ICT, new insights on old problems

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In our research works (Maschietto, 2004; Trouche, 2004a), we used to look at the way in which *artefacts* become, for teachers as well as for students, *instruments* of their mathematical work. The questions of tools and technologies in mathematical education (Barzel and al., 2006) are now widely considered in our communities. The 100th anniversary of the creation of ICMI is the occasion to go back to history, to think on the possible contribution to mathematics learning of old (but not dead) artefacts to mathematics learning and to the instruments's geneses.

1. About tools: what could be learnt from the history of ICMI

1.1 From the beginning (1908 and before)

From the reading of "L'enseignement mathématique" (which will be the official journal of ICMI since its creation in 1908) we could distinguish, about tools integration, three main elements:

- Integrating tools in mathematics teaching appears as interesting both from a *practical* and *pedagogical* point of view:

> « Non seulement quelques-uns de ces dispositifs peuvent devenir d'un précieux secours d'un point de vue pratique, *au prix de quelques perfectionnements dont l'avenir se chargera sans doute*, mais, en outre, l'exposé des principes sur lesquels reposent ces moyens est souvent de nature à *frapper l'esprit*, à *fixer la mémoire*, et à *concentrer l'action de l'élève sur certaines théories qui deviennent ainsi plus visibles*, pour ainsi dire. Il y a là, *d'un point de vue pédagogique*, un ensemble de questions dont les professeurs auraient tort de se désintéresser » (Petrovitch, 1899).

- Common tools could be transformed in efficient mathematical tools:

> « Les papiers rayés et quadrillés sont d'un usage aussi courant que le papier blanc. Comment les algébristes n'ont-ils pas songé à s'en servir comme d'une espèce d'abaque pour abréger les multiplications et divisions algébriques en n'opérant que sur les coefficients ? » (Berdeillé, 1902).

- A strong link appears between the use of a *great diversity of tools* and an *experimental approach in mathematics teaching*, and it seems to be a controversial question. For example, in 1908, a very important paper, founding the program of ICMI, was published in "L'enseignement mathématique". We can read, in the second part of this program ("The modern tendencies of mathematics teaching"), in chapter 4 ("Teaching methods"), some traces of discussions among teachers in schools:

> « Bien des écoles ont consacré *de longues discussions* à la part que l'on doit attribuer aux considérations d'ordre pratique et expérimental:
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> - dans l'enseignement élémentaire, on peut mentionner, par exemple, le plissage du papier, le travail de plein air, l'usage des instruments simples de mesure, la géométrie d'observation, etc., le calcul pratique et approximatif (degré d'approximation, logarithme à un nombre varié de décimales, usage de la règle à calcul, etc.), la question général des graphiques en algèbre, l'usage plus répandu du papier quadrillé.
>
> - il a été question ces dernières années de *laboratoires mathématiques*. Qu'a-t-on fait dans ce sens et quels en sont les résultats ? Modèles mathématiques confectionnés par les élèves, le rôle des collections de modèles ».

1.2 The first ICMI study on computers in mathematics teaching (1985)

This study focuses on the *influence of computers and informatics* on mathematics and on its teaching. From the introductory document (Churchhouse and al., 1985), we can see a sort of continuity of the ideas of 1908:

- the influence of technology which allows better things to be done *more quickly*, and in different ways (the computer *as an aid* to the teaching and learning of mathematics);

- computers have suddenly greatly increased our possibilities for *observation* and *experimentation* in mathematics (p. 7).

There are also some major differences with the ideas of 1908:

- questions of teachers' training are addressed, but the integration of new tools in mathematics teaching appears quite natural (not controversial);

- questions of using and transforming simple tools, or combining old tools and new ones are not evoked (the study is limited to the curriculum and teaching at university and pre-university level).

1.3. The second ICMI Study concerning computers (2006)

Instead of computers, the study focuses on *digital technologies* (including handheld calculators, software and Internet). Its point is the necessity of *rethinking the terrain*.

As it was the case in 1985, new opportunities for experimentation emerge (the idea of "mathematics laboratory" appears again, as in 1908). "What kind of pre-service education and professional development programs are appropriate to prepare teachers to use technology in their mathematics classrooms": this is a question already addressed in 1985, the new point, in 2006, is the idea of *ongoing use*.

From the discussion document (Hoyles and Lagrange, 2006), we could distinguish some major differences with the previous study:

- all the learning levels are taken into account;

- the *impact* of technology is questioned, distinguishing between *actual* and *potential* use, focusing on access, equity and social-cultural issues;

- a dialectic understanding of technology *influence* appears (technology can *shape teaching and learning mathematics*, while reciprocally *being shaped* by its use);

- the technology is not considered as given, but the questions of *design* are addressed, as a central challenge;

- the teacher is no longer considered as alone in his/her classroom, but the ideas of communities (learners, teachers), network, collaboration strongly appear;

- lastly, the document claims for new theoretical approaches, well fitted to the complexity of new learning environments.

These theoretical needs appear also in (Hoyles and Noss, 2003; Lagrange and al., 2003). Among these new theoretical frameworks, the *instrumental approach* (Guin and al., 2005) stresses the importance of *geneses* of instruments and the constitution of *systems of instruments*. This approach considers that a genesis, from one artefact to an instrument, is made of two interrelated processes: an *instrumentation* process (the artefact shaping a user's activity) and an *instrumentalization* process (the artefact shaped by the users' activity). These elements are also taken into account in the theoretical framework developed by Bartolini Bussi and Mariotti (in press), where the authors discuss a *semiotic approach* to the learning and the teaching processes with artefacts from a Vygotskian perspective and analyse the notion of tool of semiotic mediation. The centennial of ICMI is a good occasion to approach these questions through an historical perspective.

2. Thinking on history lead to think on interest of “old” technology

In this part we intend to contribute to the discussion about the question “Do new technologies transform old ones or erase them?”, with the starting point that an *old* technology is not a *dead* technology. The place of history in the teaching and learning of mathematics is a relevant topic, not only for general debates and educational research projects, but also to deepen the question of technology integration. The ICMI Study *History in mathematics education* (Fauvel and van Maanen, 2000), as well as conferences and summer schools (the most recent is the *5th European Summer University on the history and epistemology in mathematics education*, in Prague at the end of July 2007, see http://class.pedf.cuni.cz/stehlikova/esu5/), attest this interest.

2.1 An example from the ICMI Study on history in mathematics education (2000)

In that ICMI Study, the introduction and the integration of history in mathematics education is analysed from different approaches. Among the several proposals for classroom implementations, in Chapter 10 (Nagaoka and al., 2000), some experiences involving non-standard media in connection with the history of mathematics to improve educational experiences and opportunities are analysed (van Maanen). These media include mechanical instruments (here, they are called ‘old technologies’) and new technologies. The use of internet and hypertexts is mainly considered in terms of accessibility to historical documents, which is a crucial question, if we consider that historical texts are not easily available in a paper form. In that chapter, the contributions of Bartolini Bussi (‘Ancient instruments in the modern classrooms’) and Isoda (‘Inquiring mathematics with history and software’) are interesting for our discussion because the proposed educational approaches consider the use of both two kinds of technology (old and new), with respect to research projects concerning only one kind of technology in the mathematics education literature. Among the discussed examples, both two papers deal with the ‘mathematical machines’ (instruments for geometry reconstructed on the basis of historical sources, in Modena; see http://www.mmlab.unimore.it), but they consider two different ways to use them.

Bartolini Bussi emphasises that “the computer is much more flexible than ancient instruments, yet the understanding of the underlying theoretical assumptions that make it possible to solve problems (approximately or rigorously?) is more difficult and hidden inside the black box”. Two main categories of activities are proposed: specific classroom activities and visiting the instruments (in reality or as a virtual visit). In the teaching experiments (in the text the author does not give more details, they are summarised in Bartolini Bussi & Maschietto, 2006), pupils are invited to explore physical instruments by the aide of worksheets and to begin a process of argumentation and proof with respect to the mathematics embedded in the used instruments. In the second kind of activities, different animations of those instruments also come into play (they are available on website).

Isoda considers the integration of traditional instruments and computers for mathematical inquiry in the classroom. In this case, the articulation between old and new technologies is based on the use of multiple representation tools. He presents a teaching experiment about the ellipse: from an historical picture of van Schooten’s ellipse-drawer to its animation by a DGS (Dynamic Geometry Software), through the construction of a copy of that instrument with LEGO (http://130.158.186.11/mathedu/forAll/kikou/lego/lego.html).

Old and new technologies are used in different ways by the two authors: Bartolini Bussi considers more the exploration of instruments (physical or virtual) already built, while Isoda fosters the construction of instruments (physical or virtual). They are supposed to involve students in different manners, at least with respect to the use of software. Indeed, the use of a DGS animation is other than the realisation of that animation. Even with a different focus, these contributions emphasise the importance of tactile experience in the construction of mathematical concepts. With specific tasks, the use of physical models fosters a semiotic activity, connected to the production of language, signs and gestures that recent studies show its importance in the process of mathematics conceptualisation (Arzarello and Edwards, 2005; Maschietto and Bartolini Bussi, 2005).

Following Bartolini Bussi, we take an example to explain what kind of articulation we are thinking about. Curve drawers permit to draw a curve as a trajectory of a point (represented by the point of a pencil) moving on the plane under certain constraints. Generally, a mathematical machine has physical features that influence its movement. For this reason, on one hand they can permit to trace only a part of a curve or make drawings inaccurate; on the other hand, they can rouse mental experiments to cross limitations and to generalise. DGS animations permit not only to solve some of the physical limitations, but also to make some elements of a chosen instrument vary (such as its parameters). The different technologies could be considered complementary as follows: they have some common elements, but each of them presents potentialities that should be deeply analysed and articulated each other.

2.2 An example from a mathematics laboratory

We consider now an example where we have tried to exploit the potentialities of two technologies in the context of the laboratory session at the Laboratory of Mathematical Machines (MMLab) of Modena (Maschietto, 2005). The MMLab is accessible to classes (mainly, secondary school level) and proposes three different topics for laboratory session: conic sections, geometrical transformations and perspective. Three stages form each session: an historical introduction to the topic by the laboratory animator, a group work for pupils and the presentation of the results found by each group. In these three stages, historical instruments and their animations (realised by CabriII Plus, Cabri 3D and Cinema4D, see http://www.mmlab.unimore.it) are used. Let us consider, for example, the conic sections topic. In the first stage, the presentation of the historical development of conic theories starts with the use of big size static cones with tightened threads (for Menecmo-Euclid and Apollonius's theories), then other models are considered. In this stage, after the presentation of a big size model, the vision and comment of its animation allows to show the limitations of the model: for instance, in Apollonius's theory, the movement of the plane cutting the cone. These animations are not only showed to pupils, but they are a means to recognize what pupils have already seen in the model. In the second stage, pupils are invited to form small groups, and they are givent a mathematical machine working on the plan and a worksheet. It is a work on physical models (little size) only. In the third stage, each group presents the studied mathematical machine to their fellow pupils. Here as well, the laboratory animator can use animations to complete pupils discourse and institutionalize mathematical contents. With respect to the two previous experiences, the animations are not used nor seen by the pupils themselves. Teachers can use a laboratory session to introduce the new mathematical content in the classroom or to delve into it.

The discussed examples allow approaching the question regarding the relationship between old and new technologies in terms of articulation between the old and the new. From our viewpoint, it entails the analysis of the potentialities (semiotic and cognitive) of a technology (alternatively, we can use the term 'artefact') compared to the other one and the design of a teaching project that takes into account this analysis. This articulation seems to be consistent with the idea of *system of instruments* (Guin and al., 2004). As van Maanen (in Nagaoka and al., 2000) highlights, "one of the main benefits of having a range of media resources available is that this enables the cognitive needs of a greater number of students to be met".

3. Thinking on history leads to think on instrumentalisation

At the beginning of the 20th century, we have seen that the integration of tools was often considered as a process of transforming common tools (§ 1.1 : « le papier rayé et quadrillé ») in order to use them as artefacts for doing or teaching mathematics. The development of sophisticated tools (DGS, handheld devices), dedicated to mathematics teaching, has often hidden this process of *instrumentalisation* through which an artefact is transformed, enriched by users, to integrate it in their own activity. This aspect of instrumental geneses is not addressed in the introducing text of our working group. Considering the history of instruments helps us to rediscover the importance of this process.

3.1 An example in « old » history

As the discussion document reports, "the development of perspective drawing in Europe by means of instruments used in artists studios, has laid the foundations for modern development of projective geometry". In the following, we intend to see some elements of the historical development of those instruments through the lens of the *instrumental genesis* (Maschietto, submitted).

In the 15th century, the first 'drawing machines' were constructed, also based on the knowledge accumulated through the use of different techniques to do sight measurements. They became popular, mainly because the application of the geometric rules underlying the realisation of a perspective drawing revealed to be difficult when the object to represent was complex. Globally, perspectographs are an example of integration of geometry, optics and exact instruments and also of integrating abstract reasoning and practical abilities. Several treatises concerned perspective and perspectographs. For instance, Dürer described four instruments in his treatise on geometry (*Underweysung der messung*, 1525). Among them, the door, presented for the first time here, "can be considered the main perspective device. It is the first device to translate all the parameters of the perspective constructions into mechanical pieces: the viewer's eye is a nail, the visual ray is a thread, and the canvas is a plane defined by the intersection of two threads within a loom" (Camerota, 2001).

An interesting treatise for this part is '*Le due regole della prospettiva pratica*' written by Barozzi and afterwards annotated by Danti (Barozzi and Danti, 1583). In particular, in the first annotation of Chapter 3 of that book, several perspectographs relating to Dürer's door are described. This presentation not only allows to discover (or recognise) some perspectographs, but also is an example of an instrumental genesis process. In his comments, Danti presented two kinds of information, related to both the *instrumentation* and *instrumentalisation* processes. The first kind addresses the reader or the user: the functioning of the instruments is explained, sometimes stressing the need of a great skill. The second kind concerns the instruments: from a perspectograph to another, Danti stressed some advantages and disadvantages that arise from the use of the chosen perspectograph, compared with the others already described. Some elements from the mathematical model of the visual pyramid were also presented. In such a way, the reader can see the transformation of the door, connected with practical needs.

For instance, Danti suggested some modification to improve Dürer's door, as the use of diottra, that substituted the thread hanging from the nail (visual ray) and permitted to "draw in perspective anything you want, as far as it can be". The Archbishop of Lerino's perspectograph, presented by Danti, represents a different version of the door: the two transversal threads, describing the virtual frame in the door, are substituted by two rods. This change solved the following problem: when "the radial thread touches the transversal threads, this can push them out of place and cause us to commit a not small error".

Danti's annotation constitutes an important cultural artefact, because it contains, on the one hand, elements for the construction of a personal utilisation scheme by the reader, and, on the other hand information about the perspectograph itself. Nevertheless, from our point of view, those descriptions imply a certain sensibility of the reader to problems and questions connected to the use of perspectograph. We can put forward the hypothesis that this sensibility derived from a real use of the perspectographs needed to take into account their limits.

3.2 From old to « recent » history

The instrumentalisation process can be considered at several levels:

- from an *institutional* point of view: as for the "papier rayé et quadrillé", some technologies (for example: the spreadsheets), widely used in society, have been recently imported in schools (that could be considered as a type of instrumentalisation), which implies a process of *computerized transposition* (Balacheff, 1994) studied in the case of spreadsheets by Haspekian (2005);
- from the *students'* point of view: following students' work (Trouche, 2004a) with calculators allows to see signs of instrumentalization processes, through different stages: a stage of discovery and selection of the relevant functions, a stage of personalization (one fits the artefact to one's hand) and a stage of transformation of the artefact, sometimes in directions unplanned by the designer: modification of the task bar, creation of keyboard shortcuts, storage of mathematical results, or game programs, automatic execution of some tasks (calculator builders' web sites or personal web sites of particularly active users often offer programs of functions, methods and ways of solving particular classes of equations etc.). The teacher has to choose how to react: s/he can say: a calculator is not made for this kind of use, so it is not allowed… Or s/he can say: how students' creativity could be integrated to develop a richer instrument? It depends on teacher's ideas on *designing* processes, whose importance is pointed out by the last ICMI study (Hoyles and Lagrange, 2006);
- from the *designer's* point of view: the designers themselves (it is certainly a new trend) try to incorporate the creativity of users in order to enrich their technological tools. It is not a *limited in time* process (designing a device, then testing it and implementing it), but an *ongoing* process. For example the French experiment aiming to integrate a new symbolic calculator (e-CoLab, 2007) shows the interest, for the designer as well for the teachers, to conceive the design process as a continuous and distributed process. This idea of *conception in use* (Béguin, 2005) is, to a certain extent, an answer to the complexity of tools: to turn an artefact into his/her own instrument, to integrate it in his/her own activity, it is necessary to incorporate to this instrument something of his/her own creativity;
- from the *teacher's* point of view. As in the beginning of ICMI (§1.1), the questions of integrating and articulating various artefacts are asked to teachers. The main problem appears to be not the computer, neither the software, but the *pedagogical resource* (Trouche, 2004b) helping teachers to put in place a given mathematical situation in a given technological environment. Recent experiments (Guin and al., 2007) put in evidence the interest to conceive such resources as *flexible* resources, allowing teachers to adjust them for their own needs, but also to integrate in their resources their own experience, to enrich them with their colleagues along a collaborative design process. It is a new point of view on instruments (instruments are here constituted by pedagogical resources), on instrumentalisation (here the process of enrichment of pedagogical resources along the usages by students and teachers) and on design process (to see more on teachers' side). It is a new manner to consider teachers' professional *documentation* (Gueudet and Trouche, to be published).

The considered examples show, in different ways, the contribution of old and new technologies in the construction of mathematical meanings, the complexity of systems of instruments, the interest of considering instrumental geneses. Ideas to be shared or discussed in Roma ICMI symposium. New insights on old problems, or old insights on new problems…

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