Charting the microworld territory: the placing of theoretical signposts

Lulu Healy

Catholic University of Sao Paulo (PUC-SP); UNIBAN, Brazil

Abstract: In this contribution, I intend to focus on the concept of computer microworlds for mathematics learning and, in particular, evolutions in the theoretical perspectives associated with this concept that have emerged within the ICMI community between two moments in its history: the dissemination of the microworld vision in Mindstorms (Papert, 1980) and Papert´s plenary lecture at the 17th ICMI study conference, *Digital technologies and mathematics teaching and learning: Rethinking the terrain*, in December, 2006. Developing ideas about three issues in particular will be considered: reciprocal relationships between mathematical infrastructures, technology and thinking; the omnipresence of the perceptuo-motor activity in mathematical thinking and learning; and the influence of innovative means of representational and communication on the affective as well as the cognitive dimension.

The vision of microworlds for mathematics learning posited by Papert (1980) was a radical one. Rather then using digital technologies as an aid to teach school mathematics, the turtle geometry microworld was originally presented as the seeds of an alternative, more learnable, mathematics. He talked of "reconstructing mathematics" and the desire to use computers as mathematically expressive media with which to design an *appropriable* mathematics fitted to the learner. In December 2006, Papert reiterated this vision in a plenary lecture at the 17th ICMI Study Conference, *Digital technologies and mathematics teaching and learning: Rethinking the terrain* in Hanoi, Vietnam, as he presented the idea of *restructurations*, a term coined jointly with Wilensky (2006) and defined as reformulations of knowledge disciplines through new representational and communicational forms.

More than a quarter of a century sits between these two moments. And it is probably fair to say that despite enormous changes in computer-based opportunities to connect and interact in a variety of ways with representations of mathematical knowledge, the practices in the world's mathematics classrooms have changed rather less. Is this because the idea of a learnable mathematics is still so radical?

To examine this question, this text presents a short and personal reflection on the theoretical signposts that have been erected on microworld territories (and on the landscape of research into technology and mathematics more generally) in the search to understand relationships between digital technologies and mathematics learning. Microworlds began as provinces of mathlands as Papert (1980; p.125) imagined accessible, evocative and engaging mathematical cultures, in which learners would become immersed, and from which they would emerge as more mathematically fluent. His conception was of computational objects, which would embed a mathematics that was not only formal but related to learners themselves, permitting an approach to mathematical sense-making that is both *body-syntonic* — it relates to learners' sense and knowledge about their own bodies — and *ego-syntonic* or coherent with learners' sense of themselves as people with intentions, goals, desires, likes and dislikes.

Microworlds, mathematics and learning

Microworlds have as their core a model of a domain of mathematical knowledge. This model is provided by the designers and represented by a formal system, a set of computational tools, whose functionality is experienced via phenomenological displays (physical, graphical, auditory etc.). A microworld evolves as the learner explores its territory, adding to the initial model by building new objects and new relationships using the given tools (Thompson, 1987; p.85). In the 1980s, it was a basically Piagetian perspective that comprised the underlying rationale for microworld learning:

> "Individual action is considered the motor for learning. Students use the software to achieve a goal and in the process they learn by co-ordinating and reflecting on the form of their interactions—by developing schemes."
>
> (Hoyles, 1995; p.203)

By the end of the 1980s and into the 1990s, the microworld vision was also to become linked with Vygotsky's view of internalisation from the social to individual plane (Vygotsky, 1978) . By focussing on the representational structures afforded by computational systems, attention was directed to understanding how tools mediate meanings and how the inclusion of a tool into activity alters the course both of the activity and of all the mental processes that enter into the instrumental act. From this point of view, tools not only facilitate mental processes, they transform, they re-organise and they shape them (Vygotsky, 1981; p.139). Since the mediational means built in microworlds are designed with students' probable zone of proximal development in mind, these tools are intended to permit learners to do things with a computer that would be impossible without it, supporting the emergence of a ZPD in which culture meets cognition. Moreover, microworlds are unfinished worlds, their tools not only shape the learners and possible mathematical practices, they are also shaped by them.

Another set of signposts is currently being erected on the terrain of technology and mathematics education, marking out the instrumental approach (Verrillon and Rabardel, 1995). In this approach, which also draws from both Piagetian and Vygotskian bases, "shaped by tools" becomes "instrumentation" and "shaping tools" "instrumentalisation". The instrumental approach also recognises how technology impacts upon the cultural practices within which they are employed, with both pragmatic and, critically, epistemic consequences (Artigue, 2002).

Alongside this increasing attention to the reciprocal relationships between tools and thinking, another set of paths to be mapped out points to a reconsideration of the grounds for cognition. These signposts emphasise, rather than formal operations on abstract symbols, the situated and embodied nature of cognition. Amongst researchers in the microworlds' field, terms such as *situated abstraction* (Noss and Hoyles, 1996) and *situated proofs* (Moreno and Sriraman, 2005) have emerged. In both cases, the qualifier 'situated' serves to illustrate the role of particular tools of an expressive media in permitting particular actions and practices.

Taken together, following the paths laid out by these different signposts seems to bring us back to the idea of reconstructing mathematics (or perhaps fast forward us to restructurations): the recognition that changing the infrastructures through which mathematics is expressed and practiced alters mathematics as a knowledge discipline suggests that we can only begin to understand the role of new technologies in mathematics learning if we accept that they necessarily change mathematics. This is no longer seen as radical, at least not within the research community (see Healy, in press). But accepting this as a premise implies either a rethinking of the mathematics we teach or that school mathematics is left as a relatively technology free-zone.

Microworlds and the body

Exploration around the ground associated with the two – perhaps now rather faded – signposts planted in Papert's 1980 vision, pointing towards mathematical infrastructures that support an interaction that is both *body* and *ego-syntonic,* perhaps still holds the key for those of us who choose the first option. The notion of body-syntoncity can be associated with an area currently undergoing extensive exploration within mathematics education in general, as well as amongst those interested in the role of technology in learning: that of embodied cognition and in particular the role of perceptuo-motor activity and in the learning of mathematics (Nemirovsky & Borba, 2004; Lakoff & Nunez, 2000; Arzarello & Robutti, 2004; Radford et al. 2005). Embodied approaches emphasise how even the most abstract of symbols have physical grounding or as Radford et al. (2005) put it that "sensorimotor activity is not merely a stage of development that fades away in more advanced stages, but rather is thoroughly present in thinking and conceptualizing."

The dynamic mathematical representations that digital technologies permit appear to magnify the lens onto the ways in which mathematical meanings come to be associated with physical grounding (as well, of course, as meditating this grounding process). Advances in connectivity promise to bring yet further changes. Both Kaput (2004) and Wilensky (2006) have described how it is becoming possible for learners to interact not only with dynamic computational agents but also alongside them during mathematical explorations, bringing a new layer to what we understand by an experiential approach to learning mathematics – and another possibility with both epistemological and cognitive repercussions.

If we accept then that body-syntonicity is at least an interesting characteristic when considering the learnability of mathematics, and that the physical senses play an important role in interpreting mathematical phenomena, an interesting question is how those without access to particular senses interpret the behaviour of mathematical objects – and how any new mathematical infrastructures might be molded to also take this into account. Learners with no access to the visual field or learners who communicate through sign rather than spoken language might be expected to become involved in rather different processes of physical grounding. By building into microworlds features designed to support the mathematical activities and expression of deaf students and blind students and by considering the particularities of their interactions with different mediation systems, our current research activities aim to contribute to the charting of this still relatively unfamiliar territory (Margalhães and Healy, 2007; Fernandes and Healy, in press)[1].

Microworlds and narratives

But what of ego-syntonicity? Rather fewer explorers as yet seem to have ventured deeply into the realm of psychological, as opposed to physical, grounding. A promising signpost, perhaps best described as currently under-construction on microworld land, points towards a central role for narrative in human cognition (Healy & Sinclair, 2007; Mor & Noss, submitted;). In Healy and Sinclair (2007), we present some tentative forays onto this terrain, with our explorations suggesting that the representational and communicational possibilities incorporated within mathematical microworlds offer material for the construction of mathematically productive stories, stories in which learners bring life to the computational beings they encounter as they engage in mathematical activities. The phenomenological characteristics of these beings (they way they move, the noises they make, the colours they produce etc.) seems to contribute to the attribution of purposes, desires and feelings to interpret their behaviour, but at the same time, the stories in which these attributions are embedded are also strongly associated with the mathematical properties and relationships which underpin, which define, and which constrain the on-screen activities of these beings. Our current research is identifying regularities in the stories which emerge across learners working on the same activities, suggesting that story-telling for meaning-making is not random – it seems that we might be able to predict the storylines associated with different microworld tasks and hence highlight possible connections to the particular mathematical relations which these stories emphasise.

A final word

These reflections upon the microworld (hi)story indicate that much ground has been covered between two special moments in time, and although the vision continues as relevant as ever today, there is still some way to go before the story ends. So what would be a happy ending? Perhaps, it would not be so different from the beginning – a *real* rethinking of school

[1] This work is supported by research grants for FAPESP (Process Nos. 2004/15109-9 and 2005/60655-4

mathematics and a legitimising of more dynamic, more evocative means of doing and expressing it.

References

Artigue, M. (2002). Learning Mathematics in a CAS Environment: The Genesis of a Reflection about Instrumentation and the Dialectics between Technical and Conceptual Work. International Journal of Computers for Mathematical Learning, Vol. 7, No. 3 pp.245-274.

Arzarello, F. & Robutti, O. (2004). Approaching functions through motion experiments, *Educational Studies in Mathematics PME Special Issue* Vol. 57 No. 3.

Fernandes, S. & Healy, L. (in press). Transição entre o intra e interfigual na construção de conhecimento geométrico por alunos cegos. *Educação Matemática Revista*

Healy, L. & Sinclair, N. (2007). If this is our mathematics, what are our stories? International Journal of Computers for Mathematical Learning, Vol. 12, No. 1, , pp. 3-21.

Healy, L. (in press). Topic Study Group 15: Technology and Mathematics Education. *Proceedings of the 10th International Congress on Mathematics Education*, Denmark.

Hoyles, C. (1995). Exploratory software, exploratory cultures? In A A DiSessa, C Hoyles & R. Noss with L. D. Edwards (Eds.), *Computers and exploratory learning*. Berlin: Springer, pp. 199-220.

Kaput (2004) Technology Becoming Infrastructural in Mathematics Education. Paper presented in *Topic Study Group 15: Technology and Mathematics Education*. 10th International Congress on Mathematics Education, Denmark.

Lakoff, G. and Núñez, R. (2000). *Where Mathematics Comes From*. N. Y.: Basic Books.

Margalhães, G.R. & Healy, L. (2007). Questões de design de um micromundo para o estudo de concepções de provasproduzidas por alunos surdos. *Anais do IX Encontro Nacional de Educação Matemática (IX ENEM)*. Belo Horizonte.

Mor, Y. & Noss, R. (submitted). Programming as mathematical narrative. International *Journal of Continuing Engineering Education and Life-Long Learning.*

Moreno, L.A & Sriraman, B. (2005). Structural stability and dynamic geometry: Some ideas on situated proofs

Nemirovsky, R. & Borba M. (2004).. Bodily Activity and Imagination in Mathematics

Noss, R. & Hoyles, C. (1996). *Windows on Mathematical Meaning: Learning Cultures and Computers* Dordrecht, The Netherlands: Kluwer Academic Press.

Papert, P. (2006). From Math Wars to the New New Math. Plenary Lecture at the 17th ICMI Study Conference, *Digital technologies and mathematics teaching and learning: Rethinking the terrain.* Hanoi, Vietnam.

Papert, S. (1980). Mindstorms: Children, Computers and Powerful Ideas. London: Harvester Press.

Radford, L., Bardini, C., Sabena, C., Diallo, P. & Simbagoye, A (2005). On embodiment, artifacts, and signs: a semiotic-cultural perspective on mathematical thinking In Helen L. Chick, Jill L. Vincent (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education*, University of Melbourne, Australia, Vol. 4, pp. 113-120.

Thompson, P. W. (1987). Mathematical microworlds and intelligent computer-assisted instruction. In G. Kearsley (Ed.), *Artificial Intelligence and Education*. New York: Addison-Wesley, pp. 83-109.

Verillon, P. & Rabardel P. (1995). Cognition and artifacts: A contribution to the study of thought in relation to instrument activity. *European Journal of Psychology in Education,* 9 (3), p. 77-101.

Vygotsky, L.S. (1978). *Mind in society.* Cambridge, MA: Harvard University Press.

Vygotsky, L.S. (1981). The instrumental method in psychology. In J.V. Wertsch, (Ed.), *The concept of activity in Soviet psychology*. Armonk, NY: M.E. Sharpe, pp. 134-143.

Wilensky (2006). Agent-based Restructurations for modeling change. Contribution to the Plenary Panel onConnectivity at the ICMI Study Conference, *Digital technologies and mathematics teaching and learning: Rethinking the terrain.* Hanoi, Vietnam.