ICT throughout the history: the retrospective gaze of the crab.

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My aim is to contribute to the discussion of this working group[1] about the use of new technologies (ICT) in the classroom. This choice depends on the fact that I took part, early as a teacher and later even as a researcher, in the debate about the use of ICT in the classroom since the 80's. My experience is mainly tied to the Italian debate. However, I think that the Italian situation on the use of ICT in the classroom may be seen as paradigmatic for many aspects, and the following reflections may be extended at least to the developed countries.

As regards Mathematics teaching and learning, I'm going to assume a perspective that pays attention to the evolution of the above debate, looking at the past but without forgetting its distance from the present time.

We can use a metaphor here. It is like copying the pace of the crab. I can go back from the present to the past in the same way of the crab. As withdrawing, it walks backward while its jutting eyes continue to look at the objects of interest, and at the new ones that come at sight. We only need some steps from the present to the past, since the history of ICT in the classroom is short, although full of events. At the beginning of this walk into the history, we find in front of us the years of the third millennium. They are described by the fact that the use of ICT in the classroom consists mostly of the two following aspects:

a) the use of CAS (*Computer Algebra System*), Dynamic Geometry Software (DGS), Spreadsheets and so on, as environments for representing and exploring mathematical objects and contents;
b) the resources available on the web for searching, obtaining and sharing information, and for e-learning as well.

Going back in time, we enter in the reign of the 90's, when the use of the web is less and less widespread into the classroom, even if present. But focus on the use of software as an environment for representing and exploring mathematical objects and contents remains relevant.

As we go back, we can also recognise that the use of ICT as an environment for representing and exploring mathematical objects and contents weaken progressively. The more we enter in the early 80's, the more the attitude is to propose an use of ICT in order to foster an algorithmic point of view of Math: in this regard, huge relevance is given to programming. At the end of the 70's and at the beginning of the 80's the use of ICT in classroom is rare, and almost the whole time focused on programming, when present.

In 1989 James Fey wrote a very interesting survey about the relation among technology, mathematics education, and the use of ICT in the classroom (Fey, 1989). In that paper Fey wrote: "In the first applications of computers to mathematics teaching and learning, students were almost always involved in writing programs […] Study and debate focused on programming and mathematics education have diminished substantially in the1980's […] The technology theme group sessions at ICME VI revealed deep interests in Logo by mathematics educators from many different countries. Many investigators are still committed to exploring the broad potential of Logo for changing the mathematics learning environment and goals in fundamental ways". In 1988, Blume and Schoen wrote: "We might expect programmers to (a) be more active and systematic in both planning and the solution stages, (b) make more use of successive approximations to solutions, (c) make more use of variables and equations, and, (d) be more likely to check for and correct errors in attempted solutions". But they conclude that "Despite the logical connections, however, studies to date have not provided strong evidence that programmers and nonprogrammers perform differently when solving mathematical problems" (Blume & Schoen, 1988). Looking at that time with nowadays eyes, we may recognise that the debate on the role and the efficacy of programming in mathematics teaching and learning casts a shadow on another aspect. As Fey wrote, "Thus such debates as the value of programming to mathematics learning or the virtues of various programming languages obscure a more fundamental challenge – revising our approaches to mathematics so that we prepare our students to apply the algorithmic methods which are essential in the use of computers" (Fey, 1989).

The paper by Fey is also interesting to remember the hopes that at the beginning of the 80's were posed into the use of ICT for programming instruction: "The initial efforts to educate computers as teachers were of two main types. The first were variations on the electronic flash card theme – drill and practice programs in which the computer posed the problems and gave students immediate feedback on their performances. These programs are still very popular in schools […] The second way in which computers have been used to stimulate teaching is as an electronic medium for programmed instruction. In the quest for a computer-based course that could operate independent of any teacher or textbook, developers have often translated programmed text to the computer screen, devoting considerable energy to the problem of teaching the computer to respond to students entries in intelligent ways. As with drill and practice, there are examples of such programmed electronic courses still in use and still being developed. But it seems fair to say that interest has largely shifted from stand-alone instructional systems that deliver, access and manage all aspects of education to development of more focused uses for computers tutors as only part of the teaching/learning environment".

In this regard, in 1988 Schoenfeld wrote that "The system would present information to the student; the student would work practice problems; the system could speed the student along when her work was going well, but could also

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[1] see: http://www.unige.ch/math/EnsMath/Rome2008/WG4/WG4.html

diagnose the student's mistakes and help when things went wrong; and it could answer the student's questions on a wide range of related issues […] In order for any tutor (machine or human) to succeed at this, it has to be (a) expert at the subject matter, (b) pretty good at figuring out what's going on the student's head, and (c) pretty good at teaching" (Schoenfeld, 1988). Then, we may say that such a system should have the properties, which belong to every good teacher, in order to be successful. Because of the structure of our educational system and for evident social issues, it seems better to invest on the pre-service and in-service training, as well as on the development of the teacher role as the one helping, guiding and observing the students in ICT teaching and learning environments. Besides, in this way it is easier to avoid the risk of forgetting emotional and affective aspects or those tied to social interaction, which are so significant in teaching and learning activities.

If we go back now from the early 80's to the 70's, we may only find the seeds of the virtual forest, we started from and we are always looking at, thanks to the typical walk of the crab. But, what about the history of ICME and ICMI as for the use of ICT at school and in mathematics education? It is at ICME 3, which took place at Karlsrhue in 1976, that we have the first serious discussion on the role of the calculator in mathematics education. The next ICME 4, at Berkeley in 1980, "emphasized the importance of taking full advantage of the power of calculators and computers at all grade level" (Nebres, 1988). In 1985, the International Commission on Mathematical Instruction (ICMI) published the first book of studies on the topic: the influence of computers on mathematics and the teaching of mathematics. An updated version of that book came in 1992 and included even five articles from the 1985 ICMI conference at Strasbourg (Cornu & Ralston, 1992). In such a book many applications of computers in mathematics teaching were proposed and discussed. For example, Antony Raltson focused attention on computer for learning and teaching discrete mathematics and calculus and to make explorations and discoveries in mathematics (Raltson, 1992). Bernard Cornu presented a list of some particular uses of the computer in the classroom, such as graphic possibilities, spreadsheets, databases, artificial intelligence as an aid in problem solving activities, hypermedia, self evaluation and individualised instruction, assessment and recording, student errors (Cornu, 1992). David Tall and Beverly West analysed the resources offered by some computer graphics software in order to give insight into mathematical concepts (Tall & West, 1992).

We can easily understand that at that time great attention was drawn to the use of computers in teaching and learning activities. However, as quoted few years before in Nebres, (Nebres, 1988), "we are still far from developing a calculator integrated curriculum. While more an more of students are become familiar with the microcomputer (especially at home), the place of microcomputer in the classroom and in the mathematics curriculum is still unclear". Why? There were a lot of reasons, but we may recognise some of the most important ones in the following list:

a) the cost of computers that, still in the early 90's, was relatively high and created obstacles to the diffusion of computers at school. Besides, as Graf and colleagues wrote in 1992: "even if suitable hardware and software are now available for ordinary schools, […] hardware availability in most schools is still dictated by the needs of computers science […] and the concentration of machines in special locations prevents or make difficult the natural, selective use of software during short episodes in the teaching process" (Graf et al., 1992).
b) For a long time, software were only thought of for professional aims, not educational. In particular, CAS (Computer Algebra Systems) were mathematical assistants, without being projected toward the aim of helping students to learn mathematics: "these packages are made by professionals. Therefore, they often do not present intermediate steps and some other didactical remain" (Graf et al., 1992).
c) For some time, the use of computer was almost identified with learning with programming. But programming requires a serious investment in time and effort both for the students and for the teacher, whereas neither all the students nor all the teachers want to invest such a time and effort.
d) The teachers' perplexities on the use of computers: "In many schools the computers are locked in special room, and it is not easy for teachers to use them. They must plan in advance, be sure the room is available […] Then they go to the computer room with pupils, and the time spent there is not generally totally integrated into the course" (Cornu, 1992b). Besides, in a computer environment teachers have to re-think the relations between practice and theory. Students are exposed to a lot of mathematical facts and the teacher has the difficult task of giving students the theoretical lenses in order to look mathematically at the computer screen. This requires not only mastering of teaching but also specific competences in mastering the organisation of learning.
e) Last but not least, the fact that for a long time the use of computers in most university mathematics classes has been completely absent. This might have had huge implications on the teachers' conceptions about computer science and its teaching in relation to mathematics teaching. For more details on this point, see Bottino & Furinghetti (1996), where an attempt is made at outlining a typology of these conceptions.

Nowadays, a lot of obstacles to the diffusion of ICT in teaching and learning of mathematics are overcome. Particularly, computers are relatively cheaper and students can find them in their daily lives, out of school. As a result, teachers do not have the problem of making students familiar with computers and may focus on a teaching and learning process more oriented towards mathematics[2]. As stated by Kaput et al. (2002), "The appearance of new computational forms and literacies is pervading the social and economic lives of individuals and nations alike. […] The real changes are not

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[2] For more details, refer to the past ICME conference and to the ICMI study 'Digital Technologies and Mathematics Teaching and Learning: Rethinking the Terrain', Study Conference held in Hanoi, Vietnam, in December 2006. Study Volume to be published by Springer in 2009 (New ICMI Study Series 13).

technical, they are cultural. Understanding them […] is a question of social relations among people, not among things. The notational systems we use to present and re-present our thoughts to ourselves and to others, to create and communicate records across space and time, and to support reasoning and computation constitute a central part of any civilization's infrastructure. As with infrastructure in general, it functions best when it is taken for granted, invisible, when it simply works".

The new technology available today allows to introduce the students to different forms of representations for the mathematical concepts: in this way it can provide students with an access to mathematical topics. As James Kaput wrote, "We are early in an exciting new era for technology in Mathematics Education. Both the representational infrastructures are changing and the physical means for implementing them are changing. We are seeing new alphabets emerging, new visual modalities of human experience are being engaged and new physical devices are emerging – all at the same time. Much work need to be done" (Kaput, 2002).

The retrospective gaze of the crab suggests us that much work has been done, but much work still need to be done, in order to have really integrate and meaningfully ICT teaching and learning environments, For example, much was done for teachers training. However in general, the beliefs teachers have about the possible use of ICT for mathematics teaching and learning are not still oriented in a positive way. Much work has been done from researchers as regards an *aware use of technology* in the classroom. It means to understand if, how and when the technological artefacts can mediate / support / carve the construction of the student's mathematical knowledge in the classroom. "To do that, one must consider ICT's in a wider setting, namely to analyse them from a multifaceted perspective: that is, from a *didactic* point of view (e.g. considering teachers' role of social interactions induced by the use of technologies, and so on); from a *cognitive* point of view (e.g. considering how technologies change the mental structures of the learners); from a *cultural* point of view (e.g. considering the framework of rationality towards which the use of technologies may push the student)" (Arzarello, Paola & Robutti, 2006). But much work need to been done by researchers in order to give deeper and more meaningful answers to the following three questions:

1. How do praxeologies change in the classroom because of the introduction of software? How the new praxeologies (if any) help (or block) students in learning mathematics?
2. What is the specificity of instrumental actions in the software environment? Which instrumented actions in the software help (or block) the students' learning processes?
3. At which extent does a new technology modify the usual multimodal behaviour of students? [3]

I want to add to such reflections a brief example, which regards the use of software to produce multiple representations of mathematical objects. It is an example of how the specificity of instrumental actions in a software environment helps students to construct meaning for the mathematical concept of function.

A very classical problem was posed to students of two different classes[4], which use two different software: Cabri Gèomètre in the class 1 and TI-nspire in the class 2[5]:

In the Zumbak Republic there are two villages that we call A and B. The first one is 4 km far from the side of a very deep and tight river and the second one is 7 km far from the same side of the river. An international company designs the construction of a pipeline: a straight tube leaving from the village A, catching up a point of the river and leaving from it, again straight on, so to catch up the village B. In this way, water reaches the two villages. The minimum of the total length for the two tubes is the unknown to be searched for.

The attention of the students working with Cabri is generally caught by the graphical windows. Even if students put measures and use sophisticated tools as the 'animation'[6] tool to observe the covariation between two variables, in Cabri the graphical aspects are predominant and risk to shadow numerical and symbolic aspects. David Pimm wrote: "<The> notion of function is actually subtly different, depending on whether it is accessed through algebraic forms, graphs or numerical tables. […] I suspect that the linking of representation is never neutral: one will predominate and will, in consequence of this privileged position, lose much of its own representational status. My candidate is the graph. The machine may be providing too much to attend to at one time: human attention is usually caught by movement. The graphical window is likely to be the winner among different displays. I predict the algebraic forms will come to be seen as merely descriptive, suggesting that the meaning is the screen graphical representation, rather than maintaining two different independent but linked representations […] The algebraic expression often labels the graphs, inviting them to be seen as the names for the graphs, and hence that the graphs are the referent, not merely another representation" (Pimm, 1995; pp. 102-103). Concerning mathematical software and, in particular, dynamic geometry software, what Pimm wrote is very intriguing: Cabri offers a very interesting dynamic numerical aspect, but it is neither organized nor structured and,

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[3] The term *praxeologies* is used in the sense of Chevallard (Chevallard, 1992); the terms *instrumented* and *instrumental* are used according to Rabardel and Verillon (see Verillon and Rabardel, 1995); the terms *multimodal behaviour* comes from the *multimodal paradigm,* arising from cognitive science (see Wilson, 2000, for a summary). In the research group coordinated by Prof. F. Arzarello at the University of Torino, we are studying, from this viewpoint, teaching and learning environments using ICT (see Arzarello, to appear, and Laiolo & Paola, 2007).

[4] The teaching experiment is described in more details in (Laiolo & Paola, 2007).

[5] TI-nspire is a software by Texas Instrument, not yet marketed in Italy. The class 2 is part of a research project (coordinate by Prof. F. Arzarello) on the use of this software in mathematics teaching and learning.

[6] This instrument allows us to connect a point with one or two degrees of freedom with a spring enabling it to move. As the animation is active, the students can observe how the figure varies without using the mouse.

besides, it is only a little window that throws light only upon a single number (although variable). Thus, if we want to stress the numerical aspect, is suitable to use another environment that is more structured and organised, as a spreadsheet. It is interesting to observe the solving process performed by the students of the class 2, using the TI-nspire environment.
In Fig. 1, you can see the point A and the point B representing the two villages, and the point P that is the point of the river where water is taken from. At the beginning students move P and see that PA+PB changes: there is no correlation with a Cartesian Graph, it is only a drawing with the measures of PA and PB. The students, with the “text” tool, write PA+PB; then, with the "calculate" tool, they estimate PA+PB and observe the variation of the number "PA+PB" according to the variation of the position of P (Fig. 1).

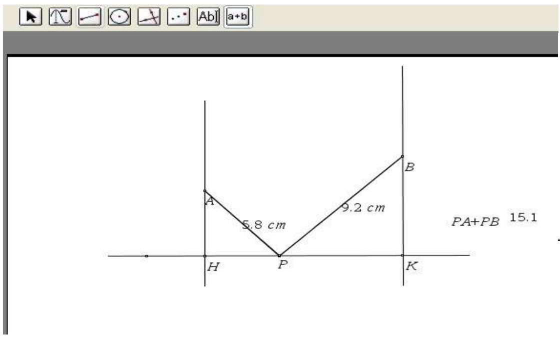


Fig. 1

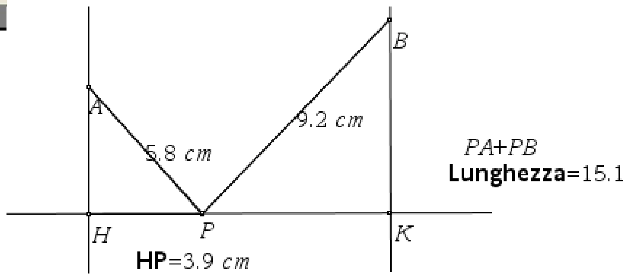
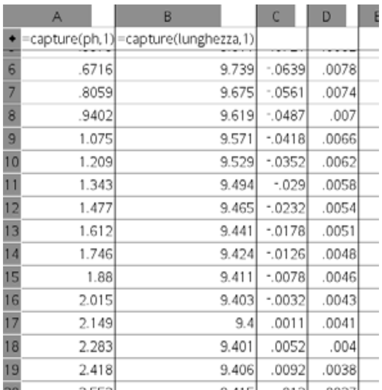


Fig. 2

In this way, the students make a conscious and explicit exploration using the numerical register. They understand that PA+PB changes with the variation of the position of P, and they choose PA+PB as the dependent variable and PH as the independent variable. Hence, they explicitly recognise a functional relation between the variables PA+PB and PH, and it is natural for them to declare the variables PH and length, that is PA+PB (Fig.2). Students can now use the TI-*nspire* resource “automatic capture of data”. The length of PA+PB and the length of PH are collected in two columns of an electronic sheet: the first column represents the independent variable and the second one the dependent variable, as shown in Fig. 3. The students collect the data automatically and estimate the first order differences of the independent variable, so to verify if it varies with a constant step. Later they try to evaluate first and second order differences of the dependent variable "length" (Fig. 3). They find not only the position of P which minimises the length with an excellent approximation, but also information on the monotony and the concavity of the function length = length (PH). Finally, they move to the diagram for seeing with their own eyes what they have already imagined (Fig. 4).



	A	B	C	D
◆	=capture(ph,1)	=capture(lunghezza,1)		
6	.6716	9.739	-.0639	.0078
7	.8059	9.675	-.0561	.0074
8	.9402	9.619	-.0487	.007
9	1.075	9.571	-.0418	.0066
10	1.209	9.529	-.0352	.0062
11	1.343	9.494	-.029	.0058
12	1.477	9.465	-.0232	.0054
13	1.612	9.441	-.0178	.0051
14	1.746	9.424	-.0126	.0048
15	1.88	9.411	-.0078	.0046
16	2.015	9.403	-.0032	.0043
17	2.149	9.4	.0011	.0041
18	2.283	9.401	.0052	.004
19	2.418	9.406	.0092	.0038
20	2.552	9.415	.013	.0037
21	2.686	9.428	.0167	.0036
22	2.821	9.445	.0203	.0035

Fig. 3

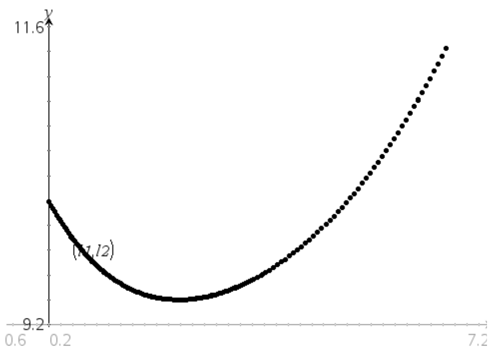


Fig. 4

If you look at the solution process of the class 2, you can observe that instrumental actions in the software allow students to shed light on the complexity and cognitive potentiality of the numerical register; in metaphorical terms, TI-nspire opens the window upon the numerical aspects of a function. Thus, the concern of Pimm about the risk that students confuse the concept of function with one of its representation, because of the predominance of graphical aspects, can be reduced. In fact, in TI-nspire the different environments (graphical, numerical and symbolic) are at the same level of the menu.

Some conclusions

New technological tools, or better particular ways of employing ICT allow the construction of mathematical curricula and activities, which *make sense*[7]. But they also entail risks and dangers. I want to conclude the reflections in this paper with a personal synthesis about some limits and potentialities of the use of ICT.

Limits, risks, dangers:

a) a purely technical use of instruments, seen as mere 'prosthesis' used to replace lacks of abilities. The great potentiality for exploration offered by ICT may inhibit the dynamic mental exploration, which is a core feature of mathematical thought.
b) Just by reason of their friendly and easy interface, ICT can become opaque and create obstacles in learning. As Radford states, tools are not neutral but they develop from a culture; a tool "is more than a gadget to economize actions. It carries in itself, in a compressed way, socio – historical experiences of cognitive activity and scientific standards of investigation. However, by taking over some of the human actions, certain aspects of the socio – historical experiences that the system holds remain 'hidden' from the individuals using it …" (Radford, 2005).
c) Some uses of ICT can contribute to lose fluency (e.g. in mental and by hand calculation). This may cause the loss of abilities very important in mathematics.
d) Giving priority to some forms of representation, the use of mathematical software may trigger the confusion between a mathematical object and one of its representations.

Potentialities:

a) ICT allows experiencing mathematical objects thanks to the resources offered by the software interface and its different representations of mathematical objects.
b) New technological tools are now actual infrastructures that are taken for granted, almost invisible. They are at students' disposal at home, not only at school; they are used in a lot of ways; in other terms, they "simply work".
c) ICT provides new modalities of interaction with students, not only in their presence, but also at a distance.

It is still necessary to underline that the use of ICT in mathematics teaching and learning does not imply a marginal or vitiated role for teachers. On the contrary, they are the warrantors of the construction of meanings. They are the warrantors of an accurate approach to theoretical and rational thinking. Teachers have to support the instrumental genesis of the tools they use in the classroom, creating the conditions through which students "see their screens mathematically"[8].

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[7] In Italy, we say *attività sensate*. The Italian word is used with three different meanings: reasonable (that is attentive to the specific possibilities and constraints of the class); linked to the natural abilities and particularly to perception; led by the intellect and specifically by a theory.

[8] Reference here is to David Pimm, who wrote: "when looking at a screen, my mathematical knowledge tells me in substantial part *how* to see what I am looking at. Without that knowledge, *must* I see these screens mathematically? (Pimm, 1995; p. 125).

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