Signal reconstruction in rough landscapes: *The BBP transition and beyond*

Chiara Cammarota

Dipartimento di Fisica, Sapienza, Università di Roma

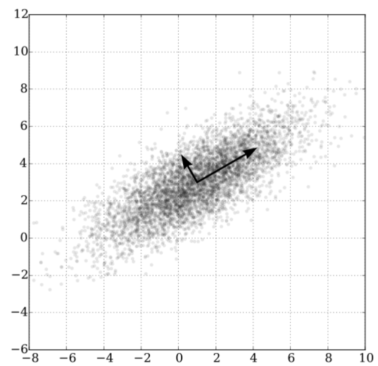
Ros, Ben Arous, Biroli, Cammarota **PRX 2019**
Sarao, Biroli, Cammarota, Krzakala, Urbani, Zdeborova **PRX 2020**
Sarao, Biroli, Cammarota, Krzakala, Zdeborova **Spotlight at NIPS 2019**
Biroli, Cammarota, Ricci-Tersenghi **J. Phys. A: Math. and Theor. 2020**
Sarao, Biroli, Cammarota, Krzakala, Urbani, Zdeborova **NIPS 2020**
Biroli, Cammarota, Ricci-Tersenghi in preparation

Random Matrices and Random Landscapes 28.07.2022

in honour of Yan Fyodorov

Two examples of signal reconstruction

MATRIX PCA, TENSOR PCA, MIXED MODELS



Estimation of rank-one k-tensor from a noisy channel(s)

Observation Corrupting noise Signal

$$T_{i_1,\dots,i_k} = W_{i_1,\dots,i_k} + v_{i_1}\dots v_{i_k}$$

Maximum likelihood estimator: minimum squared distance

$$H_k = -\sum_{(i_1,\dots,i_k)} (T_{i_1,\dots,i_k} - x_{i_1}\dots x_{i_k})^2 \propto -\sum_{(i_1,\dots,i_k)} J_{i_1,\dots,i_k} x_{i_1}\dots x_{i_k} - rN\left(\sum_i \frac{x_i v_i}{N}\right)^k + const$$

with $J_{i_1,\dots,i_k} \propto W_{i_1,\dots,i_k}$ and r signal to noise ratio

..also MIXED matrix / tensor models

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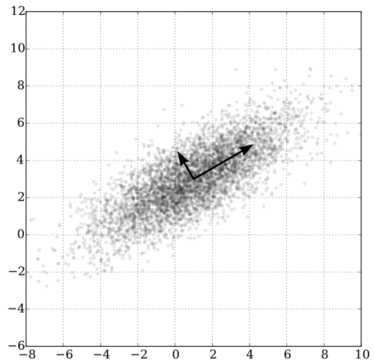

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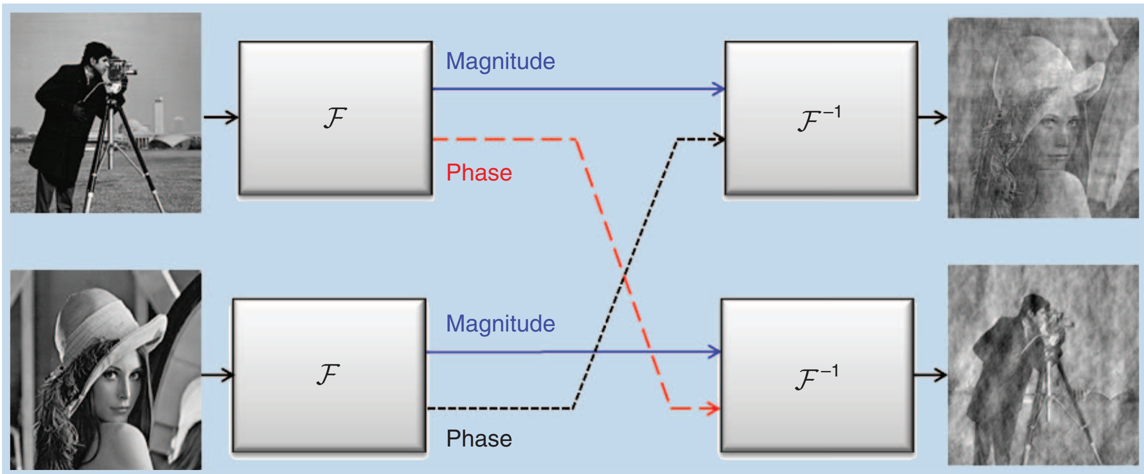

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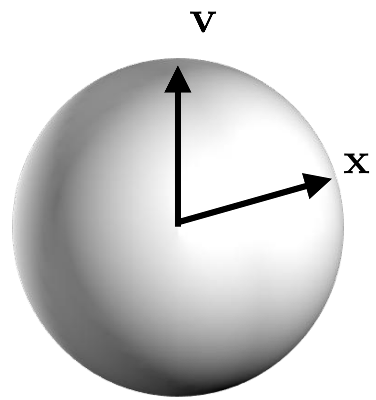

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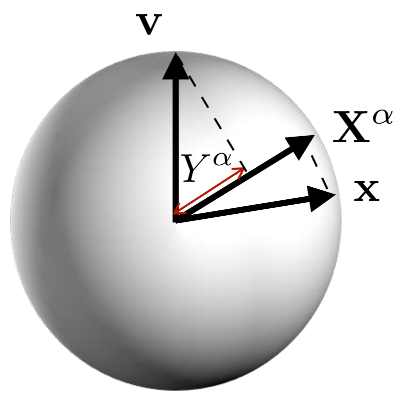
PHASE RETRIEVAL

Retrieval of an N-dimensional vector $\mathbf{v}$ from the knowledge of its projections Y^m on αN random vectors $\mathbf{X}^m$

Cost function

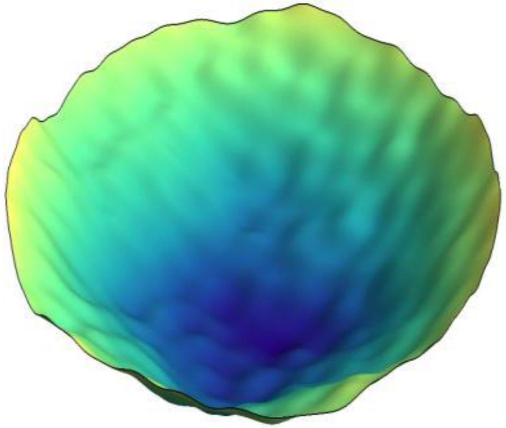
$$\mathscr{L}(\mathbf{x}) = \frac{1}{2}\sum_m^{\alpha N} \frac{(Y^{m2} - \langle \mathbf{x}, \mathbf{X}^m \rangle^2)^2}{g(Y)}$$

with α signal to noise ratio

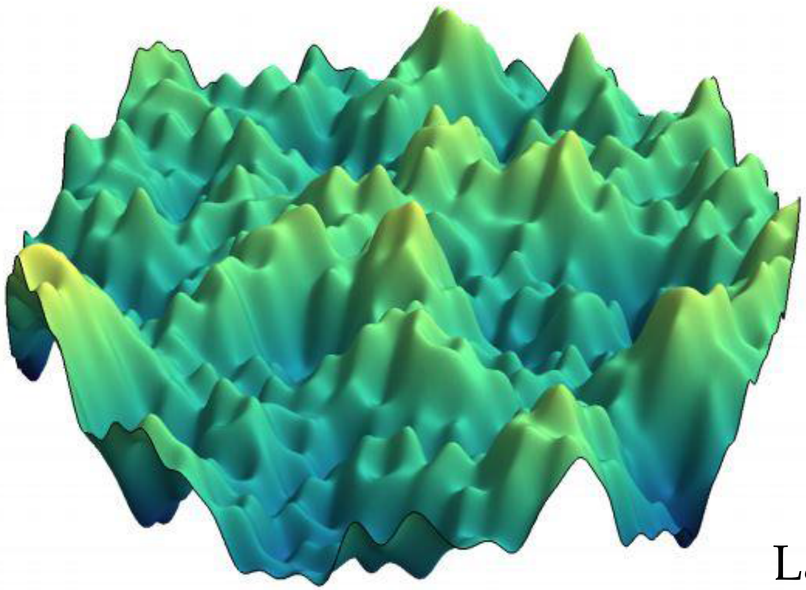


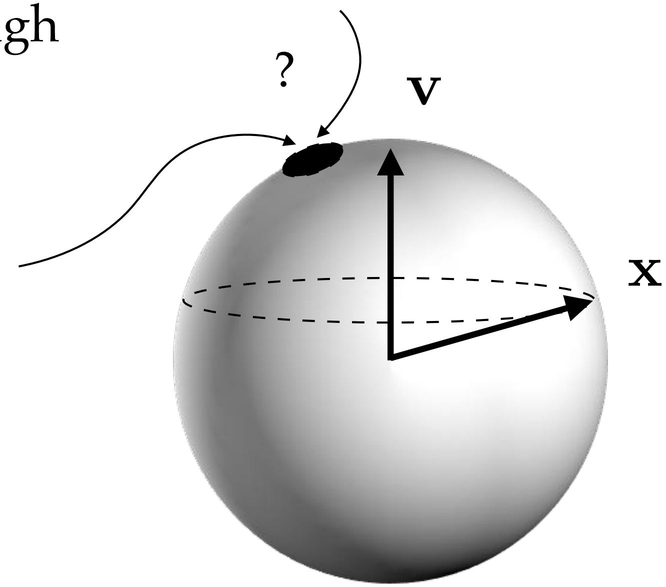
Signal reconstruction in rough landscapes

$$\dot{\mathbf{x}} = -\nabla_{\mathbf{x}}\mathcal{L}(\mathbf{x}(t)) + \mu(t)\mathbf{x}(t)$$



Minimisation via gradient flow on the sphere from random initial condition, where likelihood / cost landscape is rough





Landscape matter: gradient, <u>Hessian</u>

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The Baik-Ben Arous-Péché (BBP) transition

Baik, Ben Arous, Péché **Annals of Prob 2005**

Change of statistics of the largest eigenvalue for non-null $N \times N$ (complex) sample covariance matrices $S = \frac{1}{M}\sum_m^M v_m v_m^T$ from multivariate Gaussian vectors v_m with covariance matrix Σ

If population covariance is Identity or its largest eigenvalue is smaller than $1 + \sqrt{N/M}$

$$\lambda_{MAX}(M) - \lambda_{MAX} \sim M^{-2/3}$$

If largest eigenvalue of population covariance is larger than $1 + \sqrt{N/M}$

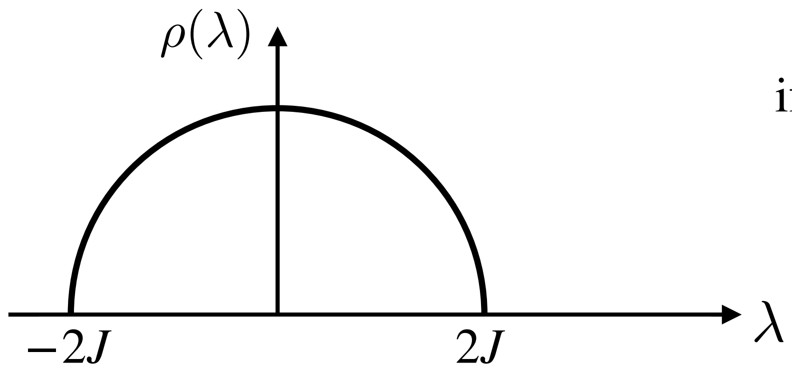
$$\lambda_{MAX}(M) - \lambda_{MAX} \sim M^{-1/2} \gg M^{-2/3}$$

Wigner 1955, Bronk **1974**

Edwards, Jones **J. Phys A 1976**

Re-derivation of Semicircular Law with replicas for (modified) GOE $N \times N$ matrix $H(r)$

with $H_{ij}(r) \sim \mathcal{N}\left(\frac{r}{N}, \frac{J^2}{N}\right)$



if $r < J$

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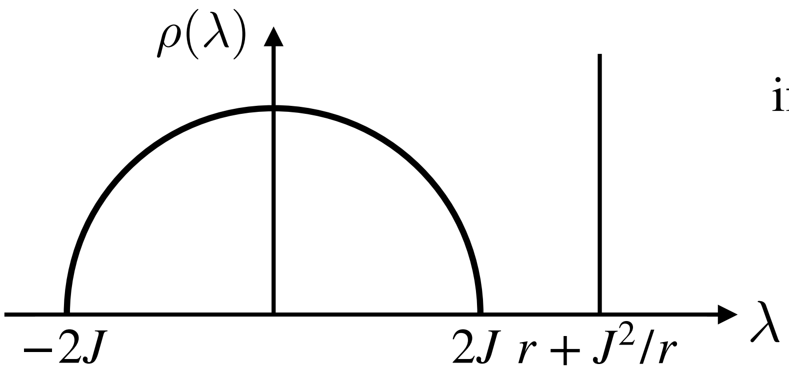


if $r > J$

The ubiquity of BBP transition (some examples)

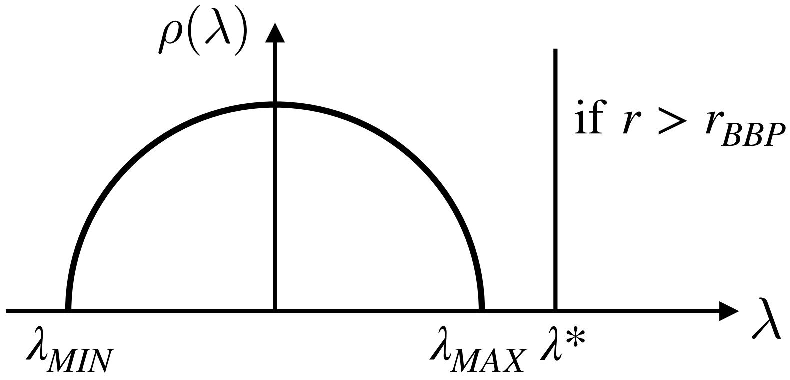
Bouchaud, Potters **The Oxford Handbook of RMT 2015**
Covariance matrices cleaning (e.g. finance..)

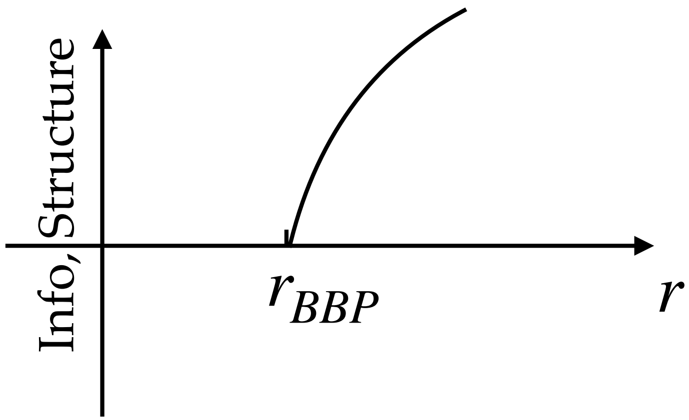
Zebari et al. **JASTT 2020**
Dimensional reduction and feature extraction in data science

Montanari, Reichman, Zeitouni **IEEE Trans. Inf. Theo. 2014**
Inference (e.g. clique reconstruction task)

Fraboul, Biroli, De Monte **arXiv:2112.06845 2021**
Evolution of ecosystems under selection pressure

General mechanism for emergence of information (or specific underlying structure)



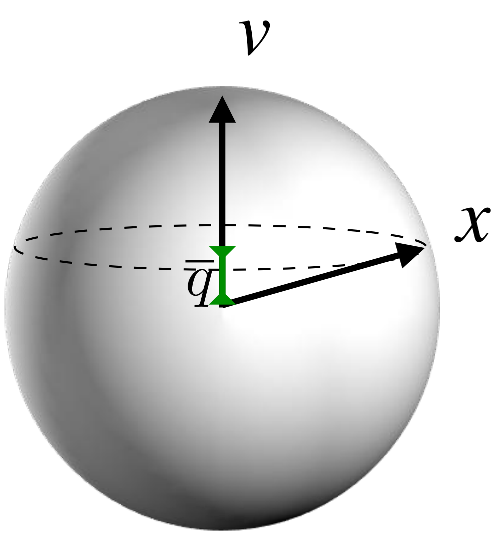


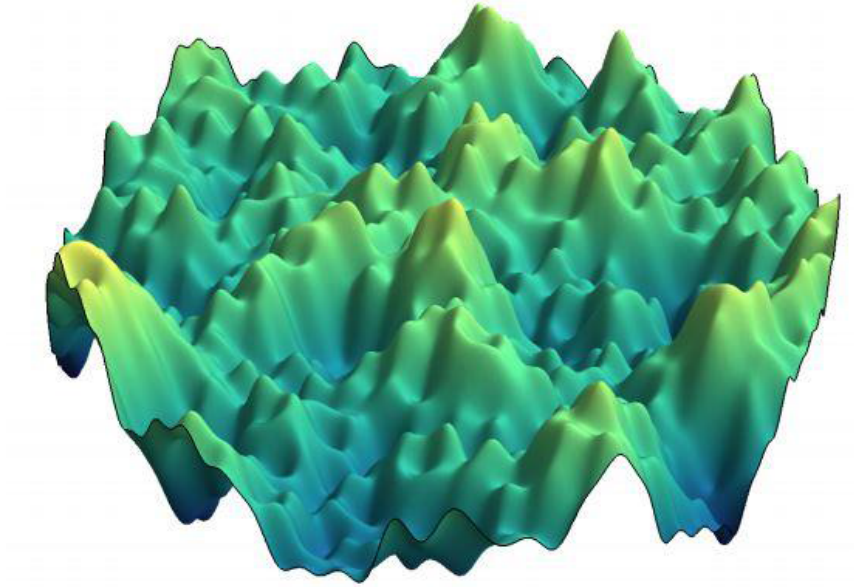
Signal reconstruction as a BBP transition

Long time limit of **gradient flow**, are stationary points stable?

Information after a **warm start**: Hessian in specific regions of the landscape

Eigenvector of the smallest eigenvalue of the initial Hessian: **spectral initialisation**





TENSOR PCA

$$\mathscr{H}(H_k)_{ij} = -\sum_{(i_3,\ldots,i_k)} J_{i,j,i_3,\ldots,i_k} x_{i_3}\ldots x_{i_k} - r\frac{k(k-1)}{N} v_i v_j \left(\sum_l \frac{x_l v_l}{N}\right)^{k-2}$$

PHASE RETRIEVAL

$$\mathscr{H}(\mathscr{L})_{ij} = -2\sum_m^{\alpha N} X_i^m X_j^m \frac{Y^{m2} - 3\langle x X^m\rangle^2}{g(Y^m)}$$

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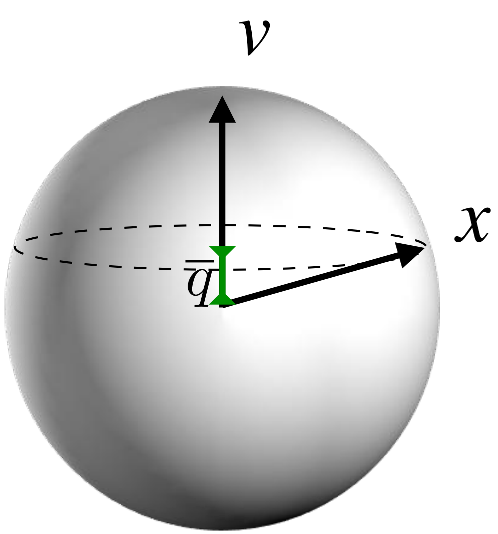

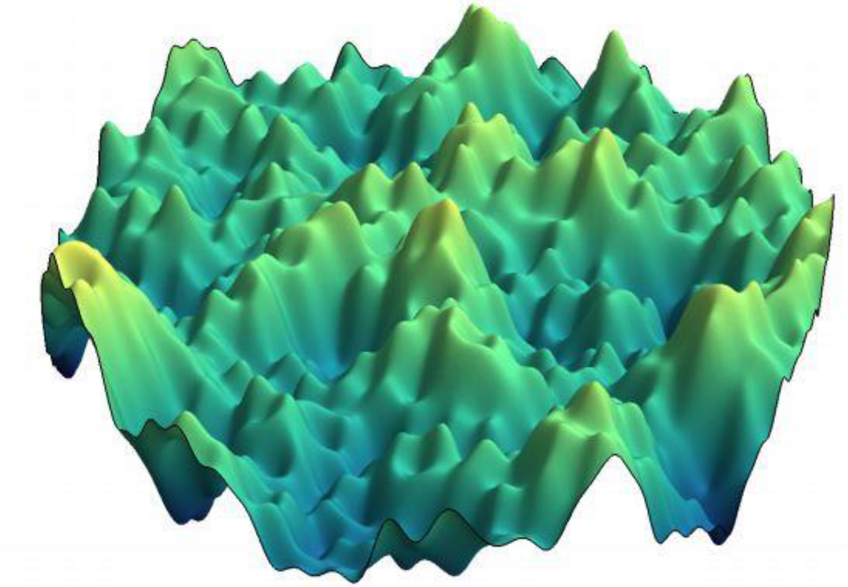

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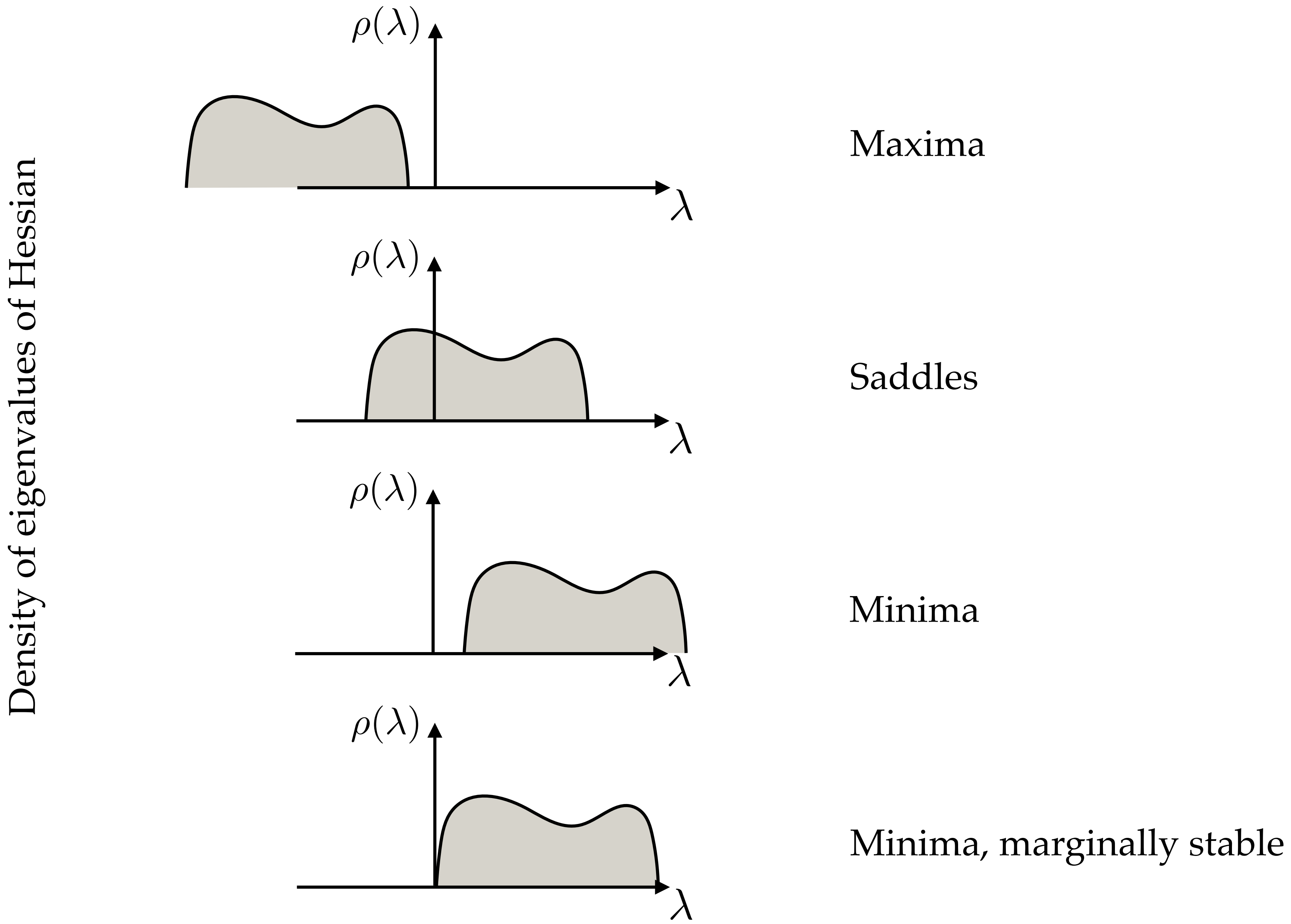
The simplest case, Matrix PCA: $\mathcal{H}(H_2)_{ij} = -J_{ij} - r\frac{2}{N} v_i v_j$

PHASE RETRIEVAL

$$\mathcal{H}(\mathcal{L})_{ij} = -2\sum_m^{\alpha N} X_i^m X_j^m \frac{Y^{m2} - 3\langle xX^m\rangle^2}{g(Y^m)}$$

A large variety of stationary points



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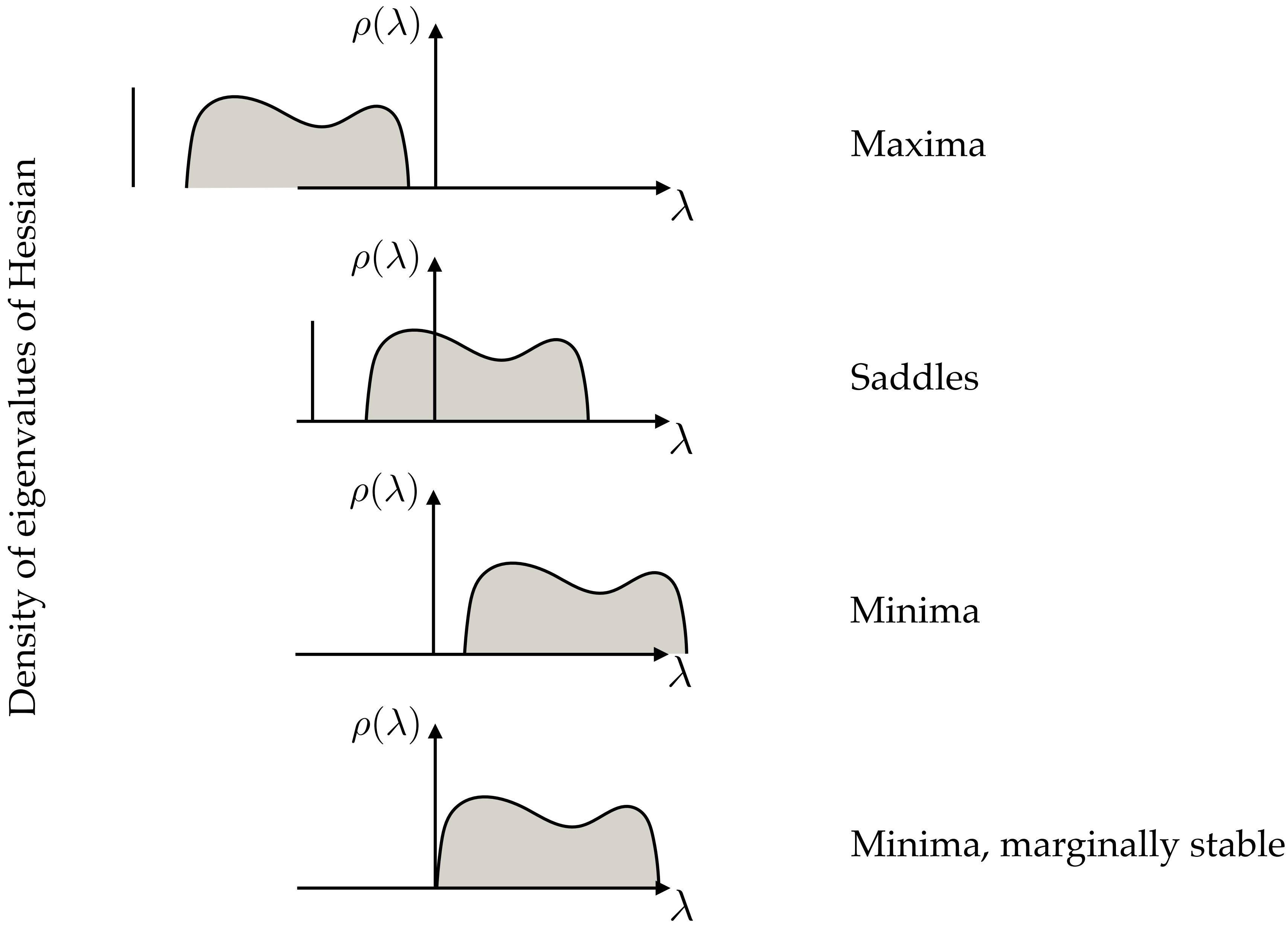

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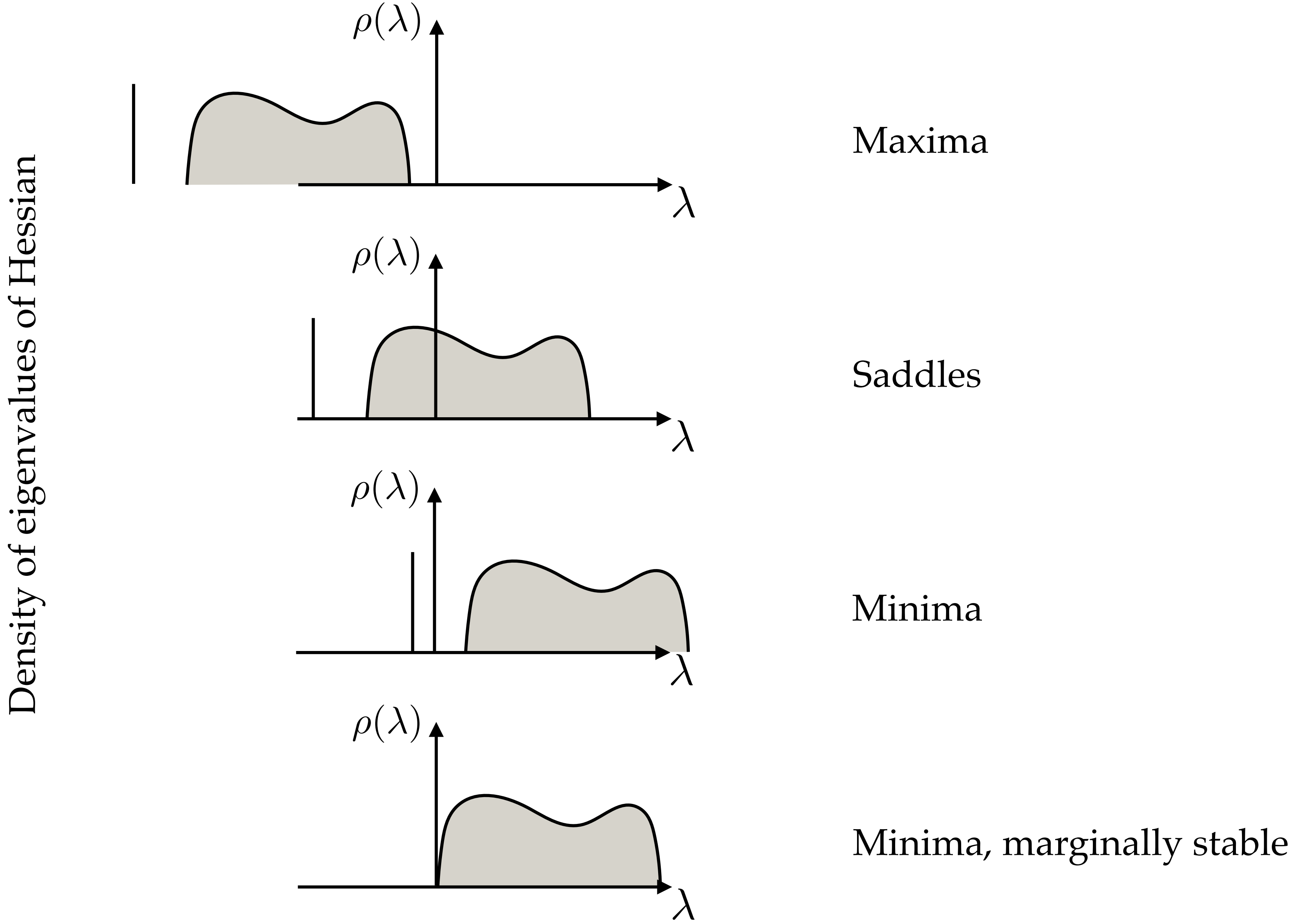

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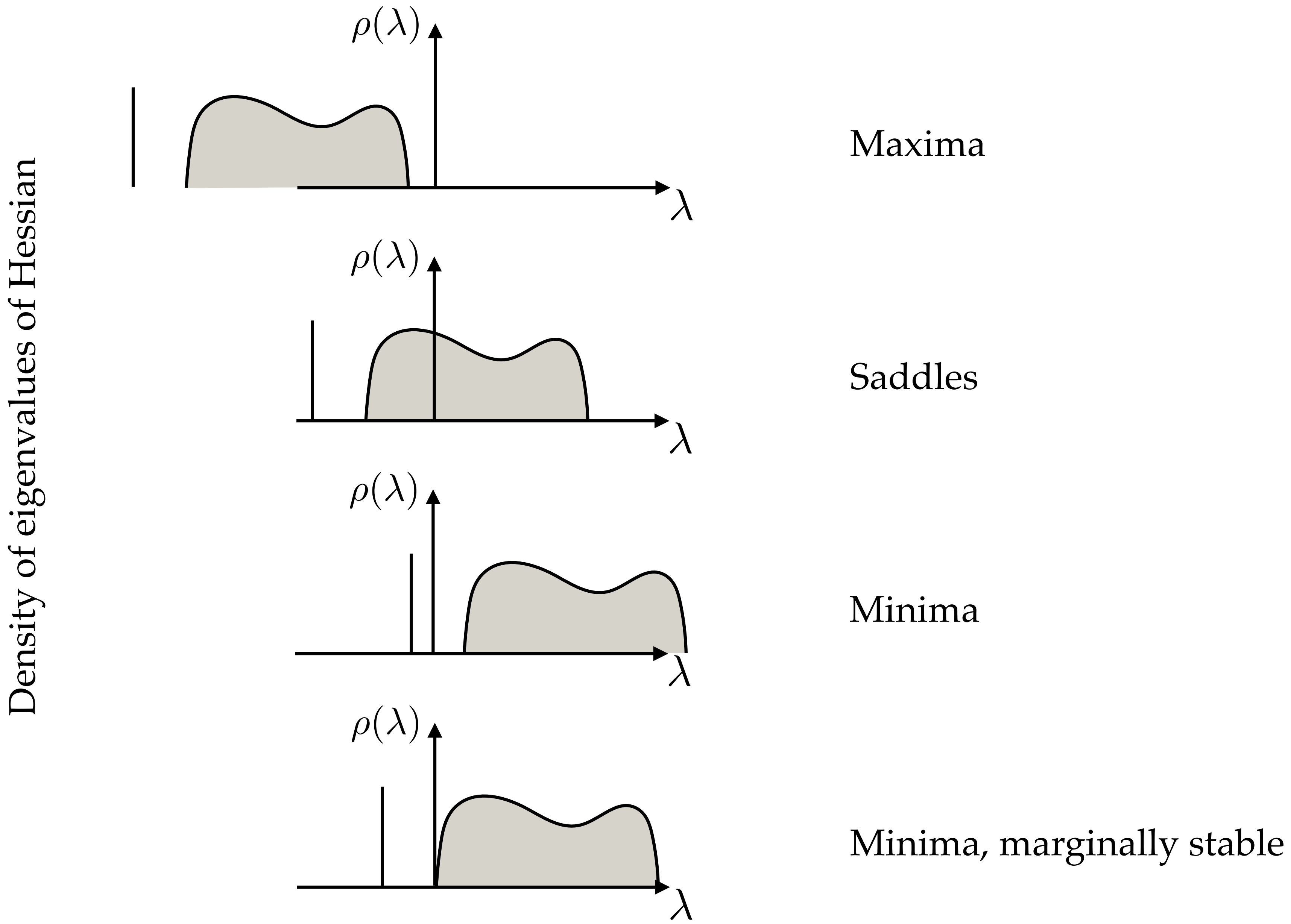

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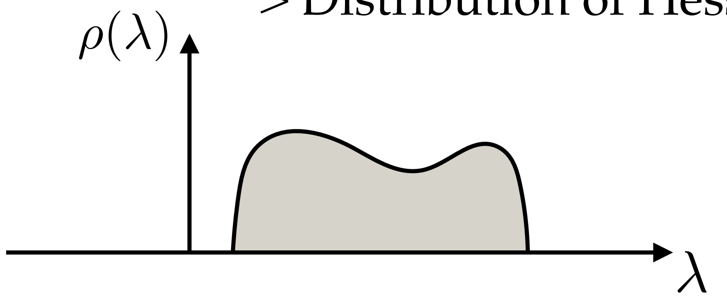
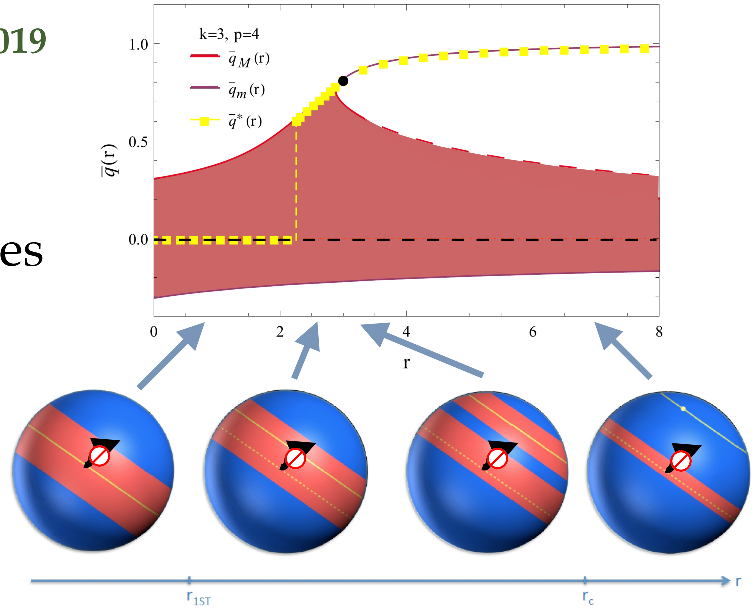
(*Fyodorov* **PRL 2004**, …,) *Ros, Ben Arous, Biroli, Cammarota* **PRX 2019**

Hessian at special stationary points in the landscape:
counting of **minima** > Structure of stationary points
> Distribution of Hessians eigenvalues





How gradient flow escapes minima? *Sarao, Biroli, Cammarota, Krzakala, Zdeborova* **Spotlight at NIPS 2019**

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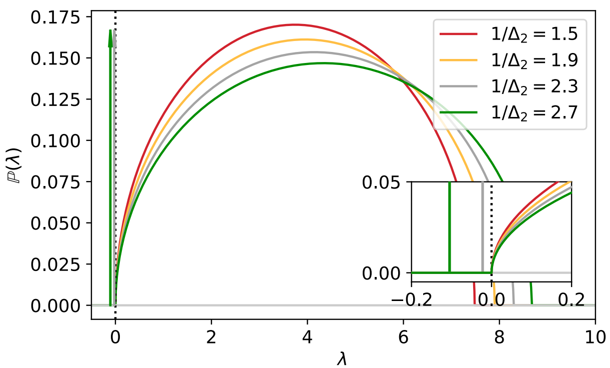

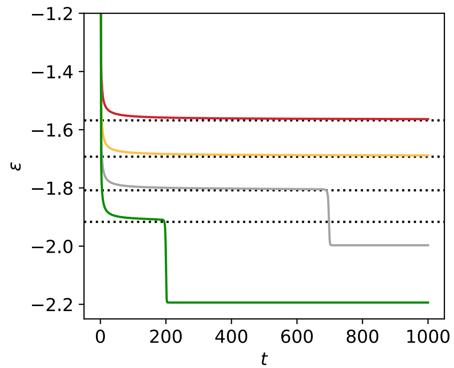

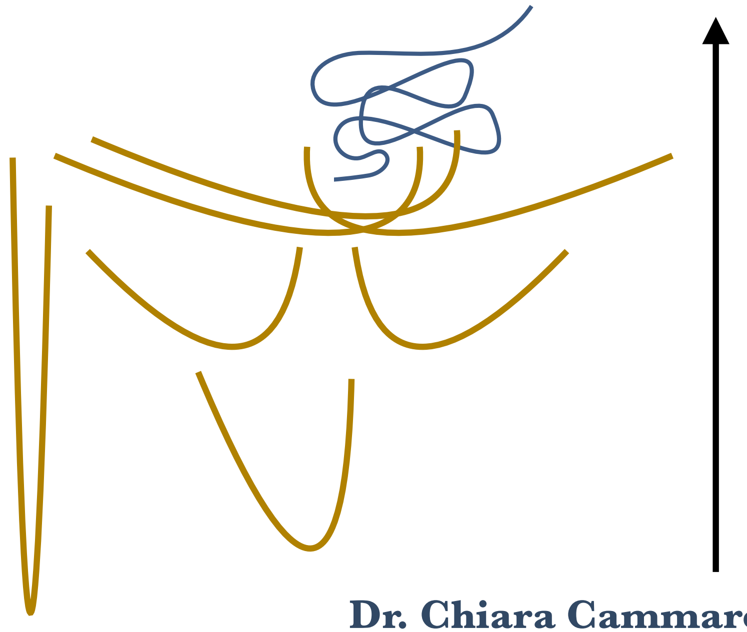

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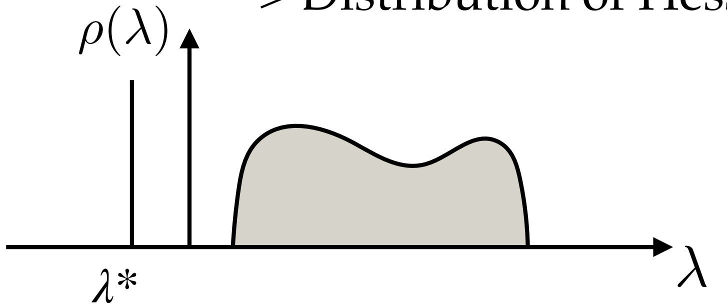

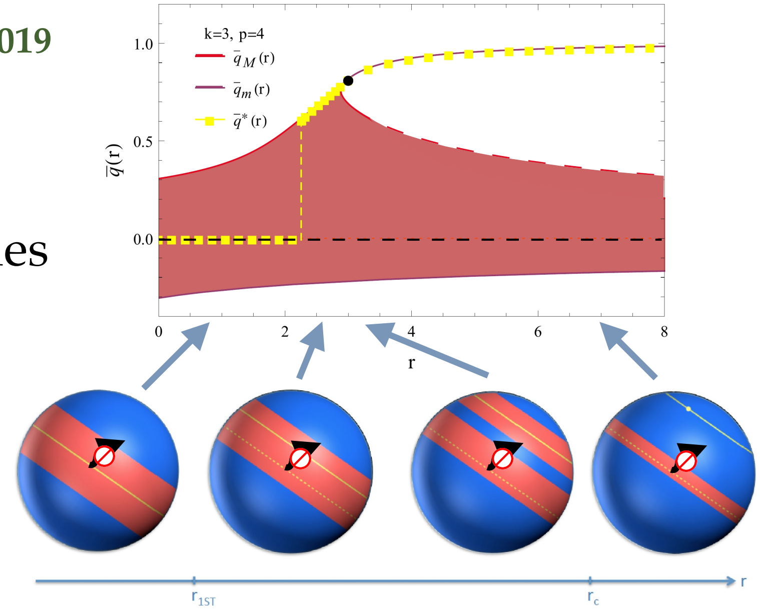

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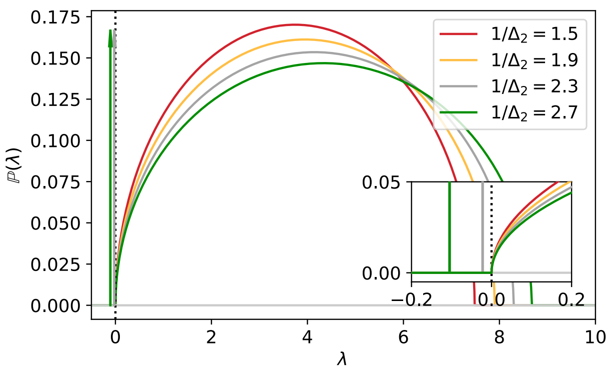

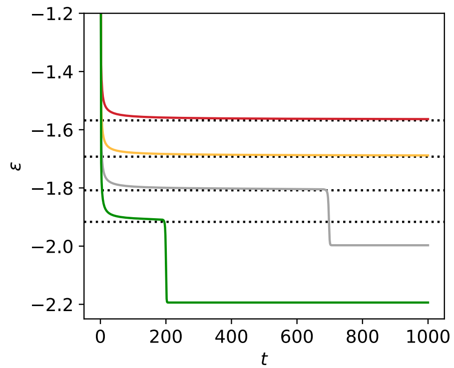

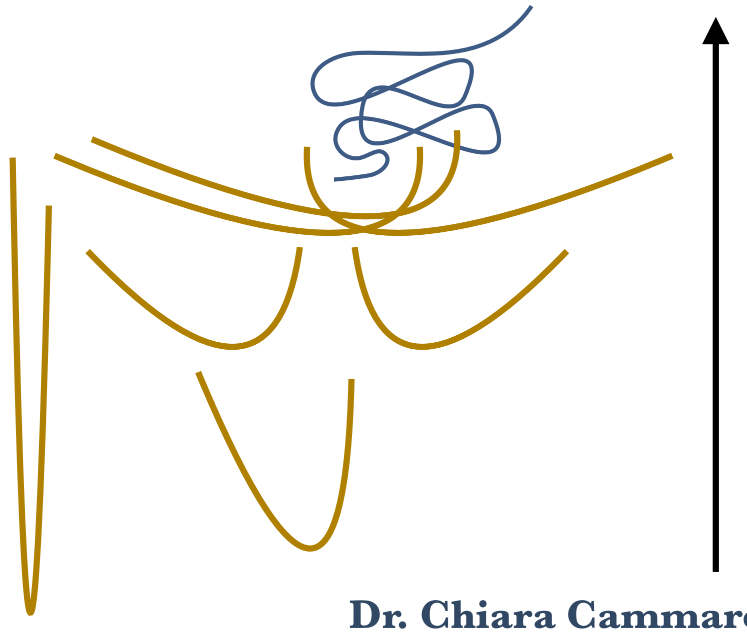

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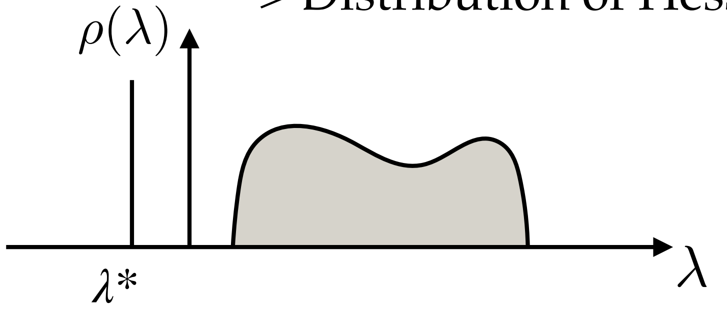

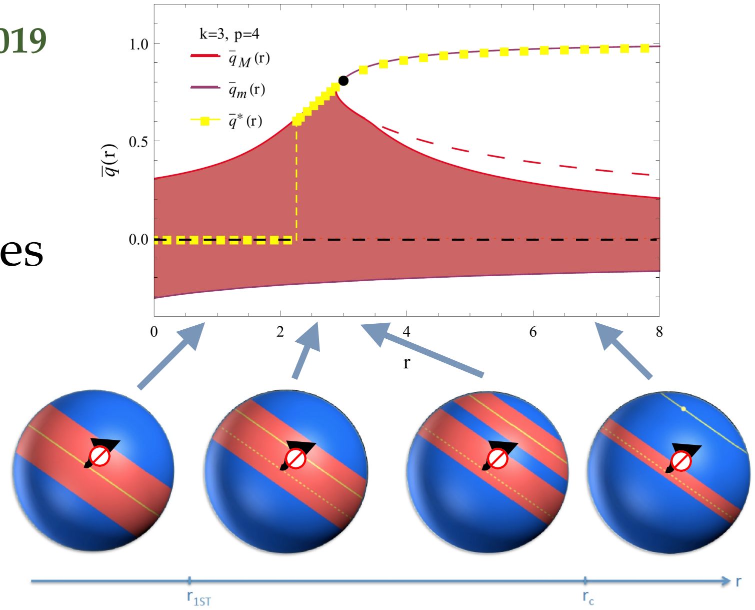

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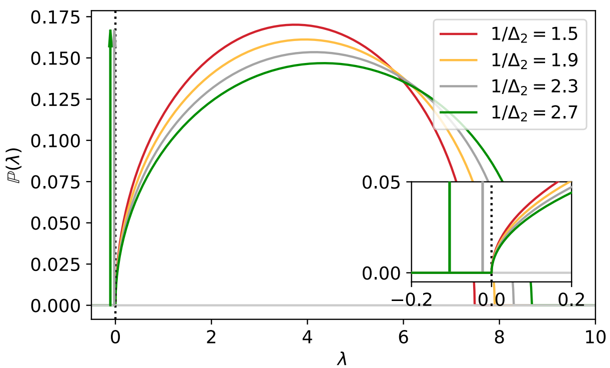

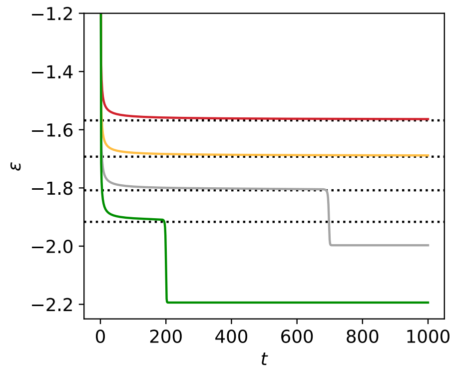

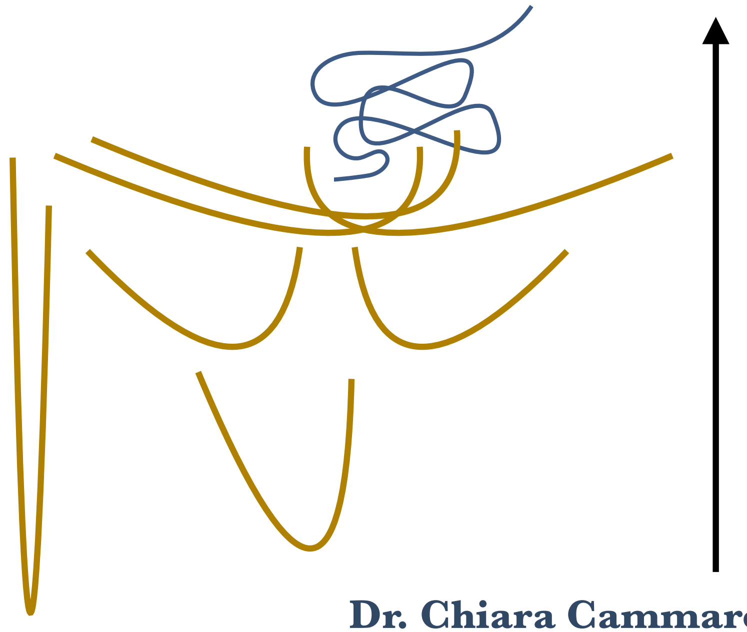

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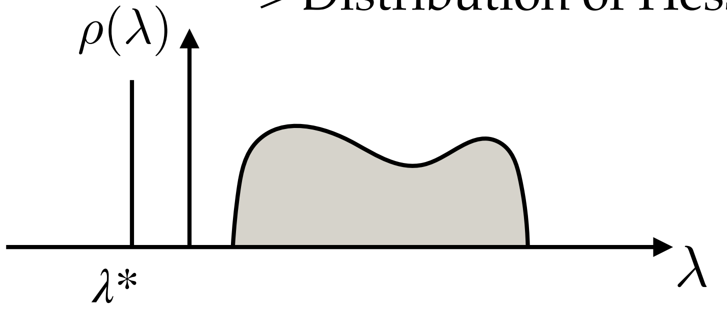

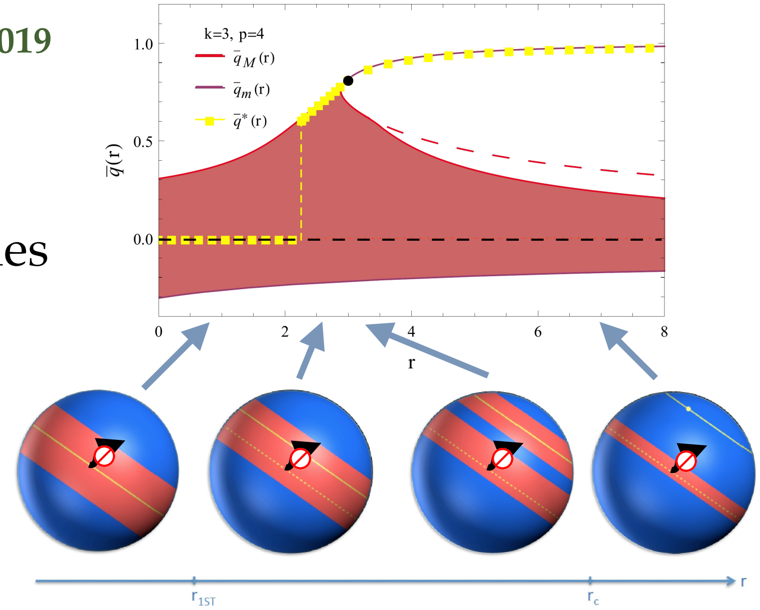

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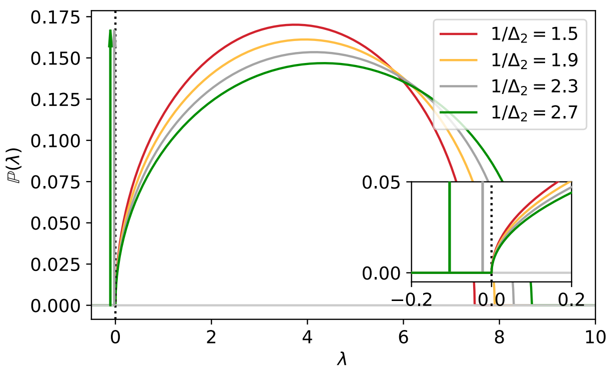

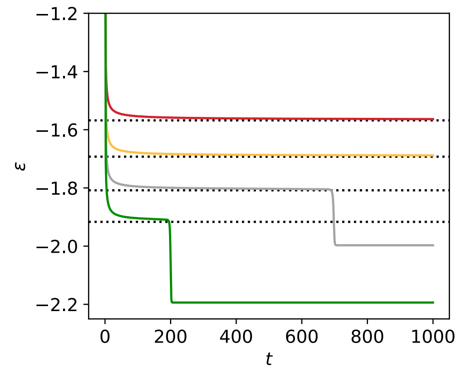

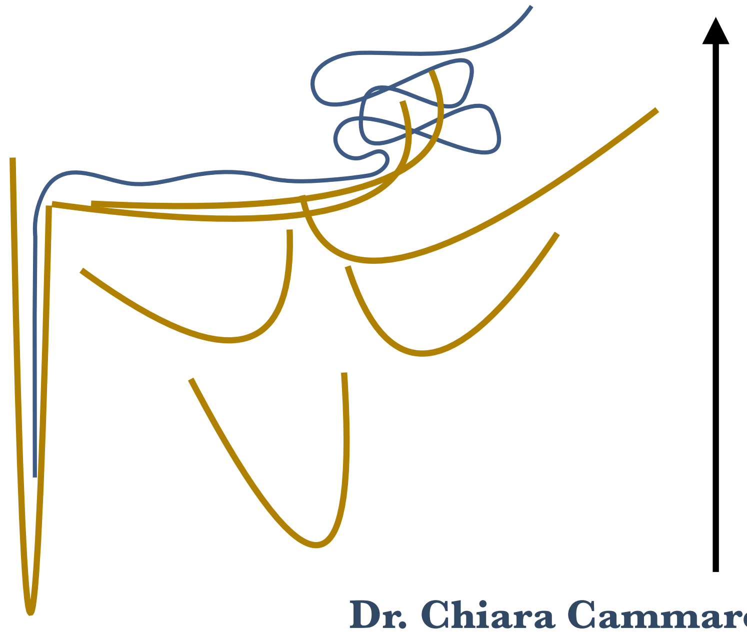

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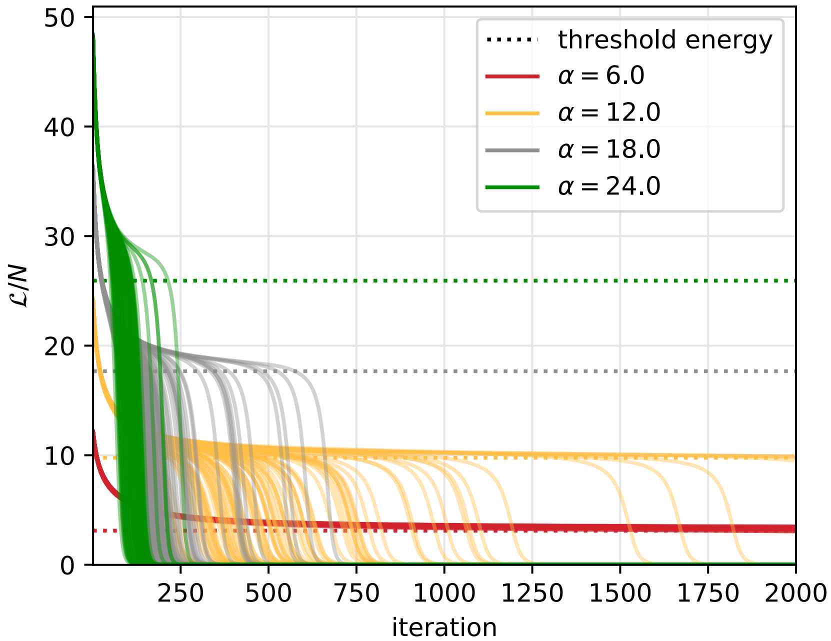
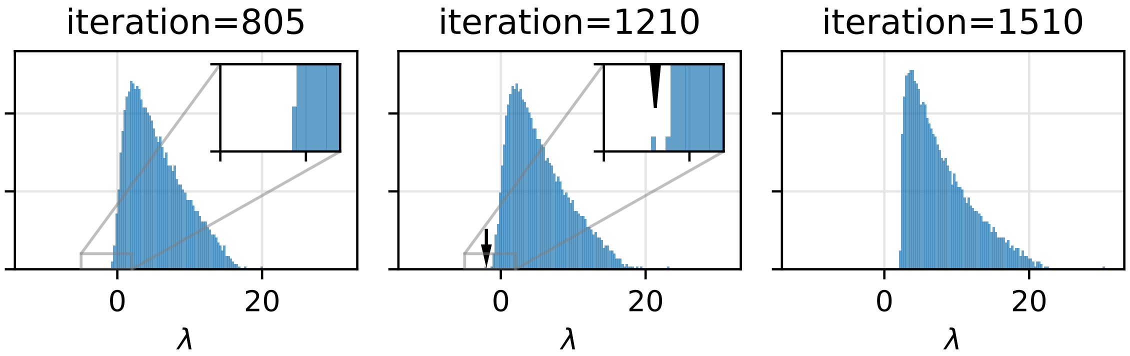
Phase Retrieval: a very similar story but…

Structure of stationary points *Maillard, Ben Arous, Biroli* **MSML 2020**

Similar mechanism to escape minima *Sarao, Biroli, Cammarota, Krzakala, Urbani, Zdeborova* **NIPS 2020**





Evidence of BBP transition but disagreement with numerical results

Different mechanism at work?

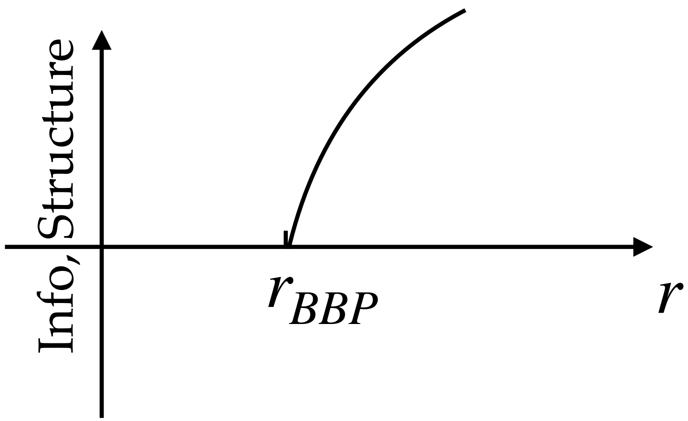
Only very strong and (favourable?) finite size effects?

Phase Retrieval: spectra initialisation and a new BBP?

Mondelli, Montanari PMLR 2017; Luo, Alghamdi, Lu 2018; Lu, Li 2019

Spectra initialisation as alternative algorithmic approach:

$$\mathcal{H}(\mathcal{L})_{ij} = \sum_{m}^{\alpha N} X_i^m X_j^m \ell(Y^m) \quad \text{with} \quad \ell(Y) = \frac{Y^2 - A}{B + CY^2}$$



$$\ell(Y) = 1 \quad \text{if } Y^2 > t; \quad = 0 \text{ otherwise}$$

$$\ell(Y) = \frac{Y^2 - 1}{Y^2 + \sqrt{\delta} - 1}$$

$$\ell(Y) = \max\left(\frac{Y^2 - 1}{Y^2 + \sqrt{\delta} - 1}, 0\right)$$

Hessian has identical structure, with special pre-processing function

$$\mathcal{H}(\mathcal{L})_{ij} = -2 \sum_{m}^{\alpha N} X_i^m X_j^m \frac{(Y^{m2} - 3\langle x X^m \rangle^2)}{g(Y^m)}$$

For some choices a **new type** of BBP transition is leading to signal reconstruction!

Bouchaud Potters A **First Course in Random Matrix Theory** *2021*

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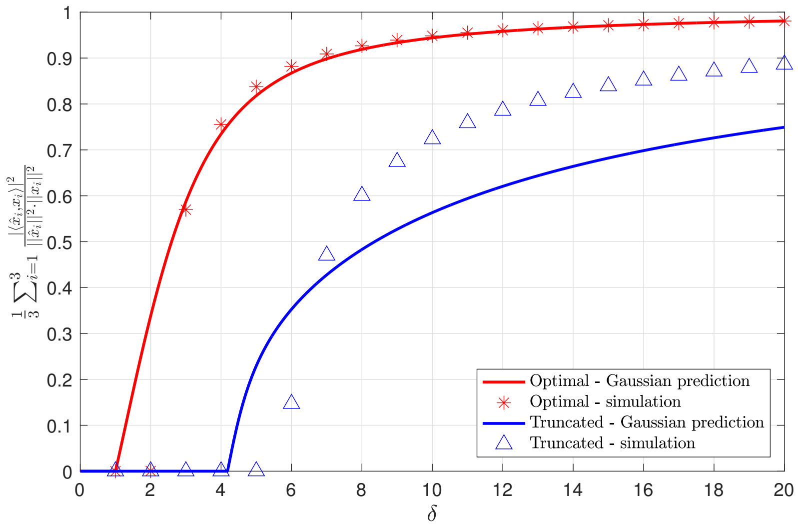
Mondelli, Montanari PMLR 2017; Luo, Alghamdi, Lu 2018; Lu, Li 2019

Spectra initialisation as alternative algorithmic approach:

$$\mathscr{H}(\mathscr{L})_{ij} = \sum_{m}^{\alpha N} X_i^m X_j^m \ell(Y^m) \quad \text{with} \quad \ell(Y) = \frac{Y^2 - A}{B + CY^2}$$



$$\ell(Y) = 1 \quad \text{if } Y^2 > t; \quad = 0 \text{ otherwise}$$

$$\ell(Y) = \frac{Y^2 - 1}{Y^2 + \sqrt{\delta} - 1}$$

$$\ell(Y) = \max\left(\frac{Y^2 - 1}{Y^2 + \sqrt{\delta} - 1}, 0\right)$$

Hessian has identical structure, with special pre-processing function

$$\mathscr{H}(\mathscr{L})_{ij} = -2 \sum_{m}^{\alpha N} X_i^m X_j^m \frac{(Y^{m2} - 3\langle x X^m \rangle^2)}{g(Y^m)}$$

For some choices a **new type** of BBP transition is leading to signal reconstruction!

Bouchaud Potters **A First Course in Random Matrix Theory** *2021*

Continuous vs discontinuous BBP transition

Biroli, Cammarota, Ricci Tersenghi, in preparation

Deformed GOE matrices and rank one perturbation

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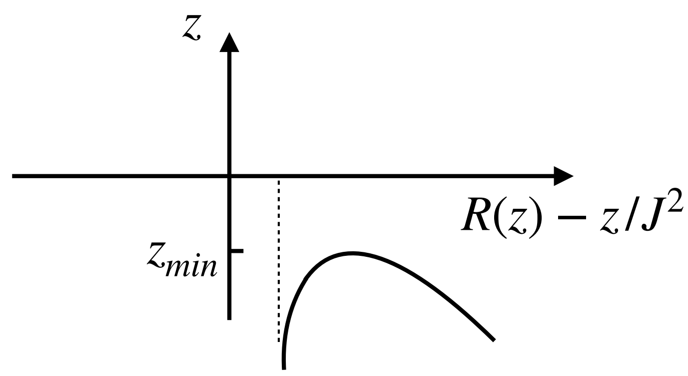
$$a_i \sim p(a) \simeq (a - a_{min})^{\phi - 1}$$



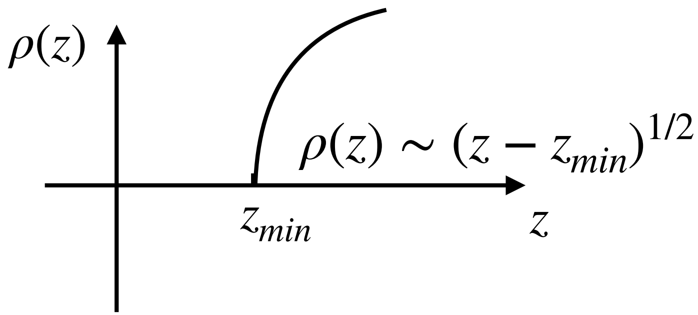
Equation for density of eigenvalues from the Resolvent

$$R(z) \left(= \lim_{N \to \infty} \frac{1}{N} \sum_i \frac{1}{z - \lambda_i} \right) = \int_{a_{min}}^{a_{MAX}} da \; p(a) \; \frac{1}{z - a - J^2 R(z)}$$

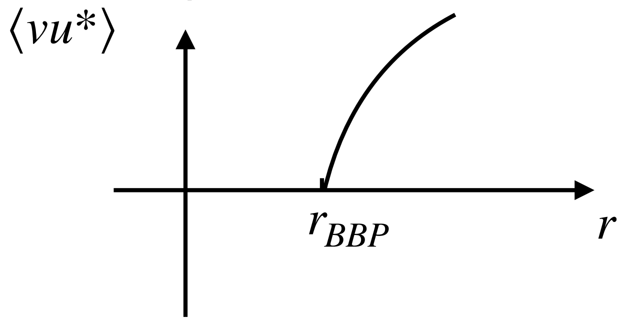
Equation for isolated eigenvalue for $z^* < z_{min} : R(z^*) = 1/r$

Continuous BBP ($\phi < 2$)

Spectrum



Projection on signal



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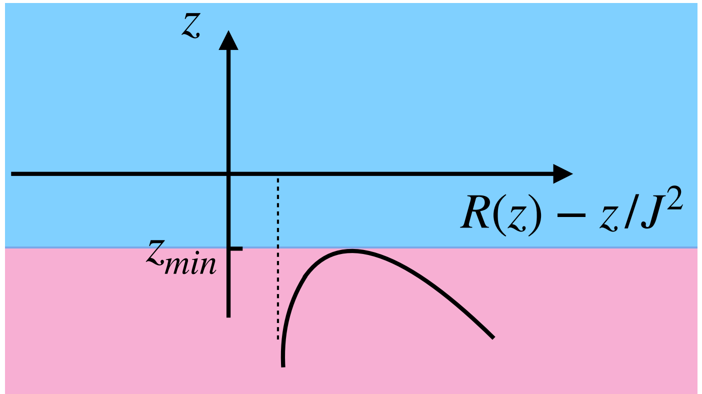

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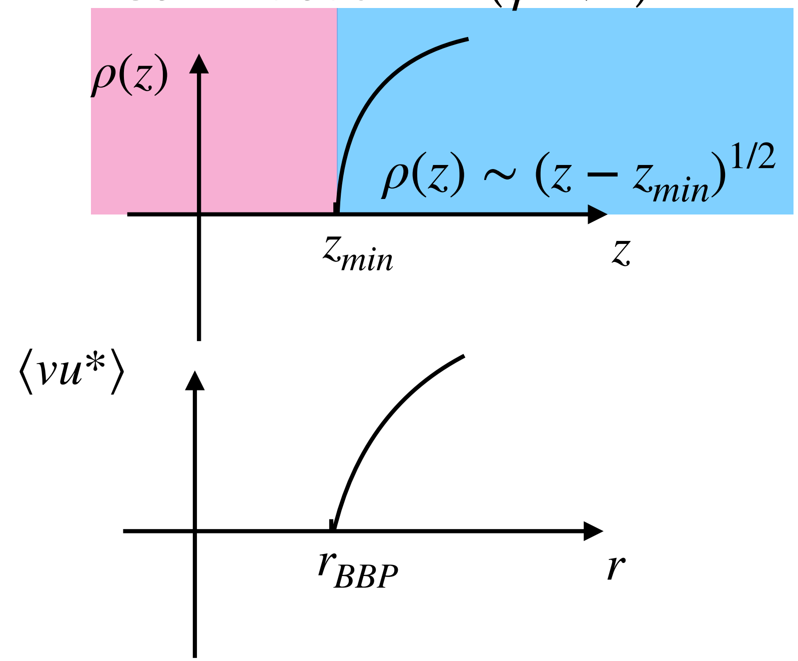

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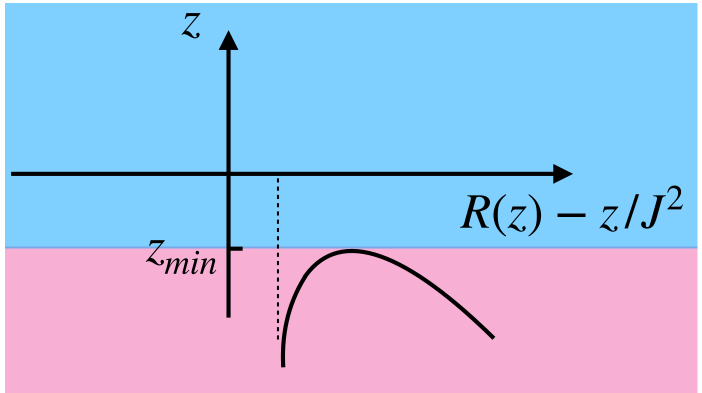

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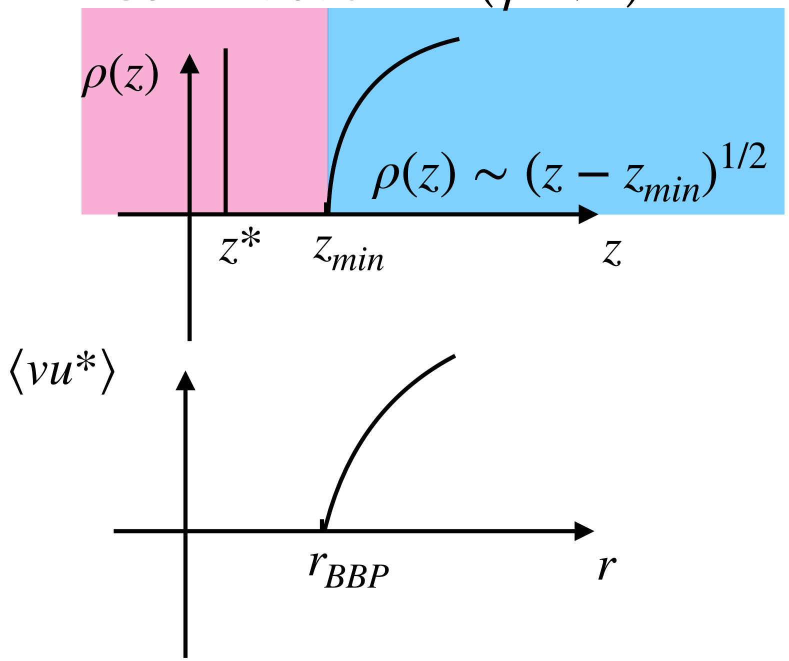

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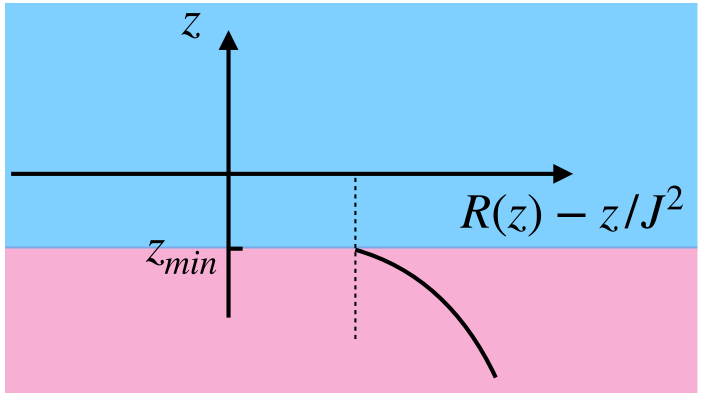

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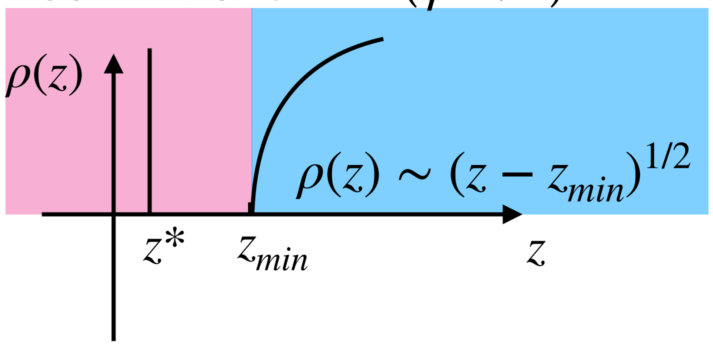

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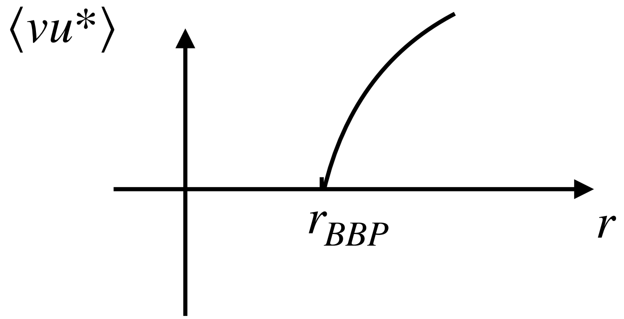

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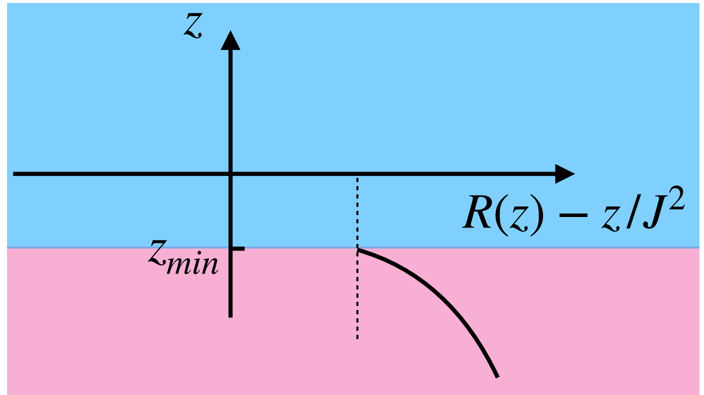

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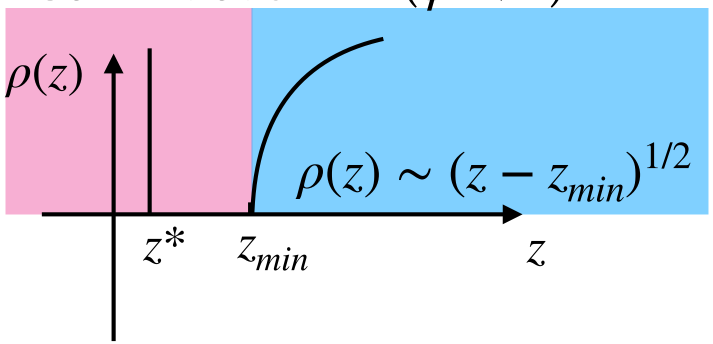
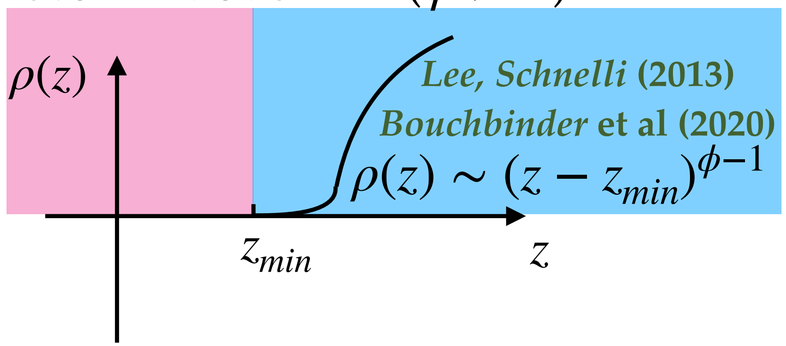
Continuous BBP ($\phi < 2$)

Discontinuous BBP ($\phi > 2$)

Spectrum





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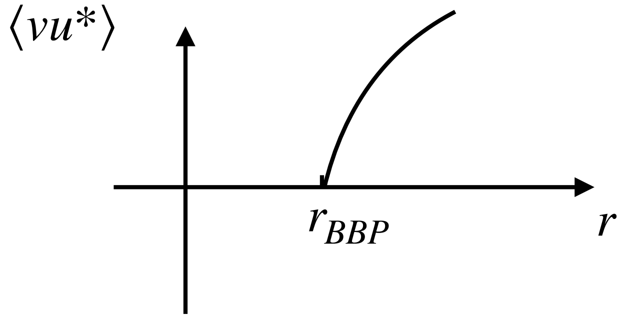

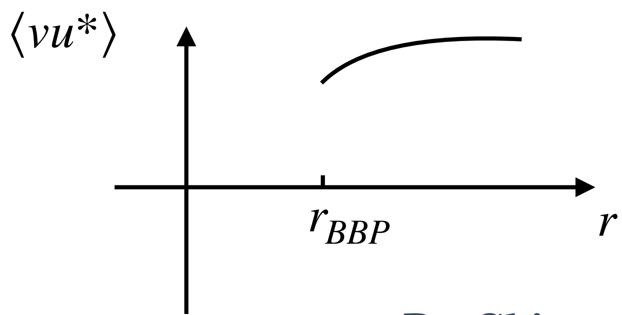

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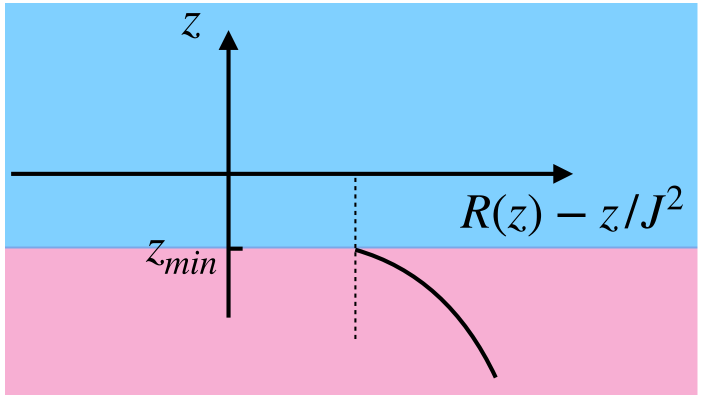

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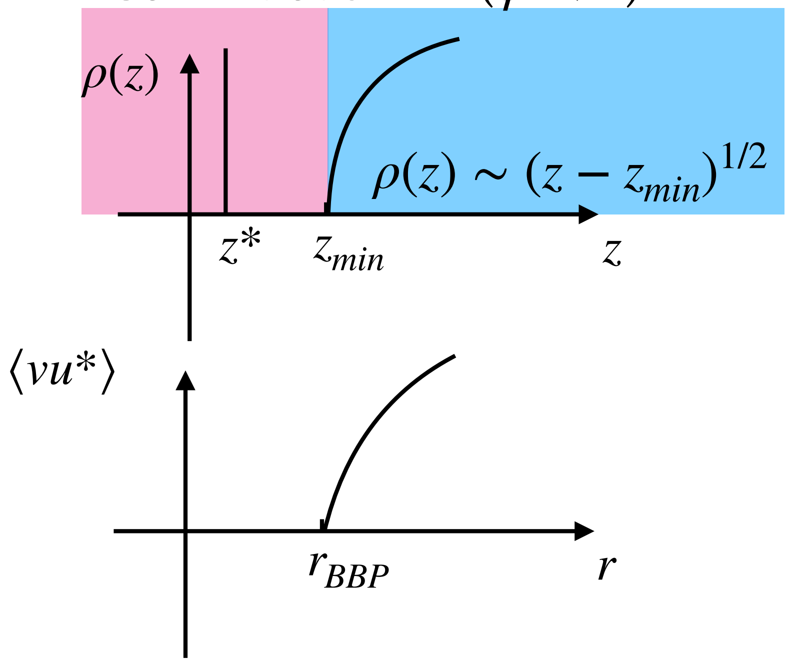
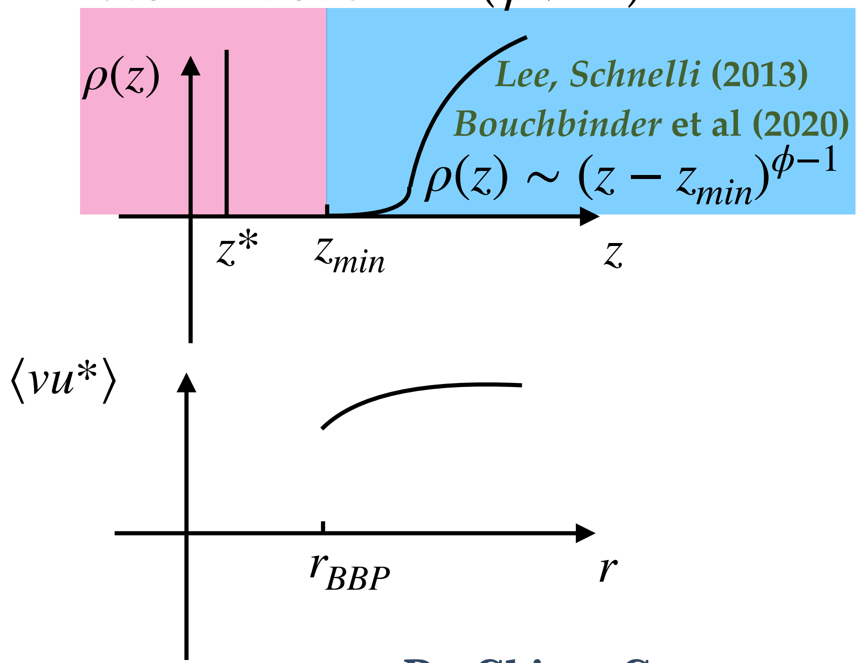


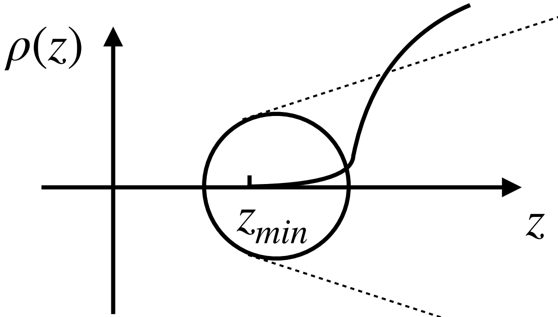
Discontinuous BBP ($\phi > 2$)

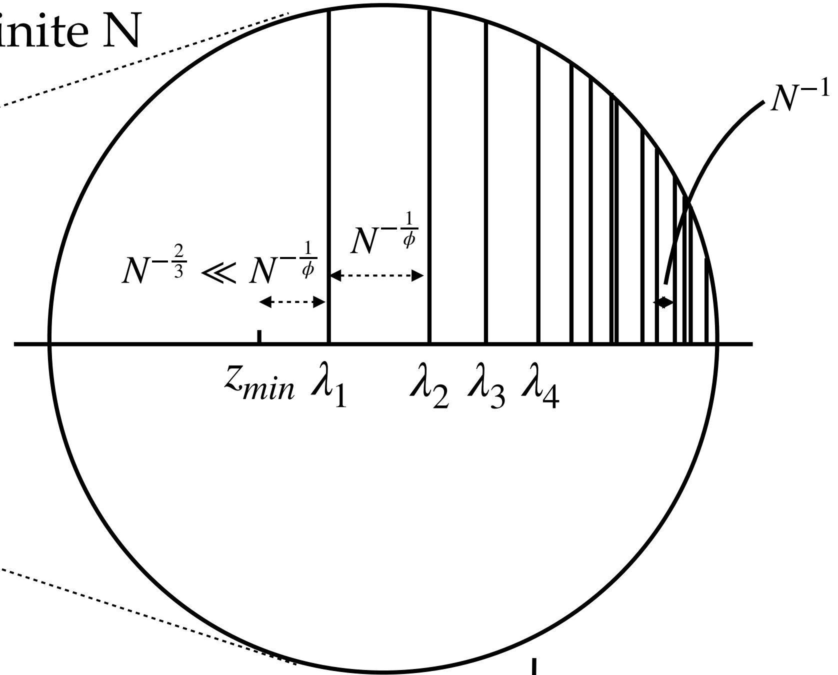


Strong finite size effects

Biroli, Cammarota, Ricci Tersenghi, **in preparation**

Spectrum and its zoom: level spacing statistics for finite N





The transition is anticipated! $r_{BBP}(N) < r_{BBP}$

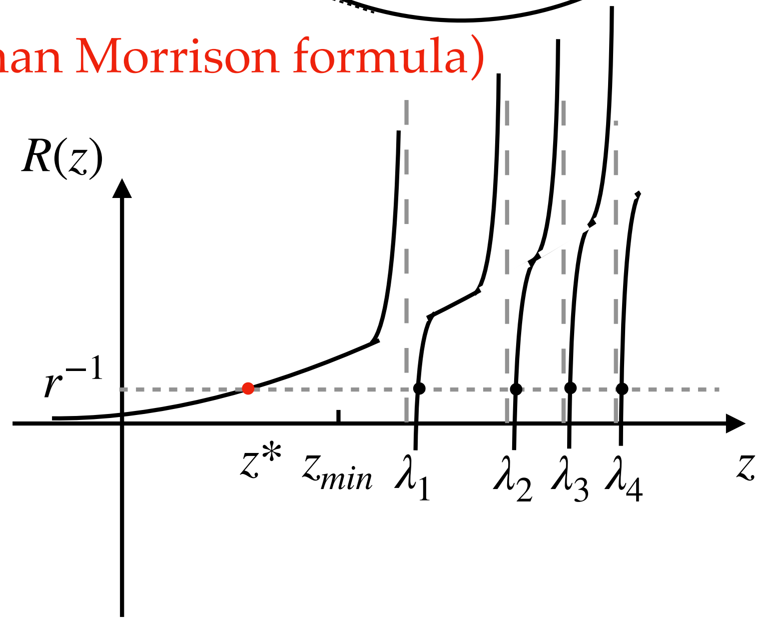
Following BBP eigenvalue z^* in the bulk (Sherman Morrison formula)

$$\frac{1}{r} = R(z) = \mathscr{P}\int \frac{\rho(\lambda)}{z-\lambda}d\lambda + \frac{1}{N}\frac{1}{z-\lambda_{j(z)}}$$

Solutions: $z^*, \tilde{\lambda}_j$

Different statistics $\delta z^* \sim 1/\sqrt{N}$ $\delta\tilde{\lambda}_j \sim N^{-1/\phi}$

BBP eigenvector reveals signal! $\langle vu^* \rangle = O(1)$



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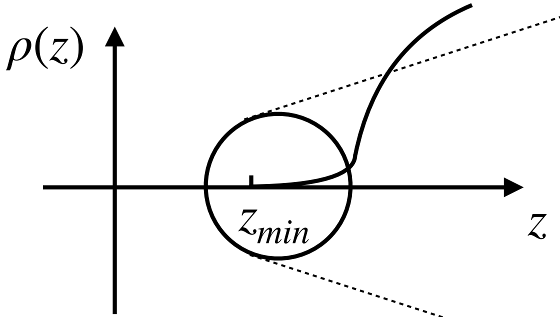

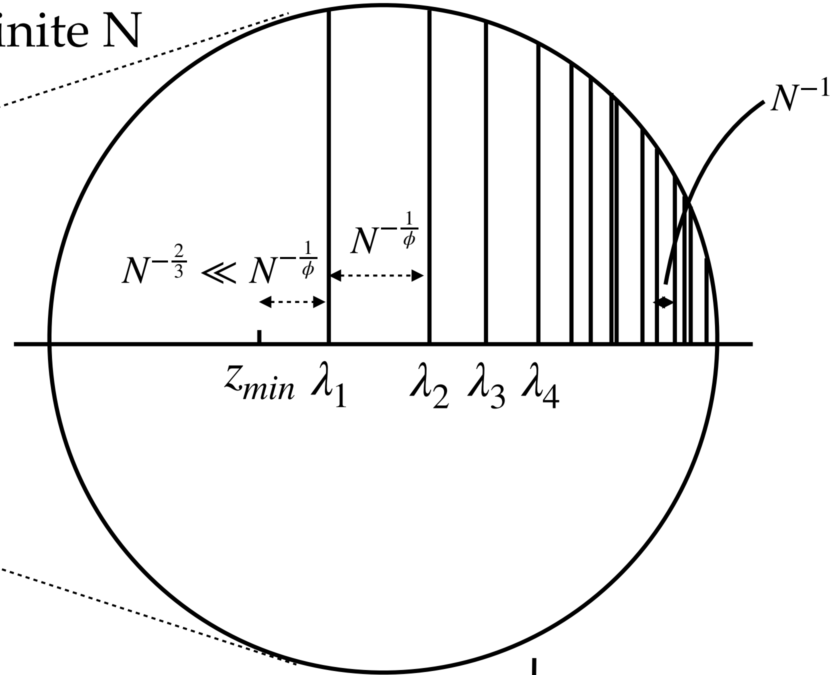

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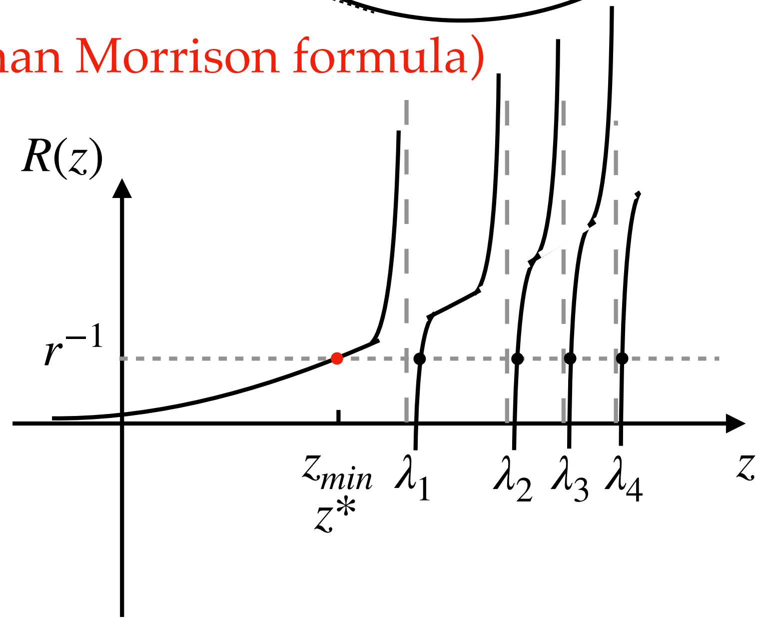

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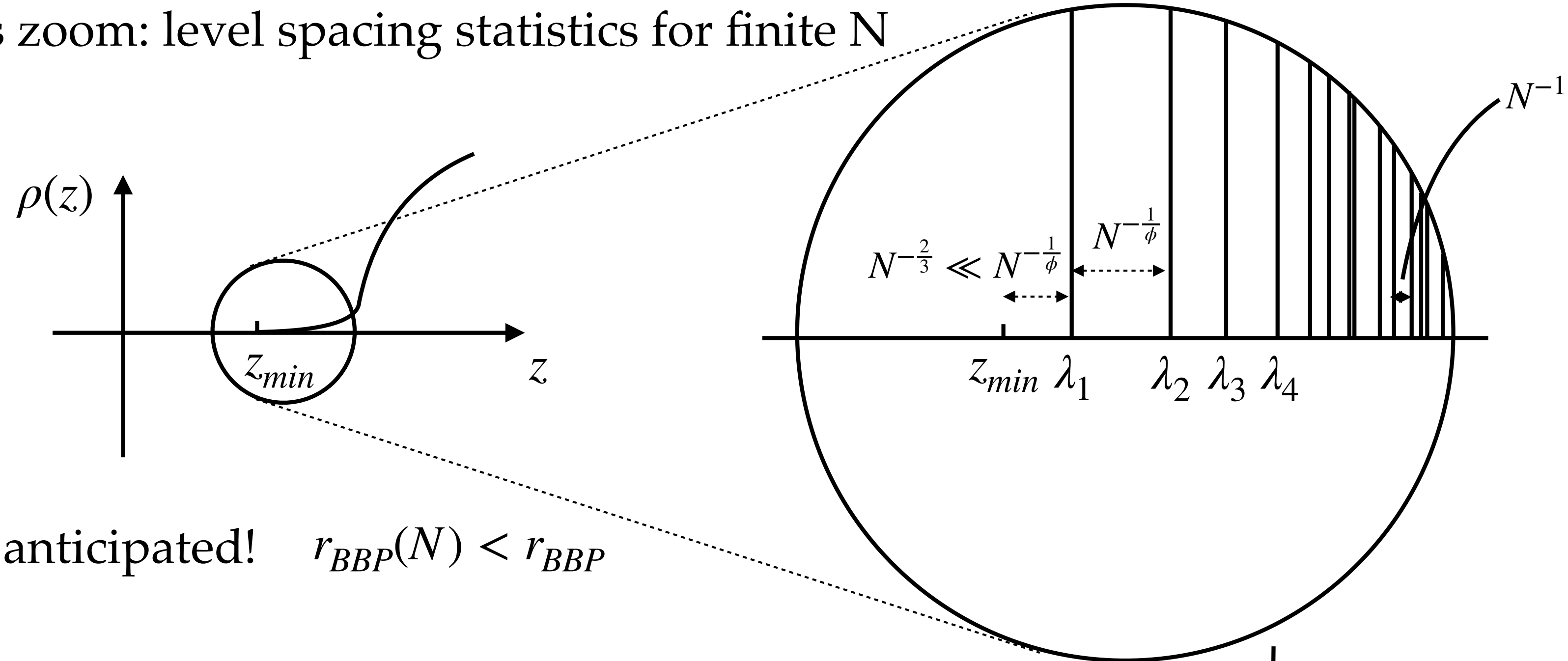

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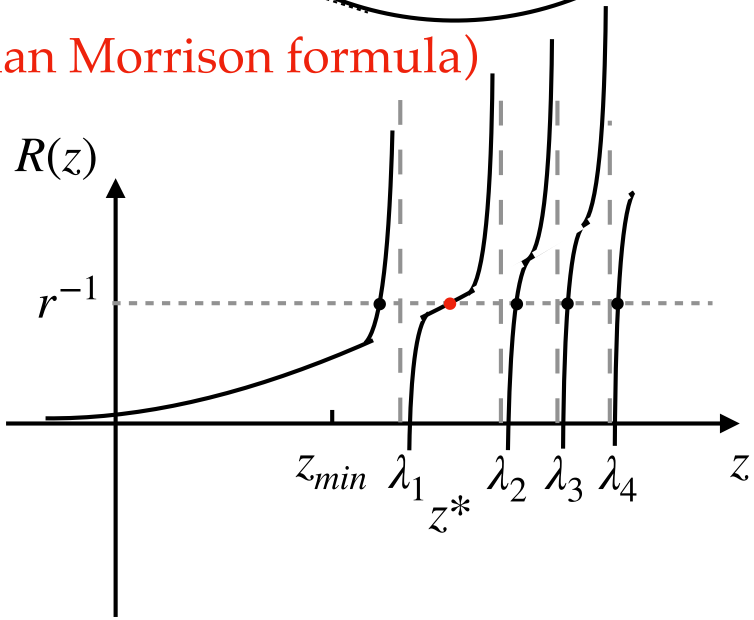

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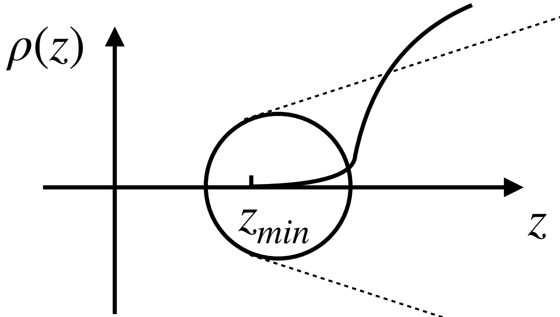

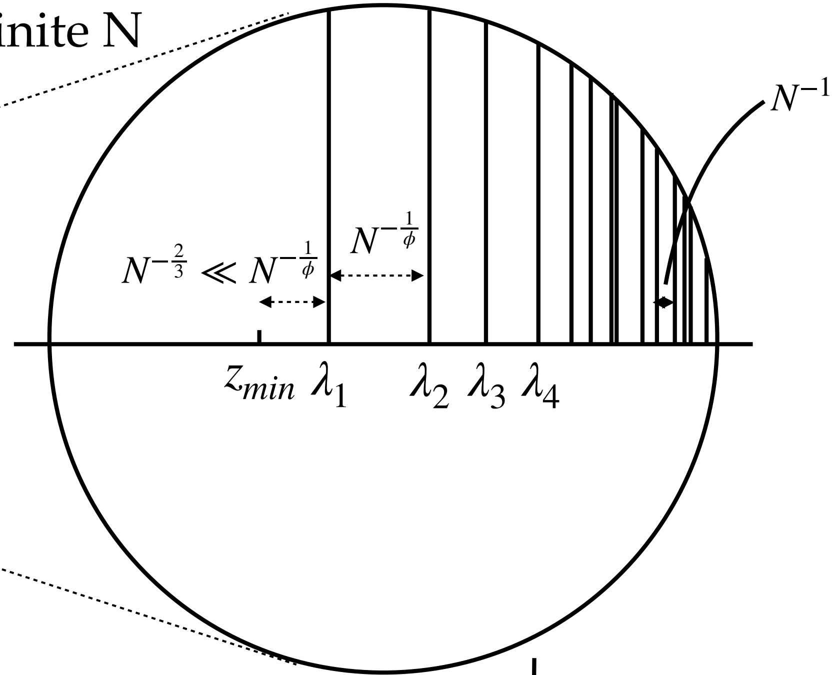

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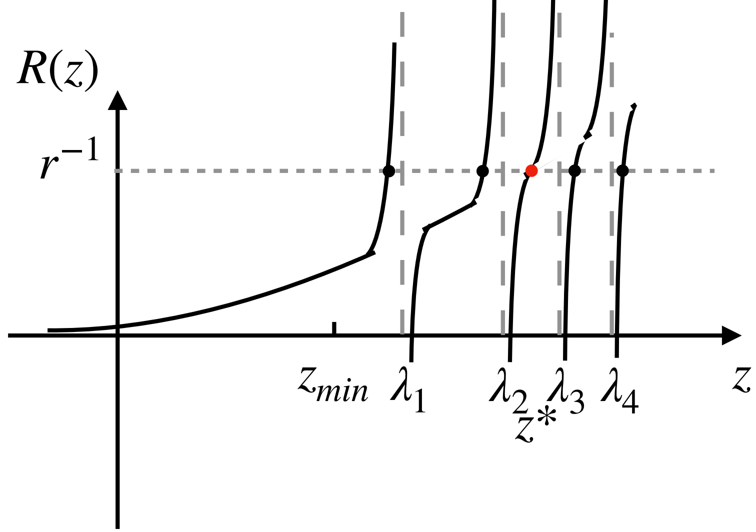

Phase Retrieval: discontinuous BBP, even stronger finite size effects

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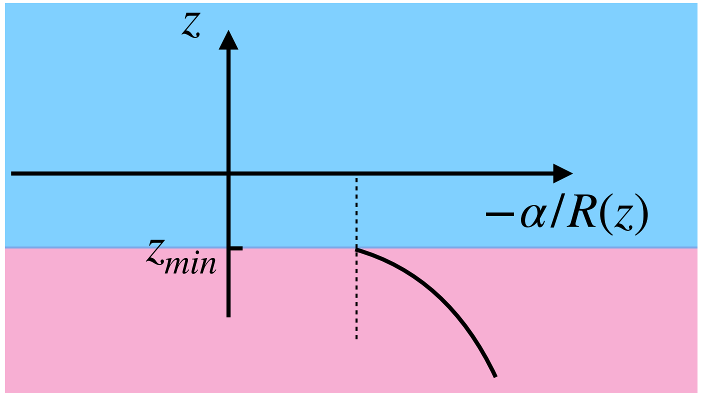

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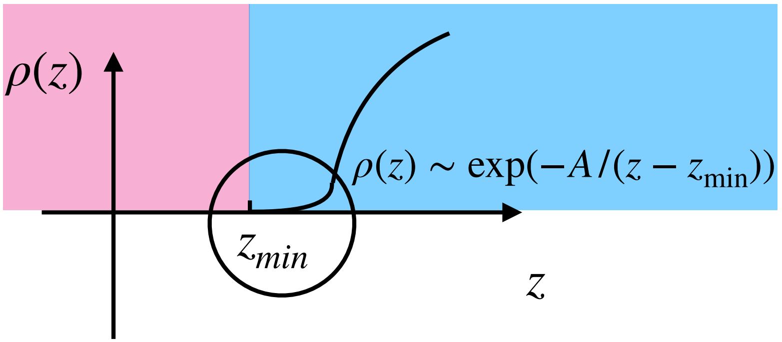
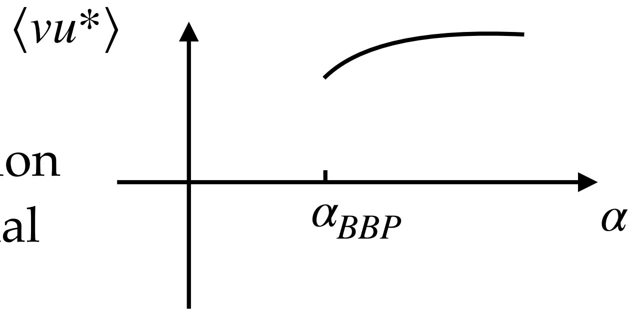


Projection on signal



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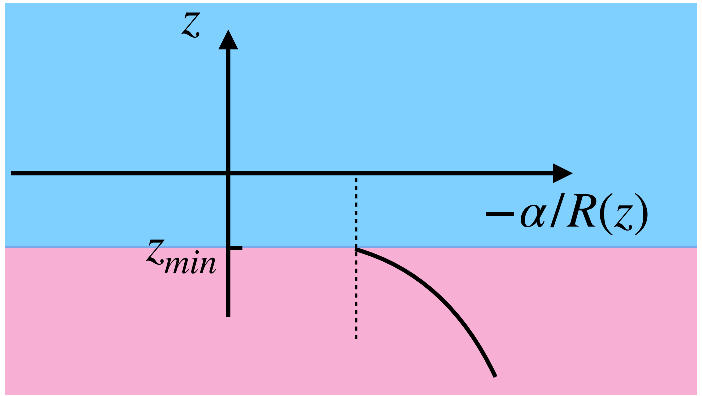

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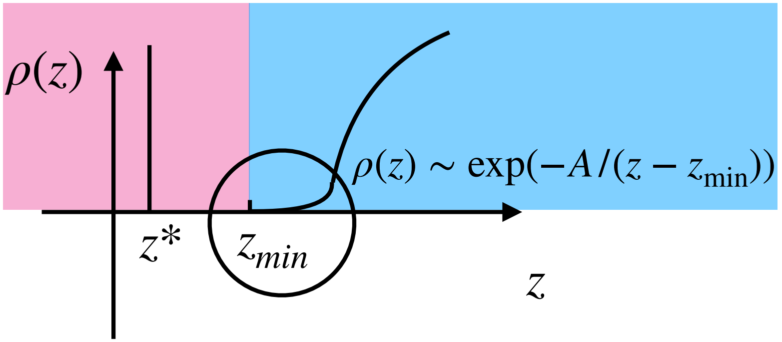

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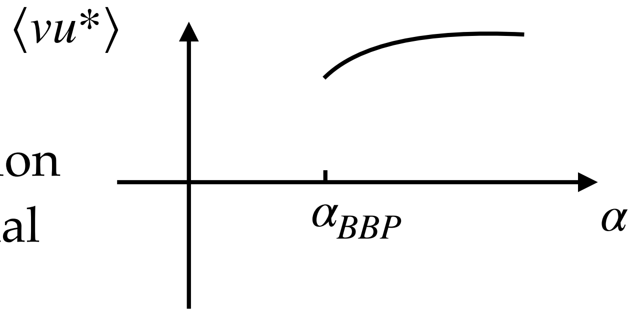

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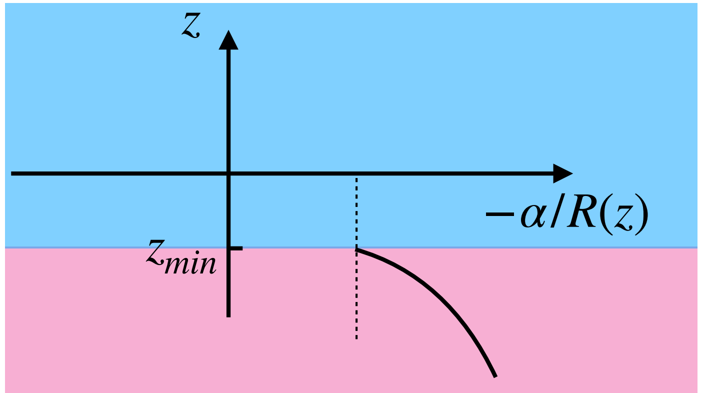

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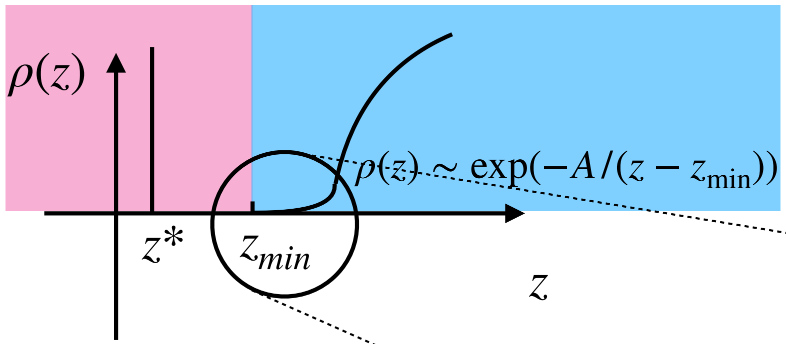

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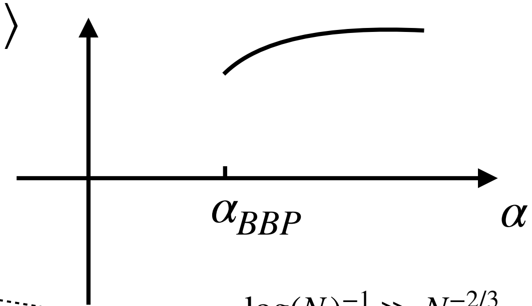

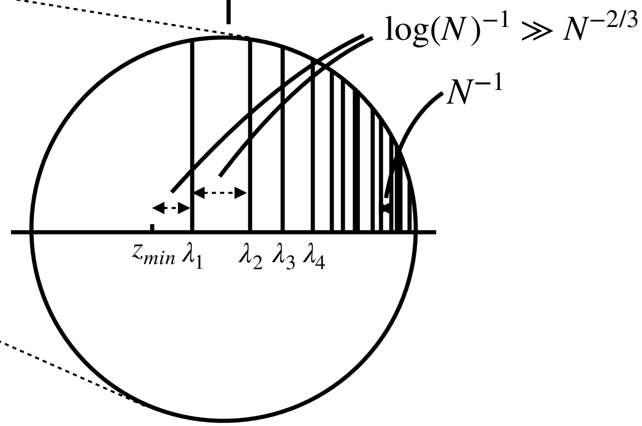

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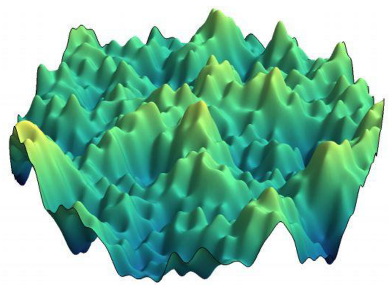
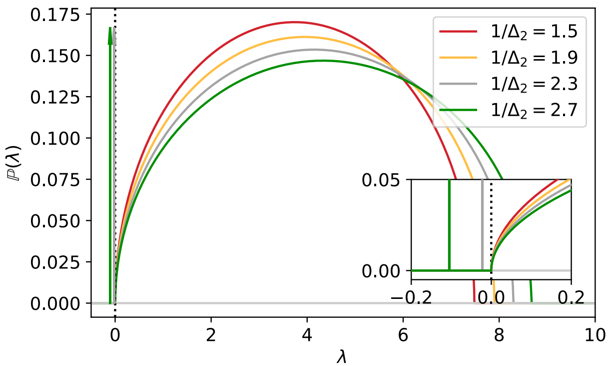
One transition, multiple results

Signal in a noisy background from Hessian of risk/cost landscapes



BBP transition: isolated eigenvalue contains information



Tensor PCA: detailed information on landscape structure

early retrieval in rough landscape

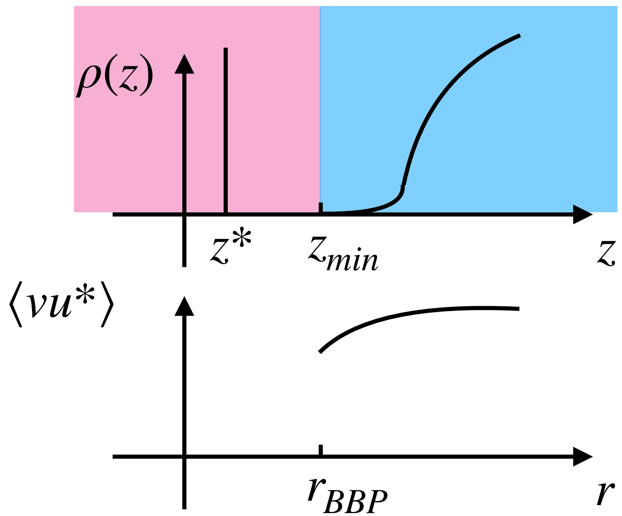
Phase Retrieval: landscape structure not fully determined

early retrieval in rough landscapes (?)

a large variety of spectral initialisation

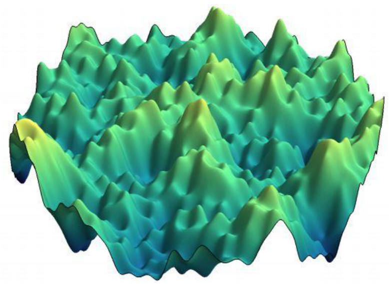
BBP transition can be discontinuous

the importance of finite size effects

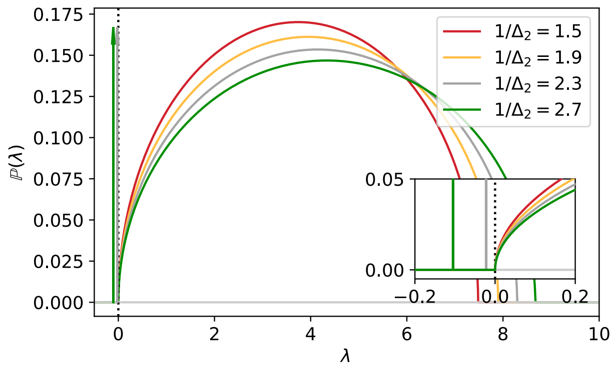


One transition, multiple results

Signal in a noisy background from Hessian of risk/cost landscapes



BBP transition: isolated eigenvalue contains information



Tensor PCA: detailed information on landscape structure

early retrieval in rough landscape

Thank you!

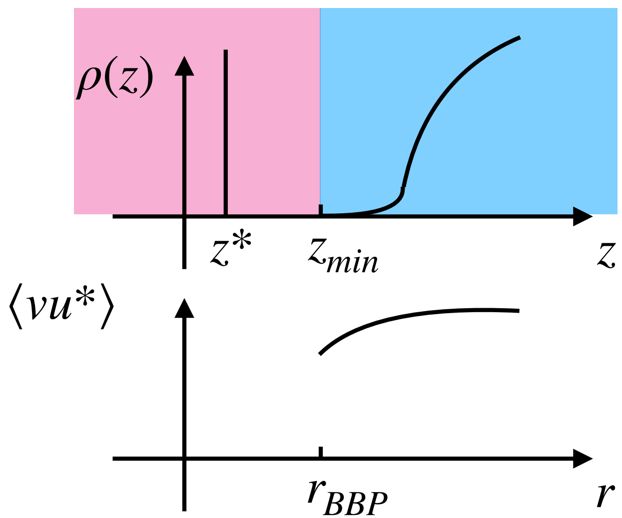
Phase Retrieval: landscape structure not fully determined

early retrieval in rough landscapes (?)

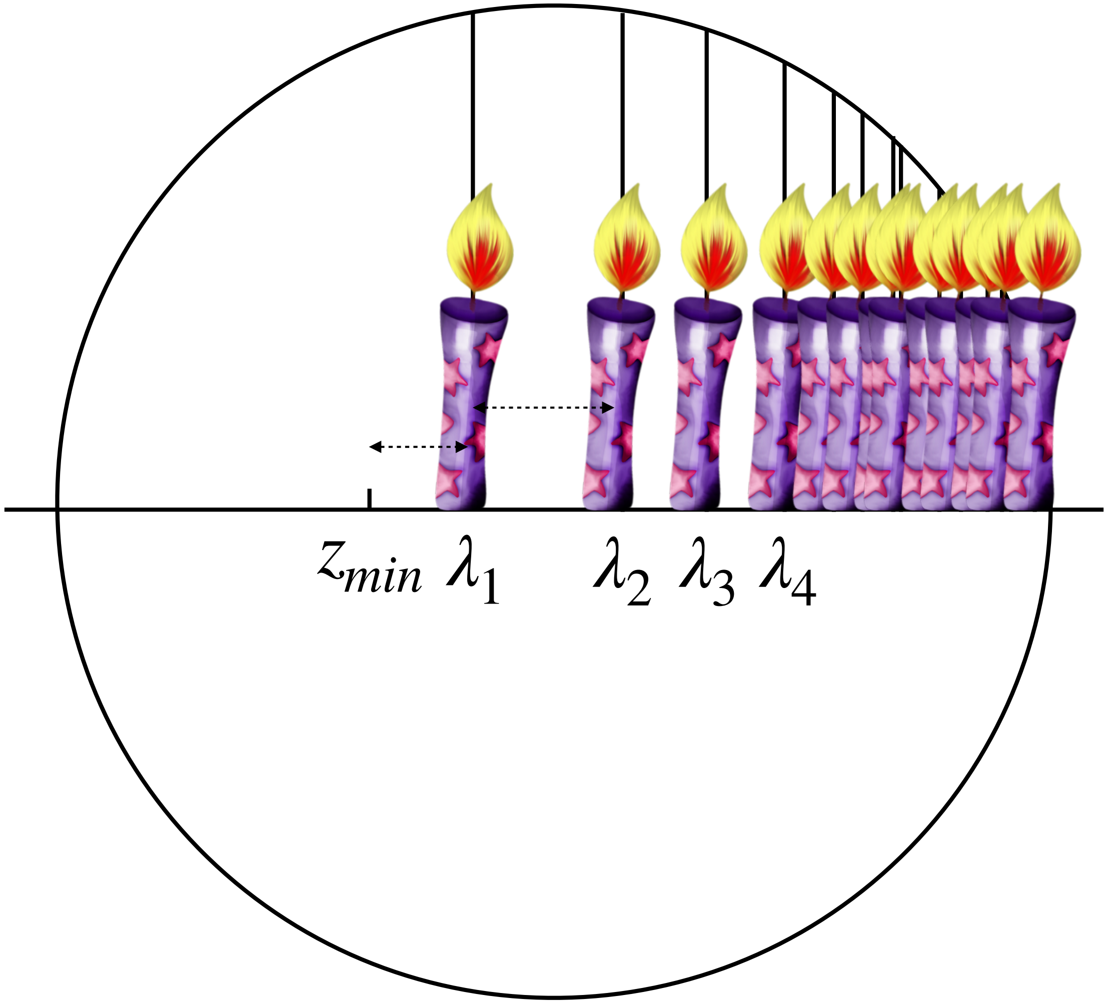
a large variety of spectral initialisation

BBP transition can be discontinuous

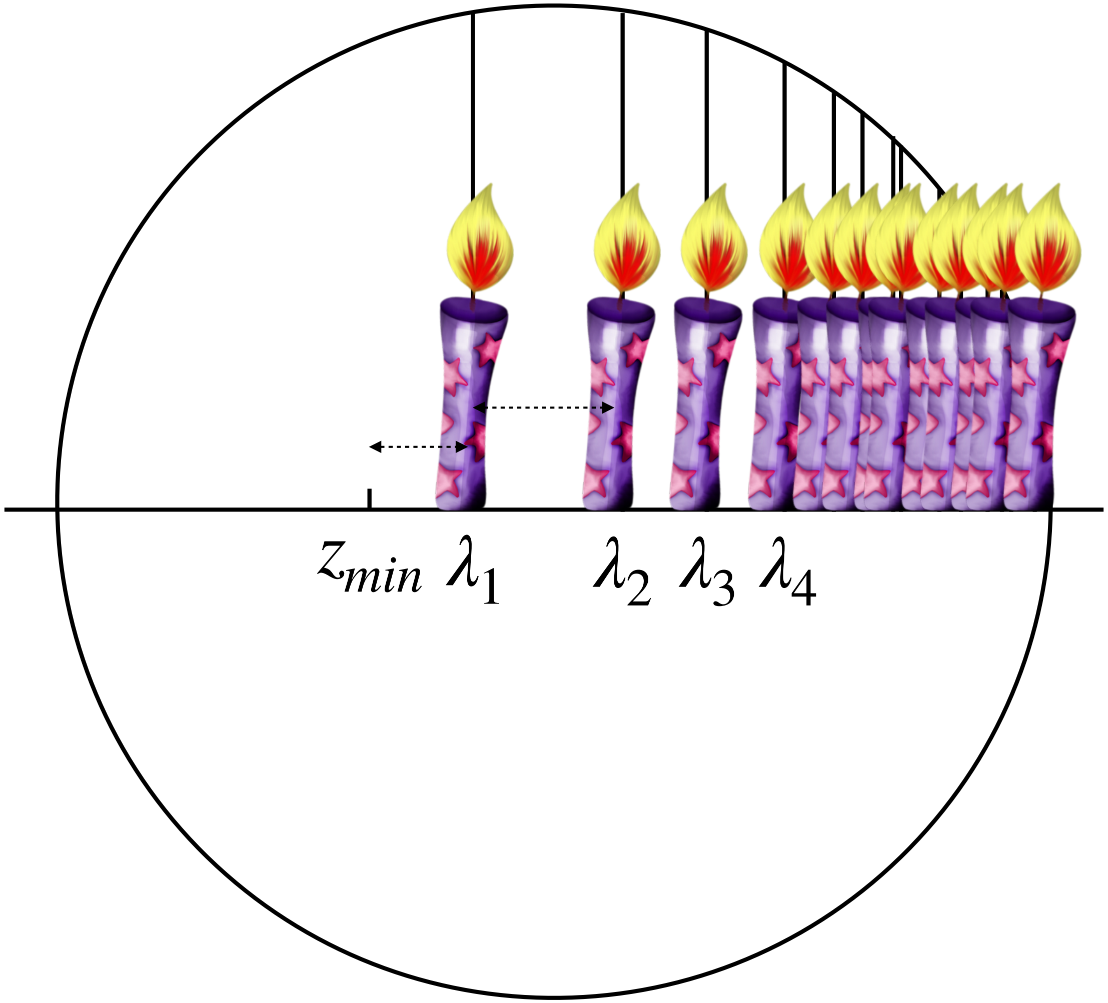
the importance of finite size effects



The birthday spectrum...



The birthday spectrum…



Happy Birthday Yan!