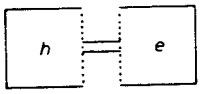


Here, however, it is left open whether *b* is a moment of *a* or of some other object.)

(11) 

might signify that the judgment *j* (the specific mental act of judgment performed by Alfredo at some specific time) is founded on a certain competence (including a knowledge of the language in which *j* is formulated), which is in turn founded on Alfredo. Alternatively it might signify that Alfredo's recovery is founded on his prior sickening, or that Alfredo's redemption is founded on his having sinned.

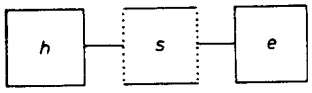
Mutual dependence

(12) 

might signify that husband Hans and wife Erna are mutually founded on each other. Alternatively it might signify the mutual dependence of the North and South poles of a magnet. The individual cells of my body, considered in and of themselves, are mutually independent; considered as functioning parts of my body, however, they are bound together in complex ways by relations of mutual foundation.

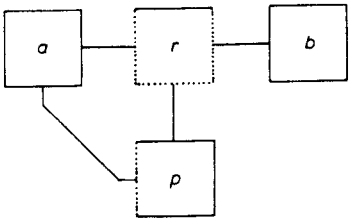
Relational dependence

A swordfight, *s*, between Hans and Erna, is a relational moment one-sidedly founded upon both members of the pair of objects which it relates:

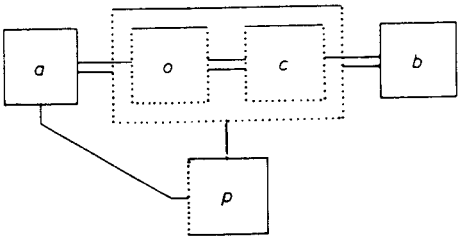
(13) 

Note that the swordfights, hits, kisses and bonds of wedlock which bind Hans and Erna to each other are material relations; thus they are to be distinguished from the relations of foundation or dependence, signified by lines emanating from non-solid frames, which are purely formal.[50]

It is not only material objects such as living bodies and lumps of cake which exhibit successive levels of articulation. Relational and non-relational moments, too, may admit the discrimination of constituent individual parts. Let us suppose that Alfredo and Bernadetto are bound together by a material relation *r*, which exists in virtue of a promise *p*, whose content has not yet been realised:

(14) 

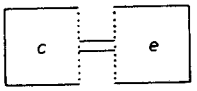
Here *r* consists, in effect, of a claim by Bernadetto on Alfredo, mutually founded on an obligation on Alfredo's part, in respect of Bernadetto. Or, in other words:

(15) 

Note that whilst the material relation *r* between *a* and *b* is merely contingent (Alfredo need not have promised anything to Bernadetto), once *r* and *p* are in place, the formal relations (represented by the lines emanating from broken frames) are relations of necessary co-existence: *o* cannot exist as a matter of necessity, unless *a* and *c*, and therefore also *b*, exist.[51]

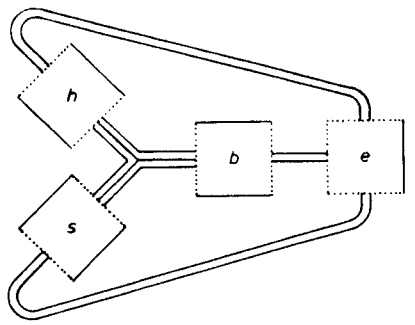
Moments of extension

A moment of colour cannot exist except as the colour of some specific moment of visual extent, which in turn cannot exist except as the bearer of some specific colour.[52]

(16) 

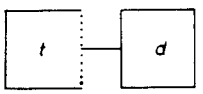
Further, every colour moment manifests constituent moments of hue, brightness and saturation. A colour-hue cannot of its nature exist, except as bound up with some specific brightness and saturation; brightness and saturation cannot exist except as bound up with some specific hue.[53] (16) can therefore be redrawn as:

(17)



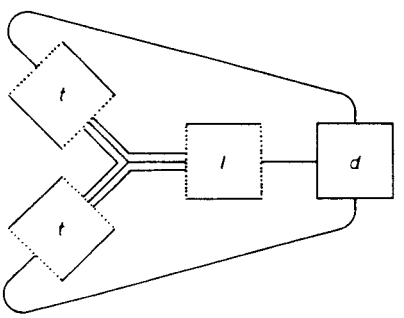
A moment of tone, similarly, depends upon some specific moment of temporal extent (every actually existing tone has some duration). The dependence involved here is one-sided rather than mutual, since a silence, too, (that is to say nothing at all) can found a temporal extent:

(18)



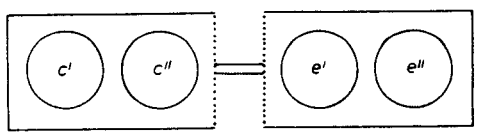
But tones also exhibit constituent moments (of pitch, timbre and loudness) mutually founded on each other. (18) can therefore be redrawn as:

(19)



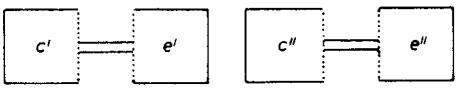
In Section 3 above we defined extensive wholes as wholes which enter into the mereological relations of discreteness, overlapping, etc., and thus admit of piecing. Those wholes which admit of arbitrary decomposition into constituent mutually independent pieces we called purely or homogeneously extensive. Spatial and temporal stretches *e* and *d* are purely extensive in this sense: they can be arbitrarily pieced, and their piecing brings about a corresponding piecing of the respective moments of colour and tone with which they are associated. (16), for example, may become transformed into:

(20)



or alternatively (where the process of piecing is carried out only in thought) into:

(21)



A movement of the hand enduring through the interval *i* can likewise be pieced, in reflection of any arbitrary piecing of *i* into constituent temporal phases. Consider, however, that movement *m* of the hand of Brutus which is his stabbing of Caesar. Considered purely as a movement, *m* is, like *c* and *n*, arbitrarily pieceable. Considered as a stabbing, however, *m* cannot be pieced, since a process of stabbing is not made up of arbitrarily delineable constituent stabbings in the way that a movement is made up of arbitrarily delineable constituent movements. We might therefore distinguish a spectrum of degrees of extensivity. At the one extreme we have pure or homogeneous extensivity, exhibited by spatial and temporal *continua* and by certain moments immediately founded on these. At the opposite extreme we have strictly non-extensive moments which are not decomposable into pieces at all. These fall into two categories. On the one hand we have killings, murderings, promisings, blessings, forgivings, etc.; processes which take place in time, but which are not decomposable into constituent processes which are themselves also killings, murderings, promisings, etc. On the other hand we have trivially non-pieceable event-*boundaries* such as beginnings and endings (Alfredo's becoming married to Carlotta, Caesar's becoming dead, and so on).[54] Between the two extremes we have material wholes (such as lumps of metal and of cake), together with, for example, processes of whistling and hitting. These can be decomposed, but only up to certain limits, into constituent parts on the same level as the original totality: a whistling may be pieceable into component whistlings, but resolves, eventually, into constituent individual whistles.

6. Epilogue on formal operations

Clearly the polygonal frames of integrity and of momenthood can be combined diagrammatically with the circular frames of Section 2 in such a way as to give rise to new families of formal ontological inference-relations after the manner of (5) above, though of a radically higher degree of complexity. Two distinct, though related problems then remain to be solved. The first consists in the specification,

within the framework of the formal ontological language, of the purely formal operations which define the meanings of its constituent constants. A solution of this (comparatively simple) problem allows us to make manifest the formal character of the concepts *moment*, *individual object*, and so on, introduced above. The second, more difficult problem is that of extending the formation and deduction rules specified for the purely Boolean fragment of the language, in such a way that they would embrace also propositional pictures incorporating polygonal frames and frame-connecting links. Neither problem is of purely parochial interest: to the extent that the frame language is, as we have claimed, a directly depicting language, the specification of its structure is part of a larger, ontological project. Our understanding of the properties of an extended Boolean language has implications also for our understanding of the structure of the world itself.

Notes

[1] Cf. Section 62 of the 'Prolegomena' to Husserl (1900/01), and also Chapters 2 and 3 of Mulligan (1980).

[2] More precisely, though also more tendentiously, we may say that logic deals with the meanings of such sentences. On the ontological distinction between meaning-entities and object-entities – which corresponds loosely to Frege's distinction between senses and (ordinary) referents – see Smith (1978), Section 2.

[3] We leave open the question whether this is a sufficiently precise demarcation of the province of logic (whether, for example, identity should be recognised as a logical concept, and whether logic should include, say, the theory of types, simple or ramified). See Hacking (1978), (1979).

[4] The literature on part-whole relations, especially from Austrian and German (and of course Polish) authors, is considerably more extensive than is normally supposed; see the bibliography to Smith (ed.) (1982).

[5] It was almost certainly Russell, in his early papers on Meinong's theory of objects, who was the first explictly to run together logical and ontological structure. Cf. his discussion of one-sided (ontological) implication on p. 25 of his (1904).

[6] Cf. Hacking (1979), Mulligan (1980, Ch. 2) and Smith (1981). The first truly coherent statement of the operational conception of logic was almost certainly made by Gentzen (1935).

[7] See, e.g., his (1901), pp. 475, 485 (1900/01), 'Prolegomena', Sections 67ff.; Investigation III, Ch. II; Investigation IV, Section 13.

[8] Cf. Dummett (1973), Ch. 2.

[9] Logic after the manner of, for example, Boole, on the other hand, has fallen entirely out of favour – and not without reason: "anyone unaquainted with Boole's works will receive an unpleasant surprise when he discovers how ill-constructed his theory actually was and how confused his explanations of it" (Dummett, 1959, p. 205). Identical criticisms can be directed against Schröder, and even against that last great representative of the Boolean logical tradition, Leopold Löwenheim (see especially the latter's 1940).

[10] See, e.g., Cocchiarella (1974) and Smith (1978).

[11] Carroll's logical writings (1977) hardly rise above the level of a formal game, though he advances a number of interesting diagrammatical innovations. The mathematical work of Clifford and Sylvester on certain algebraic analogues of the two-dimensional notation of chemistry, on the other hand, work which has its origins in the same logico-algebraic tradition, contains important and seminal ideas in what has subsequently come to be called graph theory.

[12] This is especially true in the case of Schröder (1966).

[13] Such rules have been provided for a linear calculus having many similarities to the formal ontological language developed here in: W. Degen, 'Sketch of a Rational Grammar', University of Erlangen (unpublished).

[14] *Tractatus*, 3.202, 4.22f, 5.55.

[15] We leave open the question whether this broad understanding of the concept 'object' can be extended to include, for example sentences and sentence-meanings (cf. Husserl's 4th *Logical Investigation*). This would imply, of course, that the subject-matters of logic and ontology overlap, though their respective treatments of their common subject-matters would be entirely different.

[16] Left-right concatenation accordingly carries no connotation of spatial or temporal proximity; it signifies, rather, the empty (formal) relation in which the elements of any arbitrary complex or list stand to each other: cf. the treatment of (what Husserl confusingly calls) 'psychical relations' in his *Philosophie der Arithmetik* (1891).

[17] The former reading should normally be presupposed in what follows, if only in order to avoid notational ambiguity. This ambiguity can of course be removed syntactically, e.g. by writing an assertion sign before a propositional picture in those contexts in which its intended reading is non-nominal.

[18] Cf. e.g. *Tractatus*, 6.124.

[19] See Husserl (1900/01), Investigation IV, Section 11; V, Section 35; (1929), Appendix I, Section 13.

[20] It is of course possible, once actual names have been substituted for the dummy expressions 'r', 's', 't', etc. in a propositional picture, to exhibit the logical form of the proposition which results, and this will sometimes involve the use of quantifiers (e.g. where 'r' and 's' in (4) signify respectively 'Britons' and 'Englishmen').

[21] The rule of inference at work here is Degen's law: from any Γ move to any Δ, where all well-formed parts of 'Δ' are parts of 'Γ'. Here 'Γ' and 'Δ' are meta-linguistic variables ranging over propositional pictures.

[22] See Reinach (1911), p. 114n (Section 19 of the English trans.).

[23] I am grateful to Kit Fine for this statement of the problem.

[24] Husserl (1900/01), Investigation III, Section 17.

[25] This is the standpoint of reism or concretism, first propounded by the later Brentano and most assiduously defended by Kotarbiński in his (1966).

[26] On the distinction between 'abstractables' and 'extractables' see Ryle (1960), p. 58.

[27] I am grateful to Prof. C. Lejewski for this information: cf. Kotarbiński, *op. cit.*, p. 431.

[28] The case for such a generalisation has been more fully presented in Smith (1981) and in Smith and Mulligan (1982).

[29] 'Moment', here, is a translation of the German '*das Moment*', normally rendered as 'element' or 'factor', and contrasted with the masculine '*der Moment*', signifying a moment or instant of time.

The neuter term has an established employment in the vocabulary of mechanics in expressions like 'moment of inertia', 'moment of force', 'moment of a couple', and so on.

[30] Husserl (1900/01), Investigation III, Section 17.

[31] Stout (1918); on the Stout/Moore debate see Künne (1982). The question whether moments exist is, of course, independent of the question of the formal character of the concept *moment*, which is our principal concern in the present paper. The latter question will however naturally become considerably more interesting if we find that the former has to be answered in the affirmative.

[32] For an account of Husserl's theory of moments, which arguably forms the indispensable ontological basis for the entire discipline of phenomenology, see Simons (1982); Smith and Mulligan (1982), and the references there given.

[33] Most explicitly in his (1906).

[34] (1964/65), Vol. II/1, Ch. VIII, Sections 40ff.

[35] E.g. his (1980), pp. 33ff.

[36] See his (1953).

[37] (1959), e.g. pp. 46f., 71, 79.

[38] (1930), Part III. On Marty, cf. Mulligan, *op. cit.*, Ch. 4, Section 2.

[39] (1974), Ch. VII, Section 1.

[40] (1978), Ch. 8.

[41] This account is defended by Tugendhat (1976), Ch. 10, pp. 168ff.

[42] Husserl, *Investigation III*, Section 11 on "The difference between these 'material' laws, and 'formal' or 'analytic' laws". See also Smith (1981) and Mulligan (1980), Chs. 2 and 3.

[43] Stout, and other Anglo-Saxon defenders of moments, normally failed to appreciate the importance of such necessary co-existence relations (as they failed also to appreciate the mereological properties of extensive moments). For a discussion of the modal logical properties of propositions expressing dependence relations see Simons (1982).

[44] Or, as in the case of the later Wittgenstein, he will attempt to exhibit their 'grammar'.

[45] The theory of moments was of course anticipated by Aristotle and the scholastics in their theories of individual accidents. See Smith and Mulligan (1982), Section 1.

[46] On the moment-structures in the world of physics see Köhler (1920) and the discussion in Smith and Mulligan (1982), Section 6.

[47] A taxonomy of types of material partition is provided by Rausch (1937); see also the discussion in Smith and Mulligan, *loc. cit.*

[48] Cf. Husserl, *Investigation III*, Sections 14ff. This statement of the definition is incomplete, since it fails to take into account the essences or natures of the objects involved: see Simons (1982) and Smith (1981).

[49] More detailed discussions of this relativisation of the concept of foundation to level or region are provided in Smith (1982) (especially in relation to the social formations investigated by Reinach in his (1913) and Simons, *op. cit.* On the concept of absolute foundation see Husserl, *loc. cit.*

[50] Vitalist theories of biological organisation seem to have rested on the view that there exists a material relation (call it 'life') binding together the molecules of a living body.

[51] On the synthetic *a priori* relations between social formations see Reinach (1913).

[52] Relations of mutual dependence between mental contents were first investigated by Stumpf in his (1873), a work of the highest significance in the history of the theory of part and whole, not only because it influenced Husserl in the development of his own theory in the 3rd *Logical Investigation*, but also because it effected a radical break with the atomistic theories of mental experience which had hitherto predominated amongst empirical psychologists: see Smith and Mulligan (1982).

[53] For an account of these relations see, e.g., Harrison (1973), pp. 68ff.

[54] The latter have obvious analogues in spatial boundaries, surfaces, etc. Candidate spatial examples of non-pieceable entities in the former category might be traffic signs, and works of visual art.

Bibliography

Armstrong, D.M.: 1978, *Universals and Scientific Realism*, Cambridge University Press, Cambridge, 2 vols.

Brentano, F.: 1930, *Wahrheit und Evidenz*, ed. by O. Kraus, Meiner, Leipzig, Eng. trans. *The True and the Evident*, Routledge, London, 1966.

Carroll, L.: 1977, *Symbolic Logic*, Harvester, Hassocks.

Cocchiarella, N. B.: 1974, 'Formal Ontology and the Foundations of Mathematics', in G. Nahknikian (ed.), *Bertrand Russell's Philosophy*, Duckworth, London.

Dummett, M.: 1959, 'Review [of writings by and on Boole]', *Journal of Symbolic Logic* 24, 203–9.

Dummett, M.: 1973, *Frege, Philosophy of Language*, Duckworth, London.

Gentzen, G.: 1935, 'Untersuchungen über das logische Schliessen', *Mathematisches Zeitschrift* 39, 176–210, 405–31; Eng. trans. in *The Collected Papers of Gerhard Gentzen*, North-Holland, Amsterdam, 1969, pp. 68–131.

Goodman, N.: 1966, *The Structure of Appearance*, 2nd ed., Bobbs-Merrill, New York.

Grossmann, R.: 1974, *Meinong (The Arguments of the Philosophers)*, Routledge, London.

Hacking, I.: 1978, 'On the Reality of Existence and Identity', *Canadian Journal of Philosophy* 8, 613–32.

Hacking, I.: 1979, 'What is Logic?', *Journal of Philosophy* 76, 285–319.

Harrison, B.: 1973, *Form and Content*, Blackwell, Oxford.

Husserl, E.: 1891, *Philosophie der Arithmetik. Psychologische und logische Studien*, Vol. I (only volume published), Pfeffer, Halle, repr. in Vol. XII of Husserl's collected works.

Husserl, E.: 1900/01, *Logische Untersuchungen*, 1st ed., Niemeyer, Halle; Eng. trans. of 2nd ed., *Logical Investigations*, Routledge, London, 1970.

Husserl, E.: 1901, 'Gebiet eines Axiomensystems/Axiomensystem-Operationssystem', first publ. in Vol. XII of Husserl's collected works.

Husserl, E.: 1929, *Formale und transzendentale Logik*, Niemeyer, Halle; Engl. trans., *Formal and Transcendental Logic*, Nijhoff, The Hague, 1969.

Ingarden, R.: 1964/65, *Der Streit um die Existenz der Welt*, 2 vols., the 2nd in 2 parts, Niemeyer, Tübingen.

Kenny, A.: 1980, *Aquinas*, Oxford University Press, Oxford.

Köhler, W.: 1920, *Die physischen Gestalten in Ruhe und im statio-*

nären Zustand, eine naturphilosophische Untersuchung, Vieweg, Braunschweig.

Kotarbiński, T.: 1966, *Gnosiology. The Scientific Approach to the Theory of Knowledge*, Pergamon, London.

Künne, W.: 1982, 'Criteria of Abstractness. The Ontologies of Husserl, Frege and Strawson Against the Background of Classical Metaphysics', in Smith (ed.), (1982), 401–37.

Löwenheim, L.: 1940, 'Einkleidung der Mathematik im Schröderschen Relativkalkul', *Journal of Symbolic Logic* 5, 1–15.

Meinong, A.: 1906, *Über die Erfahrungsgrundlagen unseres Wissens*, Springer, Berlin; repr. in Vol. V of the *Gesamtausgabe* of Meinong's works.

Mulligan, K.: 1980, *Representation and Ontology in Austro-German Philosophy*, Dissertation, University of Manchester.

Rausch, E.: 1937, 'Über Summativität und Nichtsummativität', *Psychologische Forschung* 21, 209–89; repr. Darmstadt: Wissenschaftliche Buchgesellschaft, 1967.

Reinach, A.: 1911, 'Zur Theorie des negativen Urteils', in Pfänder (ed.), *Münchener Philosophische Abhandlungen*, Barth, Leipzig; as repr. in Reinach, *Gesammelte Schriften*, Niemeyer, Halle, 1921, pp. 56–120; Eng. trans. in Smith (ed.), (1982), 315–77.

Reinach, A.: 1913, 'Die apriorischen Grundlagen des bürgerlichen Rechts', *Jahrbuch für Philosophie und phänomenologische Forschung* 1, 685–847.

Russell, B. A. W.: 1904, 'Meinong's Theory of Complexes and Assumptions', Part I, *Mind*, n.s. 13, 204–19, as repr. in Russell, *Essays in Analysis*, Allen and Unwin, London, 1973, pp. 21–37.

Ryle, G.: 1960, 'Letters and Syllables in Plato', *Philosophical Review* 69, as repr. in Ryle's *Collected Papers*, I, Hutchinson, London, 1971, pp. 54–71.

Schröder, F. W. K. E.: 1966, *Vorlesungen über die Algebra der Logik*, 2nd ed., Chelsea, New York.

Simons, P. M.: 1982, 'The Formalisation of Husserl's Theory of Wholes and Parts', in Smith (ed.), (1982), 113–59.

Smiley, T. J. and Shoesmith, D. J.: 1978, *Multiple Conclusion Logic*, Cambridge University Press, Cambridge.

Smith, B.: 1978, 'An Essay in Formal Ontology', *Grazer Philosophische Studien* 6, 39–62.

Smith, B.: 1981, 'Logic, Form and Matter', *Aristotelian Society* (Supplementary Volume), 55, 47–63.

Smith, B.: 1982, 'Introduction to Adolf Reinach, On the Theory of the Negative Judgment', in Smith (ed.), (1982), 289–313.

Smith, B. (ed.): 1982, *Parts and Moments. Studies in Logic and Formal Ontology*, Philosophia, Munich.

Smith, B. and Mulligen, K.: 1982, 'Pieces of a Theory', Introduction to Smith (ed.), (1982), 15–109.

Stout, G. F.: 1918, 'Do Finite Individuals Possess a Substantive or an Adjectival Mode of Being?', *Aristotelian Society* (Supplementary Volume), 55, 127–48.

Strawson, P. F.: 1959, *Individuals. An Essay in Descriptive Metaphysics*, Methuen, London.

Stumpf, C.: 1873, *Über den psychologischen Ursprung der Raumvorstellung*, Hirzel, Leipzig.

Tugendhat, E.: 1976, *Vorlesungen zur Einführung in die sprachanalytische Philosophie*, Suhrkamp, Frankfurt.

Williams, D. C.: 1953, 'The Elements of Being', *Review of Metaphysics* 6, 3–18 and 171–93.

Wittgenstein, L.: 1961, *Tractatus Logico-Philosophicus*, Routledge, London.