Local Marchenko-Pastur law

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28/7/2022

THE ROYAL SOCIETY

Square Sample Covariance Matrices

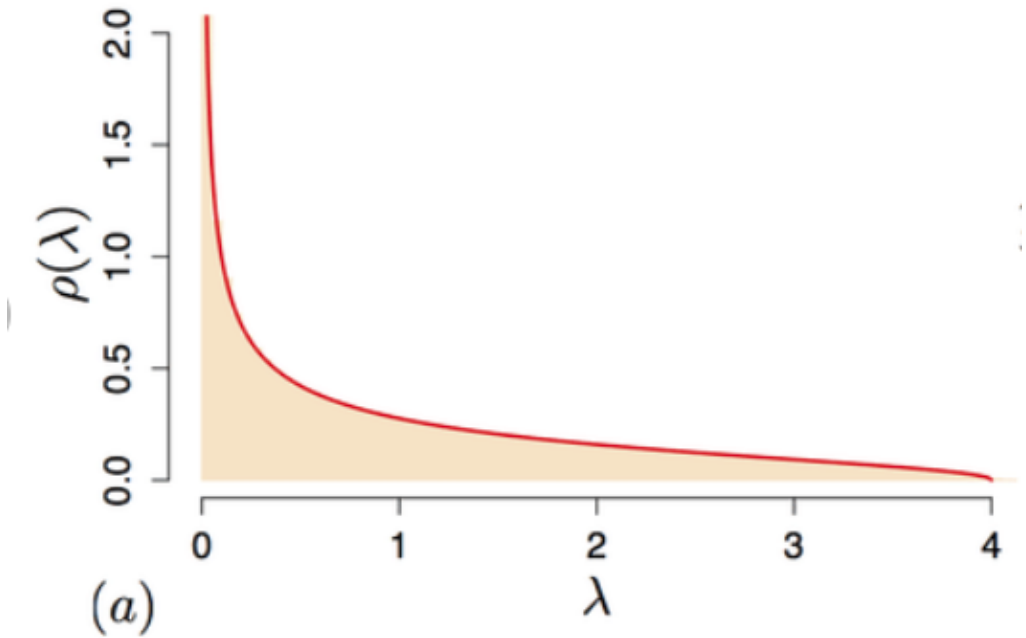
We study the behaviour of eigenvalues of large random covariance matrices, which are $N \times N$ matrices of the form $H_N = X_N^* X_N$, where X_N is a random $N \times N$ matrix whose entries are independent complex random variables with $\mathbb{E}x_{ij} = 0$ and $\mathbb{E}|x_{ij}|^2 = 1$.

We are interested in the asymptotic analysis of the empirical spectral measure of the matrix $X_N^* X_N$ for $N \to \infty$.

This limiting measure is called the Marchenko-Pastur distribution and we will consider its local analysis.

Local laws

By the nature of the eigenvalues in the limiting case, local laws tend to arise, which means that we can zoom in an interval of a constant amount of eigenvalues and still not "lose" the image of this limiting distribution.



Our results

- Optimal scale of zooming
- Removal of logarithmic and polynomial corrections in the local laws
- Results about the rate of convergence to the M-P distribution
- Eigenvalue rigidity properties

Our assumptions

- Finite fourth moments of the initial random matrix entries
- Proper truncation of the initial random matrix entries

Our techniques

- Use of the Stieltjes transformation
- Resolvent identities and inequalities
- Bounds on quadratic forms
- Bootstrapping to local scales
- Plemelj's integration formula

Thank you and happy birthday to Yan!