Dynamical Methods in Spectral Theory of Self-Similar Groups and Graphs
2-6 March 2026, G.IST Geneva
Abstracts

Mini-Courses:

Laurent Bartholdi (Geneva, ENS-Lyon).

Title: *Bounded semigroups and post-critically finite fractals.*

Abstract: I will relate, in this 2-hour minicourse, a variety of topics: in fractal geometry, the "post-critically finite fractals" of Kigami; in group theory, the bounded self-similar groups of Bondarenko and Nekrashevych, and their extension to inverse semigroups; and in graph theory, a family of recursively-defined graphs that are both the adjacency graphs of tiles in the fractals as well as Schreier graphs of the action. I will explain decidability results on the logic of these graphs.

Nguyen-Bac Dang (Paris-Saclay University).

Title: *Iterating rational maps and spectral problems.*

Abstract: This mini-course will be focused on specific well-known examples studied by Bartholdi-Grigorchuk and Grigorchuk and Sunik for which the spectrum of a self-similar group was successfully computed. The common denominator of their computation relies a close relationship between a rational map F and the spectrum.

Roughly, the spectrum is the intersection with a horizontal line in $\mathbb{C}^2$ of the preimage of a specific line under F^n.

- In the first lecture, I will introduce the basic spectral problems as well as give the general roadmap that one follows to tackle these problems.
- In the second lecture, I will present the main invariants related to general rational mappings and how these relate to our spectra.
- In the third lecture, I will present some further properties and techniques used in holomorphic dynamics to tackle these problems.

Volodymyr Nekrashevych (Texas A&M University).

Title: *Self-similar groups and operator algebras.*

Abstract: The minicourse will be a survey of results on C*-algebras related to self-similar groups and dynamical systems. Self-similarity of the group naturally defines a homomorphism from the group algebra to the algebra of matrices over the group algebra. This homomorphism is the matrix recursion used in the computations of spectra of Schreier graphs. A natural algebra associated with it is the direct limit of the iterations of the wreath recursion. This C*-algebra can also be defined as the Cuntz-Pimsner algebra of a natural Hilbert bimodule. If the self-similar group is contracting, then there is also a natural Cuntz-Pimsner algebra of the limit dynamical system. There is an interesting duality between these two constructions. I will describe one of the interpretations of this duality: a spectral triple.

Rongwei Yang (University at Albany).

Lecture 1: Linear algebra in several variables

Lecture 2: Projective joint spectrum: basic properties and examples

Lecture 3: The operator theory of self-similar groups: a progress report.

Abstract: This series of lectures presents a spectral theory for non-commuting operators. In the finite-dimensional case, this constitutes a multivariable extension of linear algebra, generalizing classical concepts such as eigenvalues, characteristic polynomials, and the Cayley-Hamilton theorem. This theory has found meaningful applications across several mathematical fields. In the infinite-dimensional setting, we focus on the projective spectrum of linear operators. In addition to reviewing fundamental properties and illustrative examples, I will provide a progress report on an application of this spectral theory to the study of self-similar groups.

Research Talks:

Mikhail Hlushchanka (University of Amsterdam).

Title: *The independence polynomial for recursive graph sequences: the dynamical perspective.*

Abstract: The distribution of zeros of partition functions on graphs is intimately related to the analyticity of physical quantities and their phase transitions. The independence polynomial of a finite graph is the generating function for the numbers of independent sets (i.e., subsets of pairwise non-adjacent vertices) of each size. It naturally originates in statistical physics as the partition function of the hard-core model for gases, where each particle occupies an exclusive region of space.

It is known that the roots of the independence polynomial are dense outside a neighborhood of the origin for the family of bounded-degree graphs. However, the overall structure of this zero locus, as well as the corresponding zero locus for concrete sequences of graphs, may be quite intricate. Jointly with Han Peters (University of Amsterdam), we develop a unified dynamical framework for the analysis of these zero sets for recursive graph sequences. In the talk, I will report on our results concerning their structure, with particular emphasis on (the absence of) phase transitions.

Anders Karlsson (University of Geneva).

Title: *Spectral zeta functions of groups and their special values*

Abstract: Special values of zeta functions is a rich topic that basically begins with Euler solving the Basel problem. We view the zetas and Dirichlet L-functions as coming from spectra of groups, graphs and Riemannian manifolds. We approach the standard functions via increasingly fine discretizations. Concerning classical special values, the discrete and continuous have a surprisingly close connection, and in this way we find new identities, for example between special values of Dirichlet L-functions. As a simplest case, Euler's values can be recovered from counting spanning forests in cyclic graphs. This topic touches a wide span of notions such as spanning trees, volumes, Verlinde sums, Gindikin-Karpelevich formulas, the Selberg zeta function and Eisenstein series. Based on joint works with Friedli, Pallich, Jorgenson, Smajlović and Müller.

Kamila Kashaeva (University of Geneva).

Title: *Recursive coverings, spectrum, and heat kernels of some infinite Cayley graphs.*

Abstract: Abstract: The heat kernel of a graph encodes information about its geometry and the spectrum of the Laplacian. F. Chung and S.-T. Yau used coverings of weighted graphs to relate their heat kernels. Further developing their method, we apply it to Cayley graphs of free products of finite groups of fixed order. In an earlier joint work with A. Karlsson, the method was applied to a Cayley graph of $\mathrm{PSL}_2 Z$. This yields explicit formulas for the heat kernel and a description of the spectrum, which may consist of intervals together with isolated eigenvalues.

Antti Knowles (University of Geneva).

Title: *Localization and delocalization in random graphs.*

Abstract: A disordered quantum system is mathematically described by a large Hermitian random matrix. One of the most remarkable phenomena expected to occur in such systems is a localization-delocalization transition for the eigenvectors. Originally proposed in the 1950s to model conduction in semiconductors with random impurities, the phenomenon is now recognized as a general feature of wave transport in disordered media, and is one of the most influential ideas in modern condensed matter physics. A simple and natural model of such a system is given by the adjacency matrix of a random graph. In this talk, I review recent results on the localization and delocalization for the Erdös-Renyi model of random graphs. In the first part of the talk, I explain the emergence of fully localized and fully delocalized phases, which are separated by a mobility edge. In the second part of the talk, I explain how optimal delocalization bounds can be obtained using a new dynamical Bernoulli flow method. Based on joint work with Johannes Alt, Raphael Ducatez, and Joscha Henheik.

Aleksey Kostenko (University of Ljubljana & TU Graz).

Title: *Laplacians on infinite graphs.*

Abstract: There are two different notions of a Laplacian operator associated with graphs: discrete graph Laplacians and continuous Laplacians on metric graphs (widely known as quantum graphs). One of our main messages is that these two settings should be regarded as complementary (rather than opposite) and exactly their interplay leads to important further insight on both sides. Based on joint work with Noema Nicolussi.

Han Peters (University of Amsterdam).

Title: *Phase transitions for the independence polynomial.*

Abstract: In this talk we will review seemingly unrelated notions from different scientific fields such as statistical physics (phase transitions), theoretical computer science (non-existence of efficient approximation algorithms), graph theory (slow decay of correlations), and dynamical systems (active parameters, bifurcations). I will argue that all these notions are not only related but occur for exactly the same parameters. The discussion will be in the context of the independence polynomial.

Luke Rogers (University of Connecticut).

Title: *Spectrum and eigenfunctions of Schreier graphs of the basilica group.*

Abstract: We obtain spectral information for the Schreier graphs of the basilica group and some related graphs via dynamics on the characteristic polynomials. The argument involves an analysis of the structure and multiplicity of the eigenfunctions and yields information about the spectrum for a generic class of fractal blowups.

Georgii Veprev (University of Geneva).

Title: *Localization of eigenfunctions in amenable unimodular random graphs.*

Abstract: Motivated by Kaplansky's conjecture, Elek showed that the existence of an eigenfunction of the adjacency operator on a Cayley graph of an amenable group implies the existence of a finitely supported eigenfunction with the same eigenvalue. Moreover, all eigenfunctions are generated by finitely supported ones. We will discuss a similar question in the context of unimodular random graphs.