Filtering of stochastic nonlinear systems with multiplicative noises

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Objectifs :

– Report ⟶ Mean square exponential stability ⟹ almost sure exponential stability (for "very large" class of stochastic systems),

– Report ⟶ Amost sure exponential stability $\not\Longrightarrow$ Mean square exponential stability(for "very large" class of stochastic systems),

– Relaxation of the stability conditions used in the literature for stochastic systems with multiplicative noises
⟶ Replace the mean square exponential stability in (literature) by the almost sure exponential stability,

– Application to observers synthesis ,

– Application to control synthesis.

Stochastic systems and notions of stability considered :

$$\mathrm{d}\,x = f(x,u)\,\mathrm{d}\,t + g(x,u)\,\mathrm{d}\,w_x$$
$$\mathrm{d}\,y = h(x)\,\mathrm{d}\,t + q(x)\,\mathrm{d}\,w_y$$

- $x \in \mathbb{R}^n$ **is the state vector,** $y \in \mathbb{R}^p$ **is the output vector and** $u \in \mathbb{R}^m$ **is the vector of known inputs,**
- $w_x \in \mathbb{R}^d$ **et** $w_y(t) \in \mathbb{R}^\ell$ **are multi-dimensional independent Brownian motions,**
- $f(x,u)$ **the drift part of the stochastic differential equation (SDE),**
- $g(x,u)$ **the diffusion part of the stochastic differential equation (SDE).**

Almost sure exponential stability : $\limsup\limits_{t\to+\infty} \frac{1}{t}\ln(\|x(t,t_0,x_0)\|) < -\alpha < 0 \qquad \forall\, x_0 \in \mathbb{R}^n$ **almost surely**

Application to the observer design

Problem statement

The Observer is gibven by

$$\mathrm{d}\,\widehat{x} = f(\widehat{x},u)\,\mathrm{d}\,t + \psi(u)(\mathrm{d}\,y - h(\widehat{x})\,\mathrm{d}\,t)$$

The filtering error $e = x - \widehat{x}$

$$\mathrm{d}\,e = (f(x,u) - f(x-e,u) - \psi(u)(h(x) - h(x-e))\,\mathrm{d}\,t + g(x,u)\,\mathrm{d}\,w_x - \psi(u)q(x)\,\mathrm{d}\,w_y$$

$\psi(u)$ is the matrix gain to determine such that the observation error $e(t)$ converge exponentially almost surely.

The almost sure exponential stability of e needs the almost sure exponential stability x.

Problem : The approaches based "Lyapunov" (literature) are reduced to the mean square exponential stability

Approach used : Stability of stochastic triangular systems

We consider a class of stochastic differential equation

$$\mathrm{d}\,x_1 = f_1(x_1,u)\,\mathrm{d}\,t + g_1(x_1,u)\,\mathrm{d}\,w \tag{1a}$$
$$\mathrm{d}\,x_2 = f_2(x_1,x_2,u)\,\mathrm{d}\,t + g_2(x_1,u)\,\mathrm{d}\,w \tag{1b}$$

and a class of stochastic differential equation "block-diagonals"

$$\mathrm{d}\,\overline{x}_1 = f_1(\overline{x}_1,u)\,\mathrm{d}\,t + g_1(\overline{x}_1,u)\,\mathrm{d}\,w \tag{2a}$$
$$\mathrm{d}\,\overline{x}_2 = f_2(0,\overline{x}_2,u)\,\mathrm{d}\,t \tag{2b}$$

Assumption1 : it exists a reel $k > 0$ such that, $\forall\, t \geqslant 0$,

$$\|f_2(x_1,x_2,u) - f_2(0,\overline{x}_2,u)\| \leqslant k\,(\|x_1\| + \|x_2 - \overline{x}_2\|)\,,$$
$$\mathrm{trace}\big((g_1(x_1,u) - g_1(\overline{x}_1,u))(g_1(x_1,u) - g_1(\overline{x}_1,u))^T\big) \leqslant k\,\|x_1 - \overline{x}_1\|^2\,,$$
$$\mathrm{trace}\big((g_2(x_1,u) - g_2(\overline{x}_1,u))(g_2(x_1,u) - g_2(\overline{x}_1,u))^T\big) \leqslant k\,\|x_1 - \overline{x}_1\|^2\,.$$

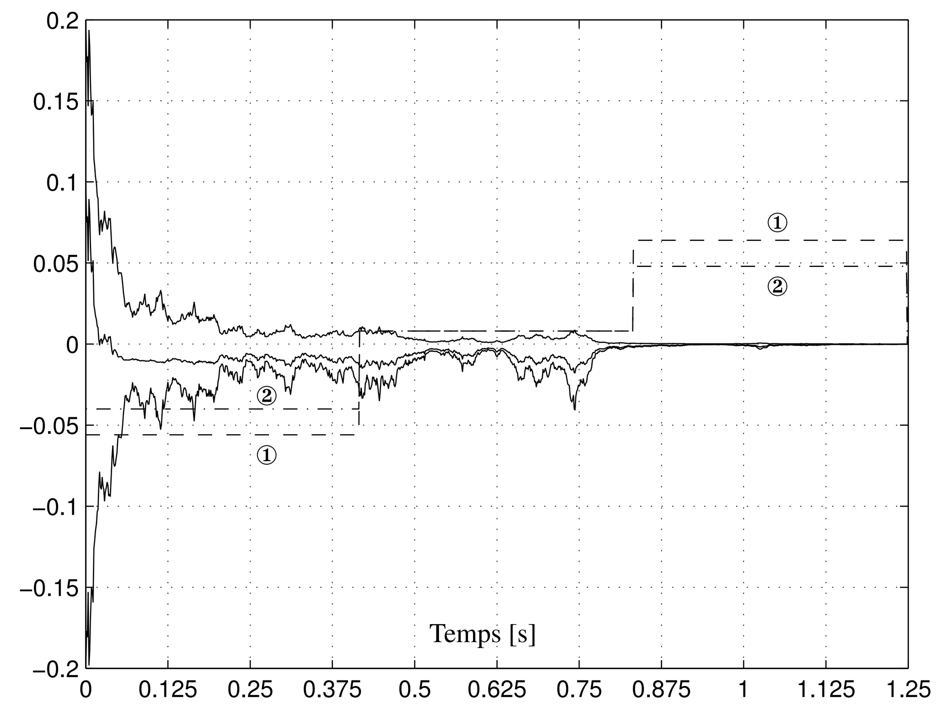
Theorem 1: With assumption 1, the equilibrium point of SDE (1) is almost surely exponentially stable if and only if the equilibrium point of SDE (2) is almost surely exponentially stable. □

Application of theorem 1 to the design observer

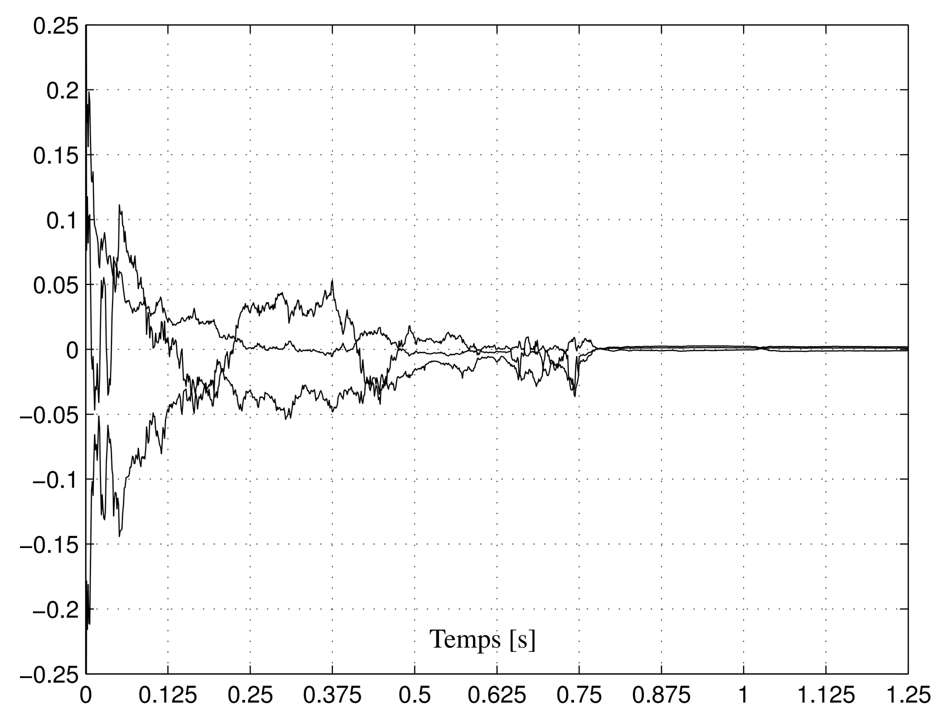
Theorem 2 : If the assumption 1 is satisfied with SDE $\mathrm{d}\,x = f(x,u)\,\mathrm{d}\,t + g(x,u)\,\mathrm{d}\,w_x$, then the system $\mathrm{d}\,\widehat{x} = f(\widehat{x},u)\,\mathrm{d}\,t + \psi(u)(\mathrm{d}\,y - h(\widehat{x})\,\mathrm{d}\,t)$ is an observer for the considered stochastic system Guaranteeing the almost sure exponential stability of the filtering error if

1. The SDE $\mathrm{d}\,x = f(x,u)\,\mathrm{d}\,t + g(x,u)\,\mathrm{d}\,w_x$ is stable exponentially almost surely,
2. it exists a matrix gain $\psi(u)$ such that theOrdinary differential equation (ODE)
$$\dot{e} = -f(-e,u) + \psi(u)h(-e)$$
is exponentially stable stable. □

Example



State of the system $x(t)$ and inputs $u_1(t)$ (①) and$u_2(t)$ (②)



Filtering error $e(t)$

Conclusion

- **Decoupling of the stability of the stochastic system from the stablity of the filtering error.**
- **Stability of the SDE of the system ⟶ Itô calculus, LMI, etc…**
- **Calcul of the gain $\psi(u)$ of the observer ⟶ literature of the observer for the nonlinear ODE ⟶ Lyapunov, LMI, great gain, etc…**