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Contextual Logic and Epistemic Contexts

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Abstract In this paper, I analyze the notion of context developed by McCarthy and Buvač (1994) and their contextual logic in order to characterize, from an epistemological point of view, a workable notion of epistemic context. This analysis contributes to showing how epistemological contextualism can be formally modeled, and how it can constitutes a general epistemological framework for epistemic normativity.

Keywords Contextual logic; Epistemic context; Epistemic normativity; Epistemic standard; Contextualism

1. Contextual Logic

The notion of context and the contextual logic originally developed by John McCarthy in the field of artificial intelligence aim at providing a solution to the problem of generality, i.e., the problem of representing ordinary knowledge and its integration into inferential processes operating on knowledge bases. The contextual logic of McCarthy and Buvač (CL_{MCB}) can be defined generally as $FOL \cup \{ist(c, \phi)\}$, where FOL is classical first-order logic and $ist(c, \phi)$ is an operator meaning that the formula ϕ is true in context c. The operator ist expresses a relation between a formula and a set of first-order true formulas which is reified as a formal object, a context. In CL_{MCB}, the completeness of FOL is preserved (Buvač and Mason 1993; Buvač, Buvač, and Mason 1995), and even though this contextual logic is not strictly speaking an epistemic logic, comparable for instance to Lemmon and Henderson (1959) or Hintikka (1962, 1975), it can be nonetheless represented in a standard multimodal logic (Buvač, Buvač, and Mason 1995).

Buvač (1996) defines the syntax of CL_{MCB} by means of the following axioms and rules[1]:

(PL) $\vdash_k \phi$, where ϕ is an instance of a propositional tautology

(UI) $\vdash_k (\forall x)\phi(x) \supset \phi(a)$

(MP) $\dfrac{\vdash_k \phi \quad \vdash_k \phi \supset \psi}{\vdash_k \psi}$

(UG) $\dfrac{\vdash_k \phi \supset \psi(x)}{\vdash_k \phi \supset (\forall y)\psi(y)}$, where x is not free in ϕ

(K) $\vdash_k ist(k', \phi \supset \psi) \supset (ist(k', \phi) \supset ist(k', \psi))$

(D) $\vdash_k ist(k_1, ist(k_2, \phi) \vee \psi) \supset ist(k_1, ist(k_2, \phi)) \vee ist(k_1, \psi)$[2]

(Flat) $\vdash_k ist(k_2, ist(k_1, \phi)) \supset ist(k_1, \phi)$

(Enter) $\dfrac{\vdash_{k'} ist(k, \phi)}{\vdash_k \phi}$

[1]Instead of $\vdash k : \phi$, I simply use $\vdash_k \phi$ to mean that a formula ϕ is provable (or assertable) in the context k.

[2]Buvač used Δ instead of D to refer to this propositional property of contexts. I shall use D in order to avoid confusion with the usual symbol for knowledge bases, Δ.

(Exit) $\dfrac{\vdash_k \phi}{\vdash_{k'} ist(k, \phi)}$

(BF) $\vdash_k (\forall v) ist(k', \phi) \supset ist(k', (\forall v)\phi)$

The first group (PL, UI, MP, UG) comprises axioms and typical rules of FOL. In the second group ($K, D, Flat, Enter, Exit$), the axioms and rules express propositional properties of contexts; axiom K is a principle of deductive closure (an analogue of the axiom K in modal logic), axiom D (which Buvač called *contextual omniscience*) permits the qualification of any information accessible from any given context, axiom $Unif$ is a principle of information preservation through contexts, and the rules $Enter$ and $Exit$ permit to access or to leave a context. Finally, in the group of quantificational properties of contexts, there is one axiom (BF) analog to the Barcan formula specifying the relation between the *ist* operator and the universal quantifier.

Classes of Contexts

Buvač (1996) makes a distinction between two classes of contexts, the *knowledge base* contexts (c_{kb}) and the *discourse* contexts (c_d). Whereas in c_{kb} predicates are univocal, in c_d predicates may be ambiguous. A c_{kb} is a set of true propositions, or facts, in a given knowledge base. A c_d is characterized by two components, a set of *epistemic states* and a set of *semantic states*. In an epistemic state, one finds typical elements of a knowledge base, i.e., facts. A semantic state sets the interpretation of a predicate by means of a relation to another predicate in a knowledge base. It is by virtue of such a relation that an ambiguous predicate in a c_d can be disambiguated.

The main motivation behind CL_{MCB} consists precisely in providing a formal framework for eliminating ambiguity.[3] This is where CL_{MCB} presents a special interest for epistemology, in particular for contextualism. Since the knowledge operator has to be interpreted as an indexical term, according to epistemological contextualism (Cohen 1987), it is an operator that requires disambiguation in function of its context of utterance, and thus an epistemic context has to be conceived as a c_d. In this view, CL_{MCB} can shed light on the dynamics at play between the interpretation of the knowledge operator and the epistemic contexts of utterance.

[3]It can also be extended to other types of contexts (Guha and McCarthy 2003).

2. Epistemic Contexts

In order to take advantage of CL_{MCB}, I will need to load the notion of c_d with some epistemological content. The notion of epistemic context (c_ε) that I will be using rests on the idea that *an epistemic context c is a context defined by an epistemic standard ε that is an introduction rule for the knowledge operator in c*. In CL_{MCB} terms, the standard ε is a subset of the axioms of the knowledge base of c (Δ_c), and to each epistemic context c_ε is associated one and only one epistemic standard. Since it is the epistemic context that determines the meaning of the knowledge operator, then an epistemic context can be envisioned as a c_d, i.e., $\varepsilon \subseteq \Delta_{c_d}$ and more specifically $\varepsilon \subseteq SemanticStates(\Delta_{c_d})$ because ε provides the *indexical content* (variable part) of the meaning of the knowledge operator. In accordance with CL_{MCB}, the complete characterization of an epistemic context depends on a twofold characterization: a characterization of its *epistemic standard* (ε) and (if any) a characterization of its *transposition rules* (τ), which are the rules that govern its relations with other c_ε.

These conceptual choices center the investigation on the conditions for context shifting and, by way of consequence, on the conditions for epistemic standard shifting. This is in line with the contextualist goal of accounting on the one hand for the dynamics observable in our epistemic exchanges, that express the variability of the epistemic standards in use, and on the other hand, for the legitimacy of these variations (i.e., they are not epistemic faults).[4] These variations in the use of epistemic standards show clearly our capacity as epistemic agents to regiment our epistemic practices accordingly to a plurality of norms in function of our epistemic needs.

One immediate consequence of the above definition of c_ε is that it entails a relativization of all contexts, including logical contexts, that is to say logical contexts are local epistemic contexts like any other epistemic contexts. This creates a difficulty of representation in CL_{MCB} since CL_{MCB} has been devised with the explicit goal of making available logical reasoning in local contexts (via *lifting*) by means of a grammar incorporating FOL. The rules PL, UI, MP and UG render accessible the resources of FOL in every local context. However, this structure cannot account entirely for contextualism, because from the contextualist point of view FOL is only one epistemic context among others, and one can imagine that in some rich and complex epistemic situations many logics, stronger or weaker than FOL, may be called upon. Consequently,

[4] And contrary to what Schiffer (1996) suggested, contextualism does not need an error theory to accommodate an indexical interpretation of knowledge attributions.

CL_{MCB} has to be amended in order to reify FOL so as to become an object of the language, which in turn requires the conversion of the rules PL, UI, MP, UG, K, and D into properties of epistemic contexts defined by logical standards.

Before considering some examples of epistemic contexts, I want to underline that the whole idea here is to give some insight into this notion of epistemic context through a (very) programmatic approach, and the proposed formalism will depart slightly from CL_{MCB} in that I make an explicit distinction among axioms between epistemic standards and transposition rules. By definition, an epistemic context will require one and only one epistemic standard, and most of CL_{MCB} grammatical rules (PL, UI, MP, UG, K, D) will be directly incorporated into contextual transposition rules. As a toy example of a set of epistemic contexts, consider the following three partial and plausible definitions of some ordinary (and common) epistemic contexts, $c_{logical}$, $c_{empirical}$ and $c_{perceptual}$:

Axioms of $c_{logical}$ (c_{log})

($\varepsilon_{log}.1$) $(\forall x)(\phi \supset K(x, \phi))$, where ϕ is an instance of a propositional tautology or of a first-order valid formula

($\tau_{log}.1$) $ist(c_{log}, \phi \supset \psi(x)) \supset ist(c_{log}, \phi \supset \forall y \phi(y))$, where x is not free in ϕ

($\tau_{log}.2$) $(\forall x)((ist(c_{log}, ist(c, K(x, \phi))) \wedge ist(c_{log}, ist(c, K(x, \phi \supset \psi)))) \supset (ist(c_{log}, ist(c, K(x, \psi)))))$

($\tau_{log}.3$) $(\forall x)(ist(c_{log}, ist(c, K(x, \phi \supset \psi))) \supset (ist(c_{log}, ist(c, K(x, \phi))) \supset ist(c_{log}, ist(c, K(x, \psi)))))$

c_{log} corresponds to the classical system of FOL. The axiom $\varepsilon_{log}.1$ is the epistemic standard defining c_{log} and it means that any instance of a propositional tautology or of a valid formula of FOL is sufficient for knowledge.[5] $\tau_{log}.1$, $\tau_{log}.2$, and $\tau_{log}.3$ are respectively the syntactic rules UG, MP, and K of CL_{MCB} expressed in terms of rules of transposition. It is worth noting that $\tau_{log}.2$ guarantees reasoning by *modus ponens* within the scope of the knowledge operator in a given and fixed context, in the very same manner $\tau_{log}.3$ preserves de-

[5]One will recognize in $\varepsilon_{log}.1$ an analogue to the rule of necessitation in modal logic.

ductive closure in a logical context.[6] According to the formulation of $\tau_{log}.3$, the epistemic context c of the antecedent and of the consequent remain fixed. Even though the problem of deductive closure escapes the limits of this paper, I shall observe nonetheless that failures of deductive closure take their origin in a confusion between distinct epistemic contexts, something for which the present proposal can account. One can easily see that any valid pattern of inference can be expressed in the form of a rule of transposition and the set of these rules could be ultimately reduced to a single axiom schema.

Axiom of $c_{empirical}$ (c_{emp})

($\varepsilon_{emp}.1$) $(\forall x)(EmpiricalControl(x, \phi) \supset K(x, \phi))$

$\varepsilon_{emp}.1$ stipulates that the condition to satisfy in order to introduce the knowledge operator in this context is some sort of empirical control made by an agent x towards the state of affairs described by a proposition ϕ. The notion of empirical control in $\varepsilon_{emp}.1$ consists only in a set of procedures providing a sufficient level of discrimination between a state of affairs described by a proposition ϕ and a state of affairs described by a proposition (or several propositions) incompatible with ϕ. In the present illustration, no transposition rule enables one to export empirical knowledge into another c_ε.

Axiom of $c_{perceptual}$ (c_{per})

($\varepsilon_{per}.1$) $(\forall xv)((See(x, v) \vee Hear(x, v) \vee Taste(x, v) \vee Smell(x, v) \vee Touch(x, v)) \supset K(x, \phi))$, where ϕ is immediately linked to v

As regards the perceptual standard, things are different since v is not a propositional content but rather a perceptual content. The knowledge operator is introduced only in virtue of a perceptual state (or a percept). The knowledge operator is in this way dependent on our physiological mechanisms and their respective limitations (think of the various perceptual biases identified by cognitive psychology for instance). No transposition rule is available in c_{per}.

The fact that neither c_{emp} nor c_{per} contain a transposition rule is determined exclusively by the definitions of the epistemic standards. A transposition rule makes possible the propagation of knowledge either within a given context or between different contexts. As opposed to the grammatical rules *Enter* and

[6] $\tau_{log}.3$ is comparable to a kind of principle of scope alteration that switches the scope of K (superior level) with the one of $\supset$ (inferior level). Such a permutation is tolerable solely in a logical order.

Exit which are only rules of access to information, the transposition rules act as qualification rules in much the same manner epistemic standards themselves do. The transposition rules of c_{log} ($\tau_{log}.1$, $\tau_{log}.2$, and $\tau_{log}.3$) are intracontextual rules of transposition. For reasons of simplicity, no such rule has been defined in c_{emp} and c_{per}. Furthermore, there is no intercontextual rule of transposition for $\left\{c_{log}, c_{emp}, c_{per}\right\}$. In c_{per}, for instance, the assertability conditions are evidently too weak to satisfy the assertability conditions of c_{log} and c_{emp}. There is no intercontextual rule of transposition between c_{per} and c_{emp}, because the satisfaction of ε_{per} does not imply the satisfaction of ε_{emp} (ε_{per} is simply too weak), and conversely, the satisfaction of ε_{emp} does not entail the satisfaction of ε_{per} (a property, for example, may be tested empirically while not being itself an object of direct perception). This shows clearly the primitive character of the notion of epistemic standard, which dictates the possibility or the non-possibility of transposition rules. As for the question whether a transposition rule can be valid *a priori*, i.e., independently of any epistemic standard, one can easily see its irrelevance within the proposed contextualist framework.

Another noticeable aspect of the previous definitions is that no intracontextual rule of transposition specifies the conditions of transmission of a knowledge item from one epistemic agent to another. One could think, for instance, that if an agent a has run an empirical control with respect to ϕ and $K(a, \phi)$, then an agent b, who knows that a has performed a test, would know by some testimonial relation that ϕ. More formally: if $\vdash_{c_{emp}} K(a, \phi)$ and $\vdash_{c_{emp}} K(b, K(a, \phi))$, then $\vdash_{c_{emp}} K(b, \phi)$. The main difficulty in the formulation $\vdash_{c_{emp}} K(b, K(a, \phi))$ can be straightforwardly isolated. If b knows that $K(a, \phi)$, then it is surely not in virtue of ε_{emp} since b is not the one who has run the test, but in virtue of another epistemic standard, namely $\varepsilon_{testimony}$. The specification of all the transposition rules for testimonial knowledge constitutes a major issue from an epistemological point of view. These rules require a fine-grained analysis that is beyond the limits of the present paper. Given that the proposed treatment aims only at presenting a workable notion of epistemic context, it is preferable on this occasion to avoid the problem of the transmission of knowledge from one agent to another.

Epistemological Theory

It seems that in our ordinary epistemic situations, the perceptual standard, the empirical standard, and the logical standard (all defined above) are representative of the epistemic resources at our disposal as epistemic agents. But the chief interest in the toy example lies elsewhere. In defining epistemic contexts by means of explicit epistemic standards, one not only gives the knowledge operator its various meanings, but one also describes a structure in which epistemic normativity is spelled out in different terms. Such a conception of epistemic normativity allows for multiple configurations of epistemic contexts, which in turn can be captured by the idea that *an epistemological theory is as a set of* c_ε. The epistemological theory presented above, say Θ, is defined as $\Theta = \left\{c_{log}, c_{emp}, c_{per}\right\}$. An epistemological theory is consequently defined by a specific set of epistemic contexts (or knowledge bases), that is to say a specific set of epistemic standards and transposition rules. The epistemological structure of the theory is given by the transposition rules that govern the inter and intracontextual relations between contexts. This definition provides a new perspective on major debates in contemporary epistemology. Foundationalism, coherentism, reliabilism, and other options based on the JTB model, may be construed as exemplifying different epistemological structures designed to meet different epistemic demands. None of them is the ultimate epistemological theory simply because all of them are instances of particular structural configurations.

The specific structure of an epistemological theory shows the relations between the different assertability conditions of the knowledge operator proper to each context. It could seem that this treatment of epistemic normativity is eluding the crucial problem of the truth conditions of the knowledge operator. Of course, this difficulty has to do with the debate between a realist and an antirealist interpretation of the knowledge operator. One merit of proposed view is its clear response: the truth conditions of K in a given epistemic context are provided by the assertability conditions of K in the given context, so that truth-conduciveness from one context to another follows assertability from one context to another. The purpose of a transposition rule is to authorize the dissemination of assertions in multiple contexts on the basis of one given context. The function of transposition rules though is to be sharply distinguished form the function of the *ist* operator, because the formula in the argument position of the operator is in mention not in use. The *Exit* rule makes explicit the genealogy, so to speak, of the truth of a formula from another context, whereas the *Enter* rule does the inverse, i.e., it encapsulates the truth into the assertability conditions of a context. For a realist, this isomorphic relation between truth conditions and assertability conditions boils down to the elimination of the truth conditions, conceived as contextually independent.

Some realists, e.g., Williamson (1996, 2000), go as far in the opposite direction as making knowledge the norm of assertion. Such a reversal in the assertability conditions does not do justice to the observable variability of epistemic standards in our epistemic practices.

These considerations lead naturally to another important difficulty that a contextualist perspective is facing. Can contextualism account for the implication between knowledge and truth, as the factivity (or veridicality) condition requires it, i.e., $K\phi \supset \phi$? This time the debate takes place between a fallibilist and an infallibilist conception of knowledge.[7] The factivity condition springs from an analysis centered on the necessary conditions for knowledge (analysis *in consequentia*). The framework developed here makes explicit only the sufficient conditions for knowledge (analysis *in antecedentia*); the epistemic standards are nothing else than introduction rules for the knowledge operator, and the antecedent of the epistemic standard may not even contain any epistemic terms, depending on the context. In the proposed view, here lies the main interest of contextualism as it constitutes a general epistemological framework within which epistemic normativity can be analyzed primarily in terms of its function rather than its content. So, in order to make explicit the characterization of some K by means of necessary conditions, the general contextualist framework has to be singularized and that process amounts to the specification of an epistemological theory, as previously defined.

According to the proposed framework, and in conformity with McCarthy and Buvač (1994), the epistemic contexts are conceived independently from the epistemic agents. This only means that the epistemic perspective of a given agent does not alter in any way the facts, or the epistemic states of Δ_{c_ε}. This property of *flatness* makes it easier to isolate the contextual variations at the level of the contexts, in other words at the level of their respective transposition rules. This reification of an epistemic context brings autonomy to the context with respect to the epistemic agents, and this accounts for the constraint that within one given epistemic context all of the epistemic agents are regimented by the very same epistemic standard and submitted to the very same epistemic demands. Certainly one could define an epistemic context with a parameter in relation to the propositional attitudes of the epistemic agents so that a context would vary as a function of the agents. But such a change would represent more than a change of epistemological theory, it would be a more radical change of logic (or grammar) since one would have

[7] In the epistemological theory Θ presented above, ε_{per} shows a high level of fallibility, compared to ε_{emp}, which is moderate, and to ε_{log} which is null.

to give up the $Unif$ axiom of CL_{MCB} in order to render possible alterations of the epistemic states of one context by means of another context. No doubt the rejection of $Unif$ would be relevant in some particular epistemological investigations, but within the limits of the proposed approach that would have the undesirable effect of concealing (at least partially) the dynamics between the epistemic standards.

Conclusion

The notion of context and the contextual logic defined by McCarthy and Buvač prove to be rich in epistemological applications. Since their formal notion of context is devised to resolve the problem sof lexical ambiguity, it furnishes by the same token an adequate framework for the indexical interpretation of the knowledge operator. By extending epistemologically this notion of context, one can provide a basis for epistemological contextualism. An epistemic context can then be defined as a set of one epistemic standard and some transposition rules, and an epistemological theory can be defined as a set of epistemic contexts. From this viewpoint, in which an epistemic standard is conceived as an introduction rule for the knowledge operator in a given context, contextualism appears to be an epistemological framework for epistemic normativity in general, rather than a particular epistemological theory in the strict sense.

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