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Learning with new technology

- some aspects of a history of Didactics of Mathematics

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1 Mathematics and its Learning: a history of tools?

The history of mathematics is full of examples showing that the availability of certain tools definitely influences, if not decides the course of the conceptual development of mathematics as a scientific discipline. As sort of an illustration, I just mention the Arabic notation of numbers in a positional system for Arithmetic and the use of ruler and (Euclidean) compass for Geometry (with the classical problems of trisection of angles and doubling a cube as prototypic tasks).

With the history and development of the scientific discipline being most influential for the teaching and learning of a related subject (for a certain exaggeration see the concept of scholarly knowledge introduced by Chevallard), the use and development of tools is also most important for the teaching and learning of a certain subject. With the "Geo-Dreieck" introduced in German Geometry teaching after World-War II, it took some time to accept that this is a tool to easily draw parallels without the tedious ruler&compass procedure from Euclid. The conceptually correct and easy trisection of angles using the "Geo-Dreieck" never became a topic in secondary Geometry teaching (and learning). It would be an interesting exercise to re-write the history of Geometry teaching and learning as a history of tools available.

The most prominent, recent and modern tool nowadays is the co-called new technology, which I prefer to discuss under the catchwords of "computer, software and communication technology". With the computer fundamentally being a mathematical machine, it does not come as a surprise that the new technology is also influential, discussed and even researched for mathematics and its teaching/learning. In some cases, it was brought forward that one can reduce the teaching and learning of mathematics with the advent of new technology, because computers and appropriate software can take over most of the mathematical tasks a person is bound to learn in general education at least in industrialised countries.

2 A case in learning with a new tool: two ICMI- studies

The International Commission on Mathematical Instruction (ICMI) after its reconstruction after World-War II soon realised the importance of the technological development of computers and mathematical software and its impact on teaching. It was the first ICMI-study, which took up the issue under the title of "The Influence of Computers and Informatics on Mathematics and its Teaching" (for the discussion document see Churchhouse et al. 1984). The issue was taken up again in the study no. 17 entitled "Digital technologies and mathematics teaching and learning: Rethinking the terrain" (short title: "Technology Revisited", for the discussion document see http://www.math.msu.edu/~mathsinc/ICMI). In order to learn about the way ICMI treated this topic, I will now take a closer at these two ICMI-studies.

2.1 ICMI-study no. 1

In the discussion document, the ICMI-study no.1 was clearly structured around three questions: "1. How do computers and informatics influence mathematical ideas, values and the advancement of mathematical science ? 2. How can new curricula be designed to meet the needs and possibilities? 3. How can the use of computers help the teaching of mathematics?" (see Churchhouse et al. 1984, p. 161).

Two features of this structure stand out: Question 1 is a clear indication for the importance of the discipline mathematics for didactics of mathematics (or: mathematics education research, I do not want to enter into this ongoing terminological debate) at this time. The development inside the scientific discipline mathematics were so important at that time, that most of the plenary sessions and one of the three working groups in the study conference were devoted to questions clearly linked to developments inside the discipline or defined by mathematical topics (like: the four-colour-theorem, discrete and continuous mathematics – also linked to the curriculum question no.2, computer algebra and logic; for a more detailed account see the rather comprehensive report on the study conference by Biehler et al. 1986). The underlying problem in these discussions was how to cope inside the discipline and within its teaching with the changing relation between experiments / explorations and proof. Is a 'brute force' approach like the computer based proof of the four colour theorem a mathematical proof – even if it can never be completely controlled by an individual mathematician? Are activities in applied mathematics and statistics with a focus on 'How can it be (best) done?' an acceptable and prototypic piece of mathematics – maybe even to be mirrored in a teaching process more oriented to exploration than to formal proof?

Question 2 somehow stands for the second important feature of the conference: In line with the then widespread focus on curricular issues, question 2 concentrates on curriculum design as a consequence of the advent of the computer and software and was subdivided into 10 questions on specific curricular issues (see Churchhouse et al. 1984, 166-168). During the conference, much time was used to discuss the inclusion of computer related content material into the teaching of mathematics and "problems of implementation" of curricula (the title of one plenary session in the conference). In this respect, the conference was an excellent example of "the era of the curriculum" (as Sfard 2005, section 6, has put it in her plenary presentation at the ICME-10-conference in Copenhagen).

Compared to these more or less detailed questions 1 and 2, question 3 on the teaching of mathematics with the help of computers is rather vague. In section 3 (loc.cit, 168-172), the discussion document comes up with 5 subsections ("general effects of computers", "objectives and modes of operation", "treatment of particular areas" as the longest subsection, "assessment and recording", "training of teachers"). Curricular issues are given special attention again, the user / learner is absent at least from the headlines, while the teacher is given at least some attention as a person to be trained for appropriate use of new technology. ICMI-study no. 1 had not yet entered the "era of the learner" or the "era of the teacher" (as Sfard, loc.cit. has put it).

On the other hand, it is worth mentioning that the curriculum area is the one where success was not really available after the study. In 1992, UNESCO edited a revised edition of the conference proceedings (Cornu&Ralston 1992), where Burkhardt&Fraser give a quite deceiving report on consequences for mathematics curricula and mathematics teaching: "The lack of progress in Domain C (the teaching and learning of mathematics, insert RS) is the major mismatch between intentions and outcomes over the last seven years. It is notable that even the use of simple calculators has not been fully integrated into the curriculum in any country in a way that realises their known potential for enhancing mathematical performance (even on traditional skills!). The reasons are less clear than is sometimes thought by those who ascribe it simply to teacher inertia and/or parental opposition" (Burkhardt&Fraser, p. 6).

Further down in the document, Burkhardt&Fraser suggest that the "work on large scale implementation should become a priority over the next decade … However, the difficulty of achieving large scale change of any kind is often underrated, or at least neglected. It clearly needs empirical study of the dynamics of change in the education system as a whole, with all the factors this brings in. We already know far more about the benefits that could flow from the use of technology … than is realised in practice. Without attention to Domain C, this mismatch will simply get worse" (loc.cit., p. 8). Even if official curricula prescribe the use of the new technology, implementation in the classroom seems far from obvious.

2.2 ICMI-study no. 17

More than 20 years later, the discussion document for the ICMI-study no.17 (see "Digital technologies …" 2006) shows a different structure. After explicitly linking the new effort to the first ICMI-study, it tries a new approach: "While we noted the first Study was largely focused on modelling mathematics, more recently work has focussed much more generally on the multitude of ways technology can shape teaching and learning mathematics, while reciprocally being shaped by its use. ... New robust paradigms for thinking about tool use in the context of mathematics education are beginning to emerge and ICMI Study 17 aims to take a further step forward in this direction" (see discussion document, p. 5). In order to follow this brief, the discussion document identified seven "themes" to "provide complementary perspectives on the use of digital technologies in mathematics teaching and learning", which were the following: "Mathematics and mathematical practices; Learning and assessing mathematics with and through digital technologies; Teachers and teaching; Designing learning environments and curricula; Implementation in curricula and in classrooms; Access, equity and socio-cultural issues; Connected and networked classrooms" (from the discussion document, p. 6).

From this citation, two changes are obvious: Developments inside the discipline mathematics are less important than in the ICMI-study no. 1, whereas ICMI-study no. 17 starts from the assumption that there are theoretical approaches and paradigms, which help with a detailed analysis of the teaching process and teachers. I deliberately have played down here the learning aspect, because Sfard's 'era of the learner' is somehow less present in the discussion document of ICMI-study 17. The "student" is mentioned only twice in the document, with the first instance talking about problems of student assessment (discussion document, p. 8) and asking for the potential of the new technology for "students with special needs" (discussion document, p. 12). The word "learner" does not show up in the Discussion Document.

According to the discussion document, the seven themes should be tackled within five "approaches" (discussion document, p. 13f), which somehow confirm the description given so far: With the approaches "impact on mathematics"", "roles of different technologies", "contribution to learning mathematics", "the role of the teacher" and "theoretical frameworks", the idea of the existence and importance of theoretical frameworks is confirmed, the teacher is clearly identified, while the learner shows up only in the respective activity. The four instances of "learn" bring us to the two places talking about "students" mentioned above. The other two places talk about "learning environments" and learning from teachers. The "contribution to learning mathematics … could be addressed in terms of cognition or affect, with regard to mathematical fields, activities and contexts at different school levels, or in contexts in and out of school" (discussion document, p. 13).

If ones looks into the plenary activities of the study conference held in Hanoi in December 2006, the general tendencies described above are confirmed: The "Keynotes" at the beginning and the end of the conference reflected on the difficulties to implement the use of new technologies in the classrooms all over the world - with the suggestion to avoid the mistakes of the introduction of the "New Math" reform and a technical solution -the "100-dollar-laptop"- offered by Seymour Papert and three "perspectives … to reflect on the potential and limitation of what has been achieved so far for thinking about the future: the theoretical perspective, the teacher perspective, the institutional and curricular perspective" (from the abstract of Michèle Artigue's keynote). Some of the papers reacting to the discussion document were presented in "parallel sessions" (in most cases 4 parallel presentations with 25 minutes for each individual paper), so one can have only a rather global idea about these contributions. Judging from the titles of these presentations (to be found in the info on the study conference on the respective website), two areas of mathematical contents were especially analysed (namely Algebra and more often: Geometry), the question of a sustainable development is still open, but there are some theoretical perspectives on the use of computers, software and communications technologies, which are now available in a way that there is even an opportunity to start comparing them (see the "TELMA"-approach and its description in TELMA 2006, contribution "c54" of the CD of the study conference).

3 Conclusion

A comparison of the two ICMI-studies implies some important lessons to be learned:

(1) The development inside the discipline mathematics has become less important for the educational use of computers, software and communication technology. The relative autonomy of the educational system, of research in didactics of mathematics and of classroom practice creates uses and rejection of new technology, which is not fully controlled by developments inside the discipline mathematics.

(2) Problems of implementation of pieces of (educational) software, learning environments and use of communication technology are far from being solved. As was already mentioned in the intermediate report by Burkhardt&Fraser, the discrepancy between intentions, suggestions and potentials to use new technology and the actual use of it is still wide. The 'royal road' to the educational use of computers, software and communication technology within mathematics teaching and learning is still to be discovered – if it ever exists.

(3) Even if this aspect was somehow neglected in the two ICMI-studies, it seems obvious that a mere analysis of the artefacts (computers, software, communication technology) is not sufficient to make this technology used in teaching and learning mathematics. "User studies" (often referred to in informatics) are an unavoidable presupposition for the implementation of new technology in the mathematics classroom. To state it in the terminology of one of the theoretical frameworks widely used in didactics of mathematics (see Rabardel 1995): The analysis of the artefact is an insufficient presupposition to introduce and understand its use. Only an analysis of the instrument, i.e. the interaction of the artefact and the utilisation schemes of its users (teachers and students), the analysis of its 'instrumental genesis' will help the implementation of computers, software and communication technology in the mathematics classroom.

(4) For individual pieces of mathematical domains, a wide range of ideas, artefacts and suggestions for their use is available. This is especially true for Geometry (for a research overview see Laborde et al. 2006). Software and suggestions for using it is also available and

well analysed in Algebra (see for instance the overview in Artigue 2002 or more recently Ferrara et al. 2006).

(5) Apart from lots of most challenging, well-designed software and suggestions to use new technology in the classroom, the most important innovation seems to be for research: different theoretical frameworks have been developed and used within research on (the use of) new technology in teaching and learning mathematics. The time is even ripe to start with comparing these frameworks in order to know more about strengths and weaknesses of the frameworks, not only of (the use of) computers, software and communication technology in teaching and learning mathematics.

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