Growth and Trade:
Fantastic Beasts and Where to Find Them*

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This preliminary version: January 2023

Abstract

Several paradoxical results in the field of international trade share the property that they can arise only if the terms of trade are sufficiently disturbed. Substitution effects, whose high values serve to dampen the required adjustment of prices to a market disturbance, thus work to prevent such results from arising. Using immiserizing growth as a salient example, we develop a multi-country multi-sector general equilibrium model to characterize the conditions under which firm heterogeneity is consequential in determining a welfare loss for a country enjoying productivity growth. A demand system with variable elasticity of substitution turns out to be crucial for this exercise as incomplete passthrough is shown to play a key role. Our approach could help rethink the terms of trade effects of various shocks in general equilibrium with firm heterogeneity.

KEY WORDS: Immiserizing Growth, Terms of Trade, Firm Heterogeneity, Variable Elasticity of Substitution, Passthrough

JEL CODES: F12, F43

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*We thank Elhanan Helpman, Sam Kortum, Eric Maskin as well as seminar participants at the University of Bayreuth and the Israel Institute for Advanced Studies for helpful comments and discussions. We are grateful to Francesco Losma and Marta Mojoli for their excellent work as research assistants.

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1 Introduction

Every field in economics has its paradoxes, phenomena which initially are thought to confound economic sense but ultimately come to be better understood (Jones (1985)). The field of international trade can claim its fair share: the examples of the Metzler paradox wherein a tariff fails to provide protection, that of immerizing growth wherein a country suffers a loss in welfare despite an improvement in production possibilities, or the transfer problem wherein in the context of many-country trade or of tax distortions donor countries gain or transfer recipients lose come easily to mind.

Behind each of these examples lies a rational explanation in which income effects prove crucial in supporting a paradoxical outcome. They share the property that paradoxes can arise only if the terms of trade are sufficiently disturbed. Substitution effects, high values of which serve to dampen the required adjustment of prices to a market disturbance, thus work to prevent paradoxical outcomes.

The aim of the present paper is to investigate whether and how firm heterogeneity affects the values of the substitution effects in a way that makes the terms of trade "sufficiently disturbed". Focusing on the example of immiserizing growth (Bhagwati (1958), Bhagwati (1968), Johnson (1967)), we show that the extent of firm heterogeneity is indeed important, and characterize the conditions under which it determines a welfare loss for a country enjoying productivity growth. We do so in the framework of a multi-country multi-sector general equilibrium Ricardian model in which sectors as well as firms within them differ in terms of total factor productivity. Sectors differ in terms of both the lower bound of the support and the density of the firm productivity distribution, and thus in terms of both the mean and the dispersion of firm productivity. As in Melitz (2003), firms are monopolistically competitive. They are homogeneous before entering the market and discover their productivity after entry. This implies the existence of a survival cutoff as some firms find out that they are not productive enough to cover costs and exit without producing. Crucially, firms face demand with variable elasticity of substitution (VES) as in Melitz and Ottaviano (2008).

Deviation from the standard assumption of constant elasticity of substitution (CES) is important because the CES demand system implies that the passthrough from the lower bound of a sector's firm productivity distribution support to the sectoral average price through the survival cutoff is one-to-one (complete) irrespective of the dispersion of firm productivity, which removes by assumption a potentially important source of disturbance for the terms of trade. Intuitively, under the empirically relevant restrictions that the elasticity of demand faced by a firm falls as the quantity it sells rises (a.k.a. 'Marshall's second law of demand), marginal firms with productivity at the survival cutoff price at marginal cost while inframarginal firms have fatter markups, the more so the father away their productivity is from the survival cutoff. At the same time, the passthrough from marginal cost to price is complete for marginal firms and less than complete for inframarginal firms. This prevents an increase in the lower bound of the productivity distribution support to be fully translated into lower prices, the more so the higher its dispersion is.

The traditional literature on immiserizing growth has highighted that the conditions under which immiserizing growth can occur are rather extreme: small ratio of domestic production to import, small elasticity to price of the demand for importables, and small elasticity to price of the supply of importables along the production possibility frontier. This conclusion has been drawn in very specific settings: small open economy taking foreign supply as given, no intra-industry trade with importable and exportable sectors, Heckscher-Ohlin comparative advantage with constant returns to scale and perfect competition, and Hicks-neutral improvement in the technology of the importable sector. We extend and update this literature by developing a multi-country multi-sector general equilibrium Ricardian model with firm herogeneity that predicts structural gravity and is suitable for counterfactual analysis (work in progress). Our main analytical insight is that firm heterogeneity has important implications for the impact of a country's technological progress on national welfare, to the extent that it may even lead to immiserizing growth in situations in which the traditional literature would rule it out such as growth biased towards the import competing sector.

The rest of the paper is organized as follows. Section 2 introduces the model and develops its properties without committing to any specific parametrization of the firm productivity distribution. Section 3 operationalizes the model in the empirically relevant case of a Pareto distrubution. For comparison with the traditional literature, Section 4 focuses on the two-country case, which is further specialized to the two-country two-sector case in Section 5. In this case Section 6 highighs the conditions under which immiserizing growth occurs. Section 7 concludes.

2 A multi-country and multi-sector economy

There are countably many countries and sectors: indexes $j = 1, ..., J$ indicate a country as a source of supply, indexes $l = 1, ..., J$ indicate a country as a source of demand, and indexes $z = 1, ..., Z$ indicate a sector. Consumption goods are traded across countries. In each country a continuum of varieties of a differentiated consumption good, indexed by $i \in [0, N_l(z)]$, is consumed; where $N_l(z)$ is the measure of varieties of goods in sector z available for consumption in country l.

Monopolistically competitive firms employ labor in one country and produce one variety in one sector with constant returns to scale. Labor is the only input, it is homogeneous, perfectly mobile across sectors but not mobile across countries. Firm entry is unrestricted but costly: producers willing to enter in a country j and sector z pay an exogenous sunk cost in terms of $f_j(z) > 0$ labor units, to develop a new technology in that country and sector pair. After this payment, a firm realizes its idiosyncratic conversion rate of labor per unit of output, as a random draw $c > 0$ from a continuous c.d.f. $G_j(c; z)$ that is specific to the country j and sector z.[1] After making a successful entry, firms producing in a country j might export to any other country l facing a sector-specific iceberg trade costs $\tau_{jl}(z) \geq 1$.

Call $N_j^E(z)$ the number of entrants in country j sector z, then $M_j(z) \leq N_j^E(z)$ is the measure of varieties produced in country j of goods in sector z, and only a subset $N_{jl}(z) \leq M_j(z)$ of them is shipped to country l. Thus, the measure of available varieties (domestic and imported) in a certain market l is given by $N_l(z) \equiv \sum_{j=1}^{J} N_{jl}(z)$.

2.1 Consumers' behavior

In every country $l = 1, ..., J$, preferences are represented by a Cobb-Douglas aggregator across sectors $z = 1, ..., Z$, while consumption bundles of varieties within a sector are

[1] The technological coefficient c is the inverse of labor productivity, i.e. it describes the ratio of units of labor per unit of output of a certain variety. Typically, the literature refers to this coefficient as marginal and average "cost" but in the present framework the wage will be determined in equilibrium, thus, the marginal and average cost at which a firm operates is endogenous, through the wage.

ranked by quadratic preferences:[2]

$$U_l = \prod_{z=1}^{Z} \left[\sum_{j=1}^{J} \left(\int_0^{N_{jl}(z)} \alpha q_{jl}^c (i;z) - \frac{\gamma}{2} q_{jl}^c (i;z)^2 \, di \right) \right]^{\beta(z)} \tag{1}$$

where $q_{jl}^c(i;z)$ is the quantity of good i from sector z produced in country j and consumed in country l and $\beta(z) \in (0,1)$ such that $\sum_{z=1}^{Z} \beta(z) = 1$ are sector-specific shares. Individual consumers in country l earning a wage $w_l > 0$ take the set of available varieties and prices as given and maximize (1) subject to the budget constraint

$$\sum_{z=1}^{Z} \sum_{j=1}^{J} \int_0^{N_{jl}(z)} p_{jl} (i;z) \, q_{jl}^c (i;z) \, di = w_l \, , \tag{2}$$

where $p_{jl}(i;z)$ is the price (at destination) of a variety i of goods from sector z produced in country j and sold to country l.

Between sectors, the marginal utility is unbounded. Therefore, every consumer of every country l will demand varieties from every sector z. Within sector, the marginal utility from consumption of a certain variety is finite. This implies that there is a choke price at which the optimal consumption of a variety is null. Thus, the first order condition evaluated at the choke price and zero consumption yields the Lagrange multiplier of the budget constraint as the ratio of marginal utility from consumption of the marginal variety over the choke price.

Let $\hat{p}_l(z)$ be the price that implies zero demand in country l for a variety in sector z. The Marshallian individual demand function in country l for a variety of sector z sold in country l at a price p, regardless where production occurs, is given by:

$$q_{jl}^*(p;z) \;=\; \frac{\alpha}{\gamma} \left(1 - \frac{p}{\hat{p}_l(z)} \right) \, , \quad \forall j. \tag{3}$$

Firm-level elasticity of demand to price is fully described by the relative price with re-

[2]Two remarks shall be noted. First, we discuss the case of a Cobb-Douglas aggregator across sectors for the sake of exposition, but the analysis goes through for every homothetic aggregator. Second, the reader can think at $\alpha \equiv 1$ without loss of generality. In fact, as taste for consumption are not heterogeneous across varieties or countries, one parameter among α or γ is sufficient to represent the taste for differentiation relative to absolute willingness to pay, i.e. the ratio γ/α, in consumer's preferences. With a richer notation, variety, sector and country-and-sector-specific parameters $\alpha_l(i;z) > 0$ and $\gamma_l(i;z) > 0$ could be used to indicate within-sector patterns of vertical differentiation and horizontal differentiation, respectively. We abstract from these sources of discrimination, by willingness to pay (*quality*) and by country of origin (*Armington*).

spect to the choke price $\varepsilon_l(p;z) = \frac{p/\hat{p}_l(z)}{1-p/\hat{p}_l(z)}$ in absolute value, and we restrict the analysis to the case $\varepsilon_l(p;z) > 1$ for price-competition to arise. The ratio $\varepsilon_l(p;z)/(\varepsilon_l(p;z)-1)$ defines the markup rate charged by a firm of sector z selling at a price p in country l, and it is given by:

$$mkp^*_{jl}(p;z) = \frac{1}{2 - \frac{\hat{p}_l(z)}{p}}, \quad \forall j, \tag{4}$$

hence, markup in a certain destination market vary with the relative price with respect to the choke price in the market. Markups are a decreasing function of the relative price $p/\hat{p}_l(z)$, thus, within a sector those varieties for which consumers exhibit greater demand are the ones sold at greater markup.

2.2 Firms' behavior

A firm located in country j, competing in sector z, endowed with a technological coefficient c, hires labor in the same country at a competitive wage w_j which is employed in a linear production function:

$$q_j(c;z) = \frac{\ell_j(c;z)}{c} \tag{5}$$

where $\ell_j(c;z)$ is the employment of labor at the firm. The marginal cost of production is w_jc. For goods that are produced in country j and shipped to a certain country l becomes the marginal cost of production and delivery is $\tau_{jl}(z)w_jc$.

Since consumers in a given country have the same income, the aggregate demand function in a certain destination l amounts to the individual demand (3) times the market size L_l. Thus, the marginal revenue in a destination l for a good of sector z at a price $p_{jl}(c;z)$ is given by $2p_{jl}(c;z) - \hat{p}_l(z)$. The equivalence of marginal revenue and marginal cost yields the price that maximizes profit for a firm of sector z producing in country j and selling to country l:

$$p^*_{jl}(c;z) = \frac{\hat{p}_l(z) + \tau_{jl}(z)w_jc}{2}. \tag{6}$$

Substituting in the Marshallian demand (3) shows that the technological coefficient that implies a zero demand in country l for a good of sector z produced in country j is

$$c_{jl}^{*}(z) \;=\; \frac{\hat{p}_l(z)}{\tau_{jl}(z)w_j}\,, \tag{7}$$

therefore, the marginal cost for producing in country j and successfully shipping to country l is bounded above $cw_j \leq \hat{p}_l(z)/\tau_{jl}(z)$. It follows that exporters serve a foreign market at a marginal cost that is lower than the local choke price. The measure of firms in country j producing in sector z that serve market l consists of the fraction of entrants $N_j^E(z)$, in that country and sector, whose technological coefficient does not exceed the export cutoff (7)

$$N_{jl}(z) = G_j(c_{jl}^{*}(z);z)N_j^E(z)\,. \tag{8}$$

The equilibrium level of price $p_{jl}(c;z)$, markup $mkp_{jl}(p;z)$, output $q_{jl}(c;z)$, employment in production $\ell_{jl}(c;z)$, revenue $r_{jl}(c;z)$ and profit $\pi_{jl}(c;z)$ associated to the shipment from country j to country l of a firm with technological coefficient c in sector z are given by:

$$\begin{aligned}
p_{jl}\,(c;z) &= \frac{\hat{p}_l(z)}{2}\left(1+\frac{c}{c_{jl}^{*}(z)}\right) \\
mkp_{jl}(c;z) &= \frac{1}{2}\left(1+\frac{c_{jl}^{*}\,(z)}{c}\right) \\
q_{jl}\,(c;z) &= \frac{L_l}{2\gamma}\frac{\alpha}{c_{jl}^{*}\,(z)}\left(c_{jl}^{*}\,(z)-c\right) \\
\ell_{jl}\,(c;z) &= \frac{L_l}{2\gamma}\frac{\alpha\tau_{jl}(z)}{c_{jl}^{*}\,(z)}\left(c_{jl}^{*}\,(z)\,c-c^2\right) \\
r_{jl}\,(c;z) &= \frac{w_jL_l}{4\gamma}\frac{\alpha\tau_{jl}\,(z)}{c_{jl}^{*}\,(z)}\left(c_{jl}^{*}\,(z)^2-c^2\right) \\
\pi_{jl}(c;z) &= \frac{w_jL_l}{4\gamma}\frac{\alpha\tau_{jl}\,(z)}{c_{jl}^{*}\,(z)}\left(c_{jl}^{*}\,(z)-c\right)^2\,.
\end{aligned} \tag{9}$$

where constant returns to scale in production and market segmentation allow all variables (including employment of labor and profit) to be assigned by origin and destination pair. Firm-level variables (9) are defined conditional on $c \leq c_{jl}^{*}(z)$, i.e. under the assumption that a firm of sector z producing in country j with technological coefficient c charges an optimal price-at-destination in market l that does not exceed the choke price $\hat{p}_l(z)$. Firms under the same circumstances but endowed with $c > c_{jl}^{*}(z)$

optimally choose not to serve market l, hence, production, employment, revenue and profits are null and there is no contribution from these firms to the set of varieties available in country l.

2.3 Equilibrium

Firm-level demand and firm-level markup are fully described by the relative price with respect to the choke price. Thus, it is useful to define the first and second moment of the distribution of prices relative to the choke price in country l sector z among goods shipped from country j:

$$\bar{p}_{jl}(z) \equiv N_{jl}(z)^{-1} \int_0^{N_{jl}(z)} \frac{p(i)}{\hat{p}_l(z)} di \quad \text{and} \quad \bar{\bar{p}}_{jl}(z) \equiv N_{jl}(z)^{-1} \int_0^{N_{jl}(z)} \left(\frac{p(i)}{\hat{p}_l(z)} \right)^2 di \, .$$

When we aggregate consumption choices, the optimal expenditure made by a consumer in country l on goods from sector z sourced from country j

$$e_{jl}(z) \equiv \int_0^{N_{jl}(z)} p(i) q^*_{jl}\left(p(i);z\right) di = \hat{p}_l(z) \frac{\alpha}{\gamma} \left(\bar{p}_{jl}(z) - \bar{\bar{p}}_{jl}(z) \right) N_{jl}(z) \, ,$$

and the maximum sub-utility enjoyed by consuming in country l goods from sector z sourced from country j

$$u_{jl}(z) \equiv \int_0^{N_{jl}(z)} \alpha q^*_{jl}(p(i);z) - \frac{\gamma}{2} q^*_{jl}(p(i);z)^2 di = \frac{\alpha^2}{2\gamma} \left(1 - \bar{\bar{p}}_{jl}(z) \right) N_{jl}(z) \, ,$$

are, indeed, characterized by moments of the relative price distribution and by the measure of varieties. Expenditure of a consumer in country l on a bundle of goods from a certain sector $\sum_{j=1}^{J} e_{jl}(z)$ sourced anywhere can be decomposed into a quantity index $\mathbb{Q}_l(z) \equiv \sum_{j=1}^{J} u_{jl}(z)$ and price index $\mathbb{P}_l(z) \equiv \sum_{j=1}^{J} e_{jl}(z)/\mathbb{Q}_l(z)$ given by:

$$\begin{aligned} \mathbb{Q}_l(z) &= \frac{\alpha^2}{2\gamma} \sum_{j=1}^{J} \left(1 - \bar{\bar{p}}_{jl}(z) \right) N_{jl}(z) \, , \\ \mathbb{P}_l(z) &= \frac{2}{\alpha} \frac{\sum_{j=1}^{J} \left(\bar{p}_{jl}(z) - \bar{\bar{p}}_{jl}(z) \right) N_{jl}(z)}{\sum_{j=1}^{J} \left(1 - \bar{\bar{p}}_{jl}(z) \right) N_{jl}(z)} \hat{p}_l(z) \, . \end{aligned}$$

The system of first order conditions of the consumer problem evaluated at the sectoral choke prices and the budget constraint across sectors determines the expenditure in goods of sector z in country l, that is $\mathbb{P}_l(z)\mathbb{Q}_l(z)L_l = \theta_l(z) w_l L_l$ where the sector-specific expenditure share is given by:

$$\theta_l(z) \equiv \frac{\beta(z)\eta_l(z)}{\sum_{s=1}^{Z}\beta(s)\eta_l(s)},$$

includes the (endogenous) coefficient

$$\eta_l(z) \equiv \frac{\mathbb{P}_l(z)}{\hat{p}_l(z)} = \frac{2}{\alpha}\frac{\sum_{j=1}^{J}\left(\bar{p}_{jl}(z) - \bar{\bar{p}}_{jl}(z)\right)N_{jl}(z)}{\sum_{j=1}^{J}\left(1 - \bar{\bar{p}}_{jl}(z)\right)N_{jl}(z)},$$

which accounts for sector and country specific patterns of the equilibrium distribution of prices relative to the choke price and number of varieties. Thus, sectoral expenditure shares in equilibrium are endogenous, they are equal to the given Cobb-Douglas shares only if price distributions and number of varieties were the same across sectors.

Welfare. Given a distribution of prices $\boldsymbol{p}_l$, hence of available varieties $\{N_{jl}(z)\ \forall(j,z)\}$, the maximum utility from consumption enjoyed by the representative consumer in country l is the Cobb-Douglas aggregator of the sectoral utility-based quantity indexes $V(\boldsymbol{p}_l, w_l) = \prod_{z=1}^{Z}\mathbb{Q}_l(z)^{\beta(z)}$. The budget constraint has to be satisfied $\mathbb{P}_l(z)\mathbb{Q}_l(z) = \theta_l(z)w_l$, that implies:

$$\mathbb{Q}_l(z) = \frac{\theta_l(z)}{\mathbb{P}_l(z)}w_l = \frac{\theta_l(z)}{\eta_l(z)}\frac{w_l}{\hat{p}_l(z)} = \frac{\beta(z)}{\hat{p}_l(z)}\frac{w_l}{\kappa_l}.$$

where the coefficient $\kappa_l \equiv \sum_{z=1}^{Z}\beta(z)\eta_l(z)$ averages the within-sector dispersion of prices relative to the corresponding sectoral choke price in country l, weighted by the sectoral shares in consumer's preferences.[3] Between-country comparisons of welfare net of within-sector concentration of prices are effectively measured by:

$$W(\boldsymbol{p}_l, w_l) = \kappa_l V(\boldsymbol{p}_l, w_l) = \prod_{z=1}^{Z}\left(\frac{\beta(z)}{\hat{p}_l(z)}\right)^{\beta(z)} w_l = \prod_{z=1}^{Z}\left(\frac{\beta(z)}{c_l^*(z)}\right)^{\beta(z)}, \tag{10}$$

where the last equality follows from the definition of choke price in equilibrium $\hat{p}_l(z) = c_l^*(z)w_l$. Therefore, individual welfare is a geometric average across sectors of produc-

[3] The coefficient κ_l summarizes the loss of utility due to sectoral price distributions that are more concentrated near the sectoral choke prices: given the same vector of sectoral choke prices and the same wage, the maximum utility $V(\boldsymbol{p}_l, w_l)$ is higher the more prices are dispersed away from the sectoral choke price.

tivity cutoffs $1/c_l^*(z)$. The expression (10) yields welfare of a consumer in country l.[4]

Resource allocation. Define $R_{jl}(z)$ the total revenue made in country l by firms producing in country j sector z and $R_l(z)$ the total revenue of firms producing in country j sector z:

$$R_{jl}(z) \equiv N_{jl}(z) \int_0^{c_{jl}^*(z)} r_{jl}(c;z) \frac{dG_j(c;z)}{G_j(c_{jl}^*(z);z)}, \quad R_j(z) \equiv \sum_{l=1}^{J} R_{jl}(z). \tag{11}$$

Under free entry (and with no intermediates) total revenue coincides with total labor income (due to both production and entry) generated by the sector $R_j(z) = w_j L_j(z)$, where $L_j(z) \equiv \rho_j(z) L_j$ is employment allocated to sector z in country j, such that

$$\rho_j(z) \equiv \frac{\sum_{l=1}^{J} R_{jl}(z)}{\sum_{s=1}^{Z} \sum_{l=1}^{J} R_{jl}(s)} = \frac{R_j(z)}{\sum_{s=1}^{Z} R_j(s)},$$

is the fraction of employment allocated to sector z in country j. Since the wage is the same across sectors and there are no intermediates, $\rho_j(z)$ is also the share of value added of sector z, hence, the share of total labor income generated by sector z.

Equilibrium conditions. In every country and sector pair (j,z) where there is entry, i.e. $N_j^E(z) > 0$, then unrestricted entry implies that the expected value of a new entry unconditional on being successful attains the entry cost:

$$\text{FEC}: \quad \sum_{l=1}^{J} \int_0^{c_{jl}^*(z)} \pi_{jl}(c;z) dG_j(c;z) = w_j f_j \qquad \forall (j,z). \tag{12}$$

such that there are no rents from firm ownership. Output market clearing requires that sales by all firms in a sector that serve a certain market l add up to the expenditure of

[4]Whether the normalization by the coefficient κ_l is innocuous or not depends on the exogenous technological distribution. But for families of technological distributions that are overwhelming in both empirical and theoretical studies (e.g. Inverse Pareto, Truncated Pareto, Lognormal, Frechet and Gumbel) the normalization comes without loss of generality. To see this, call $\tilde{c} \equiv c/\hat{c}(z)$ the random variable describing the inverse of labor productivity relative to an arbitrary sectoral cutoff, in a certain country and sector. Given the exogenous c.d.f. $G(c;z)$ call $\tilde{G}(\tilde{c};z)$ the c.d.f. of the inverse productivity relative to the sectoral cutoff. If moments of $\tilde{G}(\tilde{c};z)$ only depend on exogenous parameters of the technology then $\kappa_l \equiv \kappa$, i.e. the maximum utility from consumption is shifted by a constant scalar (thus, it does not affect comparative statics within country) which is the same between countries (thus, it does not affect cross-country comparisons).

country l in sector z:

$$\text{OMC}: \quad \sum_{j=1}^{J} R_{jl}(z) = \theta_l(z) w_l L_l \qquad \forall (l,z). \tag{13}$$

Due to free entry, sales made by by firms producing in country j in every destination correspond to the aggregate labor income (from production and entry) made by workers in country j employed in sector z. In country j and sector z, for a given wage w_j and sectoral employment $L_j(z) = \rho_j(z) L_j$, the sectoral labor market clearing conditions hold and imply aggregate labor market clearing:

$$\text{LMC}: \quad \sum_{l=1}^{J} R_{jl}(z) = \rho_j(z) w_j L_j \quad \forall (j,z) \implies \sum_{z=1}^{Z} \sum_{l=1}^{J} R_{jl}(z) = w_j L_j \qquad \forall j. \tag{14}$$

Definition of equilibrium. Given a set of preference parameters $\{\alpha, \gamma, \{\beta(z)\}_{z=1}^{Z}\}$, market sizes $\{L_j\}_{j=1}^{J}$, entry costs $\{f_j\}_{j=1}^{J}$, a distribution of technological coefficients $\{G_j(c;z)\}_{j=1,z=1}^{J,Z}$ and a set of bilateral sector specific trade costs $\{\tau_{jl}(z)\}_{j=1,l=1,z=1}^{J,J,Z}$, the equilibrium of the model within the cone of diversification consists of:

a) a vector of wages $w_l > 0$ for every country $l = 1, 2, ...J$

b) a vector of choke prices $\hat{p}_l(z) = c_l^*(z) w_l > 0$ for every country $l = 1, 2, ...J$ and sector $z = 1, 2, ..., Z$

c) a vector of measures of entrants $N_j^E(z) > 0$ for every origin country $j = 1, 2, ...J$ and sector $z = 1, 2, ..., Z$

that satisfy

i) the system of $J \times Z$ free entry conditions (12),

ii) the system of $J \times Z$ output market clearing conditions (13),

iii) the system of J aggregate labor market clearing conditions (14),

once substituting for the export cutoff (7), the measure of exporters (8) and the definitions of firm-level profit $\pi_{jl}(c;z)$ and revenue $R_{jl}(z)$ are understood, from (9) and (11). Without loss of generality, we use labor in country $j = 1$ as our numeraire, such that $w_1 = 1$, before and after any change in the fundamentals of the economy.

2.4 Gravity equation

We now derive the gravity equation implied by the equilibrium. Notice that $R_{jl}(z) = e_{jl}(z)L_l$ is the value of imports of country l from country j in sector z. Changing the notation, define expenditure of country l in goods of sector z sourced from country j

$$X_{jl}(z) \equiv R_{jl}(z) = \frac{\alpha \hat{p}_l(z) L_l}{\gamma} \left(\bar{p}_{jl}(z) - \bar{\bar{p}}_{jl}(z)\right) N_{jl}(z) \, ,$$

and let $X_l(z) \equiv \sum_{j=1}^{J} R_{jl}(z)$ be the aggregate expenditure of country l in goods of sector z sourced from anywhere. Aggregating $X_{jl}(z)$ by country of origin yields total expenditure of country l in sector z, that is $X_l(z) = \frac{\alpha \hat{p}_l(z)}{\gamma} L_l \sum_{j=1}^{J} \left(\bar{p}_{jl}(z) - \bar{\bar{p}}_{jl}(z)\right) N_{jl}(z)$. Substituting for $\alpha \hat{p}_l(z) L_l / \gamma$ yields

$$X_{jl}(z) = \frac{\left(\bar{p}_{jl}(z) - \bar{\bar{p}}_{jl}(z)\right) N_{jl}(z)}{\sum_{m=1}^{J} \left(\bar{p}_{ml}(z) - \bar{\bar{p}}_{ml}(z)\right) N_{ml}(z)} X_l(z) \, .$$

By output market clearing (13) we have $X_l(z) = \theta_l(z) w_l L_l$. Aggregating sectoral labor market clearing conditions (14) across sectors yields $X_l(z) = \theta_l(z) Y_l$, where $Y_l \equiv w_l L_l$ is income in country l. This completes the derivation of the gravity equation in terms of the measure of imported varieties:

$$X_{jl}(z) = \frac{\left(\bar{p}_{jl}(z) - \bar{\bar{p}}_{jl}(z)\right) N_{jl}(z)}{\sum_{m=1}^{J} \left(\bar{p}_{ml}(z) - \bar{\bar{p}}_{ml}(z)\right) N_{ml}(z)} \theta_l(z) Y_l \, .$$

Substituting for $N_{jl}(z) = G_j(c_{jl}^*(z); z) N_j^E(z)$ yields the gravity equation in terms of the measure of entrants in each country and sector:

$$X_{jl}(z) = \frac{\left(\bar{p}_{jl}(z) - \bar{\bar{p}}_{jl}(z)\right) G_j(c_{jl}^*(z); z) N_j^E(z)}{\sum_{m=1}^{J} \left(\bar{p}_{ml}(z) - \bar{\bar{p}}_{ml}(z)\right) G_m(c_{ml}^*(z); z) N_m^E(z)} \theta_l(z) Y_l \, .$$

Free entry (12) implies that the cost of entry in a country j sector z, that is $N_j^E(z) w_j f_j$, equals total profit in that sector and country, which we call $\Pi_j(z) = \delta_j(z) R_j(z)$, thus, describing total profit in a sector as a fraction $\delta_j(z) \in (0, 1)$ of total revenue in the sector $R_j(z) \equiv \sum_{l=1}^{J} R_{jl}(z)$. Finally, sectoral labor market clearing (14) yields $R_j(z) = \rho_j(z) w_j L_j$. This completes the characterization of the gravity equation:

$$X_{jl}(z) = \left(\frac{\left(\bar{p}_{jl}(z) - \bar{\bar{p}}_{jl}(z)\right) G_j(c_{jl}^*(z); z) \delta_j(z) \rho_j(z) L_j / f_j}{\sum_{m=1}^{J} \left(\bar{p}_{ml}(z) - \bar{\bar{p}}_{ml}(z)\right) G_m(c_{ml}^*(z); z) \delta_m(z) \rho_m(z) L_m / f_m} \right) \theta_l(z) Y_l \, , \qquad (15)$$

where the expression in brackets is the fraction of expenditure in goods of sector z that country l sources from country j.

3 (Inverse) Pareto distribution

Assume that the distribution of technological coefficients is an Inverse Pareto with country and sector specific concentration parameter $c_j^{max}(z) > 0$ and sector specific dispersion parameter $k(z) > 1$ such that $G_j(c;z) = (c/c_j^{max}(z))^{k(z)}$. This choice has implications for the distribution of relative prices and for aggregate variables.

The choke price in sector z country l is such that the firm-level demand is null, which corresponds to $c = c_{jl}^*(z)$ for a firm producing in any country j. The relative price is given by $\frac{p_{jl}(c;z)}{\hat{p}_l(z)} = \frac{1}{2}(1 + c/c_{jl}^*(z))$ and it is distributed over the support $c \in [0, c_{jl}^*(z)]$ according to the truncated Inverse Pareto $G_{jl}^*(c;z) = (c/c_{jl}^*(z))^{k(z)}$. As a consequence, the first and second moment of the relative price distribution

$$\bar{p}_{jl}(z) = \frac{2k(z)+1}{2(k(z)+1)} \equiv \mu_1(z) \quad (16)$$

$$\bar{\bar{p}}_{jl}(z) = \frac{2k(z)^2+4k(z)+1}{2(k(z)+2)(k(z)+1)} \equiv \mu_2(z) \quad (17)$$

only depend on technological concentration, through the parameter $k(z)$, but not on country characteristics. Consequently, both sectoral expenditure share and price dispersion do not depend on country characteristics but only on the sectoral concentration of technology:

$$\theta(z) \equiv \frac{\beta(z)\eta(z)}{\sum_{s=1}^{Z}\beta(s)\eta(s)} \quad \text{and} \quad \eta(z) = \frac{2}{\alpha}\frac{\mu_1(z)-\mu_2(z)}{(1-\mu_2(z))}\,, \quad (18)$$

which also implies that cross-country comparisons of welfare (10) correspond to comparison of indirect utilities, since within-sector price dispersion is the same everywhere.

Demand side. Expenditure and utility due to individual consumption in country l on goods from sector z sourced from country j are:

$$e_{jl}(z) = \frac{\alpha}{\gamma}(\mu_1(z)-\mu_2(z))\,\hat{p}_l(z)N_{jl}(z) \quad (19)$$

$$u_{jl}(z) = \frac{\alpha^2}{2\gamma}(1-\mu_2(z))\,N_{jl}(z)\,. \quad (20)$$

Quantity index, price index, number of varieties in a certain country l and sector z are:

$$\mathbb{Q}_l(z) = \sum_{j=1}^{J} u_{jl}(z) = \frac{\alpha^2}{2\gamma}\left(1-\mu_2(z)\right) N_l(z) \tag{21}$$

$$\mathbb{P}_l(z) = \frac{2}{\alpha}\left(\frac{\mu_1(z)-\mu_2(z)}{1-\mu_2(z)}\right)\hat{p}_l(z) \tag{22}$$

$$N_l(z) = \frac{\gamma}{\alpha}\frac{\theta(z)}{\left(\mu_1(z)-\mu_2(z)\right)c_l^*(z)}, \tag{23}$$

as a function of the local cost cutoff $c_l^*(z) = \hat{p}_l(z)/w_l$.

Supply side. Aggregate sales $R_{jl}(z) \equiv N_{jl}(z)\int_0^{c_{jl}^*(z)} r_{jl}(c;z)\frac{dG_j(c;z)}{G_j(c_{jl}^*(z);z)}$ and aggregate profit $\Pi_{jl}(z) \equiv N_{jl}(z)\int_0^{c_{jl}^*(z)} \pi_{jl}(c;z)\frac{dG_j(c;z)}{G_j(c_{jl}^*(z);z)}$ are given by

$$R_{jl}(z) = \zeta_X(z)N_j^E(z)[\tau_{jl}(z)w_jc_j^{max}(z)]^{-k(z)}L_l\hat{p}_l(z)^{1+k(z)} \tag{24}$$

$$\Pi_{jl}(z) = \zeta_\Pi(z)N_j^E(z)[\tau_{jl}(z)w_jc_j^{max}(z)]^{-k(z)}L_l\hat{p}_l(z)^{1+k(z)}$$

where $\zeta_X(z) \equiv k(z)\left(\frac{1}{k(z)} - \frac{1}{k(z)+2}\right)\frac{\alpha}{4\gamma}$, $\zeta_\Pi(z) \equiv k(z)\left(\frac{1}{k(z)} - \frac{2}{k(z)+1} + \frac{1}{k(z)+2}\right)\frac{\alpha}{4\gamma}$ and we have substituted for the fraction of entrants in country j that become exporters to country l, that is $N_{jl}(z) = (c_{jl}^*(z)/c_j^{max}(z))^{k(z)}N_j^E(z)$, hence for the corresponding export cutoff $c_{jl}^*(z) = \frac{\hat{p}_l(z)}{\tau_{jl}(z)w_j}$. Thus, aggregate profits are a constant fraction $\Pi_{jl}(z) = \delta(z)R_{jl}(z)$ of aggregate revenue, where $\delta(z) \equiv \zeta_\Pi(z)/\zeta_X(z)$. This also simplifies the expression for the sectoral employment share

$$\rho_j(z) = \frac{N_j^E(z)/\delta(z)}{\sum_{s=1}^{Z} N_j^E(s)/\delta(s)}, \tag{25}$$

whose country variation only depends on the measure of entrants.

Equilibrium, with Pareto. Evaluating the free entry condition (12) given the inverse Pareto distribution of technological coefficients yields:

$$\text{FEC*}: \quad \sum_{l=1}^{J}\frac{L_l/L_j}{\tau_{jl}(z)^{k(z)}}\left(\frac{w_l}{w_j}\frac{c_l^*(z)}{c_j^{aut}(z)}\right)^{1+k(z)} = 1 \;\; \forall(j,z), \tag{26}$$

$$\text{where } c_j^{aut}(z) \equiv \left(\frac{c_j^{max}(z)^{k(z)}}{\zeta_\Pi(z)L_j/f_j}\right)^{\frac{1}{1+k(z)}}$$

is the cutoff cost in sector z of country j in autarky, that is for $\tau_{jl}(z) \to \infty$ for every $j \neq l$. Output market clearing (13) evaluated with a Pareto distribution yields:

$$\text{OMC*}: \quad \sum_{j=1}^{J} \frac{L_l/L_j}{\tau_{jl}(z)^{k(z)}} \left(\frac{w_l}{w_j} \frac{c_l^*(z)}{c_j^{aut}(z)} \right)^{1+k(z)} w_j f_j N_j^E(z) = w_l f_l N_l^{E\ aut}(z) \quad \forall (l,z)\ . \tag{27}$$

where $N_l^{E\ aut}(z) = \theta(z)\delta(z)\frac{L_l}{f_l}$

is the measure of entrants in sector z of country l in autarky. Evaluating the system of sectoral labor market clearing conditions $\sum_{l=1}^{J} R_{jl}(z) = \rho_j(z) w_j L_j$, that implies (14), given the inverse Pareto distribution of technological coefficients yields:

$$\text{LMC*}: \quad \frac{N_j^E(z)}{\delta(z)} \sum_{l=1}^{J} \frac{L_l/L_j}{\tau_{jl}(z)^{k(z)}} \left(\frac{w_l}{w_j} \frac{c_l^*(z)}{c_j^{aut}(z)} \right)^{1+k(z)} = \rho_j(z)\frac{L_j}{f_j} \quad \forall (j,z) \implies$$

$$\sum_{z=1}^{Z} \left(\frac{N_j^E(z)}{\delta(z)} \sum_{l=1}^{J} \frac{L_l/L_j}{\tau_{jl}(z)^{k(z)}} \left(\frac{w_l}{w_j} \frac{c_l^*(z)}{c_j^{aut}(z)} \right)^{1+k(z)} \right) = \frac{L_j}{f_j} \quad \forall j. \tag{28}$$

Gravity, with Pareto. The specification of the distribution of technology implies that the following variable do not depend on country specific characteristics, hence they do not affect distance in within-sector trade flows: moments of the relative price distribution, such as $\bar{p}_{jl}(z)$ and $\bar{\bar{p}}_{jl}(z)$, expenditure shares, such as $\theta_l(z)$, profit to sales ratios, such as $\delta_j(z)$. The structural gravity equation simplifies to:

$$X_{jl}(z) = \left(\frac{\left(\tau_{jl}(z) w_j c_j^{max}(z)\right)^{-k(z)} \rho_j(z) L_j / f_j}{\sum_{m=1}^{J} \left(\tau_{ml}(z) w_m c_m^{max}(z)\right)^{-k(z)} \rho_m(z) L_m / f_m} \right) \theta(z) Y_l\ . \tag{29}$$

4 Discussion of the equilibrium

At the sector level, the system of sectoral labor market clearing and free entry conditions shows that the relationship between sectoral employment share, i.e. $\rho_j(z)$, and firm entry, i.e. $N_j^E(z)$, is

$$\frac{\rho_j(z)}{N_j^E(z)} = \frac{f_j}{\delta(z) L_j} \tag{30}$$

fixed by sectoral profitability, i.e. $\delta(z)$ hence ultimately technological concentration $k(z)$, and market size relative to entry cost, i.e. L_j/f_j. Average employment of labor per entrant is the same in open economy or in autarky, where $\rho_j(z) \equiv \theta(z)$. Since under Pareto, total revenue, profit and labor cost are proportional, then also the average revenue and average profit per entrant do not depend on trade openness. Therefore, if market size, entry cost and technological concentration are unchanged, a tougher selection corresponds to greater average employment, average revenue and average profit among incumbent firms.

At the country level, the entry of firms is bounded by market size and technological characteristics only, independently on the degree of trade openness. This can be seen by substituting FEC* in the aggregate LMC*, that yields an upper bound to the measure of entrants at the country level:

$$\sum_{z=1}^{Z} \frac{N_l^E(z)}{\delta(z)} = \sum_{z=1}^{Z} \frac{N_l^{E\ aut}(z)}{\delta(z)} = \frac{L_l}{f_l} \quad \forall l. \tag{31}$$

Therefore, if changes in trade openness lead to more entry in one sector, this must be compensated by less entry in other sectors.

The previous insights suggest the predictions of the model for comparative advantage. As a measure of revealed comparative advantage, consider the ratio of employment share in sector z relative to consumption share of sector z in the same country. To measure these components, rewrite the sectoral labor market clearing as an *export equation*

$$N_j^E(z) \sum_{l=1}^{J} \int_0^{c_{jl}^*(z)} r_{jl}(c;z) dG_j(c;z) = \rho_j(z) w_j L_j \ ,$$

and rewrite the output market clearing as an *import equation*

$$\sum_{m=1}^{J} N_m^E(z) \int_0^{c_{mj}^*(z)} r_{mj}(c;z) dG_m(c;z) = \theta(z) w_j L_j \ .$$

Taking the ratio of the two conditions

$$\frac{N_j^E(z) \sum_{l=1}^{J} \int_0^{c_{jl}^*(z)} r_{jl}(c;z) dG_j(c;z)}{\sum_{m=1}^{J} N_m^E(z) \int_0^{c_{mj}^*(z)} r_{mj}(c;z) dG_m(c;z)} = \frac{\rho_j(z)}{\theta(z)}$$

shows that $\rho_j(z)/\theta(z)$ corresponds to the ratio of total sales of country j in sector z (to itself and to the rest of the world) divided by total purchase of country j in sector z (from the world including the country itself). Under Pareto, we have shown that

$\rho_j(z) = \frac{N_j^E(z)/\delta(z)}{L_j/f_j}$, thus

$$\frac{N_j^E(z)\sum_{l=1}^{J}\int_0^{c_{jl}^*(z)} r_{jl}(c;z)dG_j(c;z)}{\sum_{m=1}^{J} N_m^E(z)\int_0^{c_{mj}^*(z)} r_{mj}(c;z)dG_m(c;z)} = \frac{\rho_j(z)}{\theta(z)} = \frac{f_j N_j^E(z)}{\theta(z)\delta(z)L_j} = \frac{N_j^E(z)}{N_j^{E\ aut}(z)} \,. \tag{32}$$

Therefore, a positive sectoral trade balance is associated with firm entry in the sector that is greater in open economy than in autarky, and more so the greater the revealed comparative advantage.

The model predicts that in an equilibrium with diversification, i.e. under the assumption $c_j^*(z) > 0$ for every country j and sector z, there are gains from trade. This can be seen first rewriting the FEC* in terms of revenue

$$\delta(z)\sum_{l=1}^{J} R_{jl}(z) = w_j f_j N_j^E(z) \quad \forall (j,z)\,, \tag{33}$$

that shows how $c_l^*(z) > 0$ for every country l and sector z implies $N_j^E(z) > 0$ for every country j and sector z. Then, rewriting FEC* in terms of cutoffs in open economy yields

$$\left(\frac{c_j^*(z)}{c_j^{aut}(z)}\right)^{1+k(z)} = 1 - \sum_{l\neq j}^{J} \frac{L_l/L_j}{\tau_{jl}(z)^{k(z)}}\left(\frac{w_l}{w_j}\frac{c_l^*(z)}{c_j^{aut}(z)}\right)^{1+k(z)} < 1 \quad \forall (j,z)\,, \tag{34}$$

that implies $c_j^*(z) < c_j^{aut}(z)$ for every country j and sector z. The last relationship also demonstrates that an equilibrium with diversification occurs when trade costs are sufficiently high, whereas it might fail, i.e. the system of free entry conditions FEC* is satisfied for some $c_j^*(z) \leq 0$ in at least one country j and sector z, when trade costs are too low. Given a vector of relative wages, a sufficient condition for an equilibrium with diversification is

$$\begin{aligned}\sum_{l\neq j}^{J} \frac{L_l/L_j}{\tau_{jl}(z)^{k(z)}}\left(\frac{w_l}{w_j}\frac{c_l^*(z)}{c_j^{aut}(z)}\right)^{1+k(z)} &= \sum_{l\neq j}^{J} \frac{L_l/L_j}{\tau_{jl}(z)^{k(z)}}\left(\frac{w_l}{w_j}\frac{c_l^{aut}(z)}{c_j^{aut}(z)}\frac{c_l^*(z)}{c_l^{aut}(z)}\right)^{1+k(z)} \\ &\leq \sum_{l\neq j}^{J} \frac{L_l/L_j}{\tau_{jl}(z)^{k(z)}}\left(\frac{w_l}{w_j}\frac{c_l^{aut}(z)}{c_j^{aut}(z)}\right)^{1+k(z)} < 1 \quad \forall (j,z)\,,\end{aligned} \tag{35}$$

thus, it requires trade costs to be sufficiently high when shipping to larger and richer countries; where the magnitude of trade costs must compensate for the autarkic patterns of comparative advantage, as measured by the ratios in autarkic cost cutoffs.

5 Solution

Define $x_l(z)$ as the trade-induced change in cost cutoff for sector z in country l, and $y_j(z)$ as the trade-induced change in the measure of entrants in sector z country j

$$x_l(z) \equiv \left(\frac{c_l^*(z)}{c_l^{aut}(z)}\right)^{1+k(z)} \quad \text{and} \quad y_j(z) \equiv \frac{N_j^E(z)}{N_j^{E\ aut}(z)} \tag{36}$$

then, collect in $T_{jl}(z)$ the trade costs in sector z from country j to country l weighted by market size, in $K_{jl}(z)$ the autarkic productivity cutoff of country j relative to country l in sector z and in $E_{jl}(z)$ the autarkic patterns of entry in country j relative to country l in sector z

$$T_{jl}(z) \equiv \frac{\tau_{jl}(z)^{k(z)}}{L_l/L_j}\ ,\ K_{jl}(z) \equiv \left(\frac{c_l^{aut}(z)}{c_j^{aut}(z)}\right)^{1+k(z)}\ ,\ E_{jl}(z) \equiv \frac{f_j N_j^{E\ aut}(z)}{f_l N_l^{E\ aut}(z)} = \frac{L_j}{L_l}\ .$$

This notation is useful to highlight the structure of system of equilibrium conditions:

$$\begin{aligned}
\text{FEC}^{**} \quad &: \quad \sum_{l=1}^{J} \frac{K_{jl}(z)}{T_{jl}(z)} \left(\frac{w_l}{w_j}\right)^{1+k(z)} x_l(z) = 1 && \forall (j,z) \\
\text{OMC}^{**} \quad &: \quad \sum_{j=1}^{J} \frac{K_{jl}(z)E_{jl}(z)}{T_{jl}(z)} \left(\frac{w_l}{w_j}\right)^{k(z)} x_l(z) y_j(z) = 1 && \forall (l,z) \\
\text{LMC}^{**} \quad &: \quad \sum_{z=1}^{Z} \theta(z) y_j(z) = 1 && \forall j.
\end{aligned}$$

Thus, the equilibrium of the model consists of the solution of a system of $J + (JZ)^2$ non-linear coupled equations in as many unknowns $\{w_j, x_j(z), y_j(z)\}$ for $j = 1, ..., J$ and $z = 1, ..., Z$.

For a given vector of relative wages, FEC** is a linear system of JZ equations in as many unknowns $x_j(z)$, thus, for a given vector of relative wages there exists a unique matrix of cost cutoffs. Furthermore, OMC** is a linear system of JZ equations in as many unknowns $y_j(z)$, given a vector of relative wages, hence, for a unique matrix of cost cutoffs. Thus, the system of FEC** and OMC** determines the unique matrix of values $y_j(z)$ describing the patterns of entry in open economy for a given vector of relative wages. To investigate the properties of the solution, rearranging the system of

FEC** and OMC** within a sector z

$$
\begin{aligned}
x_m(z) &= 1 - \left(\frac{w_1}{w_m}\right)^{1+k(z)} \sum_{l \neq m} \frac{K_{ml}(z)}{T_{ml}(z)} \left(\frac{w_l}{w_1}\right)^{1+k(z)} x_l(z) && \forall (m,z) \\
y_m(z) &= \frac{1}{x_m(z)} - \left(\frac{w_m}{w_1}\right)^{k(z)} \sum_{j \neq m}^{J} \frac{K_{jm}(z) E_{jm}(z)}{T_{jm}(z)} \left(\frac{w_1}{w_j}\right)^{k(z)} y_j(z) && \forall (m,z)
\end{aligned}
$$

shows that trade-induced changes in cost cutoffs $x_m(z)$ are increasing in own relative nominal wage w_m/w_1, and trade-induced changes in the measure of entrants $y_m(z)$ are decreasing in own relative nominal wage w_m/w_1.

The remaining system of equations, i.e. labor market clearing conditions LMC**, solves for the vector of relative wages, with trade-induced change in the measure of entrants $y_m(z)$ for every sector z of the same country m being decreasing in own relative nominal wage w_m/w_1. Therefore, a numerical solution of the model is obtained under standard approaches commonly used in quantitative trade models, such as Caliendo and Parro (2015), hence, by starting with a guess for the vector of relative wages, augmenting the relative wage for countries with too much entry $\sum_{z=1}^{Z} \theta(z) y_j(z) > 1$ and decreasing the relative wage for countries with not enough entry $\sum_{z=1}^{Z} \theta(z) y_j(z) < 1$. Existence of a numerical solution in the case of an arbitrary number of countries and sectors can be verified ex-post, whereas uniqueness raises the same concerns as in quantitative trade models based on Eaton and Kortum (2002). However, in the 2-country model existence and uniqueness can be proved, thus, we focus on this scenario to illustrate the properties of the equilibrium.

5.1 Solution in a 2-country model

Consider a 2-country model, for $j = 1,2$. In each sector z the system of country-specific free entry conditions yields:

$$
x_1(z) + a_{12}(z) \left(\frac{w_2}{w_1}\right)^{1+k(z)} x_2(z) = 1 \tag{37}
$$

$$
a_{21}(z) \left(\frac{w_1}{w_2}\right)^{1+k(z)} x_1(z) + x_2(z) = 1 \tag{38}
$$

where $a_{jl}(z) \equiv \frac{K_{jl}(z)}{T_{jl}(z)}$, thus $a_{12}(z)a_{21}(z) = [\tau_{12}(z)\tau_{21}(z)]^{-k(z)} < 1$. The system of free entry conditions yields $x_1(z)$ and $x_2(z)$ as a function of the relative wage:

$$x_1(z) = \frac{1 - a_{12}(z)\left(\frac{w_2}{w_1}\right)^{1+k(z)}}{1 - a_{12}(z)a_{21}(z)}$$

$$x_2(z) = \frac{1 - a_{21}(z)\left(\frac{w_1}{w_2}\right)^{1+k(z)}}{1 - a_{21}(z)a_{12}(z)}$$

The following result holds:

Existence FEC. A necessary condition for the existence of an open-economy equilibrium with diversification, such that $x_j(z) > 0$ for every country $j = 1,2$ and sector $z = 1, ..., Z$, is given by:

$$[\tau_{12}(z)\tau_{21}(z)]^{-k(z)} < a_{21}(z)\left(\frac{w_1}{w_2}\right)^{1+k(z)} < 1 \qquad \forall z = 1, ..., Z. \tag{39}$$

Comparative statics show that in an equilibrium with diversification, $x_1(z)$ is increasing in w_1/w_2 and $x_2(z)$ is increasing in w_2/w_1 for $a_{12}(z)a_{21}(z) < 1$, for every sector z.

Before proceeding with the inspection of the output market clearing conditions, it is useful to rearrange the free entry conditions as follows:

$$\left(\frac{1}{x_1(z)} - 1\right)\left(\frac{1}{x_2(z)} - 1\right) = a_{12}(z)a_{21}(z) = [\tau_{12}(z)\tau_{21}(z)]^{-k(z)} < 1\,. \tag{40}$$

Thus, holding trade costs and technological concentration constant, any change in sectoral trade-induced cost cutoffs of a country must be compensated by a change of opposite sign in the other country.

The system of country-specific output market clearing conditions yields:

$$x_1(z)y_1(z) + a_{21}(z)\frac{L_2}{L_1}\left(\frac{w_1}{w_2}\right)^{k(z)} x_1(z)y_2(z) = 1$$

$$a_{12}(z)\frac{L_1}{L_2}\left(\frac{w_2}{w_1}\right)^{k(z)} x_2(z)y_1(z) + x_2(z)y_2(z) = 1$$

Substituting for $x_1(z)$ and $x_2(z)$ as implied by the free entry condition in the system of country-specific output market clearing conditions yields the solution for $y_1(z)$ and

$y_2(z)$ as a function of the relative wage:

$$y_1(z) = \frac{1}{1 - a_{12}(z)\left(\frac{w_2}{w_1}\right)^{1+k(z)}} - \frac{[\tau_{12}(z)\tau_{21}(z)]^{-k(z)}\frac{L_2}{L_1}}{a_{12}(z)\left(\frac{w_2}{w_1}\right)^{1+k(z)} - [\tau_{12}(z)\tau_{21}(z)]^{-k(z)}} \tag{41}$$

$$y_2(z) = \frac{1}{1 - a_{21}(z)\left(\frac{w_1}{w_2}\right)^{1+k(z)}} - \frac{[\tau_{12}(z)\tau_{21}(z)]^{-k(z)}\frac{L_1}{L_2}}{a_{21}(z)\left(\frac{w_1}{w_2}\right)^{1+k(z)} - [\tau_{12}(z)\tau_{21}(z)]^{-k(z)}} \tag{42}$$

where $y_1(z)$ is decreasing in w_1/w_2 and $y_2(z)$ is decreasing in w_2/w_1. A necessary condition for the existence of an equilibrium with diversification is:

Existence FEC & OMC. A necessary condition for the existence of an open-economy equilibrium with diversification, such that $x_j(z) > 0$ and $y_j(z) > 0$ for every country $j = 1,2$ and sector $z = 1, ..., Z$, is given by:

$$\frac{[\tau_{12}(z)\tau_{21}(z)]^{-k(z)}\left(1 + \frac{L_1}{L_2}\right)}{1 + [\tau_{12}(z)\tau_{21}(z)]^{-k(z)}\frac{L_1}{L_2}} < a_{21}(z)\left(\frac{w_1}{w_2}\right)^{1+k(z)} < \frac{1 + [\tau_{12}(z)\tau_{21}(z)]^{-k(z)}\frac{L_2}{L_1}}{1 + \frac{L_2}{L_1}} \tag{43}$$

for every sector $z = 1, ..., Z$.

Visual inspection is sufficient to conclude that (43) is a more restrictive requirement than (39). Finally, the labor market clearing condition $\sum_{z=1}^{Z} \theta(z) y_j(z) = 1$ for either country $j = 1,2$ determines the relative wage w_2/w_1. Since $y_2(z)$ is decreasing in w_2/w_1 for every sector z, the 2-country equilibrium with diversification exhibits a unique relative wage.[5] This implies that condition (43) is necessary and sufficient for existence and uniqueness of an equilibrium with diversification.

5.2 Existence of an equilibrium with diversification

Existence of an open-economy equilibrium in which every country produces in every sector (43) depends, first, on trade barriers and, secondly, on country-differences in technology (including both marginal costs and sunk entry costs). A third characteristic, i.e. technological concentration, distorts the effect of the first two channels.

[5] A comparative static exercise on the function $y_j(z)$ for $j = 1,2$, after noticing that $a_{jl} \propto 1/a_{lj}$ for $j,l = 1,2$, reveals that $y_j(z)$ is increasing in $a_{jl}(Z)$ and decreasing in w_j/w_l. The level of $y_2(z)$ as a function of the relative wage w_2/w_1 is higher the greater the parameter $a_{21}(z) \propto (c_1^{max}(z)/c_2^{max}(z))^{k(z)}$. Thus, the model remains tractable enough to explain the consequences of a resource shock on welfare with simple comparative statics exercises; that we conduct in the next section.

Trade barriers and concentration. Consider the limit case of free trade, i.e. $\tau_{12}(z)\tau_{21}(z) \to 1$. Under these circumstances, there exists no well-defined equilibrium with diversification.[6] Indeed, a minimum level of trade barriers is necessary for existence of an equilibrium with diversification. On the other hand, with high trade costs, i.e. $\tau_{12}(z)\tau_{21}(z) \to \infty$ an open-economy equilibrium with diversification exists for $0 < a_{21}(z)\,(w_1/w_2)^{1+k(z)} < L_1/(L_1+L_2) < 1$ for every sector $z = 1, ..., Z$.[7] Moreover, the range in which (43) applies is wider the greater the extent of trade barriers. This insight clarifies the main connection between our framework and the classical Ricardian model: in the absence of trade barriers, international trade would lead to a complete specialization.

Technological concentration works in the direction of dampening the role of trade barriers on the existence domain of an equilibrium with diversification: greater concentration of firms similar to the worst producer unconditional on entry, e.g. high $k(z)$, lowers the impact of trade barriers, e.g. $\tau_{12}(z)$ or $\tau_{21}(z)$, and with a power rate. We conclude that existence of an equilibrium with diversification occurs when sectors are either *protected and concentrated* or *liberalized and dispersed*.

Finally, the effect of trade barriers and technological concentration is mediated by differences in market sizes. Looking at the range in which an equilibrium with diversification exists, and holding the technological coefficient $a_{21}(z)$ and concentration constant, the lower bound for the foreign relative wage w_1/w_2 is decreasing in L_1/L_2, while the upper bound is decreasing in L_2/L_1. Thus, a greater foreign market size L_1 relative to the domestic market size L_2 widens the existence domain of an equilibrium with diversification.

Technological differences and concentration. To examine the role played by technological differences in the existence of an equilibrium with diversification recall that $a_{21}(z) \equiv \frac{f_1}{f_2}\left(\frac{c_1^{max}(z)}{\tau_{21}(z)c_2^{max}(z)}\right)^{k(z)}$, which, substituted in (43), implies:

$$\frac{\tau_{12}(z)^{-k(z)}\left(1+\frac{L_1}{L_2}\right)}{1+[\tau_{12}(z)\tau_{21}(z)]^{-k(z)}\frac{L_1}{L_2}} < \frac{f_1}{f_2}\left(\frac{c_1^{max}(z)}{c_2^{max}(z)}\right)^{k(z)}\left(\frac{w_1}{w_2}\right)^{1+k(z)} < \frac{\tau_{21}(z)^{k(z)}+\tau_{12}(z)^{-k(z)}\frac{L_2}{L_1}}{1+\frac{L_2}{L_1}}$$

[6] Analytically, the case in which $a_{21}(z)\,(w_1/w_2)^{1+k(z)} = 1$ for every sector $z = 1, ..., Z$, by a trivial chance, can be studied and leads to indeterminacy in the cost cutoffs and number of entrant firms.

[7] Note that this is not equivalent to the equilibrium in autarky. In the latter, both systems of FECs and OMCs are not coupled by country, so characteristics of the foreign market do not matter at all for the existence of an equilibrium in autarky.

for every sector $z = 1, ..., Z$. This yields a range for the relative wage:

$$\frac{w_1}{w_2} > \max_z \left\{ \left(\frac{c_2^{max}(z)}{c_1^{max}(z)} \right)^{\frac{k(z)}{1+k(z)}} \left(\frac{f_2}{f_1} \frac{\tau_{12}(z)^{-k(z)} \left(1 + \frac{L_1}{L_2}\right)}{1 + [\tau_{12}(z)\tau_{21}(z)]^{-k(z)} \frac{L_1}{L_2}} \right)^{\frac{1}{1+k(z)}} \right\}$$
$$\frac{w_1}{w_2} < \min_z \left\{ \left(\frac{c_2^{max}(z)}{c_1^{max}(z)} \right)^{\frac{k(z)}{1+k(z)}} \left(\frac{f_2}{f_1} \frac{\tau_{21}(z)^{k(z)} + \tau_{12}(z)^{-k(z)} \frac{L_2}{L_1}}{1 + \frac{L_2}{L_1}} \right)^{\frac{1}{1+k(z)}} \right\}.$$

Country differences in real entry costs magnify the role played by trade barriers and market sizes, with $f_2 > f_1$ implying a shift to the right (higher values) and a wider support for the foreign relative wage w_1/w_2. Country-sector differences in the unconditional distribution of real marginal cost determine the patterns of comparative advantage, with the foreign relative wage bounded between the lowest and the highest relative productivity across sectors, adjusted by sectoral concentration.

This is clearly reminiscent of the chain of comparative advantage in Dornbusch et al. (1977), with the fundamental difference that fixed technological coefficients in the classical formulation are replaced in our model by the upper support in the unconditional cost distribution, e.g. $\left(c_2^{max}(z)/c_1^{max}(z)\right)^{k(z)/[1+k(z)]}$, and not only the central moment of the technological distribution matters but also the dispersion.

Technological concentration distorts the patterns of comparative advantage based on comparisons in average productivity: greater $k(z)$, thus a more concentrated sector, inflates comparative disadvantage of the domestic country, if $c_2^{max}(z) > c_1^{max}(z)$, and smooths comparative advantage of the domestic country, if $c_2^{max}(z) < c_1^{max}(z)$; the opposite effects occur in sectors characterized by more technological dispersion. Furthermore, the contribution of entry costs, trade barriers and market sizes tends to vanish for sectors that are highly concentrated, as $1/(1 + k(z)) \to 0$. Under the same circumstances, comparative advantage patterns are fully described by the average real cost, as in the classical Ricardian model with no firm heterogeneity, which applies in the limit case with large concentration, as $k(z)/(1 + k(z)) \to 1$.

6 Immiserizing growth

Consider country $j = 2$ as the domestic country, trading with the rest of the world, i.e. country $j = 1$. Holding trade barriers, market size and technological concentration constant, assume that a positive resource shock in the domestic country favors growth in a given sector z. Under which conditions a positive resource shock for the domestic country in a given sector leads to a worsening of welfare in the domestic country?

6.1 A positive resource shock

To answer the question that opens this section we define a *positive resource shock* in a sector z of country $j = 2$ as an exogenous, unanticipated and permanent change in the support of the cost distribution in country 2 sector z, such that $\hat{c}_2^{max}(z) = c_2^{max}(z)\xi(z)$, with $\xi(z) \in (0, 1)$; where we indicate with a *hat* the variables after the resource shock. Given the Inverse Pareto distribution, the average cost of the exogenous distribution is proportional to the support, hence, the interpretation of the shock is straightforward: e.g. a resource shock that makes the unconditional average of a producer in sector z of the domestic country 10% more efficient corresponds to $\xi(z) = 1 - 10\% = 0.90$.

6.2 Consequences for relative wage and labor reallocation

Note that the sequence of functions $\{y_j(z)\}_{z=1}^{Z}$ can be interpreted as an aggregate labor demand function. More precisely, the labor market clearing condition shows that the component of labor reallocation that matters in equilibrium is the extensive margin, due to the entry of firms. Under this interpretation we will assess the consequences of a positive resource shock for relative wage and labor reallocation.

Holding the other parameters of the model constant, the labor demand function in the domestic country $j = 2$ tilts upward, since the component $y_2(z)$ does, while no change occurs in the other sectoral components $y_2(s)$ for every $s \neq z$. Thus, the consequences of a positive resource shock in sector z of the domestic country $j = 2$ can be summarized as follows:

1 the equilibrium relative wage such that $\sum_{s=1}^{Z} \theta(s)y_2(s) = 1$ after the shock has to be higher, that is $\hat{w}_2/\hat{w}_1 > w_2/w_1$;

2 the sector hit by the shock experiences more firms paying the entry cost and greater allocation of labor $\hat{y}_2(z) > y_2(z)$;

3 symmetrically, the increase in relative wage, decreases the measure of potential entrants and the demand for labor in the other sectors of the same country $\hat{y}_2(s) < y_2(s)$ for every $s \neq z$.

These results can be assessed by looking at the labor market clearing condition in equilibrium and isolating the contribution of sector z

$$\theta(z)\left(\frac{1}{1-a_{21}(z)\left(\frac{w_1}{w_2}\right)^{1+k(z)}}-\frac{a_{12}(z)L_1/L_2\left(\frac{w_2}{w_1}\right)^{k(z)}}{1-a_{12}(z)\left(\frac{w_2}{w_1}\right)^{1+k(z)}}\right)=$$

$$1-\sum_{s\neq z}^{Z}\theta(s)\left(\frac{1}{1-a_{21}(s)\left(\frac{w_1}{w_2}\right)^{1+k(s)}}-\frac{a_{12}(s)L_1/L_2\left(\frac{w_2}{w_1}\right)^{k(s)}}{1-a_{12}(s)\left(\frac{w_2}{w_1}\right)^{1+k(s)}}\right)$$

which shows that the demand for labor in country 2 is decreasing in the coefficient $a_{12}(z)$ and increasing in the coefficient $a_{21}(z)$; thus, the demand for labor in country 2 is decreasing in $c_2^{max}(z)/c_1^{max}(z)$. We conclude that the labor market clearing for country 2 describes a decreasing relationship in the wage ratio w_2/w_1, that shifts upward when country 2 experiences growth in a sector z, such that $\hat{c}_2^{max}(z) < c_2^{max}(z)$.

The relative wage that satisfies the new equilibrium with diversification is necessarily higher than before $\hat{w}_2/\hat{w}_1 > w_2/w_1$. The fact that an increase in the endowment of resources in a given sector determines an increase in relative wage is intuitive, since labor is complementary to factors of production embedded in what we generically call sector-specific resources, whose endowment, hence employment, has increased.

In sectors of the domestic country that are not hit by the shock the labor demand function does not change, therefore, its equilibrium level has to fall in response to a higher relative wage in the domestic country, i.e. $\hat{y}_2(s) < y_2(s)$ for every $s \neq z$. Given full employment, labor market clearing implies a greater employment of labor in equilibrium in the sector experiencing the positive resource shock $\hat{y}_2(z) > y_2(z)$.

6.3 Consequences for selection

Welfare is the geometric average of the inverse of sectoral cutoff costs, see (10). Thus, determining how the positive resource shock affects sectoral cutoff costs is necessary to understand the consequences for welfare. We start with sectors $s \neq z$ that are not hit by the shock. We have shown that the relative cost cutoff $x_2(s) = c_2^*(s)/c_2^{aut}(s)$ is increasing in the relative wage w_2/w_1 for every $s = 1, ..., Z$. Thus, an increase in the relative wage of country 2 is a necessary and sufficient condition for $\hat{x}_2(s) > x_2(s)$ in every sector s that did not experience a resource growth shock. This also implies higher cutoff costs in those sectors, i.e. $\hat{c}_2^*(s) > c_2^*(s)\ \forall s \neq z$, since nothing has changed for the corresponding autarkic cutoffs.

Now we continue the analysis with the sector hit by the shock. The system of free entry conditions shows that a necessary and sufficient condition is that the relative wage does not fall as much as the support of the exogenous distribution of cutoff costs:

$$\hat{x}_2(z) > x_2(z) \iff \frac{\hat{w}_2/\hat{w}_1}{w_2/w_1} > \left(\frac{\hat{a}_{21}(z)}{a_{21}(z)}\right)^{\frac{1}{1+k(z)}} = \frac{c_2^{aut}(z)}{\hat{c}_2^{aut}(z)} = \left(\frac{c_2^{max}(z)}{\hat{c}_2^{max}(z)}\right)^{k(z)/(1+k(z))},$$

and this is always satisfied, since relative wage increases while the support shrinks. However, the condition $\hat{x}_2(z) > x_2(z)$ is necessary but not sufficient to determine an overall worsening of the productivity distribution of incumbent firms in the sector that experiences the resource shock, i.e. $\hat{c}_2^*(z) > c_2^*(z)$, as shown by rewriting the system of free entry conditions before and after the shock:

$$\left(\frac{\hat{c}_2^*(z)}{c_2^*(z)}\right)^{1+k(z)} = \underbrace{\left(\frac{1-\left(\hat{a}_{21}(z)^{\frac{1}{1+k(z)}}\frac{\hat{w}_1}{\hat{w}_2}\right)^{1+k(z)}}{1-\left(a_{21}(z)^{\frac{1}{1+k(z)}}\frac{w_1}{w_2}\right)^{1+k(z)}}\right)}_{\hat{x}_2(z)/x_2(z)}\left(\frac{\hat{c}_2^{aut}(z)}{c_2^{aut}(z)}\right)^{1+k(z)}.$$

Rearranging and substituting for the coefficients $\hat{a}_{21}(z) \equiv \frac{L_1/L_2}{\tau_{21}(z)^{k(z)}}\left(\frac{c_1^{aut}(z)}{\hat{c}_2^{aut}(z)}\right)^{1+k(z)}$ and $a_{21}(z) \equiv \frac{L_1/L_2}{\tau_{21}(z)^{k(z)}}\left(\frac{c_1^{aut}(z)}{c_2^{aut}(z)}\right)^{1+k(z)}$ yields:[8]

$$\left(\frac{\hat{c}_2^*(z)}{c_2^*(z)}\right)^{1+k(z)} = \frac{\hat{c}_2^{aut}(z)^{1+k(z)} - \frac{L_1/L_2}{\tau_{21}(z)^{k(z)}}c_1^{aut}(z)^{1+k(z)}\left(\frac{\hat{w}_1}{\hat{w}_2}\right)^{1+k(z)}}{c_2^{aut}(z)^{1+k(z)} - \frac{L_1/L_2}{\tau_{21}(z)^{k(z)}}c_1^{aut}(z)^{1+k(z)}\left(\frac{w_1}{w_2}\right)^{1+k(z)}}. \tag{44}$$

In the sector experiencing the growth shock the (general equilibrium) effect of an increase in wage $\hat{w}_2/\hat{w}_1 > w_2/w_1$ is contrasted by the (direct) improvement in unconditional average productivity due to the shock $\hat{c}_2^{max}(z) < c_2^{max}(z) \implies \hat{c}_2^{aut}(z) < c_2^{aut}(z)$. Thus, the consequences of a positive resource shock in sector z of the domestic country $j = 2$ on the selection of firms can be summarized as follows:

4 sectors not hit by the shock are characterized by a weaker selection than before,

[8]The condition (44) holds for every sector $z = 1, ..., Z$; indeed, it can be seen clearly that for sectors not hit by the shock $\hat{c}_2^{aut}(s) \equiv c_2^{aut}(s)$ any increase in the relative wage w_2/w_1 is sufficient to predict greater cutoff costs, while in the sector hit by the shock the increase in wage must be large enough to compensate for $\hat{c}_2^{aut}(z) < c_2^{aut}(z)$ for that to happen.

i.e. $\hat{c}_2^*(s) > c_2^*(s)$ for every $s \neq z$;

5 selection in the sector experiencing a positive resource shock is the outcome of two contrasting forces: the direct consequence of the shock is a lowering of the cost cutoff, but, the increase in the relative wage restores the room for producers selling at higher price.

It shall be noted that the increase in relative wage is associated with an anti-competitive effect of the resource shock. This can be intuitively understood by thinking at the reason why firms exit. There are no (exogenous) fixed costs of production, so firms exit if they would charge a price above the (endogenous) choke price. A higher relative wage in the domestic country works in the direction of an increase of the choke price.

6.4 Consequences for welfare

For the sake of exposition, consider a two-sector economy, with sector z hit by the shock while sector s does not. Welfare change in the domestic country is given by:[9]

$$\frac{\hat{W}_2}{W_2} = \left(\frac{c_2^*(z)}{\hat{c}_2^*(z)}\right)^{\beta(z)} \left(\frac{c_2^*(s)}{\hat{c}_2^*(s)}\right)^{\beta(s)} \tag{45}$$

with $\beta(z) + \beta(s) = 1$. We have already established that sectors not hit by the shock are responsible for a worse distribution of costs, thus, producing a channel that decreases welfare. Welfare gains might arrive only from the sector that directly experiences the positive resource shock, but, provided that the increase in relative wage of the domestic country w_2/w_1 is not *too high*. And even in that case, welfare gains from $c_2^*(z) > \hat{c}_2^*(z)$ must more than compensate welfare losses from the other sector $c_2^*(s) < \hat{c}_2^*(s)$.

Given the expression (44), substituting for $c_j^{aut}(z)$ as a function of $c_j^{max}(z)$, recalling that the shock can be expressed as $\hat{c}_2^{max}(s)/c_2^{max}(s) = \xi(s)$ with $\xi(s) = 1$ for $s \neq z$ and

[9]Notice that it is trivial to conclude that resource shocks affecting a sector that is marginal in consumer's preferences have no effect. In fact as $\beta(z) \to 0$ then welfare is not affected by the change in cost cutoff of sector z; but also $\theta(z) \to 0$, hence, there is no impact on the relative wage as well, so also the cost cutoff of the other sector does not change.

$\xi(z) \in (0,1)$, the change in cost cutoffs can be written as

$$\frac{\hat{c}_2^*(z)}{c_2^*(z)} = \left(\frac{\hat{x}_2(z)}{x_2(z)}\right)^{\frac{1}{1+k(z)}} \frac{\hat{c}_2^{aut}(z)}{c_2^{aut}(z)} = \left(\frac{1-\hat{g}(z)}{1-g(z)}\right)^{\frac{1}{1+k(z)}} \xi(z)^{\frac{k(z)}{1+k(z)}}$$
$$\frac{\hat{c}_2^*(s)}{c_2^*(s)} = \left(\frac{\hat{x}_2(s)}{x_2(s)}\right)^{\frac{1}{1+k(s)}} \frac{\hat{c}_2^{aut}(s)}{c_2^{aut}(s)} = \left(\frac{1-\hat{g}(s)}{1-g(s)}\right)^{\frac{1}{1+k(s)}}$$

where we have introduced a change of variable

$$g(i) \equiv a_{21}(i)\left(\frac{w_1}{w_2}\right)^{1+k(i)} \quad \text{and} \quad \hat{g}(i) \equiv \hat{a}_{21}(i)\left(\frac{\hat{w}_1}{\hat{w}_2}\right)^{1+k(i)} \quad , \; i = z, s$$

that will be convenient in what follows. The coefficients $g(i)$ are an indicator of "*gross potential revealed comparative advantage*" of the domestic country in sector $i = z, s$, where: "gross" refers to the comparison of entry costs in addition to the classical comparison in marginal cost; and "potential" refers to the marginal cost that is measured in terms of the exogenous cost support of the technological distribution.[10]

Given the equilibrium change in foreign relative wage expressed in terms of the endogenous coefficients $g(z)$ and $g(z)$, by substituting in the expression for the change in welfare yields:

$$\frac{\hat{W}_2}{W_2} = \left(\frac{1-g(z)}{1-\hat{g}(z)}\right)^{\frac{\beta(z)}{1+k(z)}} \left(\frac{1-g(s)}{1-\hat{g}(s)}\right)^{\frac{\beta(s)}{1+k(s)}} \xi(z)^{-\frac{\beta(z)k(z)}{1+k(z)}}. \tag{46}$$

Thus, the direct effect of an increase in productivity in sector z, i.e. $\xi(z)^{-\frac{k(z)}{1+k(z)}} > 1$, is followed by the endogenous response of average productivity in the sector hit by the shock, i.e. $(x_2(z)/\hat{x}_2(z))^{\frac{1}{1+k(z)}} > 1$, and - also - by the endogenous response of average productivity in the other sector, i.e. $(x_2(s)/\hat{x}_2(s))^{\frac{1}{1+k(s)}} < 1$. We conclude that:

6 (*Weak Solow's paradox*) The change in welfare is necessarily lower than the overall increase in average productivity in the sector hit by the shock, as measured by the direct effect plus the endogenous adjustment caused by firm entry discounted by the sector's share in welfare.

The intuition is that the increase in relative wage for the domestic country that experi-

[10]To see this more clearly, recall that $a_{21}(i) = \frac{f_1}{f_2}\left(\frac{c_1^{max}(i)}{\tau_{21}(i)c_2^{max}(i)}\right)^{k(i)}$. Thus, $g(i) = a_{21}(i)\left(\frac{w_1}{w_2}\right)^{1+k(i)} = \frac{f_1 w_1}{f_2 w_2}\left(\frac{c_1^{max}(i)w_1}{\tau_{21}(i)c_2^{max}(i)w_2}\right)^{k(i)}$ is the product of a comparison across countries in entry costs $\frac{f_1 w_1}{f_2 w_2}$ and in marginal costs measured at the upper bound of the cost support $\frac{c_1^{max}(i)w_1}{\tau_{21}(i)c_2^{max}(i)w_2}$.

ences a positive resource shock allows for a higher choke price in the sectors that are not hit by the shock; in fact in that sector the cutoff cost did not shrink, while the relative wage has increased.

6.5 Labor market equilibrium

The equilibrium of the model reduces to an equilibrium of the labor market, once endogenous cost cutoffs and firm entry has been expressed in terms of the relative wage; as shown in the previous section. The latter can be expressed by two conditions, that are conveniently described as functions of the sectoral comparative advantage indices of the domestic country $g(z)$ and $g(s)$.

First, the labor market clearing condition LMC** of the domestic country $j = 2$ written in terms of the coefficients $g(i)$ for $i = z, s$ is given by

$$\theta(z)\left(\frac{1}{1-g(z)} - \frac{\phi(z)\frac{L_1}{L_2}}{g(z)-\phi(z)}\right) + \theta(s)\left(\frac{1}{1-g(s)} - \frac{\phi(s)\frac{L_1}{L_2}}{g(s)-\phi(s)}\right) = 1 \tag{47}$$

that passes through the point that corresponds to the values $g(z)$ and $g(s)$ that set the parenthesis to 1; in which the patterns of firm entry are equivalent to those in autarky. Total differentiating yields:

$$\frac{dg(s)}{dg(z)} = -\frac{\theta(z)}{\theta(s)}\left(\frac{1-g(s)}{1-g(z)}\right)^2 \left[\frac{1+\frac{\phi(z)\frac{L_1}{L_2}(1-g(z))^2}{(g(z)-\phi(z))^2}}{1+\frac{\phi(s)\frac{L_1}{L_2}(1-g(s))^2}{(g(s)-\phi(s))^2}}\right]$$

thus, the LMC is a decreasing relationship in the space $\{g(z), g(s)\}$, whose graph depends on trade costs, technological concentration and expenditure shares. The slope is null as $g(s) \to \phi(s) \in (0,1)$ and $g(s) \to 1$, whereas it is vertical as $g(z) \to \phi(z) \in (0,1)$ and $g(s) \to 1$. Therefore, the function exhibits S-shape among these extremes.

Second, free mobility of labor between sectors implies the same wage in the same country between the two sectors

$$\frac{w_1}{w_2} = \left(\frac{g(z)}{a_{21}(z)}\right)^{\frac{1}{1+k(z)}} = \left(\frac{g(s)}{a_{21}(s)}\right)^{\frac{1}{1+k(s)}},$$

that describes an increasing relationship in the space $\{g(z), g(s)\}$, spreading from the origin, whose graph depends on differences in technology:

$$g(s) = a_{21}(s)\left(\frac{g(z)}{a_{21}(z)}\right)^{\frac{1+k(s)}{1+k(z)}}. \tag{48}$$

We refer to this as *Edgeworth* equilibrium condition of the labor market (as it is reminiscent of the famous "box"). Clearly, there exists a unique intersection between (47) and (48). However, the second condition exists only if the necessary condition (43) holds, here written in terms of the coefficients $g(z)$ and $g(s)$

$$0 < \frac{\phi(z) + \phi(z)\frac{L_1}{L_2}}{1 + \phi(z)\frac{L_1}{L_2}} < g(z) < \frac{1 + \phi(z)\frac{L_2}{L_1}}{1 + \frac{L_2}{L_1}} < 1 \tag{49}$$

$$0 < \frac{\phi(s) + \phi(s)\frac{L_1}{L_2}}{1 + \phi(s)\frac{L_1}{L_2}} < g(s) < \frac{1 + \phi(s)\frac{L_2}{L_1}}{1 + \frac{L_2}{L_1}} < 1. \tag{50}$$

Once the equilibrium of the model is solved in terms of comparative advantage coefficients, before $\{g(z), g(s)\}$ and after $\{\hat{g}(z), \hat{g}(s)\}$ a shock, welfare changes can be assessed rearranging the expression (46)

$$\ln\left(\frac{\hat{W}_2}{W_2}\right) \approxeq \frac{\beta(z)}{1+k(z)}\frac{\hat{g}(z) - g(z)}{1-\hat{g}(z)} + \frac{\beta(s)}{1+k(s)}\frac{\hat{g}(s) - g(s)}{1-\hat{g}(s)} + \frac{\beta(z)k(z)}{1+k(z)}\left(1 - \xi(z)\right) \tag{51}$$

where the approximation is implied by $\ln(1+x) \approxeq x$ for $x \in (-1, 1)$.

6.6 Finding immiserizing gowth

The key insight for examining the possibility of immiserizing growth is to determine an upper bound the the change in welfare. Note that:[11]

$$\hat{g}(z) + g(z) < 1 \implies \frac{1 - g(z)}{1 - \hat{g}(z)} < \frac{\hat{g}(z)}{g(z)}$$

$$\hat{g}(s) + g(s) > 1 \implies \frac{1 - g(s)}{1 - \hat{g}(s)} < \frac{\hat{g}(s)}{g(s)}.$$

[11] We know that $\hat{g}(z) > g(z)$ and note that $\frac{1-g(z)}{1-\hat{g}(z)} < \frac{\hat{g}(z)}{g(z)}$ if and only if $\hat{g}(z) - g(z) > \hat{g}(z)^2 - g(z)^2 = (\hat{g}(z) + g(z))(\hat{g}(z) - g(z))$. Thus $\hat{g}(z) + g(z) < 1$ implies $\frac{1-g(z)}{1-\hat{g}(z)} < \frac{\hat{g}(z)}{g(z)}$. Symmetrically, $\frac{1-g(s)}{1-\hat{g}(s)} < \frac{\hat{g}(s)}{g(s)}$ if and only if $\hat{g}(s) - g(s) > \hat{g}(s)^2 - g(s)^2 = (\hat{g}(s) + g(s))(\hat{g}(s) - g(s))$. We know that $\hat{g}(s) < g(s)$, thus $\hat{g}(s) + g(s) > 1$ implies $\frac{1-g(s)}{1-\hat{g}(s)} < \frac{\hat{g}(s)}{g(s)}$.

We have already established that the reallocation toward the sector hit by the shock implies $\hat{g}(z) > g(z)$ and $\hat{g}(s) < g(s)$. Therefore, $\hat{g}(z) < 1/2$ and $\hat{g}(s) > 1/2$ are sufficient (not necessary) conditions to look at an upper bound to the change in welfare:

$$\frac{\hat{W}_2}{W_2} < \left(\frac{\hat{g}(z)}{g(z)}\right)^{\frac{\beta(z)}{1+k(z)}} \left(\frac{\hat{g}(s)}{g(s)}\right)^{\frac{\beta(s)}{1+k(s)}} \xi(z)^{-\frac{\beta(z)k(z)}{1+k(z)}} = \left(\frac{\hat{g}(s)}{g(s)}\right)^{\frac{1}{1+k(s)}} \xi(z)^{-\frac{2\beta(z)k(z)}{1+k(z)}}. \quad (52)$$

Combining the constraints on the change in $g(s)$ shows that for a given $\xi(z) \in (0,1)$ a set of sufficient conditions for immiserizing growth is given by:

$$\hat{g}(z) < \frac{1}{2}, \quad \hat{g}(s) > \frac{1}{2}, \quad \xi(z)^{\frac{k(z)(1+k(s))}{1+k(z)}} < \frac{\hat{g}(s)}{g(s)} \leq \xi(z)^{2\beta(z)\frac{k(z)(1+k(s))}{1+k(z)}}. \quad (53)$$

together with the necessary and sufficient conditions for the existence of an equilibrium with diversification and the corresponding relative wage, that we repeat here for convenience:

$$\begin{aligned}
&\frac{\phi(z) + \phi(z)\frac{L_1}{L_2}}{1+\phi(z)\frac{L_1}{L_2}} < g(z) < \hat{g}(z) < \frac{1+\phi(z)\frac{L_2}{L_1}}{1+\frac{L_2}{L_1}} \\
&\frac{\phi(s) + \phi(s)\frac{L_1}{L_2}}{1+\phi(s)\frac{L_1}{L_2}} < \hat{g}(s) < g(s) < \frac{1+\phi(s)\frac{L_2}{L_1}}{1+\frac{L_2}{L_1}}, \\
&\hat{g}(z) = \hat{a}_{21}(z)\left(\frac{\hat{g}(s)}{a_{21}(s)}\right)^{\frac{1+k(z)}{1+k(s)}}.
\end{aligned}$$

Given an equilibrium with diversification, i.e. under (50) and (**??**), the sufficient (not necessary) condition of immiserizing growth (53) holds if the following parametric restrictions are satisfied.

First, **after the resource shock, the relative productivity of the domestic country (i.e. $j = 2$) in the sector hit by the shock gross of trade costs, i.e. $c_1^{max}(z)/[\tau_{21}(z)\hat{c}_2^{max}(z)]$, should be sufficiently low in comparison to the one in the other sector, conditional on market size and trade barriers**, i.e. $\hat{a}_{21}(z)$ should be sufficiently smaller than $a_{21}(s)$, and this implies an upper bound to the positive resource shock, i.e. $1 - \xi(z)$:

$$\hat{g}(z) < \frac{1}{2} \iff \hat{g}(s) < \left(\frac{1}{2} \frac{a_{21}(s)^{\frac{1+k(z)}{1+k(s)}}}{\hat{a}_{21}(z)} \right)^{\frac{1+k(s)}{1+k(z)}}$$

$$\hat{g}(s) < \frac{1+\phi(s)\frac{L_2}{L_1}}{1+\frac{L_2}{L_1}} \leq \frac{a_{21}(s)}{(2\hat{a}_{21}(z))^{\frac{1+k(s)}{1+k(z)}}} = \frac{a_{21}(s)}{(2a_{21}(z))^{\frac{1+k(s)}{1+k(z)}}} \xi(z)^{k(z)\frac{1+k(s)}{1+k(z)}}$$

$$\implies \xi(z)^{\frac{k(z)(1+k(s))}{1+k(z)}} \geq \frac{(2a_{21}(z))^{\frac{1+k(s)}{1+k(z)}}}{a_{21}(s)} \frac{1+\phi(s)\frac{L_2}{L_1}}{1+\frac{L_2}{L_1}} \tag{54}$$

Second, **the other sector should be exposed to sufficiently low trade barriers**:

$$g(s) > \hat{g}(s) > \frac{\phi(s)+\phi(s)\frac{L_1}{L_2}}{1+\phi(s)\frac{L_1}{L_2}} \geq \frac{1}{2} \implies \phi(s) \geq \frac{1}{2+\frac{L_1}{L_2}} \tag{55}$$

Third, to create the room for a change in $g(s)$ that could support immiserizing growth, **expenditure share in the sector hit by the shock should be sufficiently low**:

$$\beta(z) < \frac{1}{2}. \tag{56}$$

Given the set of parametric restrictions (54)-(56), immiserizing growth occurs under the sufficient condition that the worsening of comparative advantage in the sector not hit by the shock, as measured by the ratio $\hat{g}(s)/g(s)$, is large enough such that:

$$\frac{\hat{g}(s)}{g(s)} \leq \xi(z)^{2\beta(z)\frac{k(z)(1+k(s))}{1+k(z)}}.$$

7 Conclusion

Several paradoxical results in the field of international trade share the property that they can arise only if the terms of trade are sufficiently disturbed. Substitution effects, whose high values serve to dampen the required adjustment of prices to a market disturbance, thus work to prevent such results from arising. Using immiserizing growth as a salient example, we have developed a multi-country multi-sector general equilibrium model to characterize the conditions under which firm heterogeneity is consequential in determining a welfare loss for a country enjoying productivity growth. A demand system with variable elasticity of substitution turns out to be crucial for this exercise as incomplete passthrough is shown to play a key role. Our approach could help rethink the terms of trade effects of various shocks in general equilibrium with firm heterogeneity.

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