Communications in Mathematical Physics manuscript No.
(will be inserted by the editor)

Erratum to: Self-Attractive Random Walks: The Case of Critical Drifts

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The date of receipt and acceptance will be inserted by the editor

We provide suitably amended versions of part of the statement and the proof of Lemma 1 of [1], which were incorrect. We also use this opportunity to add a couple of comments.

Corrections to Lemma 1. The statement that A is super-multiplicative and the resulting upper bound (8) are incorrect. We should instead consider first the function

$$H(x) \stackrel{\triangle}{=} \sum_{\gamma:0\to x} a(\gamma)\,\mathbf{1}_{\{\ell_\gamma[x]=1\}}.$$

Since H is super-multiplicative,

$$\xi_H(x) \stackrel{\triangle}{=} -\lim_{n\to\infty}\frac{1}{n}\log H(\lfloor nx\rfloor)$$

is well-defined and $H(x) \leq \mathrm{e}^{-\xi_H(x)}$. Moreover, the elementary bound $H(x) \leq e^{-\phi(1)\|x\|}$ shows that ξ_H is a norm on $\mathbb{R}^d$.

The existence of ξ, as stated in Lemma 1, follows from the identity $\xi = \xi_H$, which is obtained along the lines of the proof of Lemma 1 in the following fashion. Let $k_0 \stackrel{\triangle}{=} \sup_{y\neq 0}\xi_H(y)/\|y\|$. Since $A(x) \geq H(x)$ and, by (9),

$$\sum_{k>2k_0} A^{(k)}(x) \lesssim e^{-k_0\phi(1)\|x\|},$$

it follows that

$$A(x) \lesssim \sum_{k\leq 2k_0} A^{(k)}(x) \leq H(x)\sum_{k\leq 2k_0} G_{\Lambda_{(k+1)\|x\|}}(x,x) \lesssim C_d(x)H(x),$$

* Supported by the Israeli Science Foundation grant 817/09.
** Supported by the Swiss National Science Foundation.

where

$$C_d(x) \triangleq \begin{cases} k_0^3\|x\|^2, & d=1,\\ k_0\log(k_0)\|x\|, & d=2,\\ k_0, & d\geq 3.\end{cases}$$

The desired identity $\xi = \xi_H$ now follows from $H(x) \leq A(x) \lesssim C_d(x)H(x)$.

Note that (8) should be replaced by

$$A(x) \leq e^{-\xi(x)+\log C_d(x)}.$$

This, however, has no impact on the coarse-graining estimates of Section 2, and consequently on the rest of the arguments in the paper, for the following two reasons: First, we are actually working with the function H rather than A in Section 2 (using first exit times from balls) for which (8) holds. Second, the coarse-graining estimates would actually go through with any uniform estimate of the type $A(x) \leq e^{-\xi(x)(1-o(1))}$.

Extension of Lemma 1. A closer look at the proof of Lemma 1 (and a slightly more involved argument) reveals that positivity of the critical Lyapunov exponent holds whenever $\phi \geq 0$ and $\phi(1) > 0$ with no additional assumption on monotonicity of ϕ. Note, however, that the monotonicity assumption on ϕ is used in an essential way in the rest of the paper.

Bibliographical complement. The fact that the *quenched* Brownian motion in Poissonian potential undergoes a first order phase transition from a collapsed phase to a stretched phase has been established in [2].

Acknowledgments. The authors would like to thank A.-S. Sznitman for pointing out reference [2].

References

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