A brief note on a 'widespread misunderstanding'

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In this brief note, I would like to concentrate on one question suggested as a key element of the seminar:

> The present technologies (ICT) are, in some respects, the heirs of this long tradition [of different tools and artefacts in mathematics education]. A widespread misunderstanding is that, due to their effectiveness, they may replace the old instruments: why use abaci when pocket calculators are available? Why refer to ruler and compass construction when powerful software is available?

What, exactly, constitutes this 'widespread misunderstanding' – the 'replacement fallacy'? It is, presumably, the belief that a new technology replaces an old one seamlessly, and that the rationale for this replacement is unproblematic. Stated in this way, the 'misunderstanding' is obvious – it fails to acknowledge the purpose for which the artefact is used. For example, it is evident that if rapid and accurate calculation is the aim, then calculators win over abaci (although informal 'competitions' reveal skilled abacus users winning over calculators in certain situations[1]); equally, if the objective is to achieve faultess geometrical constructions, then dynamic geometry systems surely will succeed much more quickly and efficiently than ruler and compass.

A slightly more sophisticated view would acknowledge that the replacement fallacy takes no account of the relevant activity system – and most importantly its goal or 'object'. It arises from the belief that the object of activity is essentially pragmatic rather than theoretical – to achieve a result in-the-world, rather than to construct knowledge about the calculation or construction. Artigue (2002) and others have distinguished between the *pragmatic* and *epistemic* roles of calculation, the former focusing on the productive potential (in terms of efficiency, cost etc.) and the latter, in terms of the contribution such calculation may make to understanding the mathematical objects concerned.

The recognition of this epistemic function privileges the pedagogic dimension. So, rather than efficient calcucation, perhaps the point is to observe, experiment with, and ultimately understand something about the structure of numbers – in which case, perhaps the abacus is just the right pedagogic tool; and if the objective of geometric construction is part of a longer-term attempt to understand why some constructions can and cannot be made with ruler and compass – and why it matters – then ruler and compass may well have, (pardon the pun), an edge over dynamic geometry.

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[1] See, for example, www.okinawa.usmc.mil/Public%20Affairs%20Info/Archive%20News%20Pages/2007/070504-soroban.html

This, then, is the 'misunderstanding': a reluctance to acknowledge that there is theoretical knowledge that may inhere (or at least be exploited) in obsolete technologies, and that these may have an explicit pedagogical value. At this level, therefore, the replacement fallacy stems from a failure to acknowledge the ways in which technologies mediate between subject and object, between people and their goals. At root, it is a special case of a general deterministic fallacy - a failure to discern human agency, ignoring a range of socio-technical questions that actually shape how technologies are used within cultures.

I want to argue, however, that there is a deeper issue involved, to do with the ways in which the roles of technologies change over time, and specifically, how the balance between pragmatic and epistemic functions evolves. Let us take as an example, how the introduction of the calculator replaced log tables. From a didactical point of view, this change – not so long ago, and certainly within the lifetime of some of us! - changed the epistemology of the log function. One heard, albeit only briefly, that the abandonment of log tables would lead to a difficulty with students' next encounter with the log function – probably as the integral of $^1/_x$ dx. But how many of that generation of students actually made a connection between the books of tables and the mathematical requirements that the integral needed to satisfy as a function?

In fact, as it rapidly turned out, disconnecting the log function from its historical role as a calculational aid had no particularly dramatic effects on the learning of mathematics, at least judging from the relative silence in terms of research studies. The explanation seems clear enough: it lies in the impoverished pedagogic connection that obtained between facility with log tables and the rest of mathematics. Paradoxically, precisely the failure to recognise the epistemic value of logarithms-for-calculation, meant that when they became obsolete, no particular effort needed to be expended to replace them. Log tables were introduced *simply* as a miraculous way to multiply and divide very large or small numbers, and not as the values of a function at all.

More generally, the balance between pragmatic and epistemic roles of a technology is in a state of constant flux: depending not only on the rise and fall of given technologies, but also on their cultural appropriation for different purposes. If 'obsolete' technologies are supported for explicitly didactical reasons, we have to recognise that this is an explicit didactical decision, and that there is sometimes a substantial price to pay, as students are being asked to work with obsolete technologies for a longer-term purpose that they may not share.

In such cases the question of legitimacy therefore surfaces. As Artigue points out, the transformations of mathematical knowledge in the presence of technology may or may not be judged as desirable from an educational point of view: it depends on who is judging the value of the transformed knowledge – for example, student or teacher? Ideas expressed with (new) technologies are routinely transformed from the ways they were hitherto expressed (an example would be computer programs 'versus' algebra), and abstractions might be expressed within novel representational infrastructures that are not immediately recognisable as legitimate from a mathematical point of view (see Noss & Hoyles, 1996).

I conclude with a final point concerning the misunderstanding. The strong implication of the opening paragraph, is that there is nothing especially important or different about digital technologies – other technologies (Dienes' blocks, video recorders) have come and gone, and it is hard to identify critical effects of their presence. It is here I come closest to agreeing with those accused of misunderstanding. Digital technologies have one supremely important potential for mathematical learning, to act as boundary objects between student and teacher (and between students), to provide an arena in which each can express what they know, how they know it, even if what is understood is contested (see Simpson, Hoyles & Noss, 2007, for an example of the role of technology in knowledge building about scientific/mathematical phenomena).

The implication I will draw at the Symposium is that as technologies become obsolete and are replaced by new ones, we have a reponsibility – not simply to preserve the obsolete in the name of epistemic function – but to design new objects and representations, and connect them as richly as we can to what we aim for students to learn.

References

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