EXERCISES TO TSACHIK'S COURSE

(1) Let $H \leq G$ be a closed subgroup. Show that Sub_H is closed in Sub_G with the Chaubuty topology.
(2) Show that $Sub_{\mathbb{R}} \sim [0,1]$.
(3) What can you say about $Sub_{\mathbb{R}^n}$?
(4) Show that $A(G)$, the collection of closed abelian subgroups is closed in $Sub(G)$.
(5) Let Gamma be a finitely generated discrete group. Show that a sequence H_n of subgroups of Gamma converge to a limit H in $Sub(G)$, iff the intersection of H_n with a ball of any given radius is the Cayley graph eventually stabilizes.
(6) Let Gamma be a finitely generated discrete group. Show that every finite index subgroup is isolated in $Sub(G)$.
(7) Let G be a locally compact second countable group. Show that $Sub(G)$ is compact (hint: Alexanders sub-basis lemma). Conclude that $IRS(G)$ is compact.
(8) Let G be a Lie group. Show that $D(G)$, the collection of discrete subgroups of G, is open in $Sub(G)$.
(9) Consider the group $\mathrm{SL}_n(Z)$ acting on the n-torus T^n with the Haar measure. Let mu be the associated IRS. Show that mu is a Dirac measure on the trivial subgroup.
(10) Construct non-trivial invariant random subgroups for the wreath product of two groups, where the base group is finite.
(11) Let T_n be a rooted binary tree of height n. To each T_n associate m_n — a point in the BS-space of (bounded degree) graphs. Describe the limit of m_n in the weak-* topology.
(12) Show that a finitely generated group is Sofic iff its Caylay graph is a BS-limit of finite graphs.

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