# INTRODUCTION TO THOMPSON AND NERETIN GROUPS
# NOTES FOR A MINI-COURSE AT UNI-GENÈVE

DAVIDE PEREGO

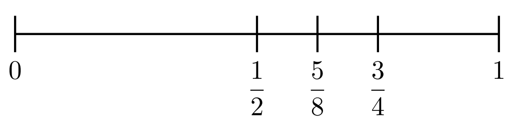
These notes serve as a record, with some minor extensions, of a mini-course the author delivered at the University of Geneva in February 2025. The course was divided into two parts: Thompson groups and Neretin groups. These groups, introduced by Richard J. Thompson and Yuri Neretin, respectively, are striking examples of infinite groups that can be naturally defined and studied through simple actions on the Cantor space. They play a significant role in geometric group theory, exhibiting many exotic properties—for instance, Thompson groups $T$ and $V$ were the first known examples of infinite, finitely presented simple groups.

The course aimed to introduce some of the fundamental techniques used in their study, with a particular focus on transitivity, supports, and commutators.

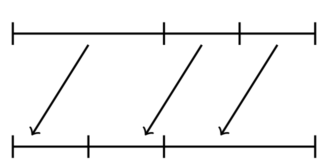
## 1. Thompson groups

The goal of this first part is to define "natural" groups that are infinite and simple, yet remarkably "well-structured". As we will see, these groups can be described from multiple perspectives, allowing us to draw on different areas of mathematics to study them.

We begin with **dyadic subdivisions** of the unit interval; intuitively, a process of repeatedly dividing the interval into halves.




We are interested in **dyadic rearrangement**: given two dyadic partitions $D$ and $R$ with the same number of intervals, we consider a piecewise linear homomorphism of the unit interval such that it maps each subinterval linearly onto its subinterval. If we want to represent the rearrangement, it is easy to see that we just need to consider a pair of dyadic subdivisions with the same number of subintervals.

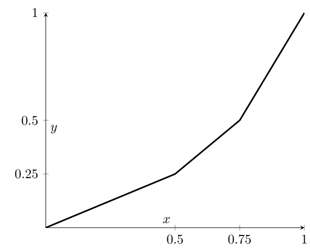


*Date*: February 27, 2025.

*Disclaimer*: These notes are meant for an audience of non-experts such as master students. The purpose is to describe the groups, not always in a formal way, and to provide an intuition of some classical methods developed to study them. The second part is heavily adapted from [GL18].

We can find the explicit forms of such maps together with their plots. For example, the map above is the following

$$x_0 := \begin{cases} \frac{1}{2}x & 0 \leq x \leq 1/2 \\ x - \frac{1}{4} & 1/2 < x \leq 3/4 \\ 2x - 1 & 3/4 < x \leq 1 \end{cases}$$




We have the following

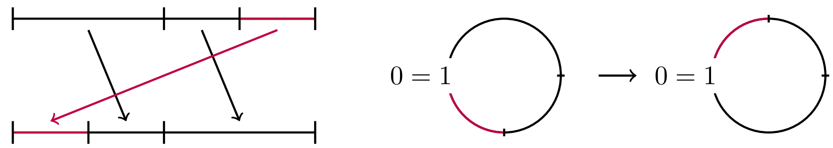
**Theorem 1.1.** *A function $f$ is a dyadic rearrangement if and only if*

(1) *every slope (angular coefficient) is a power of 2;*
(2) *break points happen at dyadic rationals $\mathbb{Z}[\frac{1}{2}]$.*

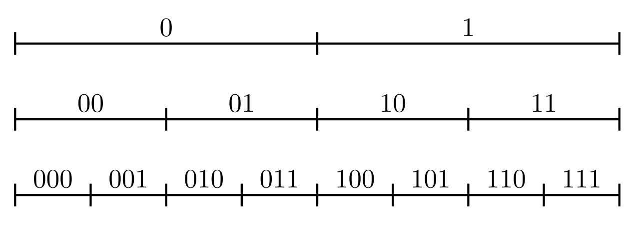
With this in mind, we are ready to define our first Thompson group.

**Definition 1.2.** The group of all dyadic rearrangements of the unit interval is called **Thompson $F$**.

To introduce the second group of this family, namely **Thompson $T$**, we need to consider the projection of the unit interval onto the unit circle realized by identifying 0 and 1. Informally, we want to keep the same behavior we had before: the main difference here is that the new point $0 = 1$ is not a global fixed point anymore. In fact, the stabilizer of such point will be isomorphic to Thompson $F$.

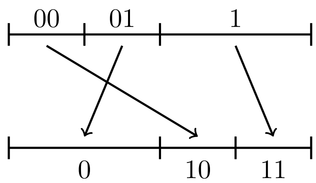



For the third group of this family, **Thompson $V$**, we need to look at one of the most common application of Alexandroff-Hausdorff Theorem[1]: the dyadic representation of the unit interval.




In this case, we do not have to take in account the topology of the unit interval or to preserve a circular order: we are acting on the infinite binary strings $\{0,1\}^{\mathbb{N}}$ and we allow any permutation we want of the subintervals, that is we are considering **prefix exchange maps**.

[1]The theorem says that any compact metrizable space is a continuous image of the Cantor set.

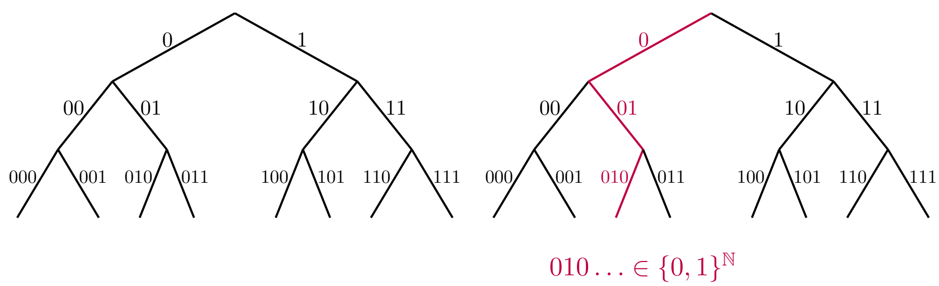



$$\begin{cases} 00 \longmapsto 10 \\ 01 \longmapsto 0 \\ 1 \longmapsto 11 \end{cases}$$

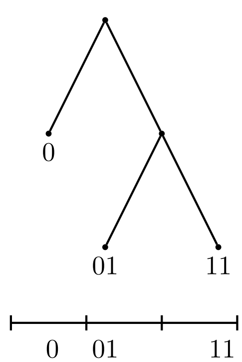
**Remark 1.3.** This way of representing elements of Thompson $V$ as prefix exchange maps draws a bridge with different areas of mathematics such as automata theory and dynamical systems.

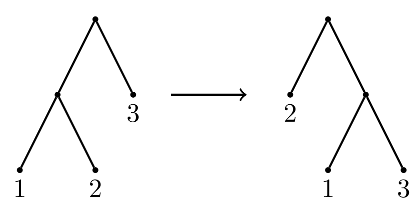
In order to formalize the "permutation of subintervals" in this setting and to ensure that the map considered is a bijection, we introduce the notion of **barrier**: a finite set $B$ of finite binary strings such that for each element $x = \sigma_1 \dots \sigma_n \dots$ of $\{0,1\}^{\mathbb{N}}$ there exists a unique $b \in B$ which is a prefix of $x$, i.e. $x = b\sigma_t \dots$ for some $t \in \mathbb{N}$. A bijective prefix echange map is a map induces by a bijection of barriers.

Among the several ways to represent elements belonging to these trio of groups, we want to use for the rest of these notes the most combinatorial representation. It come directly from the graphical representation of the finite binary strings $\{0,1\}^*$ as a rooted regular tree.

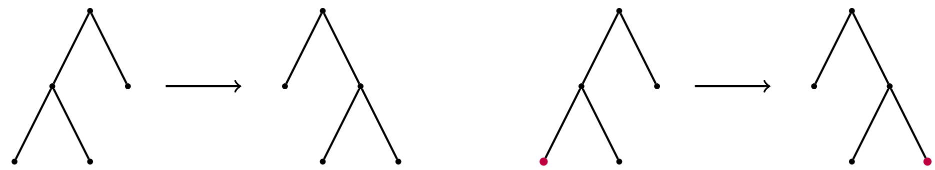



In this way, we can consider **labeled tree pair diagrams** as the one depicted in the next figure which represent the element of $V$ we described above. Usually the tree on the left is called **domain tree** and the tree on the right is called **range tree**.

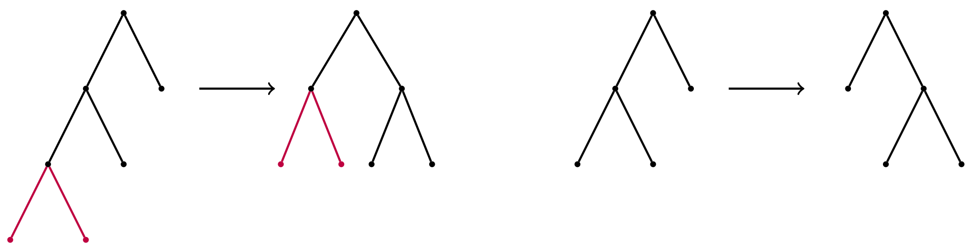


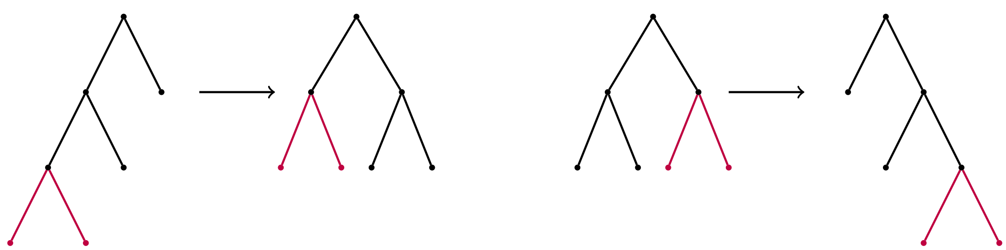

We can ask ourserlves if there is a simplified version for elements of $F$ and $T$. Based on what we have seen before, the answer is yes. Namely, for Thomspon $F$, we do not need any labels, while for Thompson $T$ we just need one.



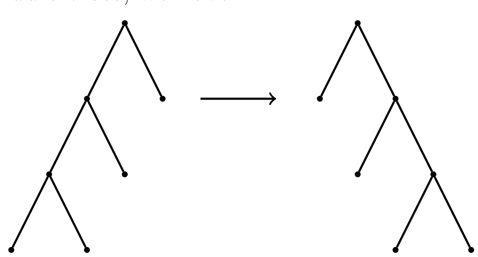
The main feature of this representation is the reduction operation. Given a tree pair diagram, it might be possible that some carets (equivalently, some dyadic subdivisions) are redundant. One can do the following:



In this way, we can consider **reduced tree pair diagrams** where no reduction is possible. It turns out that such diagrams are in a one-to-one correspondence with the elements of the group. Moreover, this feature gives rise to a well-defined notion of composition. Indeed, it is always possible to find a common tree for the range of the first element and the domain of the second, so that they become compatible.



Simplifying the middle trees, we have

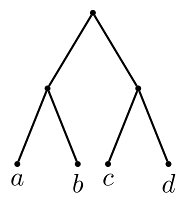


We conclude this part by stressing the ubiquity of these trio giving a curious representation for the smaller one.

**Remark 1.4.** Thompson group $F$ is the group of associative laws. A very simple way to convince ourselves is to look at the element $x_0$ described above as

$$(ab)c \longmapsto a(bc).$$

More precisely, given a tree pair diagram with $n$ leaves (per each tree), one can consider a word of length $n$ and put the parentheses so that they reflect the binary subdivision.

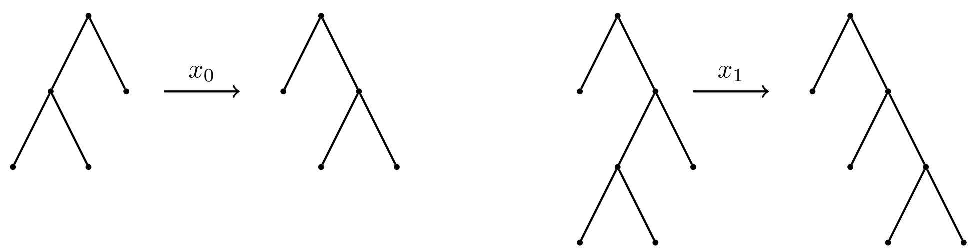


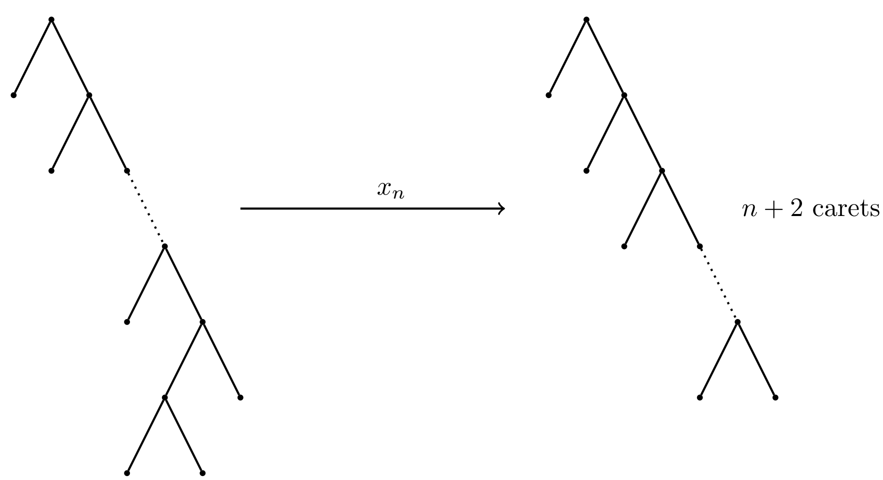

$(ab)(cd)$

1.1. **Generators.** As already claimed, Thompson groups are finitely presented (actually $FP_\infty$, see [Bro87]), a stronger notion, we will not investigate here). In this subsection, we will convince ourselves that they are finitely generated and we will do so by giving a very nice sets of generators, namely

$$F = \langle x_0, x_1 \rangle \qquad T = \langle x_0, x_1, c \rangle \qquad V = \langle x_0, x_1, c, \pi_0 \rangle \quad \text{(see e.g. [CFP96]).}$$

In order to describe these sets, we start from an infinite presentation for $F$, leading to an infinite family of generators $\{x_i\}_{i=0}^{\infty}$.

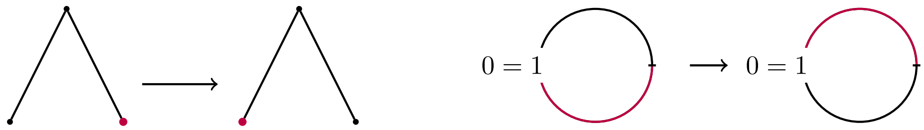


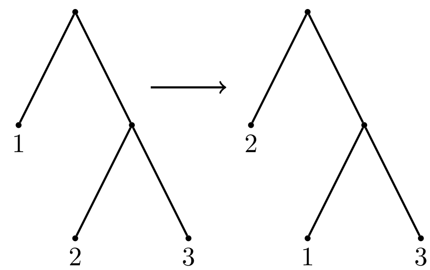

In this way, $F = \langle x_0, x_1, \ldots, x_n, \ldots \mid x_j x_i = x_i x_{j+1},\ i < j \rangle$ (see e.g. [CFP96]).

**Exercise 1.5.** Verify that $x_j x_i = x_i x_{j+1}$.

From this presentation, we deduce that $x_{n+1} = x_0^{-1} x_n x_0$ and hence $F = \langle x_0, x_1 \rangle$. Let's now discuss the additional elements $c$ and $\pi_0$, which enable us to generate the groups $T$ and $V$. First of all, we need a map that breaks the rigidity of $F$ and respect the circular order: a simple rotation of 180° will do the job.




While for $V$, we use $\pi_0$ to allow any possible permutation on the leaves. This curious notation reveals the fact that this element also belongs to an infinite family (for more details [CFP96]).




We end this subsection by making some notes about these elements and the torsion of these groups.

**Remark 1.6.**

(1) The generators $x_0$ and $x_1$ have infinite order. In fact, the group $F$ is a well-known example of torsion-free group.
(2) The other generators $c$ and $\pi_0$ have order 2 (they switch two leaves). On the very opposite of what happens with $F$, it is easy to see that $T$ contains all cyclic groups and $V$ contains all symmetric groups, hence all finite groups. A way to determine whether an element is torsion or not is to find a tree pair diagram having the exact same tree as a domain and range (see [Bur+05]).
(3) The set of generators provided are not minimal. Thompson $T$ and $V$ are actually 2-generated (see [LS97] and [BQ17], respectively).

1.2. **Simplicity.** The study of simplicity and commutator subgroups is a central theme in the research on Thompson groups and their many generalizations, along with their generation properties.

**Theorem 1.7.** *Let $G$ be $F$, $T$ or $V$. Then the commutator subgroup $[G, G]$ is simple.*

One of the main strategy to prove this theorem is making use of the *Epstein's trick*. There are several different versions of the same reasoning leading to the simplicity of commutator subgroups for group acting "nicely" on compact Hausdorff spaces. We will see one of these versions in the next section.

We now that $F/[F, F]$ is isomorphic to $\mathbb{Z}^2$ (see e.g. [CFP96]) and that $T$ and $V$ are perfect, i.e. the commutator subgroup is the whole group. In view of this fact, Thompson groups $T$ and $V$ were the first examples of infinite finitely presented simple groups.

1.3. **Generalizations.** The family of Thompson groups no longer comprises just the original trio. In the past decades, a plethora of generalizations has been proposed. There are many reasons why these generalizations are important: some add specific exotic behaviors or focus on a property of the original trio, others mix these groups together with other well-known families. The result of all this working is that different areas of mathematics already involved strengthen their bonds and other where added to the count. There are essentially two types of generalizations: the ones that change the action and the ones that change the space on which they acts. Examples of group acting on the Cantor space are Higman-Thompson groups [Bro87], topological full groups of edge shifts [Mat15], irrational slopes Thompson groups (especially golden ratio $F_\tau$ - [Cle00; BNR21]) and forest-skein Thompson groups [Bro22]. Examples of groups combining different classes are braided Thompson groups [Deh06; Bri07] and Scott-Roever-Nekrashevych groups [Sco84; Röv99; Nek04]. Groups acting on a different space are higher dimensional Brin-Thompson groups [Bri04] and rearrangement groups of fractals [BF19].

The most simple generalization of all are Higman-Thompson groups $G_s^r$. The difference is that the coding for the Cantor set is not the collection $\{0,1\}^{\mathbb{N}}$ of all binary infinite strings anymore, but the set $R\Sigma^{\mathbb{N}} := \{\rho\sigma_1\sigma_2\dots \mid \rho \in R,\ \sigma_i \in \Sigma \text{ for all } i \in \mathbb{N}\}$ with $R$ and $\Sigma$ two finite sets such that $|R| = r$ and $|\Sigma| = s$. Later in these notes, we are going to use the following result.

**Theorem 1.8.** *Consider the Higman-Thompson group $G_s^r$.*

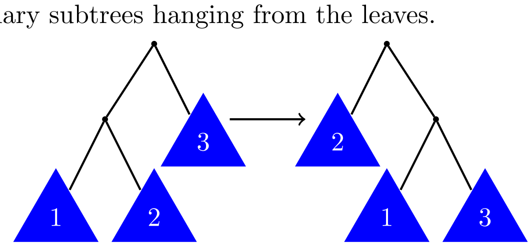
$s$ **is even:** *Then $G_s^r$ is simple.*
$s$ **is odd:** *Then $[G_s^r, G_s^r]$ is simple and has index 2.*

The proof is again based on the Epstein's trick, see e.g. [GL18].

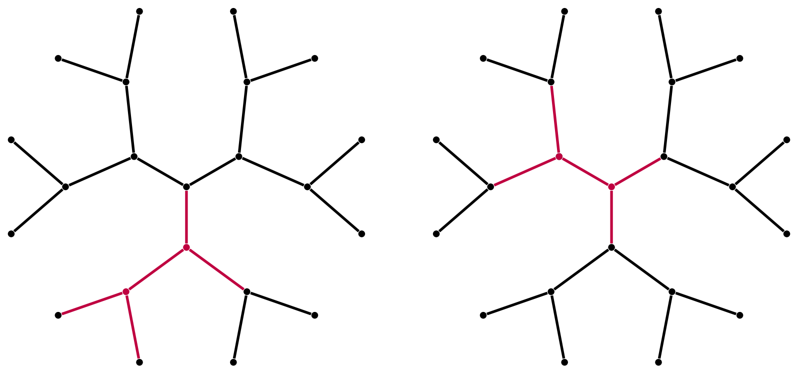
## 2. Neretin Groups

All the examples of "Thompson-like" groups we provided in the previous section were discrete. In this second part, we want to analyze one of the most famous example of *totally disconnected locally compact (TDLC)* groups, the so-called **Neretin groups**, which are constructed in the same fashion as Thompson $V$ and we will see locally resemble Higman-Thompson groups.

The main idea of Thompson $V$ was to take the infinite rooted binary tree, remove two different finite rooted subtree with the same number of leaves and permute the infinite rooted binary subtrees hanging from the leaves.




As before, we are dealing with the same combinatorial/geometric setting of a regular tree, where the arity determines the group. The main difference is that we do not fix any root, though the idea remains the same: remove two finite subtrees and permute the remaining subtrees.



Let $T$ be an $s$-regular tree and let $F$ be a finite subtree. Then we denote with $T_F$ the forest $T - F$.

**Definition 2.1.** Let $F_1$ and $F_2$ be two finite subtrees with the same number of leaves and let $\phi : T_{F_1} \to T_{F_2}$ be an isomorphism of forests. The induced map $\phi^* : \partial T \to \partial T$ on the boundary of the tree is called a **spheromorphism**.

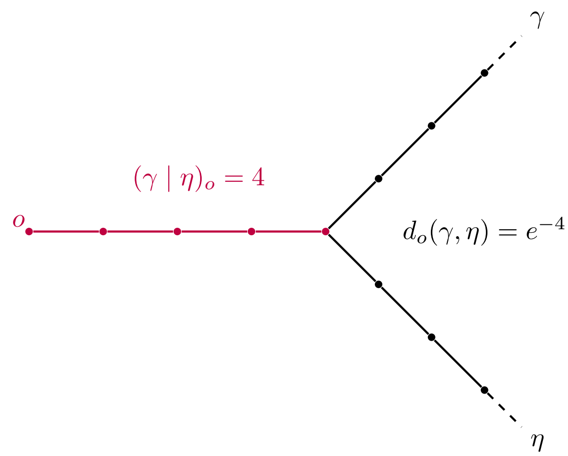
Note that, as in the Thompson case, we can refine the spheromorphism by increasing the number of edges of $F_1$ and $F_2$ accordingly. Again, this feature helps us defining the composition.

**Lemma 2.2.** *The collection of all spheromorpshisms of $T$ forms a group $N_s$.*

**Remark 2.3** (A brief note on the topology)**.** The group $N_s$ is a subgroup of the compact open group $Homeo(\partial T)$, via this embedding it inherits a totally disconnected topology which is not locally compact. Usually, this group is studied by profinite and TDLC group theory by extending the topology on $Aut(T)$. Indeed, $Aut(T)$ is a TDLC subgroup of the Neretin group and there is a way to consider a locally compact topology on $N_s$ starting from the one on $Aut(T)$.

2.1. **Visual metrics and local similarities.** Similar to what happens for Thompson groups, we have an equivalent way to define Neretin groups.

Let $o$ be a vertex of $T$. Then one can see the boundary $\partial T$ of $T$ as the collection of all rays starting from $o$. In the following, we will denote $T_o$ the same tree $T$ rooted in $o$. In this way, one can define the **Gromov product** $(\gamma \mid \eta)_o$ of two rays $\gamma, \eta \in \partial T$ as the length of the longest common prefixes between $\gamma$ and $\eta$. We are interested in the so-called **visual metric** on $\partial T$, that is $d_o := e^{-(\gamma|\eta)}$ with the convention that $e^{-\infty} = 0$.




**Remark 2.4.** The visual metric is an ultrametric. Moreover, balls are clopen and of the form $\partial T_v$ with $diam(\partial T_v) = e^{-|ov|}$ where $T_v$ is the subtree of $T_o$ rooted in the vertex $v$ and $|ov|$ is the length of the path connecting $o$ to $v$.

Starting from a isomorphism between two rooted regular trees, we can always induce an isometry of the boundaries. More precisely, if $\phi : T_v \to T_w$ is an isomorphism (mapping $v$ to $w$), then it induces an isometry $\partial\phi : \partial T_v \to \partial T_w$. More generally, this holds for an isomorphism $\phi$ of the unrooted tree $T$ when restricted to the suitable rooted subtrees.

A similarity is a homeomorphism between metric spaces that uniformly scales the distances between points, either by stretching or contracting them. In order to characterize spheromorphisms from a metric point of view, we need a maps having this local behavior.

**Definition 2.5.** A **local similarity** between two metric spaces $(X, d_X)$ and $(Y, d_Y)$ is a homeomorphism $\Phi$ with the following property: for each $x \in X$ exists a neighborhood $U$ and a constant $C_U$ such that for all $x_1, x_2 \in U$ it holds $d_Y(x_1\Phi, x_2\Phi) = C_U \cdot d_X(x_1, x_2)$.

We are ready for the equivalent definition of $N_s$.

**Theorem 2.6.** *A map $\Phi : \partial T \to \partial T$ is a spheromorphism if and only if it is a local similarity with respect to the visual metric.*

*Proof.* Let $\phi : T_{F_1} \to T_{F_2}$ be the isomorphisms related to $\Phi$. We may assume that $F_1$ and $F_2$ contain $o$ as internal vertex (this is a consequence of the expansion/reduction feature). Now $T_{F_1} = T_{v_1} \sqcup \cdots \sqcup T_{v_k}$ and hence

$$T_{F_2} = T_{v_1}\phi \sqcup \cdots \sqcup T_{v_k}\phi = T_{v_1\phi} \sqcup \cdots \sqcup T_{v_k\phi}.$$

We need to show that the restriction $\Phi_i : \partial T_{v_i} \to \partial T_{v_i\phi}$ is a similarity. Let $\gamma$ and $\eta$ be two boundary points in $\partial T_{v_i}$. Then

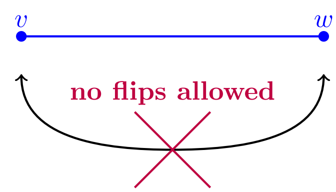
$$\begin{aligned}(\gamma\Phi \mid \eta\Phi)_o &= (\gamma\Phi \mid \eta\Phi))_{v_i\phi} + |ov_i\phi| = \\ &= (\gamma \mid \eta)_{v_i} + |ov_i\phi| = (\gamma \mid \eta)_o - |ov_i| + |ov_i\phi|,\end{aligned}$$

from which we see that the similarity constant is $e^{|ov_i| - |ov_i\phi|}$.

On the other side, if $\Phi$ is a local similarity on $\partial T$, by compactness and the fact that $d_o$ is an ultrametric, there is a finite covering of clopen balls $\partial T_{v_i}$ on which the restrictions of $\Phi$ are similarities and such that $\partial T_{v_i}\Phi = \partial T_{w_i}$. Now the restrictions preserve the ball inside $\partial T_{v_i}$, meaning that there exists a rooted level-preserving isomorphism $\phi_i : T_{v_i} \to T_{w_i}$. To conclude, we put together the $\phi_i$'s giving rise to an isomorphism of forests. □

2.2. **Generators.** In this brief subsection, we explore a method for generating Neretin groups, which not only serves as a foundation for the next subsection but also offers insights and connections to Thompson groups. As already mentioned, Neretin groups are locally Thompson groups.

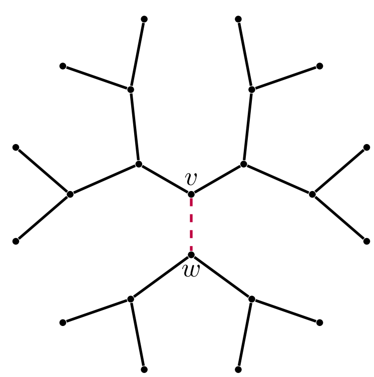
We start by introducing one family of generators. We denote by $Aut^+(T)$ the group of **type preserving automorphisms** i.e. all the $\phi \in Aut(T)$ such that if $\phi$ stabilizes the edge $\{v, w\}$ then $\phi \in Stab(v) \cap Stab(w)$. In this case we say that $\phi$ pointwise stabilizes the edge $\{v, w\}$.




The following theorem follow from the fact that pointwise edge stabilizers act transitively on the edges of $T$ and from [Tit70].

**Theorem 2.7.** *The group $Aut^+(T)$ is generated by pointwise edge stabilizers and it is simple.*

For the second family, we need an embedding of the Higman-Thompson group $G_s^2$ into $N_s$. We choose an edge $\{v, w\}$ in $T$ and we remove it from $T$, notice that we are not removing the vertices $v$ and $w$, but just the edge. In this way, we get two rooted regular trees $T_v$ and $T_w$.




Recalling the definition of $G_s^2$ it is easy to see that there is an action on $\partial T_v \cup \partial T_w$. Let $\iota : T_v \cup T_w \hookrightarrow T$ be the natural embedding and let $\iota^*$ be the induced map on the boundaries. We can define a map

$$G_s^2 \longrightarrow N_s$$

$$g \longmapsto (\iota^*)^{-1} g \iota^*$$

which is exactly the embedding we are looking for. Note that such map obviously depends on the edge we are removing, so we actually have an infinite number of embedded copies of $G_s^2$. It is not hard to see that two such embedded copies are conjugated via elements in $Aut^+(T)$.

**Theorem 2.8.** *Neretin group $N_s$ is generated by $Aut^+(T)$ and an embedded copy of $G_s^2$. In fact, it suffices to consider $Aut^+(T)$ and $[G_s^2, G_s^2]$.*

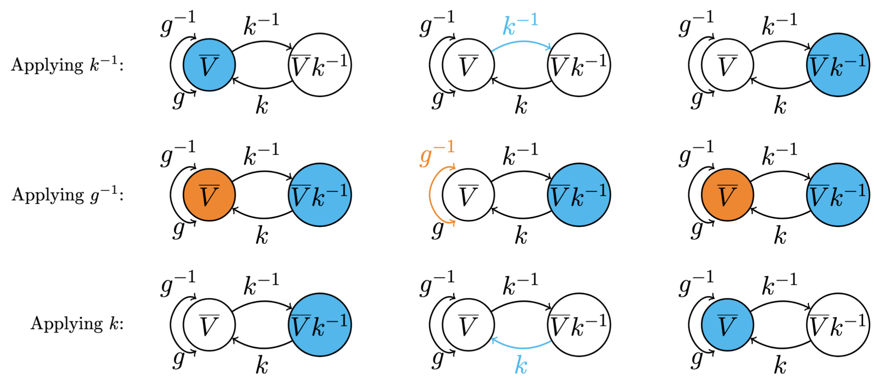
*Proof.* Let $\phi : T_{F_1} \to T_{F_2}$ be an isomorphisms of forests such that $\{v, w\}$ is a common edge between $F_1$ and $F_2$. Note that given a spheromorphism such a $\phi$ always exists by exploiting the the expansion/reduction feature. If $(G_s^2, \iota)$ is the Higman-Thompson copy related to $\{v, w\}$, then there exists a spheromorphisms $\psi^*$ belonging to $(G_s^2, \iota)$ such that $\psi : T_{F_1} \to T_{F_2}$ induces the same bijection on the trees as $\phi$. Now $\phi\psi^{-1} : T_{F_1} \to T_{F_1}$ preserves the trees in the forest $T_{F_1}$, so one can consider the automorphism $\widehat{\phi\psi^{-1}}$ of $T$ that pointwise stabilizes $F_1$ (hence $\{v, w\}$). Finally, $\phi^* = \widehat{\phi\psi^{-1}}\psi^*$ where $\widehat{\phi\psi^{-1}} \in Aut^+(T)$ and $\psi^* \in (G_s^2, \iota)$. □

2.3. **Simplicity.** In this final subsection, we will use the discussion of the simplicity of Neretin groups as an opportunity to delve into what is known as *Epstein's trick*. This technique is a powerful tool for establishing the simplicity of many (commutator) groups that arise from actions on compact Hausdorff spaces with desirable transitivity properties.

We fix $X$ to be a compact Hausdorff space and $\mathcal{U}$ a basis for $X$. We also fix $G$ to be a group acting faithfully on $X$ and transitively on $\mathcal{U}$. The following lemmas constitute the trick for Neretin's setting. They will involve the *support* of a map, together with some commutators. The **support** of an element $g$ is the subspace $supp(g) := \{x \in X \mid xg = x\}$. To fix the notation, we recall that $[g, h] = h^{-1}g^{-1}hg$ and $g^h = g^{-1}hg$.

**Lemma 2.9.** *Let $H$ be a non-trivial normal subgroup of $G$ and let $V \in \mathcal{U}$. For all $g \in G$ such that $supp(g) \subseteq V$ there exist $h \in H$ such that $g_{|V} = h_{|V}$.*

*Proof.* Let $k$ be a non-trivial element of $H$. By faithfulness, there exists $x \in X$ such that $xk^{-1} \neq x$ and separation there exists a neighborhood $\overline{V}$ of $x$ such that $\overline{V} \cap \overline{V}k^{-1} = \emptyset$.
Case $V = \overline{V}$. It suffices to put $h = [g, k]$. Indeed, $h = k^{-1}k^g \in H$ and $(g^{-1})^k$ acts trivially on $\overline{V}$.




Case $V \neq \overline{V}$. Transitivity ensures that there exists $\overline{g} \in G$ such that $V\overline{g} = \overline{V}$ and now one can apply the previous case to $\overline{g}g\overline{g}^{-1}$. $\square$

The second lemma will be proven in the same fashion as the previous. In particular, it will be an iteration of the same argument displayed above on $\overline{V}$ and on $X - \overline{V}$.

**Lemma 2.10.** *Let $H$ be a non-trivial normal subgroup so that there exist $k_1, k_2 \in H$ such that for some $x \in X$ the points $x$, $xk_1$ and $xk_2$ are distinct. Let $V \in \mathcal{U}$. For all $g_1, g_2 \in G$ such that $supp(g_1) \subseteq V$ and $supp(g_2) \subseteq V$ there exist $h_1, h_2 \in H$ such that $[g_1, g_2] = [h_1, h_2]$.*

*Proof.* Reasoning as in the previous proof, we have that $V$ is such that $V$, $Vk_1$ and $Vk_2$ are pairwise disjoint and $h_i = [g_i, k_i] \in H$ for $i = 1, 2$.
By assumption on the supports and by applying Lemma 2.9, we have $[g_1, g_2]_{|V} = [h_1, h_2]_{|V}$. Since $supp(g_i) \subseteq V$, then on $X - (V \cup Vk_i)$ we have that $h_i = k_i^{-1}k_i = id$ and hence $supp(h_i) \subseteq V \cup Vk_i$ for $i = 1, 2$. Moreover, $h_i$ preserves the sets $V$, $Vk_1$ and $Vk_2$ for $i = 1, 2$. Finally, using the same argument as before we get that $[h_1, h_2]$ is the identity on $X - V$ and we conclude the proof. $\square$

We are now ready to prove our main result by simply combining what we have gathered.

**Theorem 2.11.** *The Neretin group $N_s$ is simple.*

*Proof.* Let $H$ be a non-trivial normal subgroup of $N_s$. By Theorem 2.8, $N_s$ is generated by $Aut^+(T)$ and $[G_s^2, G_s^2]$. If we prove that they are contained in $H$, we are done. By Theorem 2.7 and Theorem 1.8, we know that $Aut^+(T)$ and $[G_s^2, G_s^2]$ are simple. Hence $Aut^+(T)$ is equal to its commutator subgroup and it suffices to show that $Aut^+(T) \cap H \neq 1$ and $[G_s^2, G_s^2] \cap H \neq 1$. It is easy to see that there exist elements in both $Aut^+(T)$ and $G_s^2$ such that their supports are contained in some $\partial T_v$, one can apply Lemma 2.10 to find non-trivial intersection between $H$ and the commutator subgroups. $\square$

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