Hundred years of technology in mathematics classrooms

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Introduction

There are many research questions within mathematics education that seems to have become persistent during the last one hundred years. One present question for quite some time has been "How will the access of modern technology affect the teaching and learning of mathematics"? The word technology or even technical aid for calculations is nevertheless a bit ambiguous. Long before the use of even the clumsiest calculator was the use of abacus for simple calculations. It may be difficult to imagine counting without numbers, but there was a time when written numbers did not exist. The earliest counting device was most likely the human hand and its fingers. Then, as larger quantities (larger than ten human-fingers could represent) were counted, various natural items like pebbles and twigs were used to help count. Merchants who traded goods not only needed a way to count goods they bought and sold, but also to calculate the cost of those goods. Until numbers were invented, counting devices were used to make everyday calculations. The abacus is one of many counting devices invented to help count large numbers.

Galileo allowed his interest in studying the moving of falling bodies to also define and use some technological devices he needed to understand what he was observing, first of all maybe just a wooden deal with a groove for the iron ball to roll in. Galileo's mathematical proof was built upon the theory that speed was a continuously changing variable, not a widely accepted assumption at that time. Galileo was familiar with continuous magnitudes from Euclidian geometry and from his work with the geometric and military compass that he designed. The compass was a tool for theoretical reasoning regarding his experiments.

At about the same time, John Napier invented the logarithms and made the ground for the first slide ruler. Consequently two main inventions for the practice of mathematics, two technological devices for the teaching, learning and practice of mathematics was invented many years before I started school. One might say that I was educated by ancient technology, while my own teaching always has been influenced by very modern technology.

Myself, I remember using a simple compass, a half circle protractor marked in degrees and a wood ruler during my whole time in compulsory school and I still remember how proud I was when I left compulsory school and started first year at the natural science program in the gymnasium (upper secondary school) at the end of the 1960ies. The first day in the mathematics class we were told to go out and by a recommended slide ruler and a thin handbook with formulas in mathematics, physics and chemistry and with tabled values for logarithms and trigonometry. I do not remember if the protractor, actually an ancient device, needed to be upgraded. I do recall that I had two slide rulers, one small and discreet for my coat pocket that I could demonstrate as an evidence of my mathematical level of proficiency and one rather large slide ruler for calculations that I carried in my bag. About the same time, during the summer between my first and second year in gymnasium, ICMI 1 took place in Lyon, France. Three of the plenary speakers, John Armitage, Edward Begle and Arthur Engel, mentioned computers in their talks to the congress, while I never laid my eyes on a calculator during my primary and secondary schooling; I depended on Galileo, Napier and the important handbook with formulas.

When I after the gymnasium went to the university to study more mathematics, the handbook was omitted from the mathematics courses or at least from the exams since we were expected to now the necessary formulas and values by heart. As far as I recall it, the slide ruler was accepted and allowed to use at exams, although I cannot remember that I used it especially much. It wasn’t of great help when studying analysis. If my memory serves me right, I used my compass and protractor much more frequently than the slide ruler.

Looking back only thirty years, it is interesting that the very first ICMI Study report: The Influence of Computers and Informatics on Mathematics and Its Teaching, was published in 1986.After the slide rulers, rather clumsy table calculators mainly used for the four rules of arithmetic existed under early 1970ies, where they were replaced by so called scientific calculators under the 1980ies and they in turn where replaced by so called graphing and symbol manipulating calculators during the 1990ies. In a parallel lane, computers evolved in a way which has made computers more and more accessible for everyday life together with better and more and more accessible computerized tools for studying and learning mathematics. This dramatically change in terms of access of tools doing and visualizing mathematics naturally has created a growing concern regarding the impact this technological evolution should have or maybe has on education in mathematics at different levels. Questions on how teaching, learning and assessment have been affected by technology have been in focus for many years in Sweden and elsewhere, between ICMI 1 and ICMI 11. Surprisingly few answers can be found so far.

Technology in the Swedish school system

When I started to teach mathematics and physics in upper secondary school in the beginning of the 1980s, the technological boom had already started. There were computer projects and new calculators every year at the gymnasium where I worked and the discussion about different operative systems and types of calculators were frequent and intensive. At that time we were not so sure about what to do with the technology. At the introduction of computers in the Swedish school system on a broad scale, programming was one of the main issues and one specific goal became how to teach students to program a computer. I still remember how important it was that I, as a teacher of mathematics at upper secondary school, should illustrate some simple BASIC programming structure for my students around 1985. With the growing complex ability in the available software of the evolving computerized environment, focus soon shifted over to the learning of word processing and spreadsheet calculations, learning to master the tools. In the beginning of 1990, the graphing calculators were released on a broad scale, even though the first graphing calculator was invented already in 1985. Consequently, ICMI 7 in Quebec became the first ICMI to focus on technology. Together with a growing concern regarding the ability that was inbuilt in the new, graphing calculators, this was also the first ICMI to acknowledge the presents of dynamic geometry tools.

Despite the fact that the Swedish society nowadays is fairly well computerized, in some sense calculators are used much more extensively in the school system compared to computers. Already in grade 7, students are allowed to use calculators to add, subtract, multiply and divide numbers which implies a view of numbers as being the equivalent of its decimal representation. That has for instance caused a rather severe loss of manipulative skills with fractions, something that is closely related to Sweden’s rather poor performance in algebra when compared to students in other nations in international tests. At the gymnasium level, I allow myself to declare that all students at theoretical programs at present time have access to a graphing calculator and also that quite a few of the students at the natural science program also have access to a symbolic manipulating calculator.

If looking at the tertiary level, the situation is much harder to evaluate. Today, most students who study mathematics in some way at both the upper secondary and the tertiary level probably have access to a private computer that probably is equipped with different sorts of computer program for calculation and visualization of mathematical processes. Over the Internet it is also quite possible to download different excellent programs for doing mathematics at your own computer or to look at so called Java applications and do some specific mathematics at some other person's computer. Altogether it means that a regular student of mathematics at upper secondary or at tertiary level has almost instant access to a very large capacity for calculations and visualization of mathematical practice. How is this richness of resources used by especially teachers of mathematics?

Some researchers have tried to get a glint over the equipment at the schools and at the same time address questions regarding the use of the available tools. Dahland reported from an investigation from 1994 that teachers in the Swedish gymnasium had the following tools available at their schools (Dahland, 1998, p. 156):

Software	**Number of schools (totally 39)**
Mathematics graphing tools	25
Derive	19
Mathematica	2
Excel	35
Word	34
Other programs	18

Research on teaching and learning mathematics in light of technology

There is no common agreement among researchers in mathematics education concerning the impact of technology on the learning of mathematics. What seems to be mutually agreed is that calculators or computers may provide significant support in the learning of mathematics if they are used in an appropriate way. The constitution of the right way is much more difficult to define. Ellington (2003) did a thorough meta-analysis of 54 calculator-based research studies published from January 1983 through March 2002, finding some evidence that calculators in general do not harm problem-solving skills. She concluded the following:

> Based on the definition used to identify a mathematical skill as a problem solving skill in preparing for this meta-analysis, little information was available on the relationship between the graphing calculator and student achievement in problem solving skills. The studies featuring the graphing calculator primarily focused on the acquisition of operational skills: consequently, the problem solving results were primarily based on basic and scientific calculators. Therefore, future research should include studies of graphing calculator use in the development of problems solving skills. (p. 458)

A calculator is, of course, a very powerful tool for a student who knows how to use it. It might very well set aside such barriers as a lack of arithmetic skill or a lack of computational skills more generally. Calculators might also, however, be tools for developing conceptions. This use is even more likely for graphing or CAS calculators. The ability to interpret the results from a calculator has always been important. Today, with their increasing mathematical power and complexity, advanced graphing and symbol manipulating calculators demand greater knowledge and insight in mathematics than ever before. A well-developed insight into the mathematics, the technique, and the methods may very well be a crucial competence that can serve as either a springboard or a pitfall. The student's competences will probably also influences his or her attitude towards mathematics as such. For the last fifteen years or so, the research on effect of technology in the teaching of mathematics has shifted from the positive optimism in the early 1980s.

During the 1990s it soon became clear that students may have difficulties in interpreting graphical representations as they appear on a screen (se for instance Goldenberg, 1987; Hillel et al., 1992; Dahland and Lingefjärd, 1996).

Dahland and Lingefjärd (1996) investigated 100 students in four different natural science classes in the Swedish gymnasium and found that when the exam allowed graphing calculators, at least two major difficulties needed to be taken into account: The student must have technical insight to be able to interpret the information given on a graphics screen, and the student must also have a sufficiently good mathematical understanding to realize the connection between the current problem and the possibilities given by the tool.

In a study by Bergqvist (1999), students were encouraged to use investigations on a graphing calculator in order to better understand the factor theorem by graphing a quadratic function together with the two linear functions of its factorization. Some evidence was found that students could benefit from investigations on a graphing calculator when trying to understand factorization and the factor theorem. Another result was that students could get a better understanding for the connection between graphical and algebraic representations of functions.

Lingefjärd (2000) investigated prospective mathematics teachers' understanding of mathematical modelling when using technology to solve a variety of problems. The framework illustrates how different sources of authority as well as conceptions and misconceptions of mathematics and mathematics modelling play different roles in the mathematical modelling process. Technology acted both as a tool and as a source of authority in this process. A transformation of authority took place in the first weeks of the course so that the students became uncritical of the results they got from the computer and graphing calculator. This happened despite the fact that they had been urged to be very cautious when using software to select models. See Lingefjärd, 2000, 2002a, 2002b, 2006; see also Holmquist & Lingefjärd, 2003; Lingefjärd & Holmquist, 2001, 2003, 2005; Lingefjärd & Kilpatrick, 1998.

At the same time technology has grown existent within different parts of ICMI. In the affiliated group ICTMA where I am active, technology is often a necessary tool for mathematical modeling. In the upcoming congress ICMI 11, there are at least a dozen topic groups that most likely have implicit views on technology, at the same time as there also is a special topic group called: New technologies in the teaching and learning of mathematics. So technology has changed its presence in the research community of mathematics education, it is so outspread and so complex that special topic groups need to structure the discussion of technology to their special needs. At the same time we have gain new insights, and new theories like for instance instrumentation is hard to imagine without new technology that requires new ways to view mathematical learning. Instrumentation is an evolutionary theory as mental schemes emerge as users execute a task. Instrumentalization is a psychological process which leads to an internalization of the uses and roles of an artifact, an organization of use-schemes, a personalization and sometimes transformation of the tool, and a differentiation between the complex processes that constitute instrumental genesis and which are critical for teachers to master (Guin & Trouche, 2002). Today a typical CAS calculator has a manual of 550 pages, while my slide ruler needed no documentation at all. Technology becomes more and more sophisticated for every year.

How much do we know about what teachers of mathematics actually do in the year 2007? I addressed two hundred teachers at all the fifty different gymnasiums within the Gothenburg area, a geographical district with a diameter of 50 kilometers, with a short survey. During the spring of 2007, 167 teachers from 46 different gymnasiums had answered the questionnaire quite throughout. The results are in some way quite surprising. Fewer than 10 % of the teachers use any computer software in their teaching, the vast majority use only graphing or CAS calculators and mainly to do calculations, sometimes to sketch graphs. Besides the calculators, the old technology is still alive and working; ruler, compass, protractor, and pencil.

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