*L'Enseignement Mathématique (2) 53 (2007), 429–436*

# COMMISSION INTERNATIONALE DE L'ENSEIGNEMENT MATHÉMATIQUE

# (THE INTERNATIONAL COMMISSION ON MATHEMATICAL INSTRUCTION)

## PERSONAL THOUGHTS ON AN ICMI STUDY: "THE TEACHING AND LEARNING OF MATHEMATICS AT UNIVERSITY LEVEL"

by Derek HOLTON

ABSTRACT. In 1997 the International Commission on Mathematical Instruction decided to organise a study into the teaching and learning of mathematics at university level. I have collected here some 'thoughts' that have occurred to me since that study. They are based on notes that I wrote at the time. I look at some personal highlights and concerns from the Study.

### 1. THE STUDY

As with all studies of the International Commission on Mathematical Instruction[1]) (ICMI), the theme of the 11th ICMI Study, on the teaching and learning of mathematics at university level, was decided by the Executive Committee of ICMI. It then appointed the International Program Committee, IPC, that took over the responsibility for conducting the Study. The first job of the IPC was to produce a Discussion Document [3] that laid out the rationale for the Study and ended with a call of interest for the Study conference. The second job was to organise a conference. This was duly held in December 1998 in Singapore. Along with plenary speakers, around 80 participants were invited from among those who responded to the Discussion Document. On the

[1]) More information on the International Commission on Mathematical Instruction can be found in [1] and [2].

*achevé de composer le 20 décembre 2007 à 09 : 08*

basis of the papers presented at this conference, a special issue of the *International Journal of Mathematics Education in Science and Technology* [4] was published early in 2000. In 2001 the Study volume [5], consisting of plenary talks and working group reports from the conference, along with commissioned papers to cover perceived gaps, was published.

## 2. Personal background

I think it is important to say that what I now want to talk about are the highlights and concerns of the Study, from my own perspective. Other participants and observers will likely see different aspects as being of importance.

Since this is a personal view, the reader should know the biases that I brought to this Study. First I am by training and inclination a pure mathematician and I am still publishing in the area of Discrete Mathematics. However, I trained (but never practised) as a secondary mathematics teacher and then entered academia almost by accident. As a result, I suppose that I have always been interested in teaching as opposed to lecturing [6]. For a long time I have been aware of the shortcomings of the traditional lecture format used in many countries and I have tried to find ways to overcome its deficiencies. I am now in a department of mathematics and statistics where many of the staff are also concerned but are unsure as to how they might change things. Like many such departments we were happier in the "good old days" when fewer, but more 'serious', students studied our subject.

## 3. Why worry?

The Study Discussion Document identified five changes that have had a profound effect on mathematics teaching at university level. These are

(i) the increase in the number of students who are now attending tertiary institutions;

(ii) the major pedagogical and curriculum changes that have taken place at pre-university level;

(iii) the increasing differences between secondary and tertiary mathematics education regarding the purpose, goals, teaching approaches and methods;

(iv) the rapid development of technology; and

(v) demands on universities to be publicly accountable.

In some universities, although there has been a doubling of the total number of students enrolled, there has either not been a similar rise in the number of students taking mathematics or this rise has not affected the number majoring or taking honours degrees in the subject. Where universities have been under financial pressure this has led to staff reductions. In some places it has also led to the closure of mathematics departments. This has been a cause of concern and worry to many academics.

At the same time, there has been a demonstrable change in the average quality of first-year students. Many staff have given up the struggle to raise them to the graduating level of the 1970s or early 1980s. But they are worried about what they see as a downward spiral in mathematics in the community. This is especially so where

secondary teachers but more especially where primary teachers are concerned. In many countries, students entering primary teacher education have very weak mathematical backgrounds and it seems that a smaller percentage of majors goes on to secondary teaching every year. There seems to be cause for concern for society at large given that so much is based on such a potentially poor mathematical foundation.

Another worry in this context in some universities is that an ageing department does not understand the basic philosophies and motivation of many undergraduates and, accordingly, finds it increasingly difficult to teach them.

There is a genuine concern for the students in our courses, for their needs and society's future needs, for mathematics itself and its future and, consequently, for what to teach and how to teach it.

## 4. Highlights of the Study

I now want to concentrate on three personal highlights of the Study. These are

(i) the efforts being made to teach mathematics;

(ii) that valuable research does not have to be big research; and

(iii) the discovery of a theory of learning.

Under the first heading I want to emphasise both the word "teach" and the word "mathematics". By "teach" I don't necessarily mean the transmission of material in the manner of a traditional lecture. Rather I mean a delivery that is based upon an awareness of the student's knowledge and needs and allows for some dialogue to take place between student and lecturer. In the process the lecturer recognises, and tries to employ, current learning theories.

Mathematics to me means more than a set of skills or a chain of theorems. I believe that we sell our subject short if we hide the human, processes, side from our students (see [6]). So I feel that some attempt needs to be made to encourage students to discover *some* mathematical results for themselves. The aim is to let students see that mathematics is more than the set skills/algorithms that we have traditionally asked them to regurgitate in examinations. But I deliberately underlined 'some' because there is not time to cover all that we want to cover by discovery alone.

It was a pleasant feeling to find that there were many people at the ICMI Study conference who not only agreed with that perspective but had gone a long way down the implementation path. Naturally I had known of the reform movement in the United States but I didn't have any idea of the extent and impact of programmes, and their research support, that were being undertaken in, for instance, France. The references [7], [8], [9], [10], [11] and [12] are in no way meant to be exhaustive but rather are meant to act as pointers into this literature.

Two aspects of the French scene that appealed to me were *la situation didactique* and *le débat scientifique.* The former involves prior analysis of students' past experiences, of what they bring to the session, and of what sorts of difficulties, pre-conceptions and misapprehensions they are likely to have or to form. Examples of tasks for the students, in order to challenge their conceptions, are chosen using what has been learned from research. So in a tutorial situation one might ask "if $f$ is continuous on $[0, \infty)$, is strictly positive and has a limit of zero at $\infty$, does $f$ necessarily have a maximum value?" Most students are likely to answer this question positively. But then

they are required to provide a proof. In the ensuing discussion, hopefully learning and understanding take place.

Under *le débat scientifique*, students are seen as participants in a scientific community whose methods of development include conjectures, proofs and regulations. Scientific debates can arise spontaneously, as when a student asks a question, or can be intentionally provoked. The guiding principles for scientific debate include:

disturbance – students must encounter and deal with conflict;

inclusiveness – everyone should have an opportunity to understand what we try to teach; and

collectivity – collective resolution of issues shows how to work with contradictions and to respect the views of others.

Turning now to research, it had always seemed to me that the problems involved in mathematics education were major ones and that worthwhile results could only be obtained by large studies over long periods of time. And how could you prove anything in education anyway? Perhaps this was just the naïve opinion of an external observer. There is no doubt there are big issues that will not be resolved quickly. These must surely exist in all disciplines. However, it is possible to gain insight into some worthwhile areas of mathematics education without a huge budget. And these insights can be extremely useful for teaching, especially at a local level. What you discover for yourself about your students can lead to the improvement of your own teaching.

Simple questionnaires, for instance, can provide valuable feedback. Anthony [13] used a questionnaire plus interview methodology to identify factors which are seen as having an important influence on first-year students' success in mathematics. Her sample consisted of 65 students and 22 teaching staff. Both lecturers and students agreed that self-motivation and study were important for success. They also agreed that lack of effort led to failure. However, in some ways the places where they significantly disagreed might be more illuminating. For instance, students placed availability of help much higher on their list of things contributing to success than did lecturers. The same was, perhaps surprisingly, true for regular attendance at lectures and taking notes during lectures. Poor study techniques were listed more highly by lecturers than by students as a factor influencing failure.

Lithner [14] worked with four students who "talked aloud" as they completed two mathematical tasks. Their work was captured on videotape. He found that when they ran into difficulties they tended to revert to what was remembered and familiar, even if inappropriate, rather than to apply mathematical reasoning to the task.

While I had some knowledge of David Tall's work in developing a theory of learning for mathematics, I had not encountered APOS theory before the conference. In [15], Gray and Tall call the combination of process and concept produced by the process that may be evoked by the same symbol, a procept. The work of Dubinsky and others, [16], links Action, Process, Object and Schema. The ideas involved in both procepts and APOS theory seemed to fit well with my understanding of mathematical learning. Clearly neither of these presents a complete picture but they have provided me with a stimulus to further reading and thinking.

## 5. Concerns

In the process of being involved in the current Study I have become concerned about a number of issues. These are

(i) the mathematician/mathematics educator divide;

(ii) technology, and

(iii) culture.

There does appear to be a feeling among some mathematicians that mathematics educators have little to contribute to lecturing or teaching at the tertiary level (and possibly at other levels too). This is not the case. However, in some places the gulf between these two groups is wide and it is not immediately clear how to bring them together.

Two schemes, one in England and Wales[2]) [17] and one in France [18], may help to ameliorate this in the long term. They are both attempts to recognise the fact that experts in a subject are not automatically even competent teachers. Both schemes provide opportunities for instruction for teaching at university level across all disciplines. The one in England and Wales is largely an in-service scheme while that in France is a pre-service one. Hopefully initiatives of this kind, both of which are backed by government, will improve the quality of teaching in universities in their respective counties. Possibly they will serve as models for other countries. With any luck they may become a catalyst to get mathematicians and mathematics educators talking to each other.

Through technology we have an ever-increasing set of tools to aid our research. Technology should also be providing us with a range of tools to aid our teaching. There is no doubt that these tools are being used but it is not clear to me that, at this moment, we are using them in ways that promote learning as efficiently as we might.

In Celia Hoyle's panel presentation at the ICMI Study conference she made the point that there was much for tertiary educators to learn from the secondary arena in the area of technology. She said that there was considerable evidence to support the computer's potential to

- ❄ foster more active learning using experimental approaches;
- ❄ provoke constructionist approaches to learning mathematics;
- ❄ motivate explanations in the face of "surprise" feedback;
- ❄ foster cooperative work, encouraging discussion of different solutions and strategies; and
- ❄ open a window on student thought processes.

But she said there was also evidence that we need to rethink

- the content and sequence of courses;
- teaching approaches; and
- the relationship between 'computer maths' and pencil and paper maths.

[2]) This has developed in two directions. For information see the Higher Education Academy (http://www.heacademy.ac.uk/) and the Maths, Stats and OR Network (http://www.mathstore.ac.uk/).

On the other hand, she noted that there are many obstacles to using computers. Among these are

- ◊ access to hardware;
- ◊ time for familiarisation with systems;
- ◊ focussing on calculation rather than reflection; and
- ◊ the potential disadvantaging of certain groups.

It seems to me that we have a long way to go before we are able to use technology for better understanding of mathematics. We need to experiment and discuss the results of our experimentation to find the best ways to use the tools that we have. I am not convinced that just giving computer packages to students will promote learning and understanding. I am also concerned about the blind rush of some administrators to the use of web sites for teaching. If those sites are no more than text-books we have made no progress.

The last point above leads me into my final concern, that of culture. At one level I feel that the Study I was involved in has, to a large extent, reflected only Western Culture. For various reasons it has not been possible to extend the papers in the Study volume as far as I would have liked. But the more I think about it the more concerned I become about the various cultures, or maybe subcultures that exist in the classroom. Certain of these are obvious. Just by looking at the students I can see that they were not all brought up in the same mainstream culture that I was. I clearly don't understand all the cultures that are represented. A couple of years ago, in an attempt to help a student from another culture, I inadvertently caused the student to withdraw from the class.

But there are less obvious cultural differences. In my classroom, middle-class culture, middle-aged culture and mathematics culture prevail. Hence I use language in ways that are different from those of many of the students I teach. I also have different views on life and learning which are not necessarily shared by all of my students. These issues are taken up in the paper by Robyn Zevenbergen [19]. It is something that I need to be more aware of. In the past when we had fewer students and many fewer overseas students, this cultural "complication" hardly existed. This is just another thing that has changed in the last 15 years or so.

## 6. Outcomes

It is very difficult to quantify the influence of a study such as this one. The Study volume and the articles in the special issue of the *International Journal of Mathematics Education in Science and Technology* have been cited in numerous publications. At ICME-9, held in Japan in 2000, there was a Working Group on Mathematics Education in Universities as well as a panel where issues related to the Study were aired further (see [20] and [21]).

The influence can also be seen at several other conferences where presentations directly related to the Study were given. For instance, at the 2nd International Conference on the Teaching of Mathematics (at the undergraduate level) held in Crete in 2002 a similar panel to that that took place at ICMI-9 was held. And the theme was maintained at ICME-10 in Denmark in 2004 with the Topic Study Group on New Trends and Developments in Tertiary Education. (Papers from this conference were also published in the *International Journal of Mathematics Education in Science and Technology* [22].)

The International Conference on the Teaching of Mathematics is now well established, as is the Delta series of conferences that are held in the Southern Hemisphere in odd years. It is not clear how much the ICMI Study has influenced these developments or is just part of a general movement towards interest in tertiary mathematics education.

## 7. Where to now?

So, given the circumstances in which universities find themselves, and given the interests and abilities of staff, and given the students who hopefully will continue to come to us for accreditation if not always for learning, and given the constraints of society, what should we teach and how should we teach for optimum learning? The current ICMI Study does not have the definitive answer to that question. Indeed, such an answer may not exist. But it does have perspectives, suggestions and pointers. Hopefully it has ideas that will stimulate its readers to enhance practice over the next several years. This will require a knowledge of

- the mathematical needs of society;
- the learning capabilities of our students;
- mathematics and especially its difficulties;
- the pedagogical use of technology; and
- how to teach mathematics so that it becomes more accessible and less aimed at an exclusive market.

To me that doesn't seem to be a very easy challenge.

## REFERENCES

[1] Niss, M. ICMI as an organisation. *ICMI Bulletin 43* (December 1997), 14–17.

[2] http://www.mathunion.org/ICMI/

[3] On the Teaching and Learning of Mathematics at University Level. *L'Enseignment Math. (2) 43* (1997), 381–390.

[4] *International Journal of Mathematical Education in Science and Technology 31 (1)* (2000). (See also http://www.tandf.co.uk.)

[5] Holton, D. A. *The Teaching and Learning of Mathematics at University Level: An ICMI Study.* Kluwer, Dordrecht, 2001.

[6] —— Teaching versus lecturing. *Teaching Mathematics and Its Applications 17 (2)* (1998), 49–54.

[7] Artigue, M. et al. (Eds.). *Vingt ans de didactique des mathématiques en France.* La Pensée Sauvage, Grenoble, 1994.

[8] Dorier, J.-L. (Ed.). *L'Enseignement de l'algébre linéaire en question.* La Pensée Sauvage, Grenoble, 1997.

[9] Dorier, J.-L., A. Robert, J. Robinet and M. Rogalski. On a research programme concerning the teaching and learning of linear algebra in the first year of a French science university. *Int. J. Math. Ed. Sci. Technol. 31 (1)* (2000), 27–35.

*achevé de composer le 20 décembre 2007 à 09 : 08*

[10] LEGRAND, M. Débat scientifique en cours de mathématiques. *Repères IREM n° 10.* Topiques Éditions.

[11] ROBERT, A. Outils d'analyse des contenus mathématiques à enseigner au lycée et à l'université. *RDM 19 (2)* (1998), 139–190.

[12] ROGALSKI, M. Teaching linear algebra : role and nature of knowledge in logic and set theory which deals with some linear problems. In : *Proceedings of the XX$^{th}$ International Conference for the Psychology of Mathematics Education*, 211–218. Univ. Valencia (Spain), 1996.

[13] ANTHONY, G. Factors influencing first year students' success in mathematics. *Int. J. Math. Ed. Sci. Technol. 31 (1)* (2000), 3–14.

[14] LITHNER, J. Mathematical reasoning and familiar procedures. *Int. J. Math. Ed. Sci. Technol. 31 (1)* (2000), 83–95.

[15] GRAY, E. M. and D. O. TALL. Duality, ambiguity and flexibility : A perceptual view of simple arithmetic. *J. Res. Math. Ed. 26* (1994), 115–141.

[16] http://www.cs.gsu./~rumec/index.htlm.

[17] MASON, J. Professionalisation of teaching in higher education in the U.K. In : D. A. Holton (Ed.), *The Teaching and Learning of Mathematics at University Level : An ICMI Study*, 501–518. Kluwer, Dordrecht, 2001.

[18] LEGRAND, M. On the training of French prospective university teachers. In : D. A. Holton (Ed.), *The Teaching and Learning of Mathematics at University Level : An ICMI Study*, 519–528. Kluwer, Dordrecht, 2001.

[19] ZEVENBERGEN, R. Changing contexts in tertiary mathematics : Implications for diversity and equity. In : D. A. Holton (Ed.), *The Teaching and Learning of Mathematics at University Level : An ICMI Study*, 13–26. Kluwer, Dordrecht, 2001.

[20] FUJITA, H., Y. HASHIMOTO, B. R. HODGSON, PENG YEE LEE, S. LERMAN and T. SAWADA (Eds.). *Proceedings of the Ninth International Congress on Mathematical Education*, 243–247. Kluwer, Dordrecht, 2004.

[21] FUJITA, H., Y. HASHIMOTO, B. R. HODGSON, PENG YEE LEE, S. LERMAN and T. SAWADA (Eds.). *Proceedings of the Ninth International Congress on Mathematical Education*, 372–373. Kluwer, Dordrecht, 2004.

[22] *International Journal of Mathematical Education in Science and Technology 36 (2–3)* (2005). (See also http://www.tandf.co.uk.)

*(Reçu le 22 octobre 2007)*

Derek Holton

Department of Mathematics and Statistics
University of Otago
New Zealand
*e-mail :* dholton@maths.otago.ac.nz