Introduction

This short paper explores the developments of ICT in ICMI’s history and the literature surveyed involves researchers and mathematics educators who have been involved in ICMI’s activities in terms of publications, presentations, posters and other ICMI forums. The four “revolutions” of ICT are asserted in this paper to explain the developments of ICT and ICMI’s history in particular, and their effects on mathematics instruction. An example of one computer program (mathematica) is provided to explain the significance and contribution of such technology to mathematics instruction. Then some researchers’ studies in ICT are highlighted to explain the achievements and constraints of using technology in mathematics instruction during ICMI’s century. At the end, some limitations of utilization of this technology are outlined and suggestions for future trends suggested.

ICT revolutions

ICMI’s history can be said to have witnessed four ICT revolutions. The first revolution was said to be comprised of films, radio, television and satellite broadcasting, while the second to comprise telecommunications and microcomputers in mathematics instruction (Paisley, 1985). The integration of telecommunications and microelectronic technology in computing was termed a “third revolution” and came to be what is called Information Technology (IT). The current trend in ICT has brought a phenomenon which can be termed a “fourth revolution” in IT. The first revolution was mainly dominated by ‘paper and pencil’ way of instruction and delivery of knowledge relied more on the teacher. Literature on ICMI presentations reveals how the teacher was the main if not sole source of knowledge in classroom instruction during these times. The teaching style came later to be termed as a “traditional” style of teaching “traditional” mathematics. Basically, the traditional way of teaching was claimed to involve imparting of knowledge by the teacher to students where students were required to memorize knowledge and reproduce it whenever required.

The second revolution can well be described when “new” or “modern” mathematics was introduced in many countries of the world. It was during this time that calculators and computers were also integrated in mathematics instruction. And it was at this time also when intense debates occurred on type of mathematics that was to be taught and proper methods of teaching the subject to be adopted. Proponents of ‘modern’ mathematics were of the view that traditional mathematics encouraged rote learning and students could not acquire long-term retention of knowledge. But also critics of modern mathematics claimed that the style of teaching did not equip students with critical thinking skills as students could not produce rapid responses and solutions as expected. Modern mathematics teaching involved more use of logic and was dominated by set theory at primary and secondary school levels. The use of calculators and computers at this time was also regarded by some mathematics educators as well as parents to promote rote learning as students were viewed to be pressing the buttons and producing answers. While some teaching strategies in modern mathematics teaching were
considered to be learner-centered and elements of constructivism were visible, critics argued that the teaching style involved guided theories where there was little opportunity for students to create their own knowledge as emphasized in constructivist theories.

The third revolution was said to promise not only a more productive person, a problem-solver and a life-long learner, but also a better informed, rational and participative citizen, a modern ‘renaissance’ person, living in the web and network of a worldwide electronic community (Papagiannis et al, 1987). This revolution was said to come at a time when there was increasing financial and economic stress in many countries of the world. At this time, there was increasing unemployment and stagnation of economic growth affecting many sectors of life and to some critics, computers were a replacement of skilled manpower with low-skilled paid jobs (Schgurenky, 1997). In the education sector, the introduction of computers in the school curricula raised many issues in both developed and developing countries. Some people were of the view that computers were a replacement of teachers for creation of unemployment and a deprivation of essential skills to students that computers could not deliver. In the early days when resources were available only through centrally controlled mainframe computer systems, some foresaw technology eventually replacing the teacher as the primary instructional delivery system (Norris, 1977).

The current “fourth revolution” in ICT has a globalization component force that has replaced other revolutions and accelerated its influence worldwide. Globalization has been described in many forms such as the intensification of interconnectedness (McGrew, 1992), a process of elimination of economic borders and increase in international exchange and transnational interaction (Dolan, 1993), a process by which peoples of the world are incorporated into a single global society (Wallerstein, 1995) and many other descriptions. In education and mathematics in particular, the delivery of knowledge using computers has influenced the design of various school mathematics curricula globally. Available current technology allows students’ interaction with the computer screen rather than with the teacher alone. Through the computer network, students are able to communicate with the teacher on the material and can discuss assignments involved. In this process students were able to attend lectures “online”. Students can now learn while at campus or outside, benefiting more distant learning students worldwide.

**Researchers’ ideas on ICT in instruction**

Some educationists consider ICT to be the only way to go if not a substitute for conventional teaching and learning resources (Broekman et al, 2002). ICT’s interactive testing and review mechanism, together with “a let-s-go-back and look-at-that-again-loop” was believed to offer the best of all worlds of learning (ICTs in learning, 2000). Studies have shown that ICT can contribute to innovative student-centered learning environment where teachers act as coaches, while remaining in firm control of the learning environment (Smeets and Mooij, 2001). ICT programs are said to promise power for students to control over their own learning, that networking replaces hierarchies and promises to give voice to learners (Broekman, 2002). There was evidence that ICT provided motivation and variety, generates enthusiasm, interest and involvement, maintains attention and enjoyment, enhances thinking and problem-solving skills (Sibiya, 2003). Current technologies in e-learning such as AulaNet provide a groupware for creation, participation and maintenance of Web-based courses emphasizing group learning where individuals share ideas online (Gay & Lantini, 1995 and Fuks, 2000). They have been described as a way to move from elite to mass education through digital media where more students get access to education for both campus and distance-
learning students (Kennedy, 2001). The success in designing and implementation of online courses has been said to require interpersonal, information collection and problem-solving interactions (Harris, 1995). These kinds of interaction provide a large source of reference materials and data required for teaching and learning.

In general, educational studies have indicated that ICT programs can be effective and versatile in instruction and the challenge throughout ICMI history has been on “how” and “where” to integrate this technology in mathematics instruction. Some generic programs i.e. general purpose programs such as word processor, spreadsheets and graphics have been developed and found to alleviate some students’ misconceptions of some mathematical and scientific ideas (Ghazali and Ismail, 1997 and Fuglestead, 1997). Various strides have been undertaken by mathematics educators and researchers whereby many computer programs have been developed and used to date in classroom instruction. For example, subject specific programs i.e. programs designed specifically for a subject area such as *derive, mathematica, logo, matlab* and others have been found to be significant in rapid delivery of knowledge in mathematics instruction than traditional ways involving “chalk and board” or “paper and pencil” (Kaino, 1994 & 1998; Maswera and Tsvig, 2002). In the following section, an example is provided where the significance of applying *mathematica* is explored, providing a challenge to the use of paper and pencil in a traditional way of teaching.

**Activity**

In this example we investigate singular, non-singular systems as well as ill-conditioned systems of linear equations in two variables considered to be solved in a ‘traditional’ way and then by *mathematica*. This area may be taught at senior secondary school level.

A linear problem with two variables $x_1$ and $x_2$ is represented by the following system:

\[
\begin{align*}
  a_{11} x_1 + a_{12} x_2 &= b_1 \\
  a_{21} x_1 + a_{22} x_2 &= b_2
\end{align*}
\]

where $a_{11}, a_{12}, a_{21}$ and $a_{22}$ are coefficients of the system.

The above equation can represent different problems in diet, manufacturing, transportation and many others. In this paper, let us consider a diet problem in which it is required to determine the unknown variables $x_1$ and $x_2$ which denote two different types of foods. Let the variables $b_1$ and $b_2$ represent two basic nutritional ingredient units which have to be achieved by a person for a balanced diet per day. Assuming that each unit of two types of foods contain $a_{11}, a_{12}, a_{21}$ and $a_{22}$ units in both nutrients respectively, then the diet problem can be represented by a system of two linear equations described above. Let us provide coefficient values of this equation and have the system as follows:

\[
\begin{align*}
  1.00 x_1 + 1.00 x_2 &= 2.00 \\
  1.00 x_1 + 1.01 x_2 &= 2.01
\end{align*}
\]

Equation (1) above has an exact solution $(x_1, x_2) = (1.00, 1.00)$ and in this case we know that no errors are encountered in the solution process. If we change coefficient $a_{22}$ from 1.01 to 1.02, the system changes to

\[
\begin{align*}
  1.00 x_1 + 1.00 x_2 &= 2.00 \\
  1.00 x_1 + 1.02 x_2 &= 2.01
\end{align*}
\]

Equation (2) also has an exact solution $(x_1, x_2) = (1.5, 0.5)$ and shows that changing the coefficient by the value of 0.01 affects the variable $x_1$ to be 1.5 times the original value as well as the variable $x_2$ by making it half the original value. Thus a change of 0.01 in the coefficient, which could be seen
as a small value, may contribute a significant change in some problems. Assuming an error of 0.01 in the coefficient $a_{22}$ of the diet problem was incurred, it would mean making the type of food $x_1$ to be 1.5 times more while reducing the type of food $x_2$ by half. Of interest, while total ingredient units $b_1$ and $b_2$ are maintained (as their values are not affected by the changes in both $x_1$ and $x_2$), the solution could bring harmful results to human health because the diet would not be balanced. Such a case can be related to a person who takes a certain fixed quantity of food daily without realizing that it did not contain appropriate nutrient components.

Consider $a_{22} = 1.01$ was changed to 0.99 and $b_2 = 2.01$ to 2.02:

$$\begin{align*}
1.00 x_1 + 1.00 x_2 &= 2.00 \\
1.00 x_1 + 0.99 x_2 &= 2.02
\end{align*}$$

Equation (3) has an exact solution $(x_1, x_2) = (4, -2)$. Compared to the former solution $(x_1, x_2) = (1.00, 1.00)$, the latter gives a big difference. If equation (3) was to represent a diet problem and assuming that the coefficients remained the same, then such a solution would be unrealistic (though correct) if assumptions $x_1 > 0$ and $x_2 > 0$ are considered. Consideration of introducing such constraints is important because realistically a minus sign in solutions of $x_1$ and $x_2$ would mean to take away that food from someone, thus contradicting the purpose.

Consider $a_{21} = 1.00$ was changed to 1.01:

$$\begin{align*}
1.00 x_1 + 1.00 x_2 &= 2.00 \\
1.01 x_1 + 1.01 x_2 &= 2.01
\end{align*}$$

Equation (4) above has no solution. Such systems with no solution need investigation of equation coefficients which is easily done by calculating the value of determinants. In this case, the determinant of the equation coefficient is zero:

$$\begin{vmatrix}
1.00 & 1.00 \\
1.01 & 1.01
\end{vmatrix} = 0$$

The above matrix has zero determinant value and the system is said to be singular. A non-singular system has a non-zero determinant value and solutions for such systems exist. Usually systems such as equation (4) are called inconsistent systems because their determinants are zero or approach zero value.

The above examples reflect the nature of sensitivity the diet problem might possess. In such systems where small changes in coefficients amounts to big different changes from exact solutions are said to be ill-conditioned systems. To model useful diet problems, ill-conditioned systems should be avoided.

Examples in equations (1)-(4) involve coefficients with few decimal places and the nature of the systems might be easily predicted. The situation becomes complicated (when solving the problem in a traditional way) when coefficients of many decimal places are involved. Let us consider the following system:

$$\begin{align*}
0.780 x_1 + 0.563 x_2 &= 0.217 \\
0.913 x_1 + 0.659 x_2 &= 0.254
\end{align*}$$

Equation (5) has an exact solution $(x_1, x_2) = (1, -1)$. If the solution is changed by 0.001, i.e. $(x_1, x_2) = (0.999, -1.001)$ the difference on the right side of the equation becomes 0.001343 and 0.001572 for the first and second equations respectively. If the output were to be approximated at two decimal
places, then there would be no differences in the values of \(x_1\) and \(x_2\). But if the output were estimated at large decimal numbers, then the differences would be significant. For example, if equation (5) were of the multiple \(10^4\) the differences would be 13.43 and 15.72 units for the two equations. If the measures are in grams, then the diet problem would be ill-conditioned. Applying \textit{mathematica} program, equation (5) can easily be identified as ill-conditioned as shown in fig. 1 below than the use of a traditional method; they are represented by the same graph. The two equations in (5) show that they are almost the same (graphically, though not symbolically) as they coincide on the same line when plotted on same axes as indicated. Visualization of the difference between two ill-conditioned equations could be a limitation using \textit{mathematica} if the nature of the system is not well recognized and axes extended

For systems of linear equations with three variables, the concept of three dimensional planes should be brought into light where we expect the intersection of three planes to obtain the solution of three unknown variables. Unlike the solution with two variables which is in 2-dimensions, the intersection of three planes (to determine the solution) as shown in Fig.2 below is not easy to visualize by \textit{mathematica}.

\textbf{Fig. 1:} Graph of two equations in (5) by \textit{mathematica}

\textbf{Summary of the activity}
Problems involving systems of linear equations can be solved using different methods such as elimination or crammer’s rule and others. Hand calculators may also be used to obtain the solution faster than paper and pencil procedures. However, the use of programs such as mathematica provides a variety of ways to obtain the solution in numerical, symbolic and in graphical forms at the same time. It further provides the opportunity to investigate various stages of the solution process to obtain required results rapidly. The limitations in the visualization of the intersection of three planes involving three unknowns in a linear system have to be observed and could be aided by illustration of physical planes in instruction. As shown in given examples, by changing values of coefficients in the equation matrix, it was possible to investigate the nature of the system dealt with and try to avoid ill-conditioned systems in practical problems. By use of mathematica, singular and non-singular matrices of the equation system can be analyzed rapidly in any modeled problem than the use of ‘traditional’ ways involving “paper and pencil”. The traditional way could easily be used in simple problems that were straight forward to obtain solutions. Unlike in sensitive problems like diet problems and others, effective and efficient methods by programs like mathematica were required. Such strides are great achievements in ICMI history.

Some challenges encountered

There have been many challenges not only on development, integration and adoptions of ICTs into mathematics curricula and classroom instruction but also involving other parameters such as availability, utilization and expertise in ICT in many education systems of the world. Studies on ICT have identified a number of reasons that slowed the adoption process of ICTs especially in developing countries. The reasons are many and include, access to computers (email and internet), affordability of computers and connectivity, telephone and electricity infrastructure, computer literacy, expertise, etc. (Davis & Danning, 2001; Oliver et al, 2001; Knowlton & Knowlton, 2001; Sibiya, 2003; Gumbo, 2003). Despite above reasons, there has been some concern that available technology has not also been fully utilized in these countries. The issue of utilizing a fraction of the capacity of existing ICT facility in developing countries (Cawthera, 2002) was related to non-availability of effective ICT policies in these nations (Kaino, 2004). Furthermore, apart from availability, utilization and expertise in ICT, the issue of positive attitudes on ICT to avoid resistance from both teachers and students in implementation has to be taken into consideration (see for example studies by OECD, 1999; Baird, 2002; Maswera and Tsvigu, 2002). In addition, access to this technology has been criticized for lack of epistemological access to students. It was essential to be aware that social construction of knowledge required the relationship between the person and the social phenomena as central to epistemological access i.e. knowledge to be constructed and developed in and through social mediation (Vygotsky, 1987). And enabling epistemological access to ICT required the consideration of developmental aspects of affinity with technology, and that affinity had to depend on confidence which develops social mediation (Broekman, 1992). Some worries of using technology to teach mathematics have been registered and emphasis to evaluate the benefits of technology and loss of student engagement in “actually doing the math” raised (Klyve et al, 2003). It is argued that some content areas that aim at developing critical thinking and reflective practice could not be satisfied by this technology, as learning entails not only of conceptual tools but also the ability to use the tools in arguments, discussions, research and practice (Broekman, 2002). Examples have been noted of students not exposed to such programs but who scored higher than students who were taught using this technology to solve mathematical problems (Robert, 1999).
Concluding Remarks
Over the century of ICMI, developed ICT programs have generally been recommended for improving conceptualization and mathematical problem solving skills among students. Their explorative and investigative nature in mathematics instruction in numeric, symbolic and graphical forms with a rapid and friendly flavor is a big achievement in ICMI history against the traditional way of “paper and pencil” instruction which has no such qualities. And it is plausible that the image of technology has shifted from replacing teachers to supplementing and enhancing teacher-based instruction. However, the findings that some programs were less suitable or unsuitable to teach particular mathematical areas or concepts provide caution that adaptation to new technology need evaluation before curriculum implementation. And curriculum implementation in ICT needs clear policies especially in developing countries to fully utilize available ICT technologies.

References


