Is cognitive neuroscience relevant to mathematics education research?

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Introduction

Mathematics education is a complex research field. While the field has its own specific questions, its complexity allows other research fields to play a role in the answers. For example, since content is important in mathematics learning and teaching, mathematics itself – with its specific nature, structure, history and status in our society – is crucial for our research. René Thom underlines the importance of the philosophy of mathematics for mathematics teaching, even if this philosophy can also be an individual one: "In fact, whether one wishes it or not, all mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics" (1973, p. 204). If instead of the philosophy of mathematics we speak of subjective theories or beliefs about the nature of mathematics, then Thom's description applies aptly to the research field of mathematics education today.

A second example is the traditional debate in philosophy regarding the nature of mathematical objects and their relationship to the symbols that represent them. Semiotics helps in the understanding of this. The status of graphical representations in mathematics learning can be discerned in these traditional philosophical investigations.

Learning is a central research field in psychology and so, naturally, discoveries in psychology have been used to explain phenomena in mathematics learning, as seen in the fact that PME was the first subgroup of ICMI.

But learning cannot be viewed solely from the perspective of the individual learner. Learning takes place in a social and cultural environment; consequently findings from social psychology are also important in understanding the learning process. Furthermore, learning – particularly mathematics learning – takes place in a specialized organisation (a school); and school learning always occurs together with teaching. Consequently, advances in pedagogy are important to mathematics education research, too.

Research into factors that influence learning has been broadening in recent decades: learning is being viewed as an increasingly complex process. Besides learning as a cognitive and social process, the importance of affect, emotions and motivation is being more widely recognized, and indeed ought to be considered if we want to understand the learning process.

With all of these concepts from other fields being used in mathematics education research, a challenging question arises. What are the essential characteristics of mathematics learning? What distinguishes mathematics learning from the learning other subjects? According to Niss (2006), the two most significant outcomes of recent research are: 1) the distinction between concept definition and concept image; and, 2) recognition of the processes involved in reification, encapsulation and complementarity.

In conclusion, the research field of mathematics education already uses concepts from many other fields. It is natural to ask: Are any other research fields relevant and necessary? In the following I will argue that neuroscience can help us better understand some phenomena of interest in mathematics education.

Neuroscience

I ought to say from the outset that my familiarity with neuroscience is one of an interested outsider and not at all of an expert in the field. My knowledge of neuroscientific results is strongly influenced by books written by neuroscientists aimed at a broad audience, which I have augmented through studying several scientific papers.

Cognitive neuroscience extends from biology to cognitive psychology, so clearly neuroscientific research takes place at a number of different levels. Of interest at the cellular level is the ongoing study of neurons. Researchers have been studying molecular biological and electrochemical processes within a single cell, particularly the synthesis of proteins, the action potential and signalling. One focus is on the interaction between neurons; i.e. synaptic transmission.

Currently, the central concern of cognitive neuroscience is the internal representation of mental events. Electrophysiological methods have made studying perception and movement possible. Also, complex cognitive processes such as attention and decision making have been shown to be correlated with activity patterns of individual groups of cells in certain regions of the brain. The neural basis of cognition starts with local operations in the brain. Indeed, studies of the consequences of lesions in the cortex have led to the knowledge that the cognitive system consists of many independently acting systems that process information. Functional brain imaging methods (PET, MRI, fMRI, EEG and MEG) have been used in the investigation of changes in the activity of neuronal systems in connection with mental processes, as well as in observing the brain systems activated in particular situations. Moreover, computers have been used to simulate the activity of neuronal populations and to study neuronal networks.

Let us consider these developments from the perspective of mathematics learning. Learning in all its forms entails that an individual has acquired knowledge of the world. This knowledge can influence the individual's behaviour. Learning is closely connected with memory. Memory means the ability of an organism to store knowledge and to retrieve this knowledge when necessary. Research in recent decades has led to the idea that various kinds of memory exist with various functions and various storage and retrieval conditions. Cognitive neuroscience today distinguishes between two basic forms of memory: the declarative or explicit memory, and the nondeclarative or implicit memory. Declarative memory is accompanied by consciousness, hence its content can be reported, whereas the content of nondeclarative memory usually cannot be. Furthermore, the two forms of memory depend on the activation of different centres in the brain. Both are subdivided into memory subtypes: the declarative memory is comprised of the episodic and semantic memory, respectively; and the nondeclarative memory is comprised of memory for skills, habits, priming, classical conditioning and nonassociative learning, respectively. However, in addition to this, emotional learning is implicit learning based on processes within the amygdala. It is important to bear in mind the neuroscientific fact that if a person has lost the ability to learn with the declarative memory, it does not necessarily mean that he or she is unable to learn with the nondeclarative memory.

For the processes of thinking and acting, the concept of working memory is important. If we consider cognitive processes such as conversing with others, thinking about a problem, mentally calculating something or actually solving a problem, we can identify subprocesses that are necessary for the success of the entire process. First, the relevant information must be identified and at least temporarily stored. Then this information must be open to manipulation, to interpretation in the light of knowledge or experiences stored in the long-term memory. The overall result of these processes should be suitable for communication in verbal or written form and for storage in the long-term memory. This means that a cognitive process requires brain systems that allow short-term storage and manipulation of (audio or visual) information; the systems are somehow connected with the long-term memory, and they are able to communicate results. All this requires overall control and monitoring.

The multi-component-model of the working memory is often used today in the study and analysis of cognitive processes. Introduced by Baddely (2003), it consists of two storage systems – the phonological loop and the visuospatial sketchpad – a central executive system for overall control and execution, and the episodic buffer. All the working memory components have a restricted capacity.

Finally, note that we ought to keep in mind that attention is a crucial prerequisite for all conscious processes, therefore it is for cognitive processes, too.

Neuroscience and mathematics learning

In this section, I will sketch some problems in mathematics learning onto which neuroscience could shed some light. This list is far from complete.

Dyscalculia: Everybody has at its disposal an innate ability called “number sense”. This allows them to analyze the external environment in terms of quantitative characteristics, through which they are able to distinguish small numbers and recognize a change in a small number of things (Dehaene, 1997). The number sense is a system for initial approximation: its output is usually not sharp, although it approximates the sharp solution. Certain regions of the inferior parietal cortex appear to be important for the number sense.

The development of symbolic number systems was an important step for humans. These systems allow us to overcome the limitations of the innate number sense and to develop “cultural mathematics”. Children have to learn number words, use these words to count things and to do calculations. But some children suffer from developmental dyscalculia, a seemingly insurmountable deficit in arithmetic acquisition. Neuroscientific studies have found hints of a physiological reason for dyscalculia: it could be due to an abnormality in the right parietal cortex (Cohen Kadosh et. al., 2007). Such discoveries could help us distinguish between different forms of dyscalculia, for instance between the physiological and developmental forms of it.

“Learning without understanding”: Mathematics teachers (at least in Austria) often say that weak learners are unable to understand mathematics. This is in accordance with common adults’ testimony of their own mathematics learning experience at school (Jungwirth et.al., 1995). Many adults reported that they always received bad marks in mathematics and were unable to understand mathematics at all. Teachers and learners often agreed that the only way to learn mathematics and pass tests was to rote learn rules and algorithms, and to practice applying these to mathematical exercises. Rote learning is not an uncommon strategy when teaching weak learners but it contradicts the view that learning – and especially mathematics learning – necessitates understanding. Teachers complained also that the knowledge that students acquired in such a learning process was often very inflexible. For instance, even

when just the signs of the variables are changed, the learner cannot solve the exercise anymore.
The effects of rote learning could be illuminated by a neuroscientific concept that is usually used to explain the phenomenon of priming – namely, the concept of a perceptual representation system (PRS) (Schacter, 1996). The PRS allows us to identify everyday objects and well-known words on printed pages (note that a PRS exists for each of the other senses, too). The PRS is specialized to process the form and structure of words and objects, while "knowing" nothing about the meaning of the words and the use of the objects. Furthermore we know from the characteristics of the PRS that the retrieval process is very strongly dependent on perceptual properties. If students use their PRS to handle mathematical tasks, we can understand why changing the symbols used to represent variables, or altering the word order of word problems, leads to an inability to solve the "new" task.

Errors in affective situations: Mathematics education research on the effect of affect in mathematics learning is concerned with long-term processes and cognitive and affective representations that determine the dynamics of learning processes; nevertheless, we need more insight into the effect of emotions during problem solving in order to explain the emergence of errors, particularly simple slip-ups. The concept of working memory can help us understand these processes. As described above, all the components of the working memory – the storage systems, the episodic buffer, as well as the central executive – are restricted in their capacity; moreover, one must bear in mind that attention is a crucial prerequisite for all conscious processes, and therefore for cognitive processes, too. In affective situations we are aware of our feelings, and therefore these feelings are manifested in the working memory and consume memory capacity. Since the working memory has limited capacity, we can imagine that there might not be enough capacity for the facts that are necessary to support the cognitive process; and that attention during the cognitive solution process is directed more towards emotions and less towards the cognitive and heuristic process components. It is understandable that in such a situation, the number of errors, particularly slip-ups, increases.

Affective categories: Research has led to the affective categories of "emotions", "attitudes", "beliefs", and "values/ethics/morals", which can be used to comprehend a complex reality. Quantitative research methods reveal stable and less intense categories, while qualitative methods are able to grasp quickly-changing and very intense reactions. Nevertheless, it seems that our research methods cannot establish a distinction between these respective sets of categories. Consequently no commonly shared definitions exist for them. Moreover, the qualities "stability" and "intensity" consolidate the description of the categories but alone are not sufficient to solve the problem of distinguishing between them. Neuroscience distinguishes between different brain systems that fulfil specific functions. So different systems exist for cognitive processes and for emotional processes. These systems are located in different parts of the brain and have their own memory system with their own specific features. The emotion system has strong connections to the body system, and an activated emotion system can lead to bodily reactions. Furthermore, although the emotion system works unconsciously, we are nevertheless able to recognize some end results of an activated emotion system – for instance, what we call "feeling".

Regarding the categories of "beliefs", "attitudes" and "values", the research methods used to investigate them are based on conscious remembrance, so the methods are controlled by the cognition even though we can see emotional reactions in interviews. However, when we use qualitative methods to observe problem-solving situations, we can see the effect of the emotion system, as well as reactions that are not completely under the control of the cognition. A similar pattern is observed when we compare values asserted in interviews to values that we can discern in situations where the emotion system is activated, which often occur in teaching situations.

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