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A micro-macro acceleration method for the Monte Carlo simulation of SDEs

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Goals:
1. accelerate weak simulation of SDEs with fast micro-dynamics and slow evolution of observables
2. investigate the assumptions underlying the method and the quality of the resulting approximations

Coarse Projective Integration

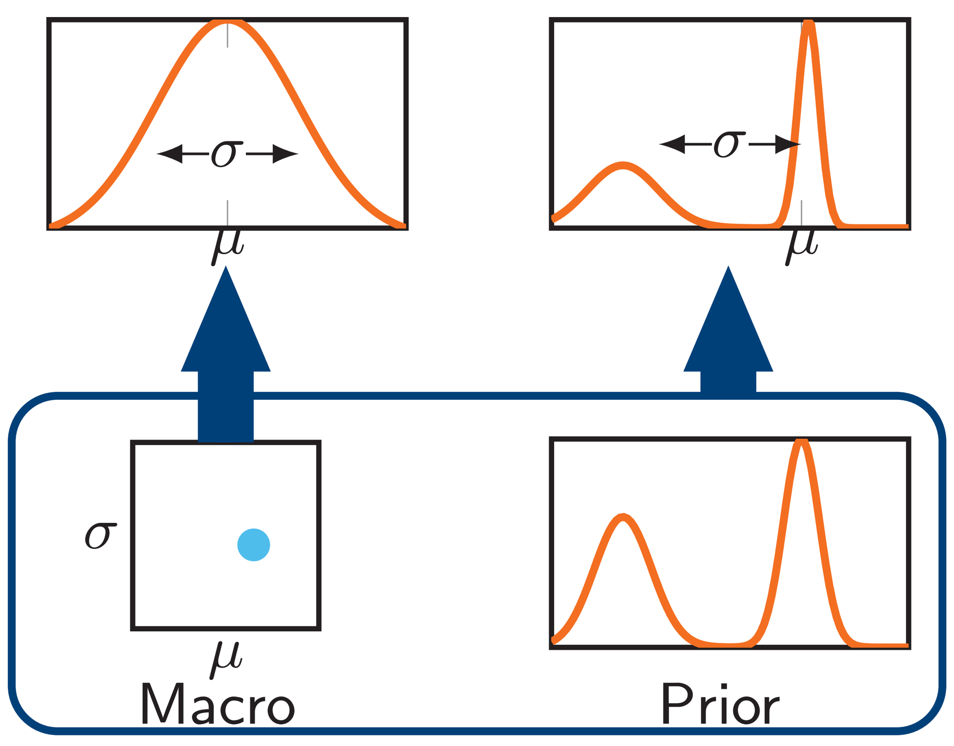
Framework to extrapolate the long time dynamic behavior of multiscale systems using appropriately initialized microscopic simulation on short time-scales

- **Lifting**
  Create ensemble of initial micro-states consistent with current macro-state
- **Simulation**
  Run the micro solver with initial data for short macroscopic time
- **Restriction**
  Compute a number of macro-states (averaging) to approximate the time derivative of macro-states

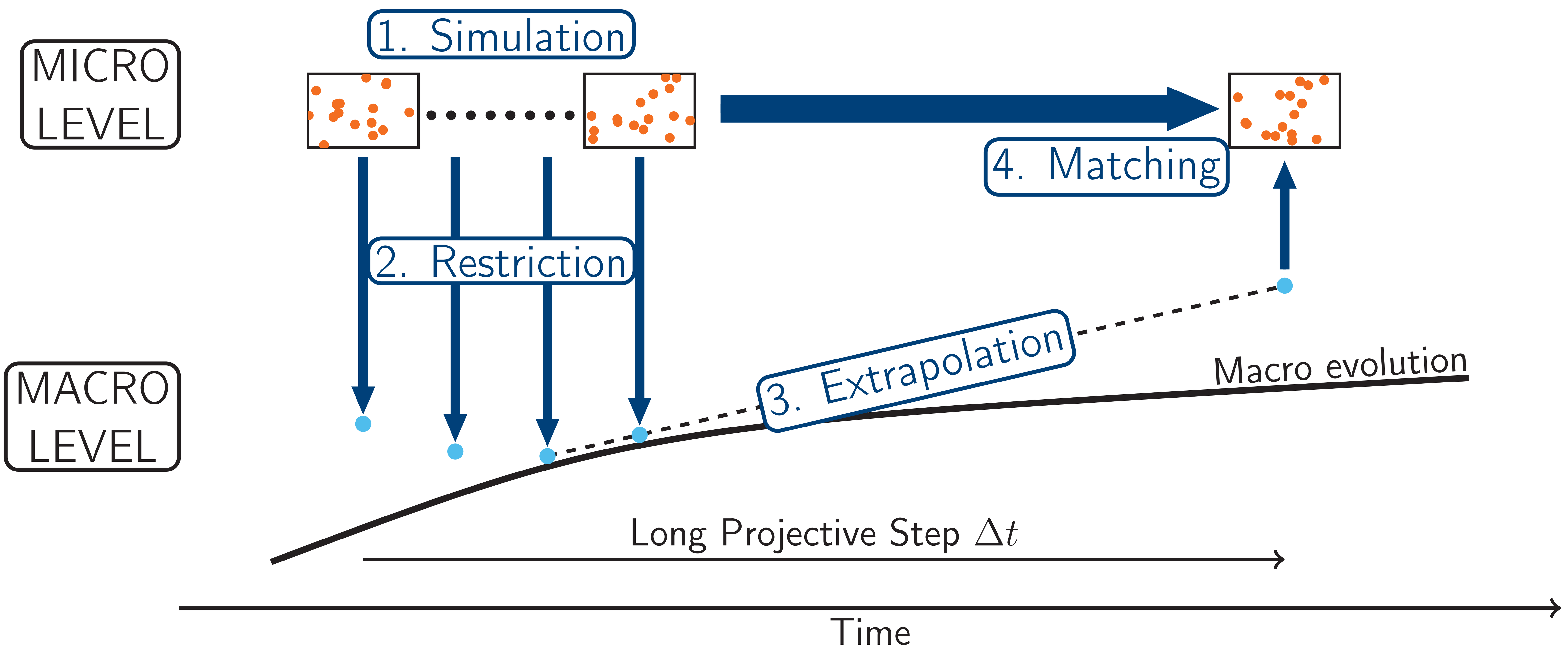
Matching – Alternative for Lifting

Idea: maximally exploit the information that is present in a (prior) microscopic state available from the previous instance of Coarse Projective Integration

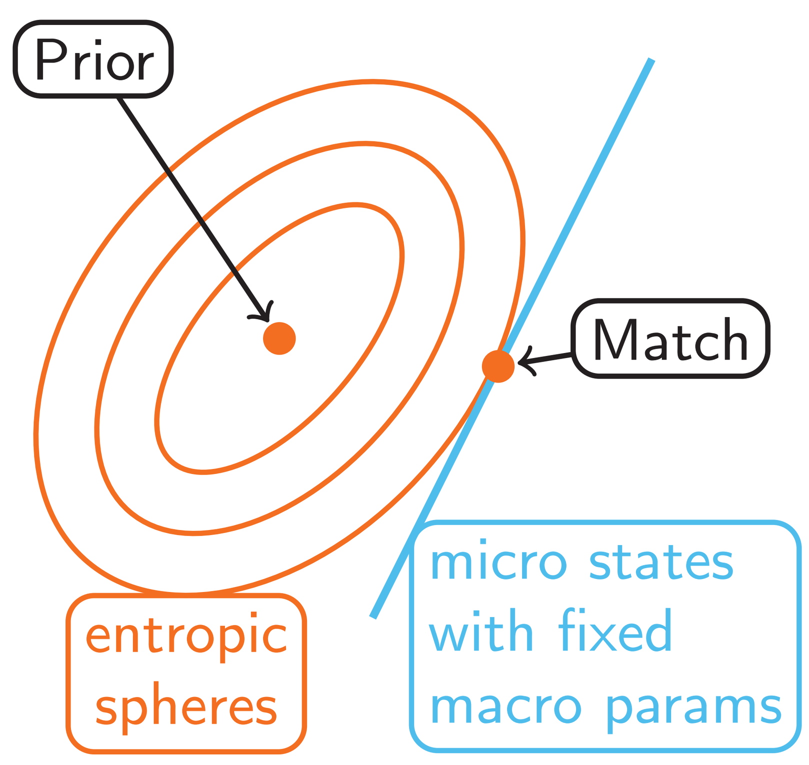
Lifting vs Matching



- **Methodology**
  Search for minimal perturbation of prior state consistent with current macro-state
- **Advantage**
  Avoids numerical modeling error associated with Lifting
- **Analysis**
  Prove weak convergence to full microscopic dynamics



Matching via Entropy Optimization



L^p norm:

$$\mathcal{I}(\phi|\pi) = \frac{1}{p}\int |\phi - \pi|^p$$

Logarithmic entropy:

$$\mathcal{I}(\phi|\pi) = \int \phi \log \frac{\phi}{\pi}$$

Hellinger distance:

$$\mathcal{I}(\phi|\pi) = \int (\sqrt{\phi} - \sqrt{\pi})^2$$

Convergence for L^2 norm

Crucial components for analysis:

- **hierarchy of macro-states**
  → parametrized by the number of variables L
  → uniquely determines the micro-states as $L \to \infty$
- **consistency of matching**
  → bounds the matching error as $\mathcal{O}(\Delta t/L)$

Global Weak Error $\sim \mathcal{O}(1/L + L \cdot \Delta t)$

Proof and example with FENE dumbbells:

K.Debrabant, G.Samaey, P.Zielinski, (2015) arXiv:1511.06171

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