BOREL SEMINAR | Kleinian groups and hyperbolic geometry
Les Diablerets, July 2014 | Misha Kapovich

Exercises

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1. Let $\gamma_1(t), \gamma_2(t)$ be two geodesics in Euclidean space $\mathbb{E}^3$. Show that the function

$$\begin{array}{rccc} d: & \mathbb{R} & \longrightarrow & \mathbb{R} \\ & t & \mapsto & d(\gamma_1(t), \gamma_2(t)) \end{array}$$

is a convex function.

2. For any $\gamma \in \text{Isom}(\mathbb{H}^n)$ and any $r > 0$, show that the set

$$U_r(\gamma) := \{x \in \mathbb{H}^n \mid d(x, \gamma x) \leq r\}$$

is convex.

3. Let Γ be a subgroup of $\text{Isom}(\mathbb{H}^n)$. Prove that the following are equivalent :
(i) Γ is a discrete subgroup of $\text{Isom}(\mathbb{H}^n)$.
(ii) For every $x \in \mathbb{H}^n$, the orbit Γx is a discrete subset of $\mathbb{H}^n$.
(iii) Γ acts properly discontinuously on $\mathbb{H}^n$.

4. Show that Γ acts properly discontinuously on Ω.

5. An elementary group cannot have both hyperbolic and parabolic elements.