MS3: Shape Optimization and Optimal Control for PDE Constrained Optimization Problems

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SPEAKERS

1. Carlos N. Rautenberg, University of Graz
   Title: Approximation Methods for Elliptic and Evolution Quasi-variational Inequalities

2. Irwin Yousept, Department of Mathematics, University of Darmstadt
   Title: Optimal control of quasilinear $H(\mathrm{curl})$-elliptic partial differential equations in magnetostatic field problems

3. Christopher Linsenmann, Department of Mathematics, University of Augsburg
   Title: The finite element-based immersed boundary method: fully implicit time-stepping scheme and optimal control of microfluidic devices

Approximation Methods for Elliptic and Evolution Quasi-variational Inequalities

Carlos Rautenberg
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We consider quasi-variational inequalities (QVIs) where the constraint set mapping determines a bound on the norm of the gradient of the state variable. The approximation of elliptic QVIs is based on a sequential minimization technique in combination of a penalization approach and a semismooth Newton iteration. The parabolic problem is handled by means of the time-step application of the elliptic solver and an appropriate stability result for the convergent behavior of the method. An account of further approximation techniques is given and methods for hyperbolic problems are discussed. Numerical tests are also provided.

Optimal control of quasilinear H(curl)-elliptic partial differential equations in magnetostatic field problems

Irwin Yousept
Department of Mathematics, University of Darmstadt, Germany

We discuss the mathematical and numerical analysis for optimal control problems governed by quasilinear H(curl)-elliptic partial differential equations. The model problem involves 3D bounded isotropic materials with magnetic permeability depending strongly on the magnetic field. Due to the physical and mathematical nature of the problem, it is necessary to include divergence-free constraints on the state and the control. The divergence-free control constraint is treated as an explicit variational equality constraint, whereas a Lagrange multiplier is included in the state equation to deal with the divergence-free state constraint. We present our recent mathematical and numerical results including the sensitivity analysis of the nonlinear control-to-state mapping, KKT optimality conditions based on the Helmholtz decomposition, regularity results, and rigorous error estimates for the finite element approximations based on the curl-conforming Nedelec edge elements. Brief numerical results illustrating the theoretical findings are presented.

The Finite Element-based Immersed Boundary Method: Fully Implicit Time-stepping Scheme and Optimal Control of Microfluidic Devices

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We use the Finite Element-based Immersed Boundary Method (FE-IBM) for simulating and optimizing the motion of elastic cells, such as red blood cells, immersed in an external viscous fluid. While the classical IBM uses Finite Differences for the spatial and temporal discretization, we employ the Taylor-Hood (P2/P1) Finite Element for the spatial discretization of the governing time-dependent Stokes equations, periodic splines in a variational setting for the representation of the elastic structure, and for the temporal discretization a semi-implicit (Forward Euler/Backward Euler, FwE/BwE) as well as a fully implicit (BwE/BwE) time-stepping scheme.

For the fully implicit scheme in fully variational formulation, we present an adaptive numerical solution technique that is based on a Newton continuation predictor-corrector method. It uses time step sizes that are determined adaptively to meet the convergence requirements of the correction step. This way, the fully implicit scheme can be realized numerically with acceptable computational effort. The feasibility of the approach is illustrated by a numerical example, cf. Fig. 1.

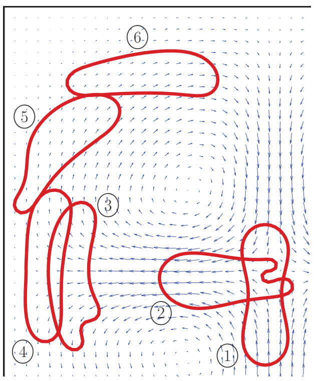

Figure 1: Snapshots of a red blood cell immersed in a fluid subject to a quadrupolar force field. The fully implicit time-stepping scheme has been used for the numerical realization.

As a motivation for the subsequent optimal control problem in the second part of the talk, we consider a cell sorting technique developed by Franke et al. to separate different cell types (e.g., red blood cells and cancer cells) from each other. A lately patented microfluidic device relies on active sorting by surface acoustic waves. These are generated by a special electrode mounted on a piezoelectric material and can be used to deflect cells on their way through a microchannel. See Fig. 2.