# Swiss Knots 2025

June 4 – 6 at University of Geneva

Abstracts

**Bucher:** *Invariant cocycles on the Furstenberg boundary*

It is well known that there exists, up to a constant, a unique measurable function $f : \mathbb{C} \setminus \{0,1\} \to \mathbb{R}$ satisfying the 5-terms equation

$$f(x) - f(y) + f\left(\frac{y}{x}\right) - f\left(\frac{1-y}{1-x}\right) + f\left(\frac{x}{y} \cdot \frac{1-y}{1-x}\right) = 0.$$

Indeed any such function is necessarily a multiple of the Bloch-Wigner dilogarithm. For smooth functions it is easy to show this with a few derivations. In the case of measurable functions, Bloch's proof relies on an identification of the measurable cohomology in degree 3 of $\mathrm{PSL}(2,\mathbb{C})$, and boils down to the claim that the latter cohomology group is the cohomology of $\mathrm{PSL}(2,\mathbb{C})$-invariant cochains on $P^1(\mathbb{C})$.

In this talk, we will be interested in possible generalisations of this claim, replacing $\mathrm{PSL}(2,\mathbb{C})$ by a semisimple connected Lie group $G$ with finite center and $P^1(\mathbb{C})$ by the corresponding Furstenberg boundary $G/P$, where $P$ is a minimal parabolic subgroup. We will see in particular that the study of the case of $G = \mathrm{PSL}(2,\mathbb{C}) \times \mathrm{PSL}(2,\mathbb{C})$ leads to invariant functions $f : (\mathbb{C} \setminus \{0,1\})^2 \to \mathbb{R}$, whose geometric interpretation remains mysterious, satisfying the corresponding 5-terms relations without being a linear combination of the Bloch-Wigner dilogarithms on the factors. We will also explore the regularity of such functions (measurable vs continuous) to deduce several new cases of a conjecture on bounded cohomology by Nicolas Monod from 2004.

This is joint work with Alessio Savini.

**Detcherry:** *A Langlands duality conjecture for character varieties of 3-manifolds*

We introduce a Langlands duality conjecture relating, for each closed 3-manifold $M$ and reductive complex group $G$, the number of characters of representations of $\pi_1(M)$ into $G$ and its Langlands dual $LG$. This conjecture is motivated by a recent Langlands duality conjecture of Ben-Zvi, Gunningham, Jordan and Safronov for $G$-skein modules, and can be considered the classical counterpart of it. We will provide many examples where our conjecture is true, and will discuss the main issues for proving it in general.

**Di Prisa:** *On rational sliceness of negative amphichiral links*

We say that a link $L$ in $S^3$ is negative amphichiral if there exists an orientation-reversing diffeomorphism of $S^3$ that sends every component of $L$ to itself with the opposite orientation. If such a map can be chosen to be an involution, then the link is said to be strongly negative amphichiral. Kawauchi proved that every strongly negative amphichiral link is rationally slice, i.e. it bounds a disjoint collection of disks in a rational homology 4-ball. In this talk, we prove that every negative amphichiral link is rationally slice, extending the aforementioned work of Kawauchi. Our proof relies on a careful analysis of the JSJ decomposition of the link complement of negative amphichiral links. This is joint work in progress with Jaewon Lee (KAIST, Daejeon) and Oğuz Şavk (CNRS, Nantes).

**Douba:** *On geometrically finite Möbius structures*

A *Möbius transformation* of the sphere $S^n$ is a composition of inversions in round hyperspheres. A *Möbius structure* on an $n$-manifold $M$ is a local modeling of $M$ on $S^n$ whose transition maps are restrictions of Möbius transformations (for $n > 2$, this is nothing but a flat conformal structure on $M$). In this talk, I will attempt to survey what is known about closed manifolds admitting Möbius structures with Zariski-dense geometrically finite holonomy.

**Flamm:** *A glimpse into non-Archimedean surfaces*

Convex projective surfaces are a generalization of hyperbolic surfaces, where the convex set is the unit disk. The study of degenerations of convex projective structures on a surface leads to replacing the reals by a non-Archimedean ordered field. The aim of this talk is to introduce convex sets over such fields, describe their geometries through several examples, and view how they arise as limits of convex projective structures. This is joint work with Anne Parreau.

**Marengon:** *Correction terms of double branched covers and symmetries of immersed curves*

We compare the signatures of a classical link in the 3-sphere with the d-invariants of its double branched cover in the spinnable spin$^c$ structures. Using a symmetry on the immersed curves description of bordered Floer homology, we define a new invariant of bordered homology solid tori, and we relate it to both the $d$-invariants and the link signatures. We deduce that when $L$ is a 2-component link and its double branched cover is an L-space, the spin $d$-invariants are determined by the signatures of $L$.

This is joint work with Jonathan Hanselman and Biji Wong.

**Marino:** *Gordian distance bounds from Khovanov homology*

The Gordian distance $u(K, K')$ between two knots $K$ and $K'$ is defined as the minimal number of crossing changes needed to relate $K$ and $K'$. The unknotting number $u(K)$ of a knot $K$ arises as the Gordian distance between $K$ and the trivial knot. Rasmussen was the first to find a connection between Khovanov homology and $u$: his invariant $s$, extracted from Khovanov homology, yields a lower bound for the slice genus and, as a consequence, for $u(K)$. In this talk, I will introduce a new invariant $\lambda$, extracted from a universal version of Khovanov homology. Although it is not connected to the slice genus, $\lambda$ is a lower bound for $u$, and in fact the inequality $|s(K)| \leq 2\lambda(K)$ always holds. Moreover, $\lambda$ displays relations to a generalization of $u$, the proper rational Gordian distance. This is joint work with L. Lewark and C. Zibrowius.

**Meilhan:** *Higher order Kirk invariants of knotted spheres in 4-space*

Paul Kirk defined in the late eighties a link-homotopy invariant of 2-component link maps, which are 'self-immersed spheres' in 4-space. Recently, Schneiderman and Teichner showed that this 'Kirk invariant' is in fact a complete link-homotopy invariant. In this talk, I will explain how to extend Kirk's construction to link maps of any number of components, in an explicit and computable way. This is a joint work with B. Audoux and A. Yasuhara.

**Mrowka:** *Instanton knot homology groups for webs*

Instanton Floer homology provides a geometric context for constructing homology group associated to webs (trivalent graphs) embedded in 3-manifolds which seems to parallel Khovanov Rozansky homology. This talk will give an overview of some aspects of this story.

**Piccirillo:** *Injectivity and surjectivity of Dehn surgery*

Dehn surgery (with a fixed slope) can be thought of as a function which takes links to closed oriented 3-manifolds. This function is well known to be surjective and highly non-injective. When one restricts the domain to knots, injectivity and surjectivity become more subtle questions. In this talk I will survey what is known about these questions, and discuss a few recent developments. These will include a proof that Dehn surgery functions for knots are never injective; this was a 1977 conjecture of Gordon from a knot theory conference in Plans-sur-Bex.

**Roig Sanchis:** *On the length spectrum of random hyperbolic 3-manifolds*

We are interested in studying the behaviour of geometric invariants of hyperbolic 3-manifolds as their complexity increases. A way to do so is by using probabilistic methods, that is, through the study of random manifolds. There are several models of random manifolds. In this talk, I will explain one of the principal probabilistic models for 3 dimensions and I will present a result concerning the length spectrum-the set of lengths of all closed geodesics- of a 3-manifold constructed under this model. If time allows, I will discuss in more detail an element in the spectrum with particular importance, the systole.