Practical output feedback tracking control for a class of stochastic system

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Abstract

This study investigates the global adaptive practical tracking for a class of nonlinear stochastic systems with dynamic uncertainties and unmeasured states via dynamic output feedback control.
We show that we can extend the work in [1] to stochastic system and generalize the work in [2]. An output feedback controller is constructed to guarantee that the closed-loop system is globally practically stable in probability and the output can be regulated to the all fixed ball almost surely.

Notations and preliminary results

Consider the following stochastic nonlinear system

$$dx = f(x)dt + g(x)dw \tag{1}$$

Where $x \in \mathbb{R}^n$ is the system state, w is an m-dimensional standard Winner process defined on the complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$. The Borel measurable functions $f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^{n\times m}$.
For any given function $V(x) \in \mathcal{C}^2(\mathbb{R}^n)$, associated with system (1), the differential operator $\mathcal{L}$ is defined as

$$\mathcal{L}V = \frac{\partial V}{\partial x}.f + \frac{1}{2}\text{tr}\{g(x)\frac{\partial^2 V}{\partial x^2}g^\top(x)\}$$

Definition 1. *[5] The solution process $\{x(t); t \geq 0\}$ of stochastic differential system (1) is said to be bounded in probability, if*

$$\lim_{t\to\infty} \sup_{0\leq t<\infty} P\{|x(t)| \geq c\} = 0$$

Theorem 1. *[6] Consider the system (1) and assume that f and g are $\mathcal{C}^1$, if there exists a function $\mathcal{C}^2$ function $V(x)$, class $\mathcal{K}_\infty$ functions β_1 and β_2, a constant $c > 0$, and a nonnegative function $W(x)$ such that*

$$\beta_1(|x|) \leq V(x) \leq \beta_2(|x|), \quad \mathcal{L}V \leq -W(x) + c \quad \textit{Then,}$$

1. *There exists an almost surely unique solution on $[0, \infty)$*
2. *The solution process is bounded in probability when $W(x) \geq \alpha V(x)$ for some $\alpha > 0$.*
3. *When $c = 0$, $f(0) = g(0) = 0$ and $W((x)$ is continuous, the equilibrium $x = 0$ is globally stable in probability and the solution $x(t)$ satisfies $P\{\lim_{t\to\infty} W(x(t)) = 0\} = 1$.*

Problem Statement and Assumptions

we consider a class of stochastic nonlinear systems in the following form:

$$\begin{aligned} dx_i &= (x_{i+1} + f_i(t,x,u))dt + g_i^\top(t,x,u)dw \quad i = 1,2,\dots,n-1 \\ dx_n &= (u + f_n(t,x,u))dt + g_n^\top(t,x,u)dw \\ y &= x_1 - y_r \end{aligned} \tag{2}$$

Where $x = (x_1, \cdots, x_n)^\top \in \mathbb{R}^n$, $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are the state, input and output, respectively. y_r is a given reference trajectory to be tracked; w is an m-dimensional standard Winner process The mapping $f_i : \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$ and $g_i : \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^m; 1 \leq i \leq n$, are unknown perturbation functions and are assumed to be continuous in the first argument and locally Lipschitz in the rest of the arguments.

0.1 Assumptions

(A1) 1. The functions f_i and g_i are contineous and locally Lipshitz.
2. There exist an <u>unknown</u> constants θ_1, θ_2 such that

$$|f_i(t,x,u)| + |g_i(t,x,u)| \leq \theta_1(1 + |y|^p)\sum_{j=1}^{i}|x_j| + \theta_2. \tag{3}$$

(A2) The reference trajectory y_r is continuously differentiable and there exists an unknown constants K such that

$$|y_r| + |\dot{y}_r)| \leq K \tag{4}$$

0.2 Problem

The objective paper is to design an adaptive output-feedback controller

$$\begin{aligned} \dot{\chi} &= \alpha(\chi, y) \\ u &= \beta(\chi, y) \end{aligned} \tag{5}$$

so that the solution process of the closed-loop system is bounded in probability and the outputs $y = x_1 - y_r$ can be regulated into a small neighborhood of the origin in probability

Controller design

0.3 Change of coordinates

$z_1 = y,\ z_i = x_i,\ i \geq 2.$

$$\begin{aligned} dz_1 &= (z_2 + f_1(t,z,u) - \dot{y}_r)dt + g_1^\top(t,z,u)dw \\ dz_i &= (z_{i+1} + f_i(t,z,u))dt + g_i^\top(t,z,u)dw \quad i = 2,\dots,n-1 \\ dz_n &= (u + f_n(t,z,u))dt + g_n^\top(t,z,u)dw \\ y &= z_1 \end{aligned} \tag{6}$$

0.4 Controller

Let $\gamma > 0$, we introduce the controller via the full-order observer

$$\begin{aligned} u_\gamma &= -\sum_{i=1}^{n}(LM)^{n-i+1}k_i\hat{z}_i \\ d\hat{z}_i &= \hat{z}_{i+1} + (LM)^i a_i(y - \hat{z}_1)dt \quad i = 1,2,\dots,n-1 \\ d\hat{z}_n &= u + (LM)^n a_n(y - \hat{z}_1)dt \end{aligned} \tag{7}$$

where $\hat{z} = [\hat{z}_1, ..., \hat{z}_n]^\top$ with the initial value $\hat{z}(t_0) = \hat{z}_0$,
gains M and L are updated by

$$\begin{aligned} \dot{M} &= -\alpha M^2 + \beta(1 + yl)M; \quad M(0) = 1 \\ \dot{L} &= \max(0, \tfrac{M}{(ML)^{2b}}(\hat{z}_1^2 + e_1^2 - \gamma^2))^2; \quad L(0) = L_0 > 0 \end{aligned} \tag{8}$$

The parameters a_i and k_i are chosen so that

$$A = \begin{pmatrix} -a_1 & 1 \dots 0 \\ \vdots & \vdots \ddots \\ -a_{n-1} & 0 \dots 1 \\ -a_n & 0 \dots 0 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \\ 0 & 0 & \dots & 1 \\ -k_n & -k_{n-1} & \dots & -k_1 \end{pmatrix} \tag{9}$$

are Hurwitz matrices.

Lemma 1. *There exists P and Q symmetric and positive definite matrices, and a positives constants c_1, c_2, c_3 and c_4, such that*

$$A^\top P + PA \leq -id_n, \qquad c_1 id_n \leq D_b P + PD_b \leq c_2 id_n, \tag{10}$$

$$B^\top Q + QB \leq -2id_n, \qquad c_3 id_n \leq D_b Q + QD_b \leq c_4 id_n, \tag{11}$$

Theorem 2. *Consider system (2) under Assumptions A1 and A2. The output-feedback controller (7) guarantees that, for any initial condition $(x_0, \hat{z}_0) \in \mathbb{R}^n \times \mathbb{R}^n$, the solution $(x(t), \hat{z}(t), M(t), L(t))$ of the resulting closed-loop system is unique and bounded on $[0, +\infty)$ a.s., and furthermore, for all $\gamma > 0$, there exists a finite time $T > 0$ so that $|x_1(t) - y_r(t)| \leq \gamma, \quad \forall t \geq T$*

1 Proof

The error dynamics and the closed-loop system

Let $e_i = z_i - \hat{z}_i$ and define the following scaling state $\zeta = (\zeta_1, \cdots, \zeta_n)^\top$ and estimation error $\epsilon = (\epsilon_1, \cdots, \epsilon_n)^\top$ as follows:

$$\epsilon_i = \frac{e_i}{(LM)^{i+b-1}} \quad \zeta_i = \frac{\hat{z}_i}{(LM)^{i+b-1}} \tag{12}$$

Where $b > 0$ is constant. Now, using (12), the closed-loop systems (6) and (7) can be expressed compactly as

$$\begin{aligned} d\epsilon &= ((LM)A\epsilon - (\tfrac{\dot{L}}{L} + \tfrac{\dot{M}}{M})D\epsilon + F(t,x,u)dt + G(t,x,u)dw(t) \\ d\zeta &= ((LM)B\zeta - (\tfrac{\dot{L}}{L} + \tfrac{\dot{M}}{M})D\zeta + (LM)a\epsilon_1)dt \end{aligned} \tag{13}$$

Where

$$a = (a_1, \cdots, a_n)^\top \qquad D = \text{Diag}\,(b, b+1, \dots, b+n-1) \tag{14}$$

and

$$F = (\frac{f_1(t,x,u)}{LM^b}, \cdots, \frac{f_n(t,x,u)}{LM^{b+n-1}})^\top \qquad G = (\frac{g_1(t,x,u)}{LM^b}, \cdots, \frac{g_n(t,x,u)}{LM^{b+n-1}})^\top \tag{15}$$

1.1 Lyapunov analysis

Consider the Lyapunov function defined by

$$V = \alpha V_1 + V_2 \tag{16}$$

Where $V_1 = \epsilon^\top P\epsilon$ and $V_2 = \zeta^\top Q\zeta$.
The remainder of the proof is omitted on grounds of space

References

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