

How Uncertainty shapes the Spatial Economy.*

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THIS VERSION OF THE PAPER IS LARGELY INCOMPLETE, COMMENTS ARE WELCOME AND PLEASE CONTACT THE AUTHORS BEFORE QUOTING.

Abstract

We develop a dynamic general equilibrium trade model in which the economy is a collection of spatially separated competitive markets and agents (workers/consumers) choose optimally where to locate and seeking for a job. Agents are heterogeneous based on (i) their pre-determined choice of a region, sector and occupation and, also, on (ii) a non-insurable risk of aging, that implies a lower option value when changing region and job characteristics.

By solving for the resulting dynamic spatial quantitative model with labor mobility under uncertainty we show that rational households behave differently compared to a setting with perfect foresight (e.g. Caliendo et al., 2019). In particular, adjustment to shocks slows down, with more people being stuck in poorer regions and higher inequality across regions and sectors in the long run. Furthermore, greater volatility in the productivity of a job (holding constant the mean) results in a strong decline in the attractiveness of those jobs. Using detailed administrative French data, we quantify the productivity-loss-equivalent of a rise in uncertainty and we document that the impact on the individual lifetime welfare of an increase in uncertainty is comparable to the one of a medium-large negative productivity shock.

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1 Introduction

A long-standing and large body of research shows that the reallocation of workers and consumers in space is a driver of welfare gains. But recently and strongly a concern emerged that an increasing number of people fail to benefit from the opportunities offered by a spatially more integrated world. This concern is nourished by the growing fraction of the population which supports protectionist and nationalistic political sentiments in elections. The same people tend to be trapped in bad locations and low-paid jobs and view globalization at large (including the mobility of workers and consumers) as a source of risk rather than one of good opportunities.¹

Both technological change and increased openness at large create incentives of reallocation between jobs and spatial units. The faster technology and openness change, the stronger the incentives to reallocate (to leave jobs and places becoming increasingly worse in relative terms). However, theories of international trade and economic geography provide rationales why the reallocation of factor owners and consumers may be imperfect. Key obstacles to such reallocation are related to technology (in the sense that factor owners may have, e.g., nontransferable job- or sector-specific abilities) or to finite moving costs between jobs or locations (associated with training costs or moving expenses) or preferences and tastes (related to both jobs and locations through amenities).²

However, there is a striking gap between the magnitude of measured changes in technology, trade costs and also moving costs, and the slow mobility of people between jobs and locations.³ Hence, there is a lacuna in our understanding reasons for inertia in multi-type and multi-location employment and residence data.

The reason beyond the standard obstacles to mobility that we focus on in this paper is uncertainty. People – workers as well as consumers – tend to be risk-averse and when prompted to change jobs and locations they are exposed to compare future uncertain returns to job with the value of their current residence and occupational status. On these premises, there is a component of inaction nested in choices made by rational and forward-looking people. Moreover, this source of inertia is associated with welfare losses rooted in the fact that economic agents “*get old*” in a broad sense. The latter means that people literally age, which involves a change in their opportunity costs, a shortening of the time horizon, a weakening of their employment capabilities as their knowledge depreciates. This process of aging entails a risk which agents cannot insure against.

To understand and quantify how *global uncertainty* (i.e. the component of aggregate uncertainty

¹ See Colantone et al. (2021) for a recent survey of the literature on the backlash against globalization.

² Artuc et al. (2010) document the slow labor market adjustment in response to trade liberalization, estimating large and heterogeneous moving costs of switching sectors. The consequence is a sharp movement of wages in local labor markets, segmented by regional, sectoral and occupational characteristics. Along these lines Caliendo et al. (2019) document the short-run response of US local labor markets to import competition from China. Furthermore, the effects of an imperfect reallocation on local labor markets remain strong in the long run, as explained in general equilibrium analysis and documented by Moretti (2011).

³ See Bilal and Rossi-Hansberg (2021) for a critique of the limits of traditional theories addressing the causes and consequences of an imperfect reallocation.

channeled through movements of goods and people across regions) affects the evolution of the spatial distribution of people and economic activity, we develop a dynamic stochastic general equilibrium open economy model with interregional and intersectoral movements of goods and people, who also might choose at any period to change occupation or enter voluntary unemployment. Within each period, the economy works as an Eaton-Kortum production equilibrium augmented with labor mobility frictions. The specific feature of our framework relies on the inter-temporal allocation of labor: Worker-consumer agents make decisions on jobs and locations under uncertainty about technology and trade policy changes happening worldwide, being aware of their expected lifetime.

As time marches on, the value of reallocation deteriorates since both the cost of changing increases and the remaining lifetime is reduced. By considering rational, aging, forward-looking, and risk-averse households in a dynamic stochastic general equilibrium model with realistic spatial linkages due to the moving costs of both goods and people, our theory provides a rich understanding of the way in which uncertainty impacts reallocation in the spatial economy, that we illustrate in a stylized simulation exercise.

Rational households behave differently in a setting with uncertainty compared to a setting with perfect foresight. In particular, adjustment to shocks slows down and a different long run spatial distribution of labor will result in more people being stuck in bad region and in higher inequality across regions and sectors. Adding volatility to the expected productivity of a given job results in a strong decline in the attractiveness of those jobs. We organize these patterns in four main results. (i) In terms of measurement of fundamentals, when ignoring existing uncertainty about future outcomes with risk-averse agents, one would inflate the role of preferences or mobility costs in order to explain residence and employment patterns in the data.

(ii) Accounting for uncertainty our model leads to a greater degree of inertia in reallocation patterns than a model without uncertainty, at given fundamentals. We show that with an increased incentive to reallocate, uncertainty is responsible for a larger number of losers, lower welfare gains (bigger welfare losses), and greater degree of dispersion in the fate of households across jobs and locations. (iii) As uncertainty mitigates reallocation in the transition phase after aggregate shocks, it reduces the extent to which worker-consumer households escape bad jobs and locations and move to good or at least better ones.

(iv) In the long-run, uncertainty raises the duration of lifetime which rational households spend in relatively bad locations and jobs, thereby reducing lifetime earnings.⁴

We bring the model to the data on the French population considering 4 lifetime spans, 2 occupational categories, 49 sectors, 22 regions of residence within France, with spatial linkages plus a rest of the world. The (preliminary) results from the quantification of our model deliver the

⁴ The latter demonstrates that fewer people end up in good jobs and places and more end up in bad jobs and places than this would be the case in a model with perfect foresight. Hence, uncertainty is a source of inertia, that generates a misallocation of individuals relative to a world with perfect foresight in response to shocks. Specifically, uncertainty generates less reallocation as a level effect, and it leads to a more sluggish adjustment to the long-run equilibrium than this would be the case in a dynamic model with perfectly-foresighted individuals.

message that the impact on the individual lifetime welfare of an increase in uncertainty, measured as a change in the volatility of total factor productivity holding the expected value constant, has a magnitude that is comparable to a medium-large negative productivity shock.

Our model builds on the central idea in Artuc et al. (2010): Labor market flows can be explained by aggregating the Euler equations of individual location problems, where each worker faces idiosyncratic shocks about the option value of being in a certain job in the future. We further develop this approach in two directions. First, in our theory locations are linked not only through peoples' mobility but also goods transactions. This is a fundamental mechanism to address global uncertainty, since the wage and price dynamics are channeled by spatial linkages. Second, we consider uncertainty about future outcomes of an aging population, meaning that agents take into account not only that moving cost change with age, but also, that after adulthood there is death. This second feature implies that agents value the opportunity of changing jobs and locations differently as their lifetime horizon becomes shorter.⁵

Investigating the role of global uncertainty in shaping the spatial economy has required advancing the typical tools for solving quantitative trade and regional models. The popularity of those frameworks stems from the fact that they can be portrayed in what has been dubbed the *hat notation* introduced by Dekle et al. (2008), where small changes in equilibrium outcomes between two periods can be specified as a function of outset observable outcomes and contemporaneous changes in fundamentals only.⁶ But the latter convenient property is lost when uncertainty of risk-averse agents about the future needs to be taken into account. Thus, our solution strategy couldn't rely on the hat algebra approach.⁷

Instead, we write the inter-temporal equilibrium of the model as a system of two difference equations: a backward Bellman equation describing the dynamic optimal control problem; and a forward Kolmogorov equation describing the evolution of the labor force distribution as implied by aggregation of individual policies. This shows that the equilibrium of the model can be described as a mean field game, and we build on finite difference solution methods for this class of problems based recently developed in the macroeconomic literature on heterogeneous agents; see Ahn et al. (2018) and Achdou et al. (2021). In doing so, we extend the existing macroeconomic applications to account for spatial dependence in the state space.

⁵ This is not the same argument than claiming that workers are of different types, which is indeed accounted for in Artuc et al. (2010). In fact, aging introduces a source of non-insurable risk which is independent on workers' type. This is important conceptually, because it allows to explain why rational agents might find themselves trapped in bad outcomes as time goes by. But it is also important for quantification purposes, otherwise, the estimates of contemporaneous certain switching costs are inflated to capture the inaction due to uncertainty on future outcomes.

⁶ The distinctive feature of this approach is that it does not require a solution of the model in levels. This is a considerable advantage, as the needs of parameter estimation in spatial models grow with curse of dimensionality. However, this advantage comes at the cost of high dependence on functional forms and constrains the validity of counterfactual exercises to within-country over-time comparisons.

⁷ Moreover, the typical challenge posed by keeping track of goods transactions with bilateral spatial linkages is further amplified by the introduction of labor mobility, with agents that are faced with a large menu of options regarding their employment and residence choices.

The paper relates to two strands of earlier work. First, we build on the modern quantitative, multi-country, multi-sector general-equilibrium literature in trade and regional economics, which started out with Ricardian models (see, e.g., Eaton and Kortum, 2002; Dekle et al., 2007; Alvarez and Lucas, 2007; Caliendo and Parro, 2015) and it is developing to include commuting, migration and a deeper understanding of spatial linkages (see, e.g. Monte et al., 2018; Adao et al., 2020). Despite their usefulness for counterfactual analysis, these frameworks are static, thus, they are not suited for addressing the dynamics intrinsically rooted in job and location decisions under uncertainty. To accomplish this goal while preserving the richness of the aforementioned quantitative models, we design a multi-region, multi-sector, multi-occupation Ricardian trade model in a dynamic programming setup.

Thus, within the quantitative trade and regional literature, our paper directly speaks to a recent line of research accounting for dynamic adjustments (see Desmet et al., 2018; Caliendo et al., 2019; Allen and Donaldson, 2020; Bilal and Rossi-Hansberg, 2021). However, existing works in this field are not designed to explain how global uncertainty affects the spatial distribution of people and economic activity.⁸ To answer this research question, in our model agents make rational forward-looking decisions under uncertainty about the future degree of openness and economic conditions in spatially linked regions.

The second stream of research that we contribute to is the quantitative structural literature on labor market adjustments following trade shocks (see Artuc et al., 2010; Dix-Carneiro, 2014; Dix-Carneiro and Kovak, 2017; Dix-Carneiro et al., 2021 and the review on local labor market adjustment by Moretti, 2011). Also in this field, existing frameworks are based on agents with perfect foresight on aggregate shocks.⁹ An exception is the work by Artuc et al. (2010), which develops a rational expectations model of dynamic labor adjustment following trade liberalizations.¹⁰

The remainder of the paper is organized as follows. In the next section we model the intertemporal allocation of labor among jobs (i.e. region, sector and occupation) as a system of individual dynamic optimal control problems augmented with a law of motion for the distribution of labor force. In the third section we describe the within-period equilibrium of the competitive multi-country multi-sector economy, given the pre-determined allocation of labor. In the fourth section we discuss the numerical solution of the model, featuring discrete time and discrete state space, and conduct a structural estimation of the model's parameters. Section five outlines the results from simulation and quantification. Section six concludes.

⁸ In Desmet et al. (2018) location choices do not depend on future economic conditions; in Caliendo et al. (2019) agents are assumed to be perfectly foresighted; in Allen and Donaldson (2020) the driving force of reallocation (together with technology) is the accumulation of amenities; and in Bilal and Rossi-Hansberg (2021) agents' location decisions respond to income shocks which are not channeled through spatial linkages.

⁹ The contributions Dix-Carneiro (2014), Dix-Carneiro and Kovak (2017) and Dix-Carneiro et al., 2021 carefully model switching costs, sector and occupation specific human capital, local amenities and search frictions in labor market, but agents face (at most) uncertainty about idiosyncratic shocks only.

¹⁰ However, Artuc et al. (2010) simulate the perfect-foresight path of adjustment following the liberalization announcement. Thus, that work does not offer a quantification of the importance of uncertainty under rational expectations.

2 Inter-temporal allocation of labor

In this section we describe a dynamic general equilibrium model with spatially segmented markets, both for products and labor. We build along the lines of Caliendo et al. (2019) to characterize the spatial equilibrium and we follow Artuc et al. (2010) to model how households of different type make forward looking decisions on their job. We extend these previous frameworks to allow for households facing aggregate uncertainty under rational expectations.

2.1 Endowments, market structure and lifetime age spells

We consider a world economy with regions $r = 1, \dots, R$. In each region a large number of perfectly competitive firms produce goods that are horizontally differentiated by region of origin and sector $s = 1, \dots, S$. Firms located in region r producing in sector s demand labor located in the same region and differentiated by occupation $k = 1, 2, \dots, K$. The triplet of region, sector and occupation defines a job $j = \{r, s, k\}$. Every job is filled by one and only one household. Following Caliendo et al. (2019), we denote sector $s = 0$ in each region as the status of unemployment. Unemployed households enjoy a value of home production and they are not distinguished by sector or occupation. Thus, the number of job types in the economy is given by $J = R \times S \times K + R$.

Time is discrete and denoted by $t = 0, 1, 2, \dots$ for an infinite time horizon. Within every period t , competitive output and labor markets open: goods are traded at their marginal cost in region-segmented output markets, and jobs are allocated at the value of their marginal product in job-segmented labor markets. Between time periods, households reallocate optimally across jobs; as we will describe below. The measure of households in each job at the beginning of a period $t+1$ is a predetermined outcome of the choice made by households before the end of period t about which region, sector and occupation locate into in the next period.

Every household goes through four age spells $a = \{b, y, o, d\}$ during her lifetime. A new born $a = b$ from households currently in a certain job does not consume or supply labor, but prepares herself to start next period as a young household $a = y$ that will supply labor in a certain job. With probability $\lambda^y \in (0, 1)$ a young household becomes old $a = o$ at the beginning of the next period. Both young and old households supply labor in the job in which they start the period, they consume by purchasing goods in the same region where their job is located (i.e. there is no commuting). By the end of each period both young and old households make a decision on the job they will start with in the next period. However, old households face a probability $\lambda^o \in (0, 1)$ to die $a = d$ and drop out of the population at zero value. Therefore, older households have a shorter expected lifetime than young households.¹¹

¹¹ New born do not account for consumption (implicitly counted in the one of their household) and by construction there is no consumption from those who die.

2.2 Benefits and costs from reallocating across jobs

Households enjoy two sources of value: utility from consumption of goods in the current period t and satisfaction from the job that they will start with in the next period $t + 1$. Preferences across consumption bundles are the same for every household. Instead, households are heterogeneous in their individual satisfaction from being in a certain region, employed in a certain sector and occupation. Households' objective is to maximize the sum of utility from consumption and job satisfaction over their lifetime.

Heterogeneity about job satisfaction captures in a reduced form not-modeled characteristics that make a difference independently on earnings, and indeed on consumption granted by a job. To model heterogeneity with respect job satisfaction, we assume that every period households draw i.i.d. realizations of taste shocks for every job, independently from other households or from other characteristics of the economy.

Idiosyncratic tastes are confronted with costs for changing job that vary by age, job of origin, job of destination and time. Households of age $a = \{b, y, o\}$ face a cost $\zeta_t^{aj,j'} \geq 0$ of changing from the current job j at time t to a future job j' at time $t + 1$. Moving costs are measured in terms of utility loss. This implies that the net benefit for a household h of moving from a job j at time t to a job i at time $t + 1$ is given by $(\varepsilon_{h,t}^i - \zeta_t^{aj,i}) - (\varepsilon_{h,t}^j - \zeta_t^{aj,j})$, hence it consists of an idiosyncratic taste and a moving cost that is common to all households with the same age moving between the same jobs.

The distribution of taste shocks, with c.d.f. $F(\varepsilon)$ and p.d.f. $f(\varepsilon)$, is invariant to time, household or job characteristics such that $\varepsilon_{h,t}^j \sim F(\varepsilon)$. Instead, moving costs evolve over time following a multivariate stochastic process $\zeta_t^{aj,j} \sim G_t^{aj,j}(\zeta)$ that combines variation along the dimensions of age and job pairs (i.e. age, region, sector and occupation).¹² Frictions captured in the moving costs are exogenous to the model, but with respect to the dimension of age we impose a restriction. In our interpretation, aging goes beyond the meaning of "getting old". It also embraces becoming less mobile across regions (e.g. because of family linkages) and facing higher costs when switching sector and occupation (e.g. because competences in the labor market are evolving while education stopped at a young age). Following this conjecture, moving costs are sorted by age spell $\zeta_t^{oj,j'} \geq \zeta_t^{yj,j'} \geq \zeta_t^{bj,j'}$ within the same job pair and time period. Without loss of generality, we normalize the cost of remaining in the same job to be null $\zeta_t^{aj,j} = 0$.

2.3 Household's inter-temporal problem

At the beginning of every period t , production starts given the current distribution of labor across jobs $L_t = \{L_t^j\}_{j \in J}$ and the current state nature of the economy $Z_t = \{Z_t^j\}_{j \in J}$, which includes the distribution of productivity across jobs and also the fundamentals of the economy that are subject

¹² An invariant distribution of idiosyncratic taste shocks is convenient for tractability. Nevertheless, the model allows for systematic patterns in the reallocation of households across jobs as channeled by the structure of moving costs.

to exogenous changes (such as moving costs). Before the end of every period t a certain household h realizes the vector of idiosyncratic tastes for all jobs $\varepsilon_{h,t} = \{\varepsilon_{h,t}^j\}_{j \in J}$. Given the endogenous state L_t , the realization of the stochastic state Z_t and the realizations of job specific shocks $\varepsilon_{h,t}$ each household makes individual and independent forward looking decision about the optimal job where to end the end the period.

Let $W^{aj}(L_t, Z_t, \varepsilon_{h,t})$ be the lifetime welfare of a household of age a in job j with a vector of idiosyncratic values $\varepsilon_{h,t}$. Let $V^{aj}(L_t, Z_t) = \mathbb{E}_h[W^{aj}(L_t, Z_t, \varepsilon_{h,t})]$ be the average lifetime value of a household in age spell a and job j at time t , where the operator $\mathbb{E}_h[\cdot]$ yields the expectation taken over the idiosyncratic realizations of the value $\varepsilon_{h,t}^j$ across all households. Let $\omega^j(L_t, Z_t)$ be the utility from current consumption when the household is in a job j and the aggregate state is $\{L_t, Z_t\}$. We assume that future values are discounted at a rate $\beta \in (0, 1)$. The lifetime value of a rational household h conditional on its age-and-job status is given by:

$$\begin{aligned} W^{bj}(L_t, Z_t, \varepsilon_{h,t}) &= \max_{n \in J} \left\{ \nu \varepsilon_{h,t}^n - \zeta_t^{bj,n} + \beta \mathbb{E}_t[V^{yn}(L_{t+1}, Z_{t+1})] \right\} \\ W^{yj}(L_t, Z_t, \varepsilon_{h,t}) &= \omega^j(L_t, Z_t) + \max_{n \in J} \left\{ \nu \varepsilon_{h,t}^n - \zeta_t^{yj,n} + \beta \mathbb{E}_t[\bar{V}^{yn}(L_{t+1}, Z_{t+1})] \right\} \\ W^{oj}(L_t, Z_t, \varepsilon_{h,t}) &= \omega^j(L_t, Z_t) + \max_{n \in J} \left\{ \nu \varepsilon_{h,t}^n - \zeta_t^{oj,n} + \beta \mathbb{E}_t[\bar{V}^{on}(L_{t+1}, Z_{t+1})] \right\} \end{aligned} \quad (1)$$

where we denoted averages across future age spells as

$$\begin{aligned} \bar{V}^{yj}(L_{t+1}, Z_{t+1}) &= (1 - \lambda^y) V^{yj}(L_{t+1}, Z_{t+1}) + \lambda^y V^{oj}(L_{t+1}, Z_{t+1}) \\ \bar{V}^{oj}(L_{t+1}, Z_{t+1}) &= (1 - \lambda^o) V^{oj}(L_{t+1}, Z_{t+1}) , \end{aligned}$$

where $\mathbb{E}_t[\cdot]$ is the expectation operation given the information available at time t . Our goal is to characterize the average of values $W^{aj}(L_t, Z_t, \varepsilon_{h,t})$ across households, by job and age groups. To this purpose, let $F(\varepsilon)$ be the c.d.f. of the i.i.d. idiosyncratic shocks $\varepsilon_{h,t}$ and $f(\varepsilon)$ be the corresponding p.d.f., then the average option value of moving when starting from a job j is given by:

$$\nu \Omega(\bar{\varepsilon}_t^{aj}) \equiv \sum_{i \in J} \int_{-\infty}^{\infty} (\nu \varepsilon + \nu \bar{\varepsilon}_t^{aj,i}) f(\varepsilon) \prod_{m \neq i} F(\varepsilon + \bar{\varepsilon}_t^{aj,i} - \bar{\varepsilon}_t^{aj,m}) d\varepsilon , \quad (2)$$

for age spells $a = \{b, y, o\}$ where

$$\begin{aligned} \nu \bar{\varepsilon}_t^{bj,n} &\equiv \beta \mathbb{E}_t[V^{yn}(L_{t+1}, Z_{t+1}) - V^{yj}(L_{t+1}, Z_{t+1})] - \zeta_t^{bj,n} \\ \nu \bar{\varepsilon}_t^{yj,n} &\equiv \beta \mathbb{E}_t[\bar{V}^{yn}(L_{t+1}, Z_{t+1}) - \bar{V}^{yj}(L_{t+1}, Z_{t+1})] - \zeta_t^{yj,n} \\ \nu \bar{\varepsilon}_t^{oj,n} &\equiv \beta \mathbb{E}_t[\bar{V}^{on}(L_{t+1}, Z_{t+1}) - \bar{V}^{oj}(L_{t+1}, Z_{t+1})] - \zeta_t^{oj,n} \end{aligned} \quad (3)$$

are the present value changes in the continuation value net of the moving cost. Taking the average

of (1) across households yields the expected lifetime value of a representative household in each group by age and job:

$$\begin{aligned}
V^{bj}(L_t, Z_t) &= \beta \mathbb{E}_t [V^{yj}(L_{t+1}, Z_{t+1})] + \nu \Omega(\bar{\varepsilon}_t^{bj}) \\
V^{yj}(L_t, Z_t) &= \omega^j(L_t, Z_t) + \beta \mathbb{E}_t [\bar{V}^{yj}(L_{t+1}, Z_{t+1})] + \nu \Omega(\bar{\varepsilon}_t^{yj}) \\
V^{oj}(L_t, Z_t) &= \omega^j(L_t, Z_t) + \beta \mathbb{E}_t [\bar{V}^{oj}(L_{t+1}, Z_{t+1})] + \nu \Omega(\bar{\varepsilon}_t^{oj})
\end{aligned} \tag{4}$$

Iterating forward, taking first difference with respect to the value of being in another job, substituting for $\nu \bar{\varepsilon}_{t+1}^{aj,n} + \zeta_{t+1}^{aj,n} = \beta \mathbb{E}_{t+1} [\bar{V}^{an}(L_{t+2}, Z_{t+2}) - \bar{V}^{aj}(L_{t+2}, Z_{t+2})]$ and taking the expectation at time t , hence substituting for $\nu \bar{\varepsilon}_t^{aj,n} + \zeta_t^{aj,n} = \beta \mathbb{E}_t [\bar{V}^{an}(L_{t+1}, Z_{t+1}) - \bar{V}^{aj}(L_{t+1}, Z_{t+1})]$ yields the Euler equation:

$$\begin{aligned}
\nu \bar{\varepsilon}_t^{bj,n} + \zeta_t^{bj,n} &= \beta \mathbb{E}_t \left\{ \left[\nu \bar{\varepsilon}_{t+1}^{bj,n} + \zeta_{t+1}^{bj,n} \right] + \left[\nu \Omega(\bar{\varepsilon}_{t+1}^{bn}) - \nu \Omega(\bar{\varepsilon}_{t+1}^{bj}) \right] \right\} \\
\nu \bar{\varepsilon}_t^{yj,n} + \zeta_t^{yj,n} &= \beta \mathbb{E}_t \left\{ \left[\omega^n(L_{t+1}, Z_{t+1}) - \omega^j(L_{t+1}, Z_{t+1}) \right] + \left[\nu \bar{\varepsilon}_{t+1}^{yj,n} + \zeta_{t+1}^{yj,n} \right] + \left[\nu \Omega(\bar{\varepsilon}_{t+1}^{yn}) - \nu \Omega(\bar{\varepsilon}_{t+1}^{yj}) \right] \right\} \\
\nu \bar{\varepsilon}_t^{oj,n} + \zeta_t^{oj,n} &= \beta \mathbb{E}_t \left\{ \left[\omega^n(L_{t+1}, Z_{t+1}) - \omega^j(L_{t+1}, Z_{t+1}) \right] + \left[\nu \bar{\varepsilon}_{t+1}^{oj,n} + \zeta_{t+1}^{oj,n} \right] + \left[\nu \Omega(\bar{\varepsilon}_{t+1}^{on}) - \nu \Omega(\bar{\varepsilon}_{t+1}^{oj}) \right] \right\}
\end{aligned} \tag{5}$$

Equation (5) is an inter-temporal optimality condition which says that for the marginal mover from a job j to another job n the total cost of moving is equal to the discounted expected benefit from being in n instead of in j in the following period.

For the sake of tractability, we assume that $\varepsilon \sim \text{Gumbel}(-\gamma, 1)$ where $\gamma \approx 0.5772$ is the Euler-Mascheroni constant. The probability that when moving from job j , a certain job i is preferred to any other job k is given by

$$m_t^{aj,i} = \frac{e^{\bar{\varepsilon}_t^{aj,i}}}{\sum_{k \in J} e^{\bar{\varepsilon}_t^{aj,k}}} \tag{6}$$

and the option value of moving starting from a certain job j is given by:

$$\Omega(\bar{\varepsilon}_t^{aj}) = -\ln(m_t^{aj,j}) \tag{7}$$

for $a = \{b, y, o\}$. By the law of large numbers, $m_t^{aj,i} \in (0, 1)$ is the fraction of households in job j who move to job i at the end of period t . Given (3), the fraction of movers of a certain age a from

job j to job i results into a function of the expected future values of being in a job:

$$\begin{aligned}
m_t^{bj,i} &= \frac{e^{\frac{1}{\nu}(\beta \mathbb{E}_t[V^{yi}(L_{t+1}, Z_{t+1})] - \zeta_t^{bj,i})}}{\sum_{k \in J} e^{\frac{1}{\nu}(\beta \mathbb{E}_t[V^{yk}(L_{t+1}, Z_{t+1})] - \zeta_t^{bj,k})}} \\
m_t^{yj,i} &= \frac{e^{\frac{1}{\nu}(\beta \mathbb{E}_t[(1-\lambda^y)V^{yi}(L_{t+1}, Z_{t+1}) + \lambda^y V^{oi}(L_{t+1}, Z_{t+1})] - \zeta_t^{yj,i})}}{\sum_{k \in J} e^{\frac{1}{\nu}(\beta \mathbb{E}_t[(1-\lambda^y)V^{yk}(L_{t+1}, Z_{t+1}) + \lambda^y V^{ok}(L_{t+1}, Z_{t+1})] - \zeta_t^{yj,k})}} \\
m_t^{oj,i} &= \frac{e^{\frac{1}{\nu}(\beta \mathbb{E}_t[(1-\lambda^o)V^{oi}(L_{t+1}, Z_{t+1})] - \zeta_t^{oj,i})}}{\sum_{k \in J} e^{\frac{1}{\nu}(\beta \mathbb{E}_t[(1-\lambda^o)V^{ok}(L_{t+1}, Z_{t+1})] - \zeta_t^{oj,k})}}
\end{aligned} \tag{8}$$

The fractions of movers (8) determine the actual reallocation of households over jobs between time t (in which some households leave their current job j) and the next period $t+1$ (in which these households are in their new job i). This information is what will be sufficient to characterize - in a deterministic way - the new spatial allocation of households L_{t+1} .

Rewriting the Bellman equations (4) making use of the equilibrium conditions (8) evaluated for the fraction of those who do not change job yields:

$$\begin{aligned}
V^{bj}(L_t, Z_t) &= -\nu \ln(m_t^{bj,j}) + \beta \mathbb{E}_t[V^{yj}(L_{t+1}, Z_{t+1})] \\
V^{yj}(L_t, Z_t) &= \omega^j(L_t, Z_t) - \nu \ln(m_t^{yj,j}) + \beta \mathbb{E}_t[(1-\lambda^y)V^{yj}(L_{t+1}, Z_{t+1}) + \lambda^y V^{oj}(L_{t+1}, Z_{t+1})] \\
V^{oj}(L_t, Z_t) &= \omega^j(L_t, Z_t) - \nu \ln(m_t^{oj,j}) + \beta \mathbb{E}_t[(1-\lambda^o)V^{oj}(L_{t+1}, Z_{t+1})]
\end{aligned} \tag{9}$$

The Bellman equations (9) generalize the spatial dynamic problem of Caliendo et al. (2019) in two directions: households have rational expectations; and they are heterogeneous with respect to a not-insurable source of uncertainty, that is aging. The form of Bellman equation (9) provides also a clear advantage in tractability. The value of a certain job is given in terms of information on the same job only. Knowledge on the fraction of households remaining in the same job is sufficient to characterize the rich underlying structure of optimal choices among a large number of jobs.

2.4 Law of motion of the labor force

Let L_t^{yj} be the population of young households in j at time t . Then $(1 - m_t^{yj,j})L_t^{yj}$ is the measure of young households that leave job j by the end of time t and $m_t^{yj,j}L_t^{yj}$ is the measure of young households that remain, but a share λ^y of them become old. Symmetrically, $\sum_{i \neq j} (m_t^{yi,j}L_t^{yi})$ is the measure of households that move to job j and only a share $1 - \lambda^y$ of them will be young at the beginning of period $t+1$. In addition, the cohort of young households at time $t+1$ in a certain job j includes new born at time t who made the decision to enter the labor market next period in job j . These “new young” are the offspring of young parents that are employed in any job $i = 1, \dots, J$ at time t . Although the relationship between parental choice and employment is an empirically

relevant aspect of job reallocation, its microfoundation is outside the scope of this paper. Therefore, we simply model a process for the birth rate that guarantees no aggregate population growth while labor allocated to each job changes endogenously over time.

Let $\lambda_t^b > 0$ be the birth rate describing the number of offspring per young households at time t . Thus, the young population in a job j at time t generates a measure of $\lambda_t^b L_t^{yj}$ new born starting next period in a certain job as young. The actual allocation of these new young across jobs is an optimal endogenous outcome. The measure of offspring generated at time t by young population with a job i making the optimal decision to start in job j at time $t + 1$ is $m_t^{bi,j} \lambda_t^b L_t^{yi}$; as implied by solving the Bellman equation (9) for a new born. Summing the reallocation of young and new born across jobs yields the law of motion for young households:

$$L_{t+1}^{yj} = (1 - \lambda^y) \sum_{i=1}^J \left(m_t^{yi,j} L_t^{yi} \right) + \lambda_t^b \sum_{i=1}^J \left(m_t^{bi,j} L_t^{yi} \right) \quad (10)$$

Accounting for the mass of old households follows the same argument, with the distinction that the population of old households consists of previous old households who decided to remain or move in job j and did not die, plus the young households who decided to remain or move in job j and became old. The law of motion for old households is given by:

$$L_{t+1}^{oj} = (1 - \lambda^o) \sum_{i=1}^J \left(m_t^{oi,j} L_t^{oi} \right) + \lambda^y \sum_{i=1}^J \left(m_t^{yi,j} L_t^{yi} \right) \quad (11)$$

The fractions of movers has to sum to 1, thus, after reallocation, the total endowment of old incumbents $L_t^o = \sum_{j=1}^J \sum_{i=1}^J (m_t^{oi,j} L_t^{oi})$ and young incumbents $L_t^y = \sum_{j=1}^J \sum_{i=1}^J (m_t^{yi,j} L_t^{yi})$ are equal to their respective initial measures. The measure of new born is proportional to the young incumbent population $\lambda_t^b L_t^y$ as $L_t^y = \sum_{j=1}^J \sum_{i=1}^J (m_t^{yi,j} L_t^{yi})$. The measure of households that die at time t is $\lambda^o L_t^o$. It follows that $\sum_{j=1}^J L_{t+1}^j = \sum_{j=1}^J L_t^j + \lambda_t^b L_t^y - \lambda^o L_t^o$. Since the measure of labor is expressed in terms of fraction of world population then $\sum_{j=1}^J L_t^j = 1$. The birth rate at which there is no growth in world population is $\lambda_t^b = \lambda^o L_t^o / (\bar{L} - L_t^o)$, where $\bar{L}$ is the aggregate population.

The aggregate measure of young households in period $t + 1$ is $L_{t+1}^y = [(1 - \lambda^y) + \lambda_t^b] L_t^y$. The aggregate measure of old households in period $t + 1$ is $L_{t+1}^o = \lambda^y L_t^y + (1 - \lambda^o) L_t^o$. The first order difference equation $L_{t+1}^o - L_t^o = \lambda^y \bar{L} - (\lambda^y + \lambda^o) L_t^o$ holds and shows that the composition of aggregate population by age group is constant over time if and only if the initial fraction of old population is equal to $L_t^o / \bar{L} = \lambda^y / (\lambda^y + \lambda^o)$ that implies $\lambda_t^b \equiv \lambda^b \equiv \lambda^y$ in every period t . We assume that this initial condition holds,

$$\sum_{j=1}^J L_0^j = \frac{\lambda^y}{\lambda^y + \lambda^o} \quad (12)$$

to avoid that our results were driven by either aggregate population growth or changes in the aggregate composition by age group. However, population size and age composition can vary by job, thus, by region, sector and occupation.

2.5 Role of uncertainty

The inter-temporal optimality condition (5) implies that for the marginal mover from a job j to a job n the cost of moving is equal to the discounted expected total gain from moving, that consists of current value plus option value. In the light of this optimal behavior, four channels describe the role of uncertainty:

1. Do not accounting for uncertainty, systematically over-estimates the speed of reallocation. Call $x_{t+1}^{a'n}$ for $a' = \{a, a+1\}$ the realization at time $t+1$ of the stochastic process faced by a household of age a in job n at time t (it contains aggregate shocks as well as the realization of the idiosyncratic age shock). Call $V^n(x_{t+1}^{a'n})$ the value of being at time $t+1$ in job n contingent on the realization $x_{t+1}^{a'n}$. Now, consider a change in the realization of the stochastic own-state of job n , for instance the TFP of producing in job n increases while it remains constant elsewhere. If the function V^n is strictly concave with respect to updates in the own-state variables then the Jensen's inequality implies:

$$\mathbb{E}_t \left[(1 - \lambda^a) V^n(x_{t+1}^{an}) + \lambda^a V^n(x_{t+1}^{(a+1)n}) \right] \leq V^n \left(\mathbb{E}_t [x_{t+1}^{a'n}] \right)$$

where the equality holds if and only if there is perfect foresight. ■

2. Not-insurable risks make the bias in assessing the speed of reallocation more severe and heterogeneous. Call z_{t+1} the realization at time $t+1$ of aggregate shocks and call $V^{an}(z_{t+1})$ the contingent value of being at time $t+1$ in age a job n . Then, the sorting in moving cost $\zeta^{oj,n} - \zeta^{yj,n} > 0$ implies that the value of being in a job for young households is greater than for old households¹³

$$V^{yj}(z_t) - V^{oj}(z_t) = \zeta^{oj,n} - \zeta^{yj,n} + \beta \{ (1 - \lambda^y) \mathbb{E}_t [V^{yn}(z_{t+1})] + (1 - \lambda^y - \lambda^o) \mathbb{E}_t [V^{on}(z_{t+1})] \} > 0.$$

As a result, the continuation value for a young household is greater than the continuation value for a old household in the same job

$$(1 - \lambda^y) \mathbb{E}_t [V^{yn}(z_{t+1})] + \lambda^y \mathbb{E}_t [V^{on}(z_{t+1})] > (1 - \lambda^o) \mathbb{E}_t [V^{on}(z_{t+1})].$$

¹³ Note that sorting in moving cost is sufficient but not necessary, since young households live longer in expectation.

To prove this we proceed by contradiction. Assume that this is not true, then

$$(1 - \lambda^y) \mathbb{E}_t [V^{yn}(z_{t+1})] + \lambda^y \mathbb{E}_t [V^{on}(z_{t+1})] < (1 - \lambda^o) \mathbb{E}_t [V^{on}(z_{t+1})]$$

and

$$(1 - \lambda^y) \mathbb{E}_t [V^{yn}(z_{t+1})] < (1 - \lambda^y - \lambda^o) \mathbb{E}_t [V^{on}(z_{t+1})].$$

For every positive value function, if $(1 - \lambda^y - \lambda^o) < 0$ the above statement is falsified because the RHS becomes negative. If $(1 - \lambda^y - \lambda^o) > 0$ then $\zeta^{oj,n} - \zeta^{yj,n} > 0$ implies $V^{yj}(z_t) - V^{oj}(z_t)$ and so in expectation $\mathbb{E}_t [V^{yn}(z_{t+1})] > \mathbb{E}_t [V^{on}(z_{t+1})]$. But then the LHS is greater than $\mathbb{E}_t [V^{on}(z_{t+1})]$ which is a contradiction with the initial statement. ■

3. During the transition that follows aggregate shocks, uncertainty smooths incentives for reallocation in a systematic way: less outflow from bad outcomes and more inflow toward bad outcomes. Jobs are characterized by different fundamentals. However, over time, the attractiveness of jobs changes because of region & sector specific shocks. Consider two jobs $\{n, j\}$ with the same fundamentals and identical shock realizations at time t .

Assume that at the turning point with time $t + 1$ a positive productivity shock occurs in job n while nothing changes in job j . Households made their reallocation choices looking at comparing expected future gains

$$\text{Under perfect foresight: } V^{an}(z_{t+1}) - V^{aj}(z_{t+1}) = \Delta > 0$$

$$\text{Under rational expectations: } \mathbb{E}_t [V^{an}(z_{t+1}) - V^{aj}(z_{t+1})] < \Delta ,$$

therefore, under rational expectations there are more households remaining in the worse region than under perfect foresight. A similar hysteresis occurs for negative shocks as well

$$\text{Under perfect foresight: } V^{an}(z_{t+1}) - V^{aj}(z_{t+1}) = \Delta < 0$$

$$\text{Under rational expectations: } \mathbb{E}_t [V^{an}(z_{t+1}) - V^{aj}(z_{t+1})] > \Delta ,$$

thus, under rational expectations there are more households reallocating to the region that becomes worse than under perfect foresight. ■

4. Looking at the long-run equilibrium after a permanent shock, uncertainty is responsible for more households making the rational choice to spend a greater portion of their life trapped in relatively bad jobs. Given an initial distribution of households across jobs, the law of motion is shaped by the Markov matrix of the fractions of movers mapping the current densities into future densities. The previous result has shown that, after a shock, the fraction

of movers is systematically lower toward jobs that becomes better and systematically greater for remaining at jobs that become worse, than under perfect foresight. Thus, after a permanent shock, the new long-run distribution of households is characterized by a greater measure of households in worse jobs under uncertainty than under perfect foresight. ■

In section 5 we will simulate a calibrated version of the model to quantify the strength of these four results.

3 Within-period spatial allocation of output

In this section we model the production side of the economy. As in Caliendo and Parro (2015), intermediate goods are traded across regions. Within each region, production in a certain sector employs intermediate goods from other regions and sectors to deliver a composite sectoral good, which is consumed by households located in the region.

3.1 Production

Firms in each sector and country produce varieties of an intermediate good hiring labor in different occupations $k = 1, \dots, K$ and employing final goods as material inputs from many sectors $s = 1, \dots, S$. Firms producing in region r employ labor located in the same region and final goods only after they are made available in that region; thus, foreign final goods are employed in the production of domestic intermediate goods at a price that includes the cost to make these goods available in the domestic region.

Total factor productivity in the production of intermediates is a composite of a sector-region *fundamental productivity* A_t^{rs} and a variety-specific *efficiency* z^{rs} ; we refer to each variety by its efficiency. The output of a variety z^{rs} of intermediate good $q_t^{rs}(z^{rs})$ produced in country r sector s is given by

$$q_t^{rs}(z^{rs}) = z^{rs} \left[A_t^{rs} \prod_{k=1}^K (l_t^{sk})^{\epsilon_{sk}} \right]^{\gamma^{rs}} \prod_{s'=1}^S (M_t^{rs,rs'})^{\gamma^{rs,rs'}}, \quad (13)$$

where l_t^{sk} denotes the labor input of occupation k and $M_t^{rs,rs'}$ denotes the input of final goods of sector s' available in region r employed in the production of varieties in region r sector s . The coefficient $\epsilon_{sk} \in (0, 1)$ governs which share of labor is of occupation k in sector s and $\sum_{k=1}^K \epsilon_{sk} = 1$. The coefficient γ^{rs} denotes the share of value added by labor as a composite input in the production of intermediate goods of region r sector s . The coefficient $\gamma^{rs,rs'} \in (0, 1)$ yields the share of value added by material inputs sourced in sector s' in the production of intermediate goods of region r sector s . In order to ensure constant returns to scale, we assume that $\sum_{s'=1}^S \gamma^{rs,rs'} = 1 - \gamma^{rs}$.

Let w_t^{rsk} be the wage earned by a worker of occupation k in sector s of region r and let P_t^{rs} be the price of material inputs in region r sector s . A firm producing intermediate goods in region r and sector s employs the input bundle whose unit price is given by:

$$x_t^{rs} = B^{rs} \left[\prod_{k=1}^K (w_t^{rsk})^{\epsilon_{sk}} \right]^{\gamma^{rs}} \prod_{s'=1}^S (P_t^{rs'})^{\gamma^{rs,r's'}}, \quad (14)$$

where $B^{rs} = \prod_{k=1}^K (\epsilon_{sk} \gamma^{rs})^{-\epsilon_{sk} \gamma^{rs}} \prod_{s'=1}^S (\gamma^{rs,r's'})^{-\gamma^{rs,r's'}}$ is a constant. The total factor productivity of a firm z producing in a region and sector pair (r, s) is given by the product $z^{rs} (A_t^{rs})^{\gamma^{rs}}$. The marginal (and average) cost to produce a variety z^{rs} is given by $x_t^{rs} / [z^{rs} (A_t^{rs})^{\gamma^{rs}}]$.

Production of intermediate goods employs labor and material inputs sourced from the same region in which production takes place. Then, variety of intermediate goods that are produced in a certain region can be traded across regions at a certain cost. Shipping goods from the region where they are produced, say r' , to another region where they are employed, say r , implies an iceberg-type trade cost, denoted by $\tau_t^{rs,r's} \geq 1$.¹⁴ The marginal cost of a variety $z^{r's}$ in sector s produced in a region r' gross of the delivery at a destination region r is given by $\left(\frac{\tau_t^{rs,r's} x_t^{r's}}{A_t^{r's} z^{r's}} \right)$. The procurement of inputs is perfectly competitive, such that in every destination region r producers source from the cheapest input supplier in the economy, gross of the bilateral trade cost. The price of a specific variety of intermediate good z^s of sector s in region r is given by the lowest marginal cost at which the good is available in the region:

$$p_t^{rs}(z^s) = \min_{r'} \left\{ \frac{\tau_t^{rs,r's} x_t^{r's}}{z^{r's} (A_t^{r's})^{\gamma^{r's}}} \right\}. \quad (15)$$

The distribution of productivity z^{rs} in a region and sector follows a Fréchet distribution with cumulative density function $\mathcal{F}(z^{rs}) = \exp(-(z^{rs})^{-\theta^s})$. The joint distribution across regions is given by $\mathcal{F}(z^s) = \exp(-\sum_{r=1}^R (z^{rs})^{-\theta^s})$. The implied distribution of prices in region r for varieties of intermediate goods used in sector s at time t is given by:¹⁵

$$G_t^{rs}(p) = 1 - \exp(-\Phi_t^{rs} p^{\theta^s}) \quad , \quad \phi_t^{rs,r's} = \left(\frac{(A_t^{r's})^{\gamma^{r's}}}{\tau_t^{rs,r's} x_t^{r's}} \right)^{\theta^s} \quad , \quad \Phi_t^{rs} = \sum_{r'=1}^R \phi_t^{rs,r's}. \quad (16)$$

¹⁴ We follow the notation in Caliendo et al. (2019), the superscript of trade costs is labeled as {destination, source}.

¹⁵ The distribution of the price at which region r' serves region r is given by $G^{r's,rs}(h) = 1 - \mathcal{F}\left(\frac{\tau_t^{rs,r's} x_t^{r's}}{A_t^{r's} h}\right)$. The distribution of the minimum price is $\Pr(\min\{..., p^{r's,rs}, ...\} \leq p)$, thus it is the distribution of the event that at least a price value out of $r' = 1, \dots, R$ is lower than p . This is the complement at 1 of the probability that all price values are greater than p , that is $\Pr(\min\{..., p^{r's,rs}, ...\} \leq p) = 1 - \prod_{r'=1}^R [1 - G^{r's,rs}(p)] = 1 - \prod_{r'=1}^R \mathcal{F}\left(\frac{\tau_t^{rs,r's} x_t^{r's}}{A_t^{r's} p}\right)$.

We call $\tilde{q}_t^{rs}(z^s)$ the quantity of intermediate good that is an input for sector s demanded in region r and sourced from the supplier of the cheapest variety, at the price $p_t^{rs}(z^{rs})$ in (15). The distribution of prices for intermediate goods (16) takes into account all intermediates varieties of goods in sector s that are available in region r , whether domestically produced or imported.

Single varieties of intermediate goods that are available in a certain region are not directly consumed or employed in production; they are aggregated into a sectoral composite good. Varieties of intermediate goods of sector a available in region r are aggregated into a local sectoral good Q_t^{rs} according with a C.E.S. technology with elasticity of substitution $\eta^{rs} > 1$

$$Q_t^{rs} = \left[\int (\tilde{q}_t^{rs}(z^s))^{1-1/\eta^{rs}} d\mathcal{F}(z^s) \right]^{\eta^{rs}/(\eta^{rs}-1)}. \quad (17)$$

The C.E.S. structure in input demand and the Frèchet distribution of productivity, imply that the price index of the composite good of sector s in region r is given by:

$$P_t^{rs} = \Gamma^{rs} \left(\sum_{r'=1}^R \left(x_t^{r's} \tau_t^{rs,r's} \right)^{-\theta^s} \left(A_t^{r's} \right)^{\theta^s \gamma^{r's}} \right)^{-\frac{1}{\theta^s}} = \Gamma^{rs} (\Phi_t^{rs})^{-\frac{1}{\theta^s}}, \quad (18)$$

where $\Gamma^{rs} = \Gamma(1 + (1 - \eta^{rs}/\theta^s))$ and $\Gamma(\cdot)$ is the Gamma function and we assume $1 + \theta^s > \eta^{rs}$. The sectoral composite good Q_t^{rs} has two uses: it is consumed by households in region r , and it is employed by producers of intermediate goods located in region r . The price index P_t^{rs} is indeed relevant for consumption choices of households in region r , and it is the price of material inputs for sector s sourced from region r .

Households allocate consumption across sectoral goods according with Cobb-Douglas preferences, such that they spend a share $\alpha^s > 0$ of their income on the consumption of local good from sector s and $\sum_{s=1}^S \alpha_s = 1$. The consumption based price index that applies to households located in a region r is given by:

$$P_t^r = \prod_{s=1}^S \left(\frac{P_t^{rs}}{\alpha_s} \right)^{\alpha_s}. \quad (19)$$

The indirect utility of a household earning a wage w_t^{rsk} is equal to w_t^{rsk}/P_t^r .

3.2 Optimal sourcing policy

Call $X_t^{rs} = P_t^{rs} Q_t^{rs}$ the sales of sectoral good s in region r . Since the output market is competitive, sales are equal to costs. Sectoral good producers combine intermediate inputs (they do not directly employ any labor), thus, total sales are equal to the total value of purchased intermediate inputs from all sectors and regions.

Let $\rho_t^{r',r}$ be the number of units of currency of region r needed to buy one unit of currency of

region r' . Thus, a price $x_t^{r's}$ denominated in the currency of region r' is equal to $\rho_t^{r',r} x_t^{r's}$ in units of currency of region r . The probability that producers of intermediates from region r' sector s (*source*) are the best supplier for sectoral good producers in region r sector s (*destination*) yields the share of total expenditure of region r sector s on intermediates sourced from region r' sector s

$$\pi_t^{rs,r's} = \frac{\left(\rho_t^{r',r} x_t^{r's} \tau_t^{rs,r's}\right)^{-\theta^s} \left(A_t^{r's}\right)^{\theta^s \gamma^{r's}}}{\sum_{r''=1}^R \left(\rho_t^{r'',r} x_t^{r''s} \tau_t^{rs,r''s}\right)^{-\theta^s} \left(A_t^{r''s}\right)^{\theta^s \gamma^{r''s}}} = \frac{\phi_t^{rs,r's}}{\Phi_t^{rs}} \quad (20)$$

The equilibrium condition (20) illustrates within-sector bilateral trade patterns between regions. Since only intermediate goods are traded, while composite sectoral goods are not, this equation fully characterizes composition and direction of trade flows.

3.3 Output market clearing

Out of total sales X_t^{rs} of a certain sectoral good, a fraction is due to its purchase as an input in the local production process of intermediates; the remaining fraction is due to its purchase as consumption good by local consumers. We now perform an accounting exercise of these two components.

Consider an arbitrary region r' and sector s' . The fraction of expenditure of region r' sector s' on intermediates produced in region r sector s' is given by $\pi_t^{r's',rs'} X_t^{r's'}$. The value of expenditure of region r' sector s' on intermediates produced in region r sector s' in terms of the currency of region r is given by $\rho_t^{r',r} \pi_t^{r's',rs'} X_t^{r's'}$. Summing over regions of destination $\sum_{r'}^R \rho_t^{r',r} \pi_t^{r's',rs'} X_t^{r's'}$ yields the global sales of intermediates produced in the sourcing region r sector s' , in currency of country r . The total cost of producing intermediates in region r sector s' is equal to global sales minus a tax $\ell^{s'} T_t^r$, where $T_t^r \geq 0$, $\ell^s \in (0, 1)$ and $\sum_{s=1}^S \ell^s = 1$, collected by the region at the expenses of local producers of intermediates. Therefore, the total cost of production in region r sector s' is equal to $\left(\sum_{r'}^R \rho_t^{r',r} \pi_t^{r's',rs'} X_t^{r's'} - \ell^{s'} T_t^r\right)$. The coefficient $\gamma^{rs',rs}$ yields the fraction of total cost in region r sector s' that pays for materials $M_t^{rs',rs'}$ sourced from the same region r and sector s ; as implied by the technology for intermediate goods (13). Thus, the value $\gamma^{rs',rs} \left(\sum_{r'}^R \rho_t^{r',r} \pi_t^{r's',rs'} X_t^{r's'} - \ell^{s'} T_t^r\right)$ is the purchase of sectoral good of region r sector s used as an input in the production of intermediates in (the same) region r and sector s' . Summing over sectors of destination yields the component of sales of the sectoral good of region r sector s due to its use as an input in the local production process of intermediates, that is $\sum_{s'}^S \gamma^{rs',rs} \left(\sum_{r'}^R \rho_t^{r',r} \pi_t^{r's',rs'} X_t^{r's'} - \ell^{s'} T_t^r\right)$.

The remaining source of sales is due to consumption in region r . Households allocate consumption across sectoral goods according with Cobb-Douglas preferences, such that they spend a share $\alpha^s > 0$ of their income on the consumption of local good from sector s . To the goal of computing total income in a region-sector pair we sum employment across occupations $L_t^{rs} \equiv \sum_k^K L_t^{rsk}$ and

define $\bar{w}_t^{rs} \equiv \frac{w_t^{rsk} L_t^{rsk}}{\epsilon_{sk} L_t^{rs}}$, such that $\bar{w}_t^{rs} L_t^{rs}$ is the labor income in region r sector s . In every region r there are L_t^{r0} units of unemployed labor.

Workers who are unemployed consume the value of home production, which we refer to as the element $\bar{w}_t^{r0}$ of the vector of wages. The value of home production can be interpreted in two alternative ways. One option is to assume that unemployed workers earn an unemployment benefit, which is income financed entirely through taxes collected in the region. The alternative option is to assume that unemployed workers consume an exogenous bundle of goods that are not produced. The value of labor income in a region is the same in both cases, the difference consists in the accounting of output market clearing. In the case of unemployment benefit, the expenditure on final goods produced in a certain region r includes the purchase by unemployed workers $\bar{w}_t^{r0} L_t^{r0}$. This source of expenditure is not present when the value of home production consists of an exogenous bundle of goods which are not produced. We will keep both cases in our notation: the indicator function $\mathbf{1}[u]$ takes value 1 in the case of an unemployment benefit (jointly with a strictly positive tax revenue in the region); instead $\mathbf{1}[u] = 0$ captures the case of $\bar{w}_t^{r0}$ corresponding to the value of an exogenous bundle of goods (jointly with no tax revenue).

Total income in region r is given by $\sum_{s'=1}^S \bar{w}_t^{rs'} L_t^{rs'} + \mathbf{1}[u] \bar{w}_t^{r0} L_t^{r0}$. There are no means of saving, therefore total purchase of consumption goods from sector s in region r is given by the expression $\alpha^s \left(\sum_{s'=1}^S \bar{w}_t^{rs'} L_t^{rs'} + \mathbf{1}[u] \bar{w}_t^{r0} L_t^{r0} \right)$. The following accounting identity (which also serves as output market clearing) holds:

$$X_t^{rs} = \underbrace{\sum_{s'}^S \gamma^{rs',rs} \left(\sum_{r'}^R \rho_t^{r',r} \pi_t^{r's',rs'} X_t^{r's'} - \ell^{s'} T_t^r \right)}_{\text{purchase as input}} + \underbrace{\alpha^s \left(\sum_{s'=1}^S \bar{w}_t^{rs'} L_t^{rs'} + \mathbf{1}[u] \bar{w}_t^{r0} L_t^{r0} \right)}_{\text{purchase as consumption good}}. \quad (21)$$

3.4 Labor market clearing

Local labor markets are spatially separated and clustered by type of occupation. Within each local labor market there is perfect competition. Therefore, total wages are equal to the component of value added by the corresponding labor types. Consider an arbitrary region r' and sector s' . The fraction of total expenditure of region r' sector s' on intermediates produced in region r sector s' is equal to $\rho_t^{r',r} \pi_t^{r's',rs'} X_t^{r's'}$, expressed in currency of region r . Summing over all regions $\sum_{r'}^R \rho_t^{r',r} \pi_t^{r's',rs'} X_t^{r's'}$ yields the global sales of intermediates produced in region r sector s' . Sales net of taxes $\ell^{s'} T_t^r$ yields the total cost of production in region r sector s' , that is $\left(\sum_{r'}^R \rho_t^{r',r} \pi_t^{r's',rs'} X_t^{r's'} - \ell^{s'} T_t^r \right)$. A fraction $\epsilon_{s'k} \gamma^{rs'}$ is the value added in region r by labor with occupation k in sector s' , that is $w_t^{rs'k} L_t^{rs'k}$. Summing over occupation yields the value added in region r sector s , that is $\bar{w}_t^{rs'} L_t^{rs'}$. The labor market clearing condition at the level of region and

sector pair is given by:

$$\bar{w}_t^{rs'} L_t^{rs'} = \gamma^{rs'} \left(\sum_{r'}^R \rho_t^{r',r} \pi_t^{r's',rs'} X_t^{r's'} - \ell^{s'} T_t^r \right). \quad (22)$$

3.5 Equilibrium

Labor markets $\{r, s, k\}_{r=1, s=1, k=1}^{R, S, K}$ and output markets $\{r, s\}_{r=1, s=1}^{R, S}$ clearing determines competitive wages and prices. Unemployed labor units $\{L_t^0\}_{r=1}^R$ do not participate in production, but they participate to the equilibrium as consumers spending non-labor income financed by tax revenue in the region.

Since the output market of each sectoral good clears in every region (and there are no means of saving), the value of expenditure by consumers in a certain region is equal to their total income, which is the sum of labor income (of those who are employed) and non-labor income (of those who are unemployed). Production costs consist of the value of labor directly employed as input, or indirectly embedded in the value of intermediate inputs. But production costs do not add up to sales, the wedge between sales and production costs consists of tax revenues collected at the regional level. This value, indeed, pays for non-labor income in the region. To complete the analysis as simply as possible, we assume that taxation takes the form of a value added tax $\ell^{rs} T_t^r = \psi^r \sum_{r'}^R \rho_t^{r',r} \pi_t^{r's',rs} X_t^{r's'}$ for an average tax rate $\psi^r \in [0, 1]$.

The equilibrium of the intra-temporal allocation of output consists of five sets of variables: wages $\{w_t^{rsk}\}_{r=1, s=1, k=1}^{R, S, K}$, sales $\{X_t^{rs}\}_{r=1, s=1}^{R, S}$, bilateral fractions of expenditure $\{\pi_t^{rs, r's'}\}_{r=1, s=1, r'=1}^{R, S, R}$, local prices of aggregate goods $\{P_t^{rs}\}_{r=1, s=1}^{R, S}$ and relative local prices of intermediate goods $\{x_t^{rs}\}_{r=1, s=1}^{R, S}$; that solve the system of five equilibrium conditions:

$$\begin{aligned} x_t^{rs} &= B^{rs} \left[\prod_{k=1}^K (w_t^{rsk})^{\epsilon_{sk}} \right]^{\gamma^{rs}} \prod_{s'=1}^S (P_t^{rs'})^{\gamma^{rs, rs'}} \\ P_t^{rs} &= \Gamma^{rs} \left(\sum_{r'=1}^R (x_t^{r's} \tau_t^{rs, r's})^{-\theta^s} (A_t^{r's})^{\theta^s \gamma^{r's}} \right)^{-\frac{1}{\theta^s}} \\ \pi_t^{rs, r's} &= \frac{(x_t^{r's} \tau_t^{rs, r's})^{-\theta^s} (A_t^{r's})^{\theta^s \gamma^{r's}}}{\sum_{r''=1}^R (x_t^{r''s} \tau_t^{rs, r''s})^{-\theta^s} (A_t^{r''s})^{\theta^s \gamma^{r''s}}} \\ X_t^{rs} &= \sum_{s'}^S \left(\frac{\gamma^{rs', rs}}{\gamma^{rs'}} + \alpha^s \right) \left(w_t^{rs'k} \frac{L_t^{rs'k}}{\epsilon_{s'k}} \right) + \mathbf{1}[u] \alpha^s \bar{w}_t^{r0} L_t^{r0} \\ w_t^{rsk} &= \frac{\epsilon_{sk} \gamma^{rs} (1 - \psi^r)}{L_t^{rsk}} \left(\sum_{r'}^R \pi_t^{r's, rs} X_t^{r's} \right) \end{aligned} \quad (23)$$

where we have substituted back for the definition $w_t^{rsk} L_t^{rsk} = \epsilon_{sk} \bar{w}_t^{rs} L_t^{rs}$, that are the price index of intermediates (14), price index of sectoral goods (18), bilateral trade patterns (20), output market clearing (21) and labor market clearing (22); given a spatial distribution of employment $\{L_t^{rsk}\}_{r=1,s=1,k=1}^{RSK}$ and $L_t^{r0} \equiv \sum_{k=1}^K L_t^{r0k}$ and aggregate productivity $\{A_t^{rs}\}_{r=1,s=1}^{RS}$.

When solving the model with unemployment, we shall discuss two approaches. If the value of home production is given in terms of unemployment benefit then the latter is financed by tax revenue and this implies the tax rate $\psi^r \in (0, 1)$ in the region, given the pre-determined allocation of labor. If the value of home production consists of an exogenous bundle of not-produced goods then $\bar{w}_t^{r0}$ is a given parameter and there is no tax to be collected, i.e. $\psi^r = 0$. Both scenarios can be handled assuming that the value of home production is a constant and exogenous fraction $h^{r0} \in (0, 1)$ of the average labor income in the same region $\bar{w}_t^{r0} = h^{r0} \sum_{s=1}^S \sum_{k=1}^K \left(\frac{L_t^{rsk} w_t^{rsk}}{\sum_{s=1}^S \sum_{k=1}^K L_t^{rsk}} \right)$.

4 Solving the model and taking the solution to the data

Given a certain distribution of labor force L_t and realizations of aggregate productivity Z_t , the system (23) solves for the intra-temporal equilibrium wages $\{w_t^{rsk}\}_{r=1,s=1,k=1}^{R,S,K}$, final prices $\{P_t^{rs}\}_{r=1,s=1}^{R,S}$, prices of intermediates $\{x_t^{rs}\}_{r=1,s=1}^{R,S}$, expenditure shares $\{\pi_t^{rs,r's}\}_{r=1,s=1,r'=1}^{R,S,R}$ and sales $\{X_t^{rs}\}_{r=1,s=1}^{R,S}$, for an arbitrary choice of a numeraire.

4.1 Solution of the inter-temporal optimal spatial allocation of labor force

The predetermined endogenous state is a column vector $L_t = \{L_t^{aj}\}_{j=1,a=o,y}^J$ with $2 \times (R * S * K + R)$ elements, where L_t^{aj} is the fraction of world population that is of age a and allocated to a certain job j at time t . The aggregate stochastic state is a column vector $Z_t = \{Z_t^j\}_{j=1}^J$ with $(R * S * K + R)$ elements, where $Z_t^j = A_t^{rs} \xi_t^k$ for $j = (r, s, k)$ is the joint stochastic process of total factor productivity in a certain region and sector A_t^{rs} and a mean-1 shock ξ_t^k to the productivity of an occupation, at time t . We assume that total factor productivity evolves according with a Markov switching process between two regimes of volatility.

Let $\varphi_{a|b}$ be the probability that the state of the economy will be “a” in the next period, conditional on the current state being “b”. Consider the Bellman equations (9). We assume that the stochastic process is stationary and Markovian, defined over a finite grid of points. Let $\varphi_{a|b}$ be the probability that the state of the economy will be “a” in the next period, conditional on the current state being “b”. Let $\{s_{i2}\}_{i2=1}^{n_s}$ be the grid of points for the exogenous state, then the

contingent value functions are given by:

$$\begin{aligned}
V^{bj}(L_t, s_{i_2}) &= -\nu \ln(m^{bj,j}(L_t, s_{i_2})) + \beta \sum_{i=1}^{n_s} \varphi_{s_i|s_{i_2}} V^{yj}(L_{t+1}, s_i) \\
V^{yj}(L_t, s_{i_2}) &= \omega^j(L_t, s_{i_2}) - \nu \ln(m^{yj,j}(L_t, s_{i_2})) + \beta \sum_{i=1}^{n_s} \varphi_{s_i|s_{i_2}} [(1 - \lambda^y) V^{yj}(L_{t+1}, s_i) + \lambda^y V^{oj}(L_{t+1}, s_i)] \\
V^{oj}(L_t, s_{i_2}) &= \omega^j(L_t, s_{i_2}) - \nu \ln(m^{oj,j}(L_t, s_{i_2})) + \beta \sum_{i=1}^{n_s} \varphi_{s_i|s_{i_2}} (1 - \lambda^o) V^{oj}(L_{t+1}, s_i)
\end{aligned} \tag{24}$$

where $m_t^{aj,j} \equiv m^{aj,j}(L_t, Z_t)$ is the fraction of households that remain in the same job (8), defined on the endogenous and exogenous state. The system (10)-(11) given the initial condition (12) define the aggregate law of motion. The coupled system of (24) and aggregate law of motion yields backward-forward mean field game of J -many agents. This is a discrete-time and discrete-state analogous to the class of models introduced in the macroeconomics literature by Achdou et al. (2021), but with aggregate uncertainty. In a recent working paper, we develop an algorithm that converges to the unique numerical solution (in the viscosity sense) for this class of models for an arbitrarily large number of agents' type, i.e. jobs J in this paper. Thus, we compute the numerical solution of the model as an application of that algorithm given the intra-temporal equilibrium allocation (23).

4.2 Taking the model to the data

In taking the model to the data we follow Caliendo and Parro (2015) and Caliendo et al. (2018), whose production equilibrium is fully consistent with the one that we develop in this paper.¹⁶ With respect to their approach, however, we must overcome a further challenge. We aim to solve the model in levels, instead their approach provides a calibration that is sufficient to solve the model in changes, but not in levels.

The critical step for solving the model in levels is to estimate the region and sector price index P_t^{rs} . This is an endogenous variable of the model which does not have a counterpart in the data, since it is an artifact price implied by the aggregate production function (17). The price index is disciplined by a Fréchet distribution $\mathcal{F}(z)$ with a certain dispersion parameter θ^s and by the elasticity of technical substitution across varieties η^{rs} . Caliendo and Parro (2015) offers a convenient methodology to estimate the dispersion in efficiency across intermediates. Caliendo et al. (2018) show how changes in the fundamental productivity A_t^{rs} , which are not observable, are related to changes in measured total factor productivity of the aggregate production function at the region and sector level. We build on their results, to estimate the moments of the distribution of aggregate

¹⁶In particular, we define trade flows and value added shares as in Caliendo and Parro (2015), sections 3.3 and 4 in their paper; and we follow Caliendo et al. (2018) in the measurement of total factor productivity, section 4 in their paper.

productivity across regions.

Measuring value added shares ϵ_{sk} , γ^{rs} , $\gamma^{rs,rs'}$. The level of observation in our data consists of NUTS2 region in France and a rest of the world (ROW), WIOD sector classifications and 2 different occupations, over twelve years, from 2005 to 2016. We observe hours of labor L_t^{rsk} and hourly wage w_t^{rsk} . The value added shares ϵ_{sk} are indeed computed as the average of the fraction of labor cost of occupation k on total labor cost by region and sector over the time horizon (excluding the minimum and maximum values to mitigate measurement error). Total employment of labor in a region and sector is $L_t^{rs} = \sum_{k=1}^K L_t^{rsk}$ and the average wage by region and sector $\bar{w}_t^{rs}$ is computed such that $w_t^{rsk} L_t^{rsk} = \epsilon_{sk} \bar{w}_t^{rs} L_t^{rs}$.

For every triplet of region, sector and year we observe value of gross output Y_t^{rs} , value added G_t^{rs} , value of imports $N_t^{rs,r's'}$ from region r' sector s' . From the Trade Analysis and Information System (UNCTAD-TRAINS) we obtain bilateral ad valorem tariff rates $r_t^{rs,r's'} \geq 0$, which are set to zero for regions within France. Regional and sectoral imports $N_t^{rs,r's'}$ are computed aggregating firm level French customs data.

The expenditure (c.i.f.) of region r sector s on goods from region r' sector s' is $X_t^{rs,r's} = (1 + r_t^{rs,r's}) N_t^{rs,r's}$. Expenditure in region r and sector s on goods from the same region and sector is given by gross output minus spending on imported goods $X_t^{rs,rs} = Y_t^{rs} - \sum_{r' \neq r} X_t^{rs,r's}$. Total expenditure of region r sector s is equal to gross output $X_t^{rs} = Y_t^{rs}$.

From WIOD and OECD input-output tables we obtain the fraction $\chi^{cs,s'}$ of spending of sector s in goods from sector s' over total spending in intermediates of sector s , for a certain country c , which coincides with a region in the case of other countries than France. For France we attribute the fraction of spending by region using the observed fraction of that region on national spending between two sectors $\sum_{r'}^R X_t^{rs,r's'} / \sum_r^{France} \sum_{r'}^R X_t^{rs,r's'}$ times the number of regions in France with positive bilateral spending of sector s from sector s' . We take the average over time periods, again excluding the minimum and maximum values to mitigate measurement error. The computed measure $\chi^{rs,s'}$ is now defined at the regional level in France and at the country level outside France.

The share of value added γ^{rs} is computed as the average of the ratio G_t^{rs}/Y_t^{rs} over the time periods (excluding the minimum and maximum values to mitigate measurement error). The share of value added for sectoral intermediate inputs is computed as the product of sectoral spending and value added in intermediates $\gamma^{rs,rs'} = \chi^{rs,s'}(1 - \gamma^{rs})$.

Estimation of the trade elasticity $\hat{\theta}^s$. The share of total expenditure of region r sector s on intermediates sourced from region r' sector s' , that is $X_t^{rs,r's}/X_t^{rs}$, corresponds to the share $\pi_t^{rs,r's}$ defined in equation (20). Substituting for the equilibrium sectoral price index (18) yields the

fraction of spending of (r, s) in goods from (r', s) :

$$X_t^{rs, r's} = \frac{\phi_t^{rs, r's}}{(P_t^{rs}/\Gamma^{rs})^{-\theta^s}} X_t^{rs} . \quad (25)$$

We use the triple ratio methodology proposed in Caliendo and Parro (2015) to obtain estimates for the trade elasticity.¹⁷ Using (25) to compute the two-ways ratio of the trade flows triplet of regions $r = i, n, h$ yields:

$$\frac{X_t^{is, ns} X_t^{ns, hs} X_t^{hs, is}}{X_t^{ns, is} X_t^{hs, ns} X_t^{is, hs}} = \left(\frac{\tau_t^{is, ns} \tau_t^{ns, hs} \tau_t^{hs, is}}{\tau_t^{ns, is} \tau_t^{hs, ns} \tau_t^{is, hs}} \right)^{-\theta^s} \quad (26)$$

which has the advantage that unobservable components (such as productivity A_t^{rs} , cost of an input bundle x_t^{rs} , price index P_t^{rs}) cancel out from the expression of trade flows, making the trade elasticity θ^s identified by bilateral tariff rates.

We model (log) trade costs as a linear function of (log) gross tariff rates $\ln \tau_t^{rs, r's} = \ln(1 + r_t^{rs, r's}) + fe^{rs} + fe^{r's} + fe^{rs, r's} + \varepsilon_t^{rs, r's}$ plus importer fixed effect fe^{rs} , exporter fixed effect $fe^{r's}$, symmetric bilateral fixed effect $fe^{rs, r's} = fe^{r's, rs}$ and an error $\varepsilon_t^{rs, r's}$ assumed to be orthogonal to tariff rates and trade policy. Substituting the modeled trade cost in (26) yields the estimating equation

$$\ln \left(\frac{X_t^{is, ns} X_t^{ns, hs} X_t^{hs, is}}{X_t^{ns, is} X_t^{hs, ns} X_t^{is, hs}} \right) = -\theta^s \ln \left(\frac{(1 + r_t^{is, ns})(1 + r_t^{ns, hs})(1 + r_t^{hs, is})}{(1 + r_t^{ns, is})(1 + r_t^{hs, ns})(1 + r_t^{is, hs})} \right) + \tilde{\varepsilon}_t^Z \quad (27)$$

which assumes that the error $\tilde{\varepsilon}^s$ is orthogonal to tariff rates and trade policy. Thus, sectoral trade elasticity (which coincides with the dispersion in efficiency) is estimated exploiting the co-variation in a triangle of bilateral trade flows and corresponding tariffs rates.

Computation of the technical elasticity of substitution $\hat{\eta}^{rs}$. Optimal factor demand functions of intermediate producers are given by:

$$l_t^{rsk}(z) = \epsilon_{sk} \gamma^{rs} \frac{p_t^{rs}(z) q_t^{rs}(z)}{w_t^{rsk}} \quad M_t^{rs, rs'}(z) = \gamma^{rs, rs'} \frac{p_t^{rs}(z) q_t^{rs}(z)}{P_t^{rs'}} .$$

Aggregating over varieties yields

$$L_t^{rsk} = \epsilon_{sk} \gamma^{rs} \frac{Y_t^{rs}}{w_t^{rsk}} \quad M_t^{rs, rs'} = \gamma^{rs, rs'} \frac{Y_t^{rs}}{P_t^{rs'}} .$$

¹⁷ For details on this approach see the discussion in footnote (38) in Caliendo and Parro (2015).

Thus, optimal factor demands can be expressed in terms of aggregate inputs:

$$l_t^{rsk}(z) = \frac{p_t^{rs}(z)q_t^{rs}(z)}{Y_t^{rs}} L_t^{rsk} \quad M_t^{rs,rs'}(z) = \frac{p_t^{rs}(z)q_t^{rs}(z)}{Y_t^{rs}} M_t^{rs,rs'} .$$

Substituting in the production function of intermediate varieties (13) yields the value of gross output in region r sector s , that we call Y_t^{rs} , which scaled by the region and sector specific price index, yields real gross output:

$$\frac{Y_t^{rs}}{P_t^{rs}} = \frac{x_t^{rs}}{P_t^{rs}} \left[\left(\prod_{k=1}^K L_t^{rsk} \right)^{\epsilon_{sk} \gamma^{rs}} \prod_{s'=1}^S \left(M_t^{rs,rs'} \right)^{\gamma^{rs,rs'}} \right] \quad (28)$$

where we have substituted for the competitive price $p_t^{rs}(z) = \frac{x_t^{rs}}{z(A_t^{rs})^{\gamma^{rs}}}$. The equation (28) shows that the model implies a region-and-sector specific production function with total factor productivity given by the input bundle price relative to the output price x_t^{rs}/P_t^{rs} . In the terminology by Caliendo et al. (2018), we call this ratio measured total factor productivity. By expressing the sectoral price index (18) using the domestic expenditure share (20), the measured total factor productivity of region r sector s is given by:

$$\frac{x_t^{rs}}{P_t^{rs}} = \frac{(A_t^{rs})^{\gamma^{rs}}}{\Gamma^{rs} (\pi_t^{rs,rs})^{\frac{1}{\theta^s}}} . \quad (29)$$

Substituting for (29) and the optimal input demand functions in the production function of intermediates (13) and then in the aggregate production function (17) yields $p_t^{rs}(z) = \frac{P_t^{rs}/z}{\Gamma^{rs} (\pi_t^{rs,rs})^{1/\theta^s}}$, where we are exploiting the fact that $Y_t^{rs} = P_t^{rs} Q_t^{rs}$ and $zp_t^{rs}(z)$ does not depend on z , as $zp_t^{rs}(z) = x_t^{rs}/(A_t^{rs})^{\gamma^{rs}}$. Therefore, prices are distributed as the reciprocal of a Fréchet random variable, indeed prices are distributed as a Weibull with shape parameter θ^s . The average price across varieties given by $\bar{p}_t^{rs} = \frac{\Gamma(1+1/\theta^s)}{\Gamma^{rs} (\pi_t^{rs,rs})^{\frac{1}{\theta^s}}} P_t^{rs}$.

The identifying assumption is that the price index P_t^{rs} is proportional to the average price among producers of intermediates in region r sector s , that is $P_t^{rs} = (\xi_t^{rs})^{\frac{1}{\theta^s}} \bar{p}_t^{rs}$, that implies

$$\pi_t^{rs,rs} = \left(\frac{\Gamma(1+1/\theta^s)}{\Gamma(1+(1-\eta^{rs}/\theta^s))} \right)^{\theta^s} \xi_t^{rs} .$$

Taking the average over time shows that the parameter η^{rs} is estimated by the unique solution to

the equation:¹⁸

$$\left(\frac{\Gamma(1 + 1/\hat{\theta}^s)}{\Gamma(1 + (1 - \hat{\eta}^{rs}/\hat{\theta}^s))} \right)^{\hat{\theta}^s} = \bar{\pi}^{rs,rs} \quad (30)$$

where $\bar{\pi}^{rs,rs}$ is the average over time of the fraction of spending of region r sector s on own goods $\pi_t^{rs,rs}$, as before we drop the minimum and maximum value to mitigate measurement error. The condition (30) implies $\hat{\Gamma}^{rs} = \Gamma(1 + 1/\hat{\theta}^s) (\bar{\pi}^{rs,rs})^{-1/\hat{\theta}^s}$.

Estimation of TFP central moments $\hat{A}_t^{rs}$. Given trade flow data and estimates of trade elasticities, we use (25) to reproduce the normalized import share as defined in in Eaton and Kortum (2002):¹⁹

$$n_t^{rs,r's} \equiv \frac{X_t^{rs,r's}/X_t^{rs}}{X_t^{r's,r's}/X_t^{r's}} = \left(\frac{\tau_t^{rs,r's} \Gamma^{rs} P_t^{r's}}{\Gamma^{r's} P_t^{rs}} \right)^{-\theta^s} \quad (31)$$

where we have substituted for $\phi_t^{rs,r's} = \left(\frac{(A_t^{r's})^{\gamma^{r's}}}{\tau_t^{rs,r's} x_t^{r's}} \right)^{\theta^s}$ and the left hand side is bounded within the positive unit interval. With estimates of trade elasticity $\hat{\theta}^s$ in hands, the left hand side can be transformed into a variable that does not depend on sectoral trade elasticity $\tilde{n}_t^{rs,r's} \equiv \left(n_t^{rs,r's} \right)^{\frac{1}{\hat{\theta}^s}}$. With respect to Eaton and Kortum (2002) the only difference consists of region-specific elasticities η^{rs} in the aggregate production function (17). However, this pattern is constant over time, as well as the component of trade costs $\tau_t^{rs,r's}$ but for tariff rates. The normalized import share net of estimated trade elasticity and observed tariff rates is the product of a region pair and sector specific constant component and time varying relative prices $(1 + r_t^{rs,r's}) \tilde{n}_t^{rs,r's} = \frac{\Gamma^{r's}/\Gamma^{rs}}{\bar{\tau}^{rs,r's}} \frac{P_t^{rs}}{P_t^{r's}}$ where $\bar{\tau}^{rs,r's} \equiv \tau_t^{rs,r's}/(1 + r_t^{rs,r's})$ is the component of trade costs that is constant over time. Substituting for the estimated price index, yields the following system of estimating equations:

$$\begin{aligned} \ln \left[\left(1 + r_t^{rs,r's} \right) \frac{\tilde{n}_t^{rs,r's}}{\hat{\Gamma}^{r's}/\hat{\Gamma}^{rs}} \right] &= f e^{rs,r's} + f e_t^{rs} - f e_t^{r's} + \zeta_t^{rs,r's} \quad \forall r' : n_t^{rs,r's} > 0 \\ \ln \left[\left(1 + r_t^{r's,rs} \right) \frac{\tilde{n}_t^{r's,rs}}{\hat{\Gamma}^{rs}/\hat{\Gamma}^{r's}} \right] &= f e^{r's,rs} + f e_t^{r's} - f e_t^{rs} + \zeta_t^{r's,rs} \quad \forall r : n_t^{r's,rs} > 0 \end{aligned} \quad (32)$$

¹⁸ The Gamma function is continuous on the positive real line, with one stationary point that is a global minimum at ≈ 1.4616 . For $\hat{\eta}^{rs} < 2\hat{\theta}^s$ the left hand side in (30) is real and positive. For $\hat{\theta}^s < 2.1664$ and $\hat{\eta}^{rs} < \hat{\theta}^s - 1$ or for $\hat{\theta}^s > 2.1664$ and $\hat{\eta}^{rs} > \hat{\theta}^s - 1$ the left hand side in (30) is bounded within the unit interval. It can be verified that there exists a range of values within $1 < \hat{\eta}^{rs} < 2\hat{\theta}^s$ that satisfies both the theoretical restriction $\theta^s > \eta^{rs} - 1 > 0$ and the field of existence for condition (30).

¹⁹ See equation (12) in Eaton and Kortum (2002).

where $fe^{rs,r's} \neq fe^{r's,rs}$ are region-and-sector pair directional fixed effect, fe_t^{rs} and $fe_t^{r's}$ are region-sector-and-time fixed effects, $\zeta_t^{rs,r's}$ and $\zeta_t^{r's,rs}$ are i.i.d. error terms. The price index that is consistent with the theory is given by the region-sector-time fixed effect $\hat{P}_t^{rs} = \exp(fe_t^{rs})$.

The price index $\hat{P}_t^{rs}$ is identified using the info on all trade flows in which the pair of region r and sector s appears as importer or exporter. Thus, we are exploiting the variation in normalized trade flows between different import origins (or export destinations) within the same year, sector and region. With region and sector specific price index $\hat{P}_t^{rs}$ in hands, with wages w_t^{rsk} and value added shares $\{\epsilon_{sk}, \gamma^{rs}, \gamma^{rs,r's'}\}$, the cost of an input bundle x_t^{rs} is computed as in (14), that is

$$\hat{x}_t^{rs} = B^{rs} \left[\prod_{k=1}^K \left(w_t^{rsk} \right)^{\epsilon_{sk}} \right]^{\gamma^{rs}} \prod_{s'=1}^S \left(\hat{P}_t^{rs'} \right)^{\gamma^{rs,r's'}}, \quad (33)$$

where $B^{rs} = \prod_{k=1}^K (\epsilon_{sk} \gamma^{rs})^{-\epsilon_{sk} \gamma^{rs}} \prod_{s'=1}^S (\gamma^{rs,r's'})^{-\gamma^{rs,r's'}}$. Substituting for (30) in (29) allows the fundamental productivity to be computed as implied by the theory:

$$\hat{A}_t^{rs} = \left[\frac{\Gamma(1 + 1/\hat{\theta}^s) \hat{x}_t^{rs}}{\hat{P}_t^{rs}} \left(\frac{\pi_t^{rs,rs}}{\bar{\pi}^{rs,rs}} \right)^{\frac{1}{\hat{\theta}^s}} \right]^{\frac{1}{\gamma^{rs}}} \quad (34)$$

given the estimated values of price index $\hat{P}_t^{rs}$, input bundle price $\hat{x}_t^{rs}$, trade elasticity $\hat{\theta}^s$ and the observed share of value added γ^{rs} and consumption of own goods $\pi_t^{rs,rs}$ in a certain region r and sector s . Given the sequence over time of computed values $\hat{A}_t^{rs}$ the stochastic process driving the fundamental productivity can be estimated.

Structural estimation of bilateral moving costs. Substituting for $\bar{\varepsilon}_t^{aj,i} = \ln(m_t^{aj,i}) - \ln(m_t^{aj,j})$ and $\Omega(\bar{\varepsilon}_t^{aj}) = -\ln(m_t^{aj,j})$ in (5) yields the Euler equation expressed in fraction of movers relative to the fraction of stayers, by group of households:

$$\begin{aligned} \ln \left(\frac{m_t^{bj,i}}{m_t^{bj,j}} \right) &= \mathbb{E}_t \left\{ \beta \ln \left(\frac{m_{t+1}^{bj,i}}{m_{t+1}^{bi,i}} \right) + \frac{\beta \zeta_{t+1}^{bj,i} - \zeta_t^{bj,i}}{\nu} \right\} \\ \ln \left(\frac{m_t^{yj,i}}{m_t^{yj,j}} \right) &= \mathbb{E}_t \left\{ \frac{\beta}{\nu} [\omega^i(L_{t+1}, s_{t+1}) - \omega^j(L_{t+1}, s_{t+1})] + \beta \ln \left(\frac{m_{t+1}^{yj,i}}{m_{t+1}^{yi,i}} \right) + \frac{\beta \zeta_{t+1}^{yj,i} - \zeta_t^{yj,i}}{\nu} \right\} \\ \ln \left(\frac{m_t^{oj,i}}{m_t^{oj,j}} \right) &= \mathbb{E}_t \left\{ \frac{\beta}{\nu} [\omega^i(L_{t+1}, s_{t+1}) - \omega^j(L_{t+1}, s_{t+1})] + \beta \ln \left(\frac{m_{t+1}^{oj,i}}{m_{t+1}^{oi,i}} \right) + \frac{\beta \zeta_{t+1}^{oj,i} - \zeta_t^{oj,i}}{\nu} \right\} \end{aligned} \quad (35)$$

Actual moving decisions are determined by the expectation on future utility gains from consumption, plus the expectation on the value of waiting; that is given by moving next period to a certain job instead of moving in the current period and remaining afterward in such a job.

The fraction of movers and real wages are observable, by age and job characteristics. The indirect utility is increasing in nominal wages $\{w_t^{aj}\}_{j=1}^J$ for $a = \{b, y, o\}$, which vary at the level of age and job, and it is decreasing in prices $\{p_t^{r(j)}\}_{j=1}^J$, which vary at the regional level. Moving costs are not observable. We model them as an AR(1) process, such that $(\beta \mathbb{E}_t [\zeta_{t+1}^{aj,i}] - \zeta_t^{aj,i}) / \nu = \theta_t^{aj} \equiv \theta_t^{r(j),r(i)} + \theta_t^{s(j),s(i)} + \theta_t^{ak(j),ak(i)}$ where $\theta_t^{r(j),r(i)}$ is region pair & time fixed effect, $\theta_t^{s(j),s(i)}$ is a sector pair & time fixed effect, $\theta_t^{ak(j),ak(i)}$ is an age class & occupation pair & time fixed effect. The parameters $\{\beta, \nu, \theta_t^{aj}\}$ can be estimated by regressing current flows of movers on future flows and future real wages with an age & job specific intercept:

$$\begin{aligned} \ln \left(\frac{m_t^{bj,i}}{m_t^{bj,j}} \right) &= \beta \ln \left(\frac{m_{t+1}^{bj,i}}{m_{t+1}^{bi,i}} \right) + \theta_t^{bj} + \mu_{t+1}^{bj,i} \\ \ln \left(\frac{m_t^{yj,i}}{m_t^{yj,j}} \right) &= \frac{\beta}{\nu} \left[\ln \left(\frac{w_{t+1}^{yi}}{p_{t+1}^{r(i)}} \right) - \ln \left(\frac{w_{t+1}^{yj}}{p_{t+1}^{r(j)}} \right) \right] + \beta \ln \left(\frac{m_{t+1}^{yj,i}}{m_{t+1}^{yi,i}} \right) + \theta_t^{yj} + \mu_{t+1}^{yj,i} \\ \ln \left(\frac{m_t^{oj,i}}{m_t^{oj,j}} \right) &= \frac{\beta}{\nu} \left[\ln \left(\frac{w_{t+1}^{oi}}{p_{t+1}^{r(i)}} \right) - \ln \left(\frac{w_{t+1}^{oj}}{p_{t+1}^{r(j)}} \right) \right] + \beta \ln \left(\frac{m_{t+1}^{oj,i}}{m_{t+1}^{oi,i}} \right) + \theta_t^{oj} + \mu_{t+1}^{oj,i} \end{aligned} \quad (36)$$

where $\mu_{t+1}^{aj,i}$ for $a = \{b, y, o\}$ is a random event revealed at time $t + 1$ such that $\mathbb{E}_t [\mu_{t+1}^{aj,i}] \equiv 0$. The theory implies that past flows of movers and past real wages are valid instruments since they are correlated with future flows and wages but they are not correlated with the residual. This is the identification argument in Artuc et al. (2010), that we build on to estimate bilateral moving costs.

5 Simulation results

In this section, we conduct simulations that serve as comparative statics exercises to illustrate what we gain from explicitly taking into account uncertainty. In particular, there are two core differences to a perfect foresight setup that can be illustrated by restating the value function of an agent in age group a living in j :

$$V_t^{aj} = u(c_t^j) + \mathbb{E}_h \left[\max_{n \in J} \left\{ \nu \varepsilon_{h,t}^n - \zeta^{aj,n} + \beta \mathbb{E}_t \left[(1 - \lambda^a) V_{t+1}^{an} + \lambda^a V_{t+1}^{(a+1)n} \right] \right\} \right] \quad (37)$$

First, there exist shocks that households can insure against by choosing optimally location, sector and occupation – these shocks are summarized in the expectation operator. Second, there exist shocks that households cannot insure against which in our setting is aging (λ^a).

In order to understand how these shocks matter, we will model uncertainty as a mean-preserving increase in volatility of region-sector-specific productivity and consider two settings:

1. Perfectly correlated uncertainty (aggregate uncertainty)

2. Uncorrelated uncertainty (uncertain versus certain jobs)

We will illustrate these shocks in a setting with two regions, ‘good’ and ‘bad’ that vary in terms of the stochastic process governing their productivity draws. In the case of aggregate uncertainty, the evolution of productivity is governed by the same stochastic process but the mean of productivity draws varies between the good and the bad region where the bad region has a mean productivity that lies below the one in the good region. The stochastic process governing the draws is, however, perfectly correlated: Hence, whenever the bad region has a draw below its expected value this is also true for the other region.

Aggregate uncertainty illustrates how the optimal behavior of agents adjusts in a general environment of increased uncertainty that affects all jobs equally. The implications are summarized in Figure 1. As can be seen from Panel A, if we do not account for uncertainty we are systematically over-estimating the speed of reallocation. The intuition is that while the cost from changing jobs and the current value from changing jobs is certain, the option value from changing jobs is lower under uncertainty as opposed to the case with perfect foresight: Call $x_{t+1}^{a'n}$ for $a' = \{a, a+1\}$ the realization at time $t+1$ of the stochastic process faced by a household of age a in job n at time t . If the function V^n is strictly concave then Jensen’s inequality implies

$$\mathbb{E}_t \left[(1 - \lambda^a) V^n(x_{t+1}^{an}) + \lambda^a V^n(x_{t+1}^{(a+1)n}) \right] \leq V^n \left(\mathbb{E}_t [x_{t+1}^{a'n}] \right)$$

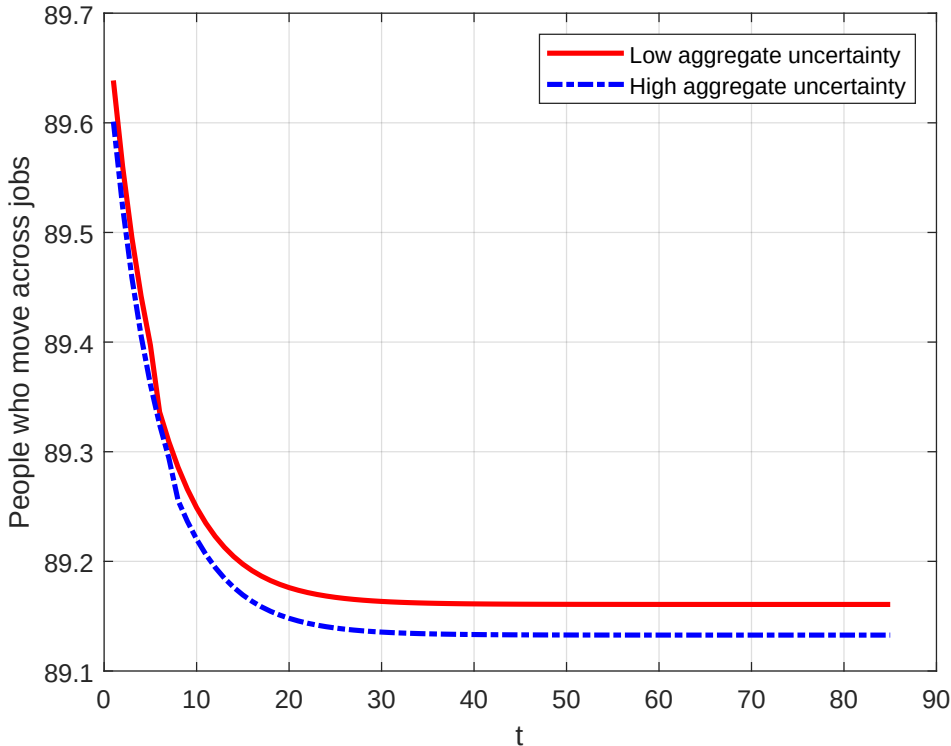
where the equality holds if and only if there is perfect foresight.

Panel B illustrates that under rational expectations there are more people remaining in the worse region and there are less people reallocating to the better region than under perfect foresight. This leads to a different spatial allocation of labor despite realizations as well as expected values of productivity being identical to a situation with less aggregate uncertainty. To see that consider two jobs $\{n, j\}$ with the same fundamentals at time t . A positive productivity shock occurs in job n while nothing changes in job j . Moving decisions under perfect foresight are governed by $V^{an}(z_{t+1}) - V^{aj}(z_{t+1}) = \Delta > 0$ and rational expectations $\mathbb{E}_t [V^{an}(z_{t+1}) - V^{aj}(z_{t+1})] < \Delta$ under rational expectations.

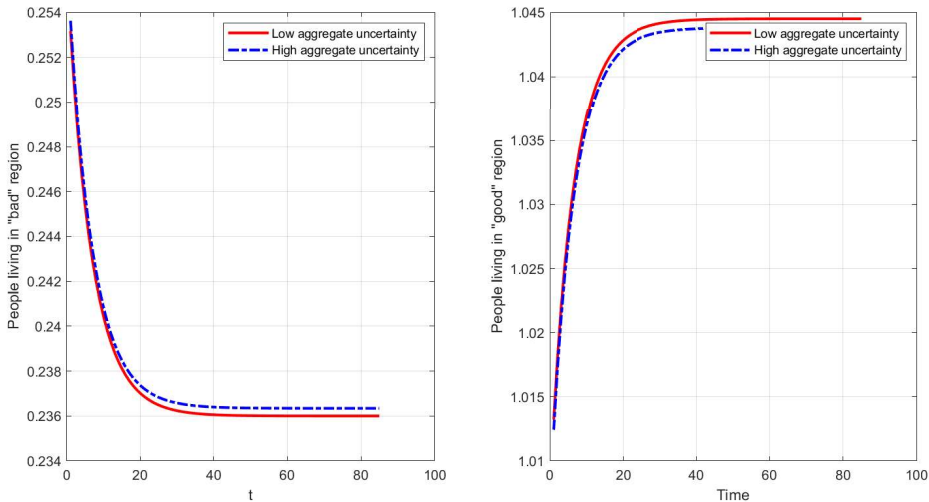
This has also long-run implications for inequality as can be seen from Panel C since the fraction of movers towards better jobs is systematically lower and greater in (remaining at) worse jobs. Therefore, the long run distribution of households across jobs is systematically different between uncertainty and perfect foresight. More people being crowded in bad regions lead to even lower wages there while they lead to even higher wages in good regions.

Finally, as can be seen in Panel D, also the distribution of age groups differs and relatively more old people end up in bad regions. This is because non-insurable risks make the cost from changing job turn suddenly higher for some agents. With heterogeneous moving costs, both the current value and the option value from changing job are lower for agents with higher costs of changing job.

Figure 1: Implications of aggregate uncertainty

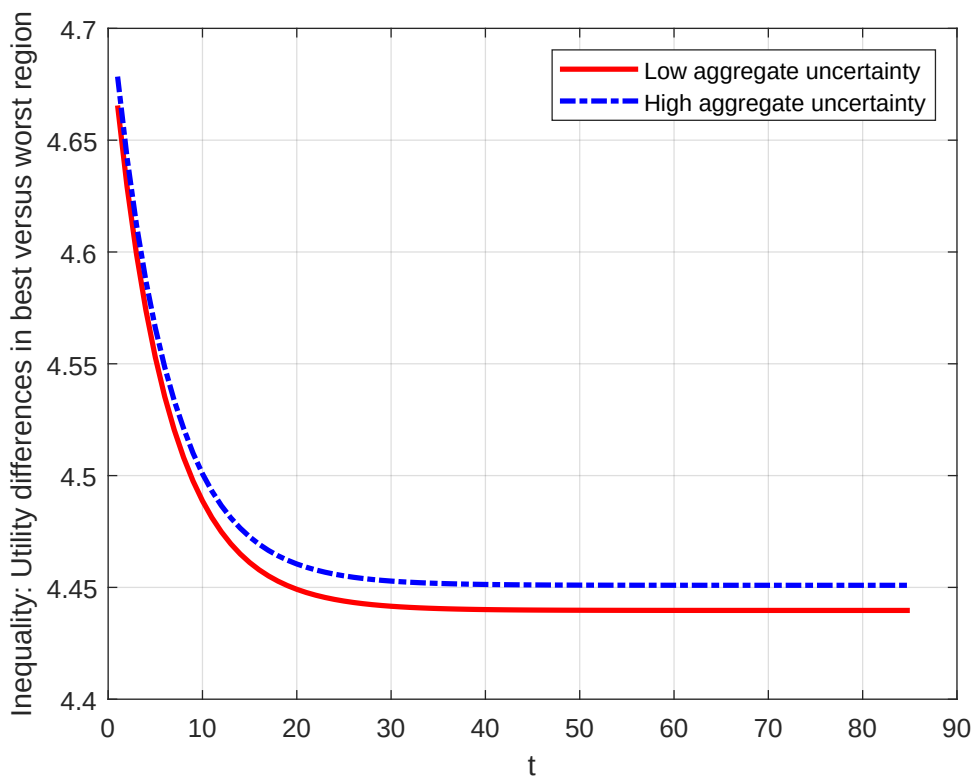


Panel A: Implication 1 – Slower adjustment

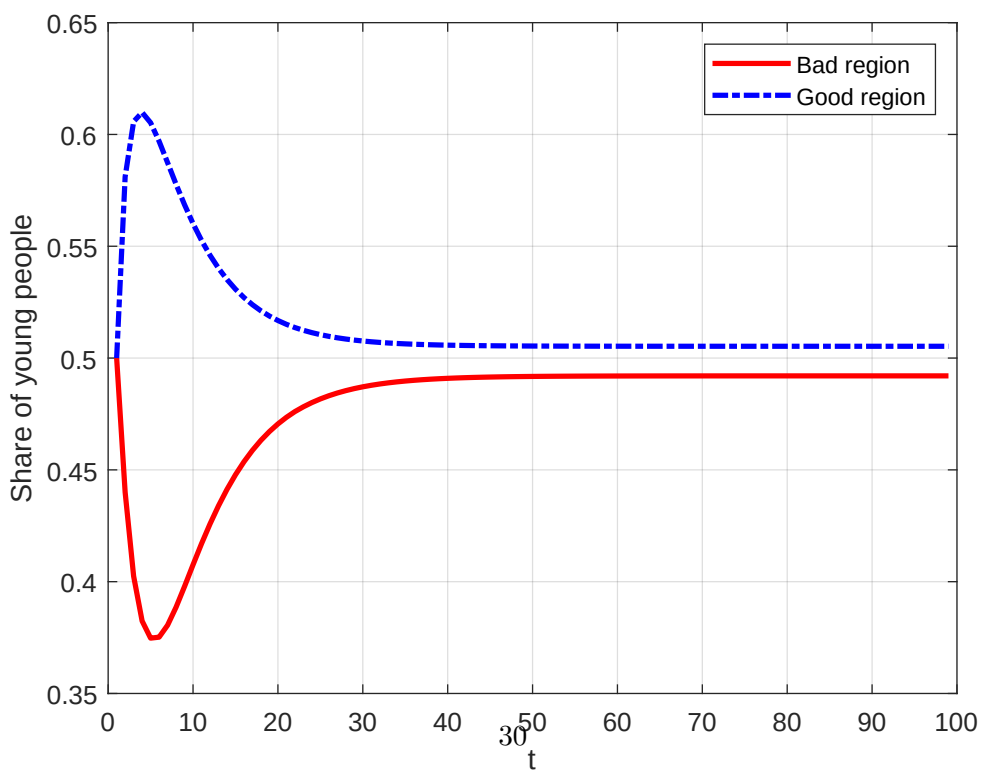


Panel B: Implication 2 – Different spatial distribution of labor

Figure 1: Implications of aggregate uncertainty (cont.)



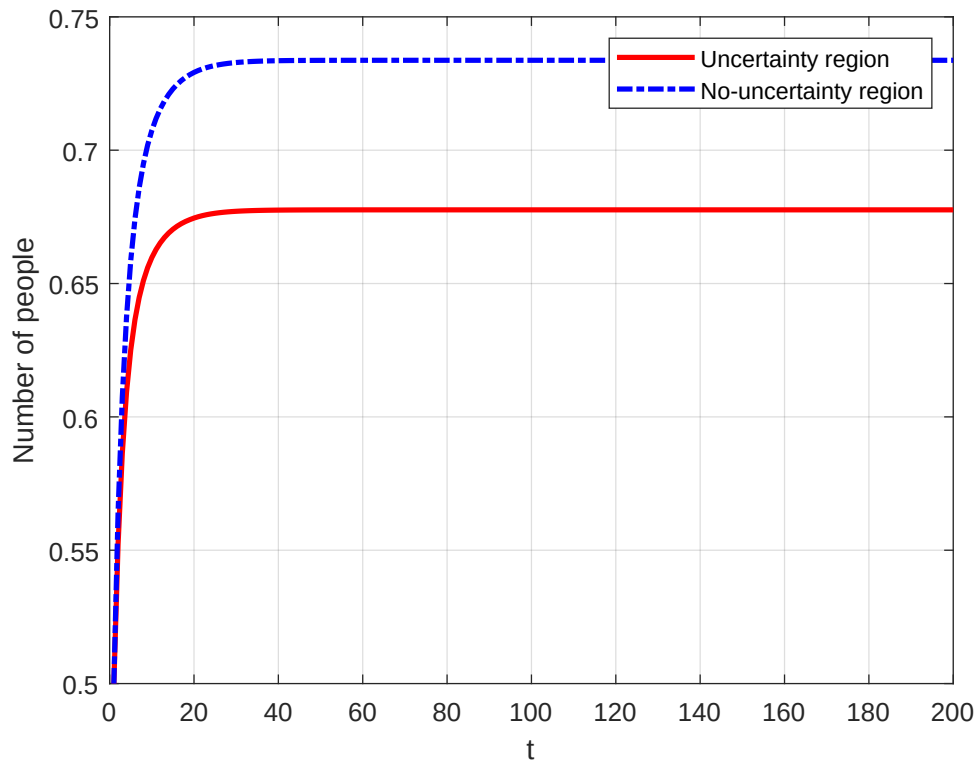
Panel C: Implication 3 – Long-run consequences for inequality



Panel D: Implication 4 – Different spatial distribution of age groups

In the second exercise, we consider two jobs that have the same expected value of productivity draws but one region is subject to a higher level of uncertainty (which implies also a potential for higher draws). As we can see this uncertainty results in a strong decline in jobs' attractiveness at the same level of productivity.

Figure 2: Implications of uncorrelated uncertainty



6 Conclusion

We deviate from the standard dynamic spatial general equilibrium model by explicitly allowing for uncertainty and risk. People – as workers as well as consumers – tend to be risk-averse and when prompted to change jobs and locations they are exposed to compare future uncertain returns to job with residence and occupational choices. On these premises, there is a component of inaction nested in choices made by rational and forward-looking people. Moreover, this source of inertia is associated with welfare losses rooted in the fact that economic agents “*get old*” in a broad sense. The latter means that people literally age, which involves a change in their opportunities, a shortening of time horizon to enjoy those and knowledge depreciation. This process of aging entails a risk which agents cannot insure against.

To understand and quantify how *global uncertainty* (i.e. the component of aggregate uncertainty channeled through movements of goods and people across regions) affects the evolution of the spatial distribution of people and economic activity, we develop a dynamic stochastic general equilibrium open economy model with interregional and intersectoral movements of goods and people, who also might choose at any period to change occupation or enter voluntary unemployment. Within each period, the economy works as an Eaton-Kortum production equilibrium augmented with labor mobility frictions. The specific feature of our framework relies on the inter-temporal allocation of labor: Worker-consumer agents make decisions on jobs and locations under uncertainty about technology and trade policy changes happening worldwide, being aware of their finite lifetime. As time marches on, the value of reallocation deteriorates since both the cost of changing increases and the remaining lifetime is reduced. By considering rational, aging, forward-looking, and risk-averse households in a dynamic stochastic general equilibrium model with realistic spatial linkages due to the moving costs of both goods and people, our theory provides a rich understanding of the way in which uncertainty impacts reallocation in the spatial economy. We quantify the role of uncertainty and risk using detailed data on the French economy.

7 Appendix

7.1 Derivations of the inter-temporal problem

7.1.1 Value functions

Rearranging the Bellman equations (1) yields

$$\begin{aligned} W^{bj}(L_t, Z_t, \varepsilon_{h,t}) &= \beta \mathbb{E}_t [V^{yj}(L_{t+1}, s_{t+1})] + \nu \max_{n \in J} \left\{ \varepsilon_{h,t}^n + \bar{\varepsilon}_t^{bj,n} \right\} \\ W^{yj}(L_t, Z_t, \varepsilon_{h,t}) &= u(c_t^j) + \beta \mathbb{E}_t [\bar{V}^{yj}(L_{t+1}, s_{t+1})] + \nu \max_{n \in J} \left\{ \varepsilon_{h,t}^n + \bar{\varepsilon}_t^{yj,n} \right\} \\ W^{oj}(L_t, Z_t, \varepsilon_{h,t}) &= u(c_t^j) + \beta \mathbb{E}_t [\bar{V}^{oj}(L_{t+1}, s_{t+1})] + \nu \max_{n \in J} \left\{ \varepsilon_{h,t}^n + \bar{\varepsilon}_t^{oj,n} \right\} \end{aligned}$$

where:

$$\begin{aligned} \bar{\varepsilon}_t^{bj,n} &\equiv \frac{1}{\nu} \left(\beta \mathbb{E}_t [V^{yn}(L_{t+1}, s_{t+1})] - V^{yj}(L_{t+1}, s_{t+1}) \right) - \zeta_t^{bj,n} \\ \bar{\varepsilon}_t^{yj,n} &\equiv \frac{1}{\nu} \left(\beta \mathbb{E}_t [\bar{V}^{yn}(L_{t+1}, s_{t+1})] - \bar{V}^{yj}(L_{t+1}, s_{t+1}) \right) - \zeta_t^{yj,n} \\ \bar{\varepsilon}_t^{oj,n} &\equiv \frac{1}{\nu} \left(\beta \mathbb{E}_t [\bar{V}^{on}(L_{t+1}, s_{t+1})] - \bar{V}^{oj}(L_{t+1}, s_{t+1}) \right) - \zeta_t^{oj,n} . \end{aligned}$$

Suppose that for a given household h the new job which maximizes the expected future value starting from j is i ; that is $i = \arg \max_{n \in J} \left\{ \varepsilon_{h,t}^n + \bar{\varepsilon}_t^{aj,n} \right\}$ and $\max_{n \in J} \left\{ \varepsilon_{h,t}^n + \bar{\varepsilon}_t^{aj,n} \right\} = \varepsilon_{h,t}^i + \bar{\varepsilon}_t^{aj,i}$. This is true if and only if $\varepsilon_{h,t}^i + \bar{\varepsilon}_t^{aj,i} \geq \varepsilon_{h,t}^m + \bar{\varepsilon}_t^{aj,m}$ for every $m \neq i$. The realizations of idiosyncratic values are assumed to be independent across jobs, and this implies that the event $\varepsilon_{h,t}^m \leq \varepsilon_{h,t}^i + \bar{\varepsilon}_t^{aj,i} - \bar{\varepsilon}_t^{aj,m}$ for every $m \neq i$ occurs with the joint probability density $f(\varepsilon_{h,t}^i) \prod_{m \neq i} F(\varepsilon_{h,t}^i + \bar{\varepsilon}_t^{aj,i} - \bar{\varepsilon}_t^{aj,m})$. Taking the expectation over the continuum of households, hence for $\varepsilon_{h,t}^i$ that takes any value ε over the real support, yields $\int_{-\infty}^{\infty} (\varepsilon + \bar{\varepsilon}_t^{aj,i}) f(\varepsilon) \prod_{m \neq i} F(\varepsilon + \bar{\varepsilon}_t^{aj,i} - \bar{\varepsilon}_t^{aj,m}) d\varepsilon$, which is the contribution of a single job to the value of $\mathbb{E}_h \left[\max_{n \in J} \left\{ \varepsilon_{h,t}^n + \bar{\varepsilon}_t^{aj,n} \right\} \right]$. Summing over all candidate jobs yields the average option value among households of age a of choosing the next period job starting from being at job j in the current period t

$$\Omega(\bar{\varepsilon}_t^{aj}) \equiv \sum_{i \in J} \int_{-\infty}^{\infty} (\varepsilon + \bar{\varepsilon}_t^{aj,i}) f(\varepsilon) \prod_{m \neq i} F(\varepsilon + \bar{\varepsilon}_t^{aj,i} - \bar{\varepsilon}_t^{aj,m}) d\varepsilon , \quad a = \{b, y, o\} .$$

7.1.2 Parametrization of the distribution $F(\varepsilon)$

Assume that $\varepsilon \sim \text{Gumbel}(-\gamma, 1)$ where $\gamma \cong 0.5772$ is the Euler-Mascheroni constant. The first moment is equal to zero and the variance is equal to $\frac{\pi^2}{6}$. The c.d.f. and p.d.f. are given by $F(\varepsilon) = e^{-e^{-(\varepsilon+\gamma)}}$ and $f(\varepsilon) = e^{-(\varepsilon+\gamma)} F(\varepsilon)$. Define the auxiliary variables $\delta_t^{aj,ik} = \bar{\varepsilon}_t^{aj,i} - \bar{\varepsilon}_t^{aj,k}$, $\lambda_t^{aj,i} = \ln \left(\sum_{k \in J} e^{-\delta_t^{aj,ik}} \right)$. The probability that when moving from a certain job j , a given job i is

preferred to any other job k is given by

$$\int_{-\infty}^{\infty} f(\varepsilon) \prod_{k \neq i} F(\varepsilon + \delta_t^{aj,ik}) d\varepsilon = e^{-\lambda_t^{aj,i}} = m_t^{aj,i} \quad (38)$$

where we proceeded with a change of variable $y = \varepsilon - \lambda_t^{aj,i}$, and $\int_{-\infty}^{\infty} e^{-(y+\gamma)} e^{-[e^{-(y+\gamma)}]} dy = 1$. By the law of large numbers, $m_t^{aj,i} \in (0, 1)$ is the fraction of households in job j who move to job i at the end of period t . The option value is given by:

$$\begin{aligned} \Omega(\bar{\varepsilon}_t^{aj}) &= \sum_{i \in J} \int_{-\infty}^{\infty} (\varepsilon + \bar{\varepsilon}_t^{aj,i}) f(\varepsilon) \prod_{m \neq i} F(\varepsilon + \delta_t^{aj,im}) d\varepsilon \\ &= \lambda_t^{aj,j} = -\ln(m_t^{aj,j}) \end{aligned} \quad (39)$$

where $\int_{-\infty}^{\infty} (y+\gamma) e^{-(y+\gamma)} e^{-[e^{-(y+\gamma)}]} dy = \gamma$ and $\int_{-\infty}^{\infty} (\lambda_t^{aj,j} - \gamma) e^{-(y+\gamma)} e^{-[e^{-(y+\gamma)}]} dy = \lambda_t^{aj,j} - \gamma$, and we have substituted back for $\lambda_t^{aj,j} = \ln\left(\sum_{k \in J} e^{-\delta_t^{aj,jk}}\right)$. Substituting for $\lambda_t^{aj,i} = \ln\left(\sum_{k \in J} e^{-\delta_t^{aj,ik}}\right)$ and then $\delta_t^{aj,ik} = \bar{\varepsilon}_t^{aj,i} - \bar{\varepsilon}_t^{aj,k}$ in the expression for the fraction of movers yields $m_t^{aj,i} = e^{\bar{\varepsilon}_t^{aj,i}} / \left[\sum_{k \in J} e^{\bar{\varepsilon}_t^{aj,k}}\right]$ for $a = \{b, y, o\}$.

7.1.3 Rewriting the value functions

Substituting for $\nu\Omega(\bar{\varepsilon}_t^{aj}) = \nu\lambda_t^{aj,j}$ in the Bellman equation (4), given the definition $\lambda_t^{aj,j} = \ln\left(\sum_{k \in J} e^{\bar{\varepsilon}_t^{aj,k}}\right)$, the equilibrium expression for $\bar{\varepsilon}_t^{aj,k}$ in (3) and the equilibrium option value (7), yields the set of Bellman equations:

$$\begin{aligned} V^{bj}(L_t, Z_t) &= \nu \ln \left(\sum_{k \in J} e^{\frac{1}{\nu} \left(\beta \mathbb{E}_t[V^{yk}(L_{t+1}, s_{t+1})] - \zeta_t^{bj,k} \right)} \right) \\ V^{yj}(L_t, Z_t) &= \omega^j(L_t, Z_t) + \nu \ln \left(\sum_{k \in J} e^{\frac{1}{\nu} \left(\beta \mathbb{E}_t[(1-\lambda^y)V^{yk}(L_{t+1}, s_{t+1}) + \lambda^y V^{ok}(L_{t+1}, s_{t+1})] - \zeta_t^{yj,k} \right)} \right) \\ V^{oj}(L_t, Z_t) &= \omega^j(L_t, Z_t) + \nu \ln \left(\sum_{k \in J} e^{\frac{1}{\nu} \left(\beta \mathbb{E}_t[(1-\lambda^o)V^{ok}(L_{t+1}, s_{t+1})] - \zeta_t^{oj,k} \right)} \right) \end{aligned} \quad (40)$$

Rewriting the Bellman equations (40) making use of the equilibrium conditions (8) evaluated for the fraction of those who do not change job yields (9).

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