CAN WE CONSIDER GRAPHIC CALCULATORS AS AN “OLD” TECHNOLOGY, COMPARED WITH A NEW ONE?

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ABSTRACT

In this paper I present some teaching experiments carried out at secondary school level and at teacher training school, with the use of graphic calculators and the TI-Navigator software, which gives a common environment where working together in the mathematics classes. The aim of the paper is twofold: to be in continuity with some ideas emerged from the last ICME Congresses and to analyse the question written in the title: may we have more support in the teaching/learning processes by this new software, with respect to the usual calculators or computers laboratory? If yes, why and how?

KEY WORDS: mathematical laboratory, modelling, function, representation, calculator, sign, meaning, semiotic approach.

THEORETICAL FRAME

According to the main issues of this ICMI Symposium, I would like to trace some lines coming from the last ICMEs in Copenhagen and in Tokyo:

1. theoretical, from the plenary lecture of F. Arzarello at ICME10;
2. methodological, from some Groups I participated at ICMEs;
3. practical for the teaching/learning processes, from the group sessions about the use of technology in mathematics at the last ICMEs.

The first line refers to a cognitive model introduced by F. Arzarello and called *APC Space*: the Space of Action, Production and Communication during a mathematical activity carried out by the students in a social way, for example working in small groups (Arzarello, 2006). This model is a way to describe the mental activity of the students, reflected by what we can observe as researchers: gestures, words, glazes, actions on the paper, in the air, on the artefacts, interactions with the teacher, and whatever sign they use to evolve in approaching the solution of a problem. Namely, it allows to consider how these signs determine the processes of learning and to describe them as important multimodal ingredients. To analyse the students’ activity the model takes into account as its main components: the body, the physical world, the cultural environment of teacher and students. And to describe them, it uses two levels of reading: the diachronic and the synchronic one, the first used to show the evolution in the construction of meanings, the second used to picture the complexity of interacting elements at a certain instant, giving a reason of multimodal aspects of the learning processes.

The second line refers to the *mathematics laboratory*, introduced at ICME10 by the Italian community through discussions in the groups (e.g. DG20), a CD-ROM, a booklet of the recent Italian mathematic education research, and a paper published on the website of ICME10 (Bartolini et al., 2004). A mathematics laboratory is a methodology, based on various and structured activities, aimed to the construction of meanings of mathematical objects. A mathematics laboratory activity involves people, structures, ideas. We can imagine the laboratory environment as a Renaissance workshop, in which the apprentices learned by doing, seeing, imitating, communicating with each other, in a word: practicing. In the laboratory activities, the construction of meanings is strictly bound, on one hand, to the use of tools, and on the other, to the interactions between people working together. It is important to bear in mind that a tool is always the result of a cultural evolution, and that it has been made for specific aims, and insofar, that it embodies ideas. This has a great significance for the teaching practices, because the meaning can not be only in the tool per se, nor can it be uniquely in the interaction of student and tool. It lies in the aims for which a tool is used, in the schemes of use of the tool itself (UMI-CIIM & MIUR, 2004).

The third line is based on the recent observations made in the last ICMEs (e.g. Thematic Afternoon at ICME 10), about the use of technological tools and materials, and collected in a book on the occasion of the last thirty years of PME (e.g. Ferrara, Pratt, & Robutti, 2005). The main streams of this book are: the attention to the interactions student-technology and the support in learning; the use of graphic representations for teaching algebra, pre-calculus and calculus; the importance of using sensors for collecting data and modelling functions; the effective support given by DGE for learning geometry, and so on.

But I would like to add some theoretical elements to these sketched above, namely: the humans-with-media approach and a semiotic-cultural analysis of students' cognitive processes. The book written by Borba and Villareal (2006) introduced a point of view that contains and enlarges the instrumental approaches that take into account only the relationship between a subject and technology. Borba focuses the attention on the community of learners (small groups, as well as the whole class or bigger groups), along with the instruments used by them. This point of view overcomes the deep-rooted dichotomy between humans and technology: humans as users of technology and technology as tool to be used. This recent perspective suggests that knowledge is always collectively produced by *humans-with-media*, and mathematics is part of this process. Learning is a social undertaking, a process of interaction among humans as a group, including tools in it. In fact media, such as computers and their evolving interfaces, reorganize mathematical thinking; insofar they are not simply substitutes for humans or supplements to them, rather they are 'actors' in a collective thinking, because they are carriers of a historical-cultural heritage and mediate the construction of knowledge. Media interact with humans, in the double sense that technologies transform and modify humans' reasoning, as well as humans are continuously transforming technologies according to their purposes.

This approach is compatible with the semiotic-cultural frame, recently discussed in various occasions (e.g. Arzarello, 2006; Radford, 2006; Robutti, 2006), which is used to analyse the students' activity for constructing mathematical meanings, looking at the signs introduced in the social development of this activity. These signs can be words, gestures, actions on the tools, and so on. Sometimes, these signs can be the signals of the introduction of a new piece of knowledge that does not exist before in this social activity, and this event has been called *objectification of knowledge* (Radford, 2006). In this approach, thinking is not only a reflexive practice, but also a mediated one, if we refer to the role played by artefacts and signs used in the activity. So learning mathematics is a matter of *being-in-mathematics*, living in a classroom as a community, working together and sharing activities and results.

Within this framework, what we study as researchers is when and how the students make visible in the group activity something that was not before, and in order to do it, we observe every kind of signs they introduce at this aim: symbols, drawings, graphics, actions, bodily movements and glances. The activities are planned following the methodology of mathematics laboratory and the signs are interpreted in their evolution during the process of meaning construction.

The importance of the role and the mediation of the artefacts should not let us think that the role of the teacher is less important that in the past, when at school there was less technology. What is requested today by the teacher are higher competences than in the past, not only in using technologies, but in planning activities with technologies, in observing students using them, in conducting discussions and in mediating the impact technologies could have in mathematics. In fact, it is commonly accepted that computers and calculators by themselves cannot bring any improvement of the quality of learning.

METHODOLOGY AND RESEARCH QUESTIONS

We carried out two teaching experiments at secondary school level (grade 10) and one at the teacher training school for secondary level (SIS Piemonte), in order to analyse the evolution in the construction of meanings in modelling activities by students working in small groups (while future teachers individually) with graphic calculators TI-84 and the software TI-Navigator, which provides wireless communication between students' graphic calculators and the teacher's computer (Trouche, 2006; Dougherty & Hobbs, 2007). In TI-Navigator, the public display is the most innovative part of the connectivity system: it consists in a common Cartesian plane (called Activity Center), to which each student and the teacher can give their personal contribution. Another environment of the software is the Capture Screen, which let be possible to capture simultaneously all the screens of the students' calculators. Both the environments can be projected on a big screen, if the teacher's computer is connected to a video-projector. In Figure 1 an example of Activity Center (almost at the beginning of the teaching experiment), with the task to trace a straight line parallel to a given line of equation *y=2x-1*. Some lines are correct, others not (showed by the arrow). In Figure 2 the Screen Capture during the task to write the equation of a line intersecting the origin of the Cartesian plane and passing through the second and fourth quadrant. One of the group made a mistake, sending the equation of a line through the first and third quadrant (showed by the arrow).

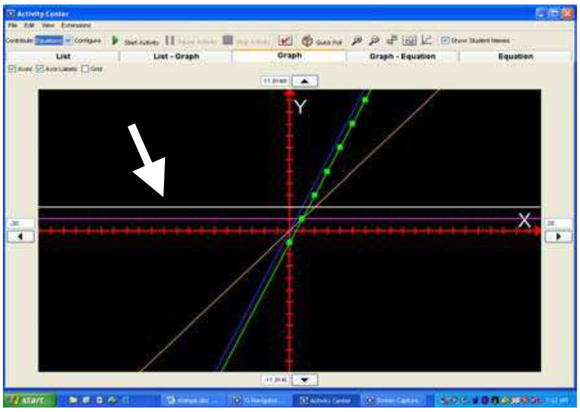

Figure 1: Activity Center

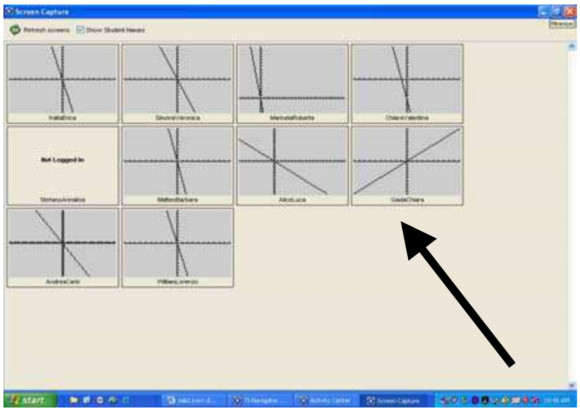

Figure 2: Screen Capture

The students work together in small groups (two or three members); each group uses a graphic calculator (TI-84) connected to a network hub, which communicates with the teacher's computer. The activities are followed by collective discussions conducted by the teacher (or by the researcher). In addition to written materials (worksheets) and data from the software, we also analysed the videos of the activities, in terms of the students' use of semiotic resources (language, signs, gestures, actions on artefacts). In Figure 3 an example of the gestures we analysed. Three people were present in the classroom: myself as participant observer, the mathematics teacher and a master degree student as participant observer and manager of the technological equipment. In the teacher training course, myself as teacher, the master degree student and an assistant.

Our study is focused on the construction of the meaning of function, starting from different representations: graphical, algebraic and numeric. The activities are centred on families of functions, principally linear, quadratic, exponential and start from modelling problems. As cognitive roots for the description of a function we choose the qualitative concept of invariance and the quantitative concept of slope and its variation, as a ratio of increments. Related to these roots, we also use other concepts, as: domain, sign, intersection, zero, parallelism, and so on.

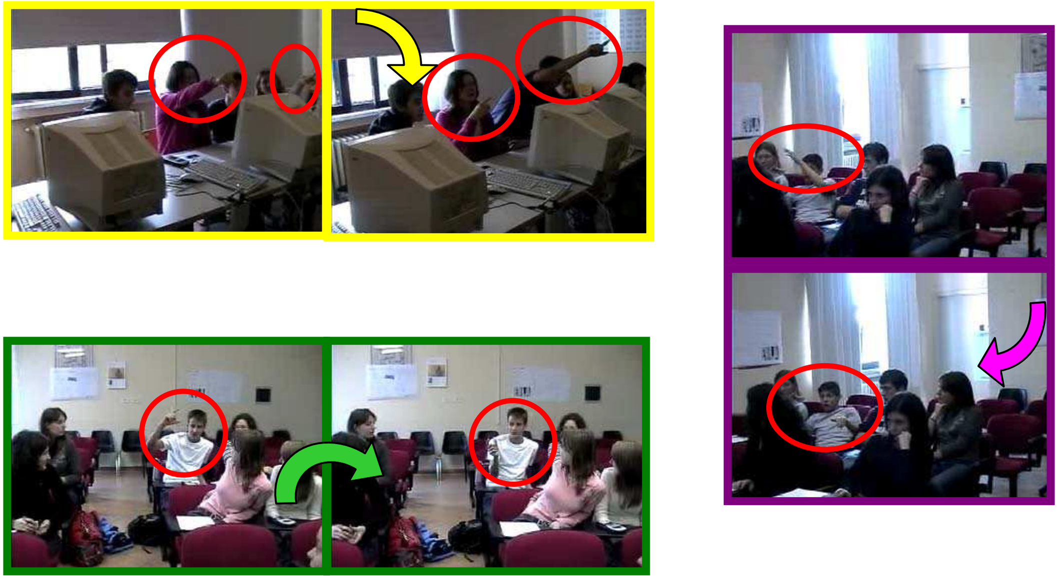

Figure 3: Examples of gestures during the activities

Our research questions are really numerous, because this new equipment is substantially different from the usual, made of computers or calculators used by singles or groups. What changes is the dynamic of the class activity. In a usual laboratory setting, in fact, each group of students follows the screen of his calculator and does not have information of what is happening in the other groups. The teacher herself, if wants to have information of the processing made by the students on their screens, has to pass from one group to the other and to look, supervise, discuss with the single group. But often the rest of the class looses this discussion. In our setting with TI-Navigator things are happening differently from both the points of view: of the students and of the teacher. In fact, each group may follow his job but simultaneously also other groups' job, looking at the big screen where all the jobs are projected. And the teacher herself may remain in a central position, following every job on the big screen, discussing with a single group or guiding a class discussion where everyone can take part, because information is shared, both through Activity Center and Capture Screen.

The general questions of our research are the following:

1. What are the potentialities and the constraints of a new environment aimed at facilitating connectivity among students?
2. Is there an "added value" to the teaching/learning processes with respect to a traditional technological equipment?

And in particular: which educational profit can be offered to students by a shared display? What are the effects on the learning process? What is the kind of mediation of this equipment in the learning processes? How is teacher's role in the classroom with this equipment? What are the new difficulties and opportunities for the teacher in designing and managing mathematical activities? How do the rhythms change with respect to a more traditional activity (with or without technology)?

EXAMPLE OF AN ACTIVITY

In Figure 4 one of the last activities of the teaching experiments (Hershkowitz & Kieran, 2001; Robutti et al., 2007):

You have three families of rectangles (A, B and C) whose sides are growing up according to different rules, as depicted in figure (Figure 4). Represent the areas of the rectangles of the three families as functions. Which family has an area greater than the others and when?

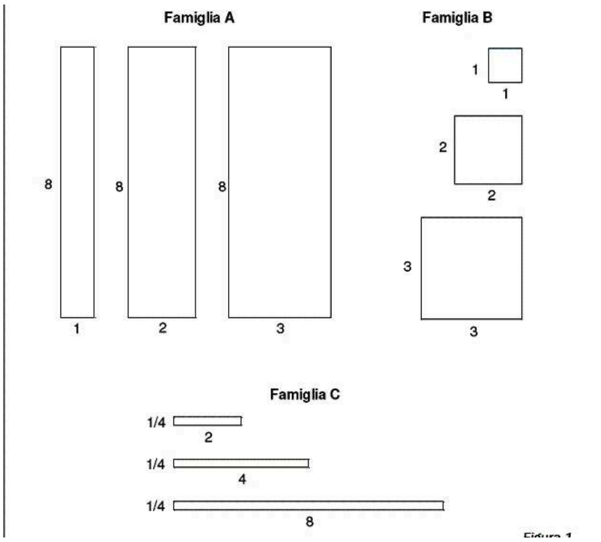

Figure 4: Activity of families of rectangles

Students here have to discuss in pairs in order to find the function modelling each area, to write it in the function environment of the calculators and to represent them on the same Cartesian graph. Then they have to compare the kinds of growing of the functions. The three functions modelling the three areas of the families are respectively linear, quadratic and exponential. While the first two are relatively simple to be found, the third presents some difficulties, because the students do not know with the exponential function. The future teachers solve the same activities of the secondary school students and comment them from a didactical point of view.

In the following we may observe the discussion in a group, referring to Family C (O means the observer):

> ***O.*:** What can you multiply by two?
> ***A.*:** The area.
> ***O.*:** What area?
> ***A.*:** The area of the two rectangles, …is always multiplied by two.
> ***O.*:** In which sense? Can you explain me?
> ***A.*:** Because … well, one side remains constant, then the height remains constant, while the basis doubles at every step, so also the area, consequently, doubles at every step.
> ***O.*:** Then to find the 184th area, how do you do?
> **C.:** You have always to find the previous one … Every time.

The difficulty in finding the right model for expressing the area of the third family as a function of the step is evident in the previous protocol, where emerges the awareness of the recursive law, but not more. Later, during the class discussion in front of the projection of all the screens of the groups, that function will appear as the “a power of 2”.

CONCLUSIONS AND OPEN PROBLEMS

Our teaching experiments have been planned taking into account the notion of mathematics laboratory from one side, and the recent results of the research in the semiotic-cultural approaches depicted above. What I found is the possibility of describing the students’ processes in constructing mathematical meanings through the model of the *APC Space* and the approaches of *humans-with-media* and *to-be-in-mathematics*. These approaches outline the importance of the students’ social activity, to let the individual subjective meanings move toward a shared and institutional one, grounded on historical/cultural bases, and the significance of all the signs used, can analyse from a semiotic standpoint, to trace an evolution in this movement.

Trying to answer to the research questions listed above, I present the following suggestions:

1. the potentialities offered by TI-Navigator may really facilitate connectivity among students, reinforce their right ideas and give support in correcting wrong results. The mediation given by this technology is particularly evident in the use and sharing of signs (e.g. equations, representations, coordinates, functions), and their meanings in different representations (numerical, graphical, symbolic). From a semiotic-cultural approach, this new technology

offers something more than a traditional one (made of calculators without TI-Navigators), because deeply influences the introduction of signs coming from every single student, but also from the sharing of the data processing, in a continuous exchange of information and feedback among humans and media.

2. It is interesting to depict the "added value" with respect calculators only:

- an added value to the students in their individual learning processes, because Activity Center is a mathematical environment where *being-in-mathematics* really together, receiving a feedback that can be positive and negative, and consequently reinforcing an idea or giving awareness of a mistake;
- an added value to the class in their collective learning processes, because Screen Capture is a sort of Big Brother of the calculator screens, where everyone can look at all the results of the other groups, comparing them and receiving a feedback that can be immediately used;
- an added value to the teacher in the teaching process, because Activity Center is an instrument for doing together, as *humans-with-media*, discussing about the activity on real time, looking at the single contribution on the common screen; and because Screen Capture is an instrument for comparing different solutions, or because it is and instrument for saving and reproducing the history of the different solution processes of each group.

For these reasons, TI-Navigator can be considered a new technology compared with the usual one. The added value described above could also be a "lost value", because, if it is true that the teacher has more time and energy to dedicate to the students (supporting them through discussions because the software gives an immediate and fast feedback, causing a rapid changes of opinion, or correction of mistakes, or comparison of more solutions), it could also happen that the teacher needs a help in the classroom for managing the technology and simultaneously guiding a discussion.

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AKNOWLEDGEMENTS. Research program supported by MIUR and by the Università di Torino and the Università di Modena e Reggio Emilia (PRIN Contract n. 2005019721) and by Texas Instruments.