Technology as Experience: Three Instrumental-Genetic Issues

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> Language molded itself, as it developed, to genetic tools already there. The reason algebra is less well aligned with genetic tools is that it was *not allowed* to align itself; it was *made* by mathematicians for their own purposes while language *developed* without the intervention of linguists. (Papert, 2002, p. 582)

Several recent research essays in the history of mathematics and in semiotics and psychology of mathematics education drive this short reflection on the role of technological tools or, more generally, information and communication technology (ICT), in students' learning of mathematics. The description for the Working Group on resources and technology throughout the history of ICMI articulates the need to acknowledge the history of ancient or classical tools in conducting, say, geometric processes, including the fundamental role of historical processes in shaping the nature of current and emerging ICT tools, considered to be heirs. There is a concern, legitimate as it is, that such ICT tools will eventually replace the old ones. Such a concern reminds me of a situation in the history of mathematics which Ivor Grattan-Guinness (2004, 2003) has labeled as the history versus heritage perspectives in accounting for the development of mathematical notions (definitions, techniques, algorithms, notations, proof methods, etc.). While a historical account seeks to explain how events in the past might explain a present mathematical situation, with attention to the changing notations, motivations, and causes that come with such a narrative telling, a heritage perspective inserts the historical only when the latter becomes appropriate. A heritage perspective favors an account of justifying why the present situation is the way it is with a special focus on the solidified knowledge in finished form (Grattan-Guinness, 2003) which, in the first place, is the central object of analysis. In the case of technological and classical tools that are used in mathematics, whereby the former ones are being used more widely than the other, the source of the tension between doing away with, say, the abacus in favor of pocket or scientific calculators or paper-and-pencil graphing and paperfolding in favor of graphing calculators and dynamic software could be reinscribed within the history-versus-heritage debate. Certainly, it is worth considering what gets lost in the translation or underemphasized once one perspective gains more ground over the other. Grattan-Guinness (2003) notes, however, the need to be "aware of both of them, and the relationships that they excite and unavoidably impose" (p. 175). Both Duval (2006) and Dörfler (2002) have addressed the difficult issue of comprehending mathematical objects. Dörfler (2002) sees the process of constructing knowledge about such objects to be primarily a function of a combined action involving ontology and epistemology. Duval (2006) foregrounds the plausibility of developing various semiotic representations that capture features of the relevant objects, that is, different representational systems bring forth differing semiotic representations of the objects for learners.

In relation to technological tools that are used in mathematical learning, I am primarily interested in determining how tools that enable virtual experimentation can assist learners to comprehend ideas in a particular intentional way and which cannot be easily accomplished with tools in the traditional context of real experimentation. Further, connecting the historical-heritage concern, there is a need to both surface and articulate disciplinary and cognitive issues when mathematical learning shifts from real to virtual experimentation as current and emerging learning tools are now capable of providing an animated, subjective, interactive simulated environment for constructing mathematical knowledge. Thus, in the rest of this paper, I raise three issues as we explore the full implications of instrumental genesis in the mathematics classroom, namely: resolving the historical-heritage tension in tool use; dealing with the ontology/ideology status of mathematical objects and truths in ICT contexts, and; extrapolating different learning and instructional mechanisms that can effectively bridge the divide between cognitive abilities and internalized mathematical knowledge.

The History versus Heritage Tension in Tool Use

While there is voluminous research on the use of technological tools in mathematical learning, we have just begun the complex task of exploring the full implications of an instrumental genetic approach to developing mathematical knowledge. From tools as enablers and amplifiers, to tools as mediating knowledge, and then to tools as generative models, we are slowly developing scientific, empirical knowledge, and so far within the methodological context of effective design experiments, of what and how it means to do mathematics that anticipate the complexities of both instrument-based and "collateral knowledge." Hoffmann and Roth (2005) trace the idea of collateral knowledge from Peirce, American semiotician, in relation to "those forms of knowledge that remain hidden though being an essential condition for a cognitive activity." Situated within and beyond constructivist views, an instrumental genetic view of mathematical learning takes into account how individuals develop instrument-related schemes that then evolve as valid mathematical notions and practices as the tools are transformed into psychological tools. Thus, mathematical knowledge is seen to be a type of internalized, collateral knowledge which is a consequence of both tool use and other relevant knowledge. For constructivists, especially radical, the relationship between subject and object (i.e., knowledge) is primary; for instrumental geneticists, subject, object, and tool are triadically interacting and knowledge is seen as distributed among, and emerging from, them. Competence with the tools is also made complicated by the fact that learners bring with them elements of their own subjectivities and prior learning experiences which influence the way they use the tools.

A consequence of an instrumental genetic view of technological tools involves deemphasizing the historical aspect of such tools in favor of emergent knowledge that learners derive when they acquire competence and facility (instrumentation) in using the tools, including how they appropriate the tools for other purposes (instrumentalization). For example, various software applications in the TI-84+, such as Cabri Jr. and Transformation Graphing, allow learners to conveniently explore mathematical relationships in a graphical representational context without being distracted by the preparatory mathematical work necessary to produce the graphs. Because such technological tools appear within learners' experiences in packaged forms, they tend to downplay the historical processes involved while increasing the importance accorded to the anticipated inherited knowledge. For example, while geometric constructions with a ruler and a compass necessitate a good theoretical knowledge of Euclidean theorems, constructing in a Cabri environment tends to skip many of the Euclidean processes that are involved which are emphasized in greater detail if classical tools are used instead. In a published research report, I documented the emergence of a graphical process for solving polynomial inequalities among a cohort of precalculus students. Such a process shifts away from traditional algebraic ways of solving inequalities in favor of knowledge that emerge from the students' developing expertise and various usage and utilization schemes with a graphing calculator, including how they overcome the limitations of the calculator which enabled the emergent graphical process to finally take shape. Worth noting are some instrument-induced notations and practices that are valid mathematically. Thus, an important issue in an instrumental genetic account of mathematical learning deals with the extent to which learners ought to be bound by the historical processes or notions despite the mathematical validity of emerging ones that could be interpreted as a consequence of inherited knowledge? For example, if construction by Cabri is seen as an inherited system, and that learners develop instrumental, mathematically valid knowledge, does the absence of a full grasp of the corresponding historical processes of Euclidean construction make the knowledge less mathematical and rigorous than otherwise?

The Ontology/Ideology Status of Mathematical Objects and Truths in ICT Contexts

My current project involves using a computational environment to teach Grade 8 children (with a mean age of 13 years) how to obtain algebraic generalizations in closed form for pattern sequences containing figural cues. Results from various studies done with young and older children both in the USA and in other countries reveal a predisposition towards developing recursive, additive formulas when they see patterns such as the Garden Tiles Pattern shown in Figure 1. In fact, very few could correctly express and justify a constructive generalization in a multiplicative format and the percentage is far fewer among children in terms of their ability to make sense of a deconstructive generalization. A constructive generalization of a figural pattern sequence involves perceiving the pattern as having an invariant structure that consists of one or more non-overlapping symmetrical parts in the pattern (see, for e.g., Figure 2). A deconstructive generalization also has an invariant structure but is oftentimes much more difficult to see because some or all parts overlap either at a vertex, an edge, or in a region (see Figure 3).

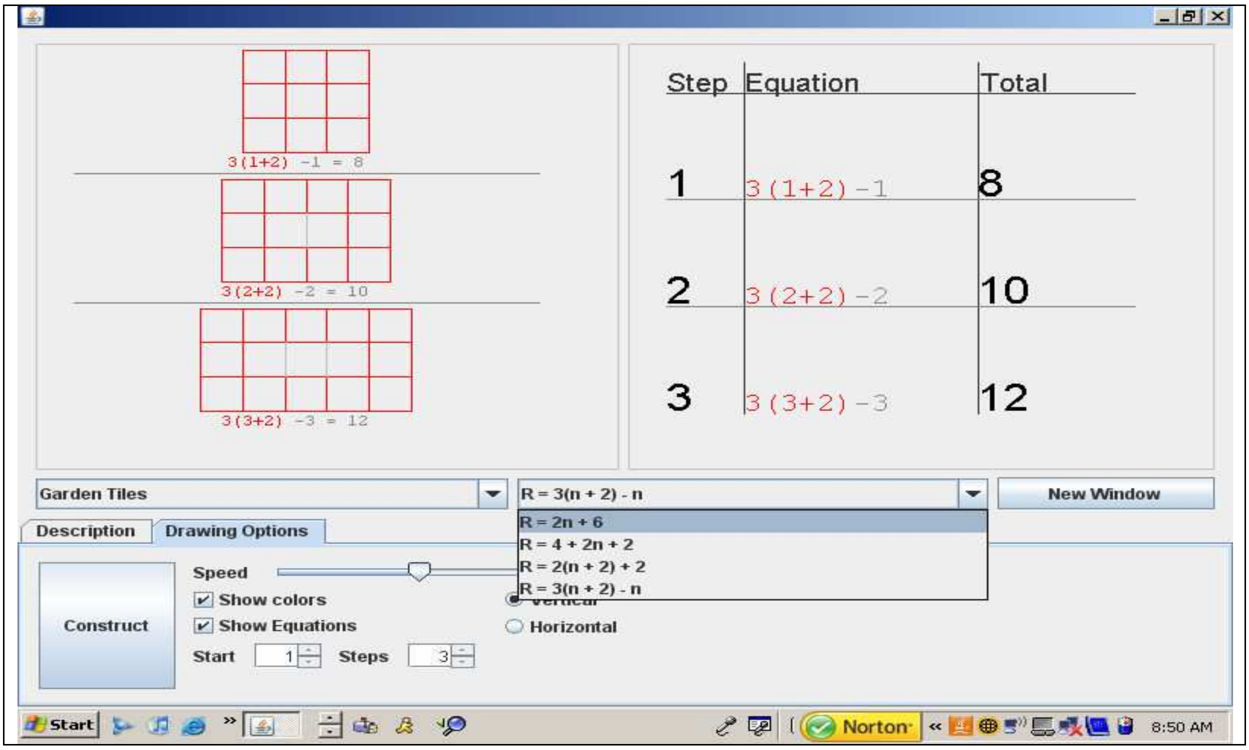

Figure 1. Exploring Formulas for the Garden Tiles Pattern

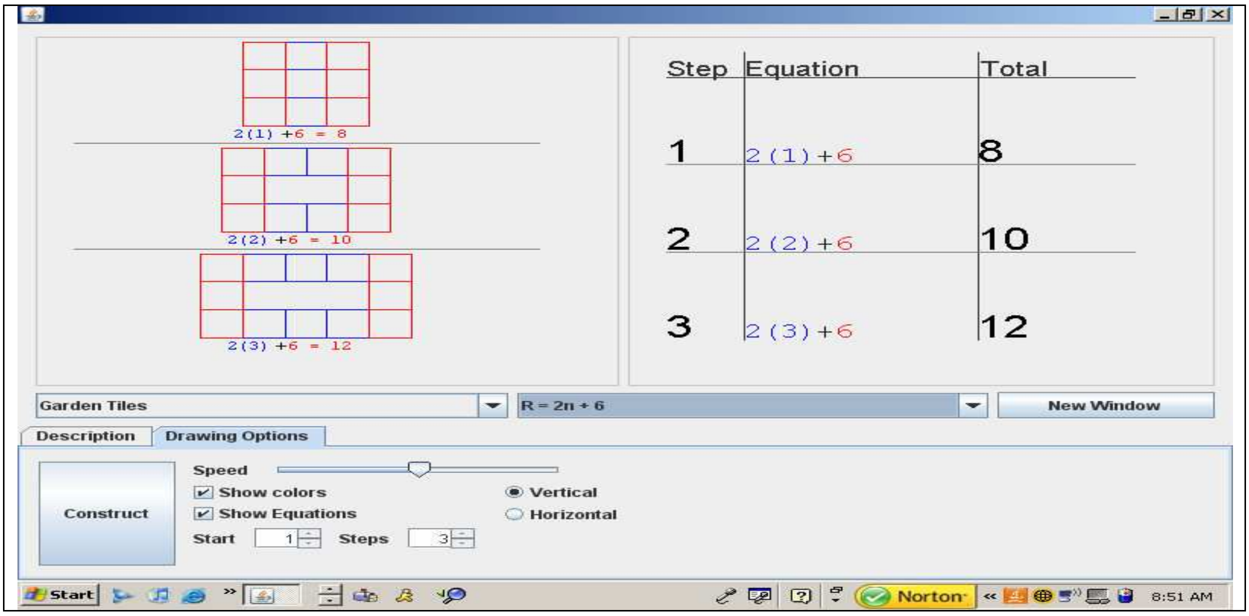

Figure2. A Constructive Generalization for the Garden Tile Pattern

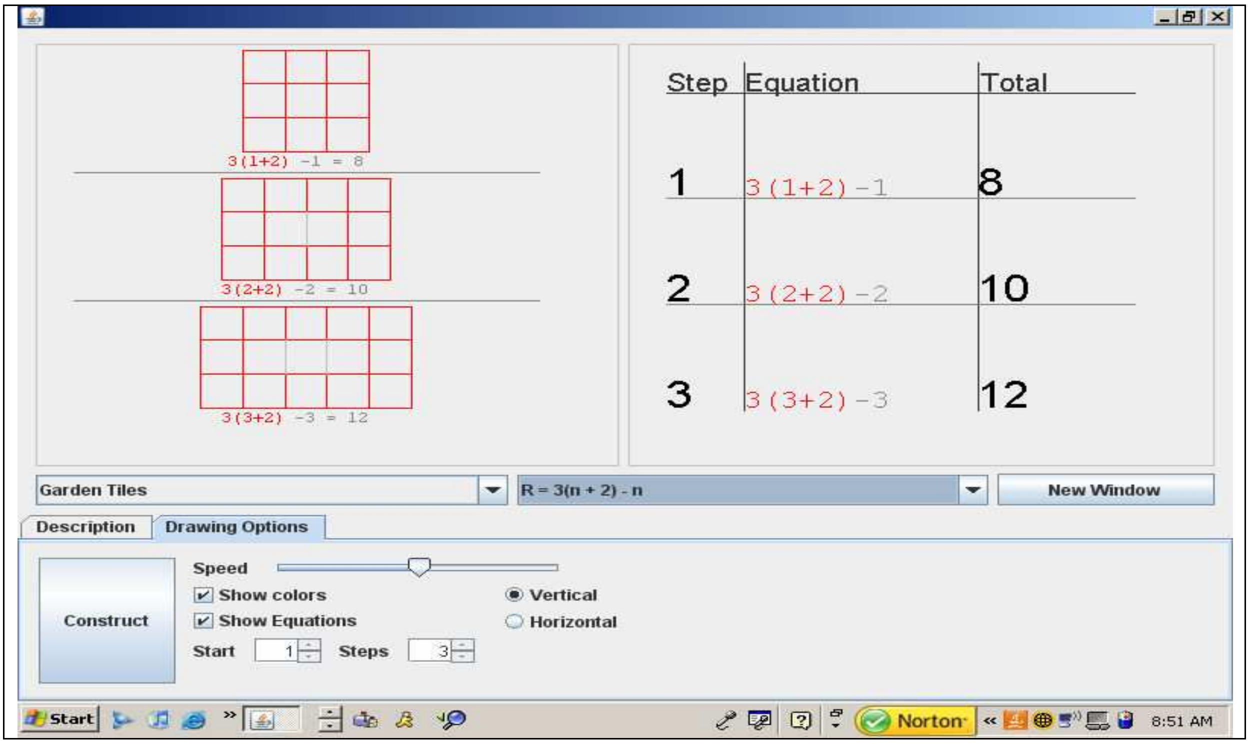

Figure 3. A Deconstructive Generalization for the Garden Tile Pattern

In developing the above computational tool, I assume that it will serve to both mediate in and generate a model for thinking about figural patterns in a particular way, in fact, in a way that scaffolds thinking towards the development of multiplicative thinking resulting in closed-form generalizations. But such a tool also implicitly affirms the validity of an unquestioned ontological commitment towards patterns in a way that values what mathematicians see, that is, their ideology or sociocultural practice. While instrumental genesis seems concerned with the development of valid, usable schemes as a result of instrumentating a tool, however, it does not fully take into account the messiness of a learner's pre-theoretic understanding in the development of the tool itself. Duval's foregrounding of learners' differing semiotic representations for mathematical objects articulates the problematic of what, and how, it means to see, grasp, or notice something about the objects in ways that are different from the institutional or intentional version. But dynamic software in geometry, graphing calculators, and other symbolic manipulation tools do not fully take into account the complexity of reference and ontological relativity, especially considering the fact that some learners do not share the same ideology as do mathematicians. Thus, emergent knowledge in an instrumental account of mathematical learning may not be fully authentically emergent since the ontology of both objects and the corresponding truth values have been framed from a particular standpoint. The ontology of everyday objects like a hammer is different from the ontology of mathematical objects like patterns; both have different pragmatic and epistemic values that enable the natural, situated character of the former type to be instrumentalized by the user in ways that he or she might find necessary and useful. While the instrumentation aspect of knowing a technological tool, which emerges from tool use to what could be reinscribed in mathematical form, may engender collateral knowledge, the action of instrumentalization may not even take place due to the inherited discursive properties of the tool which seeks to mediate in a particular way reflective of its sociocultural history.

From Instrumental Genesis in the Development of Cognitive Abilities to Internalized Mathematical Knowledge: In Search of Stable Models

Hoffmann (2007) characterizes cognitive abilities as being prior to (implicit or collateral) knowledge. Knowledge pertains to abstractions drawn, and is independent, from concrete activities, while cognitive abilities depend on the concrete activities that give them meaning. For example, in my own research with thirty-four 13-year-old students who were learning late middle- to early secondary-school algebra, I observed that while all of them had the cognitive ability to use algeblocks in performing operations involving polynomials, however, some still had difficulty when asked to combine certain polynomials in the absence of algeblocks. In fact, I noticed a similar action taking place earlier when those same kids were asked to use algeblocks to, say, obtain -4 - 2 on a basic mat consisting of positive and negative regions (Figure 4). That

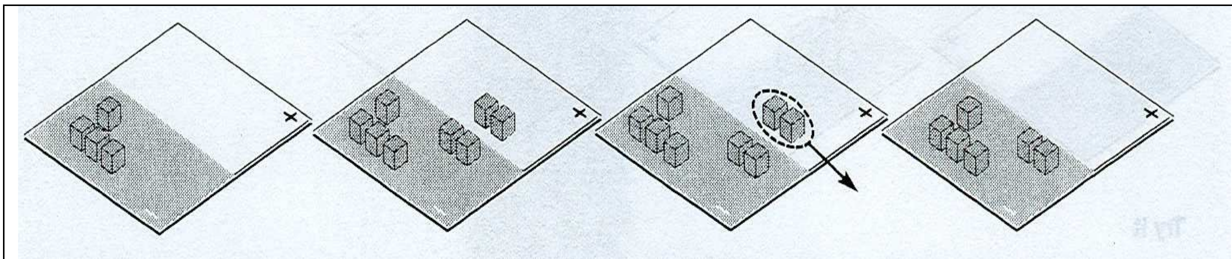

Figure 4. -4 – 2 Using Zero Pairs in an Algeblock Context

is, while they showed a cognitive ability in setting up zero pairs in order to take away the required number of negative unit cubes leading to the correct difference, they had difficulty obtaining the difference between, say, -47 and -25 which was resolved only much later when they developed knowledge of some relevant rule for subtracting opposites with the help of smaller numbers. Thus, in relation to instrumental genesis research, there is a need to develop cognitive or extra-cognitive (say, linguistic) mechanisms that enable learners to shift from cognitive ability to knowledge within what Hoffmann (2007) defines as a cognitive system which comprises of objects and relations that are situated within the ability-knowledge continuum. Morever, such mechanisms can become truly powerful if proposed models can be shown to hold across learner- or grade-level groups. Hoffmann (2007) suggests one such possible model, that is, through internalization based on Vygotsky's empirical work. Hoffmann modifies Vygotsky's model and proposes the following three processes (instead of two) in his model: the social plane that introduces the relevant signs and/or tools in the context of an anticipated cognitive ability; the individual plane in which a learner uses the sign or tool to regular his or her own cognitive processes, and; the stage of intramentation in which the individual manipulates at the level of implicit, collateral knowledge in the absence of the sign or tool. Considering my own research on computational media and patterning in relation to the object of algebraic generalization (as described in the previous section), I see internalization of knowledge among learners to be dealing not so much with the artifact-to-instrument transition by way of mediation but as an emergent phenomenon, that is, internalization as being about assisting them to develop unique ways of doing mathematics of their own which they find meaningful and at the same time valid from the point of view of institutional practices. Perhaps favoring a heritage view of mathematical practices, the above computational environment on patterning that I constructed for learners was not meant to be an unproblematic internalization of ways to establish an algebraic generalization for a pattern; I also wanted them to, in Papert's (2002) words, "create for the first time the possibility of freely exploring the infinitely open-ended variety that forms of knowledge and their learning can take" (p. 585). What these forms are, how they are acquired through learning, and how to negotiate between personal and institutional forms, still need further research.

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