McGill Computational Science and Engineering Seminar

Friday April 14th at 14:00 in Burnside Hall room 1205

Convergence of GMRES

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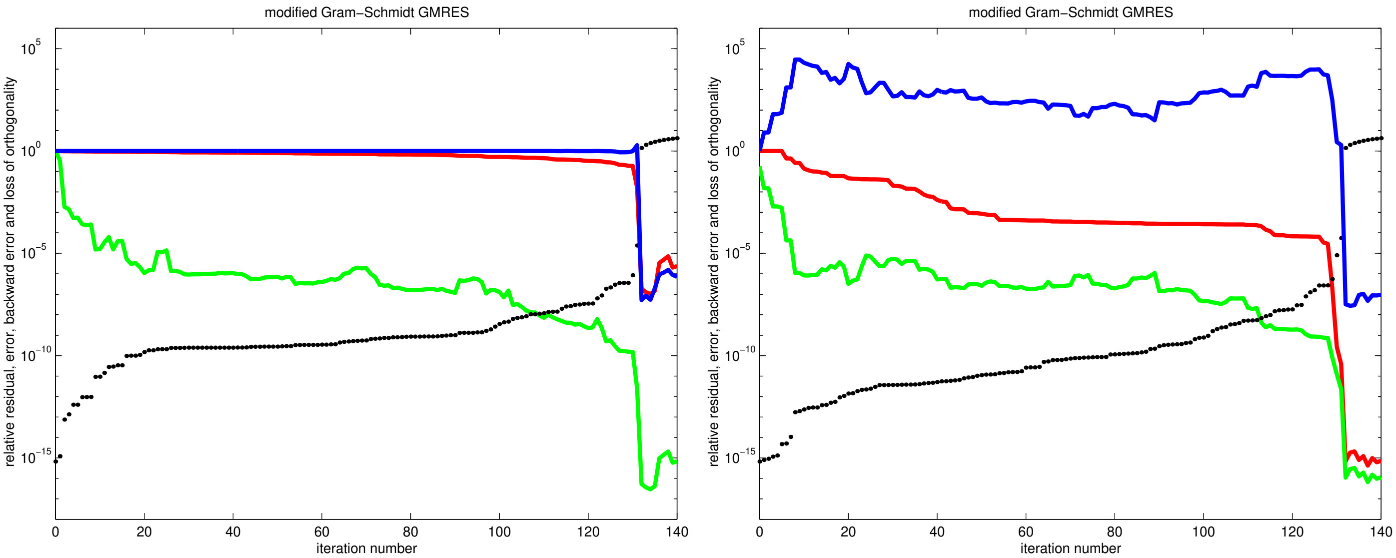
Consider a linear algebraic system $Ax = b$ where A is a given n by n nonsingular matrix and b an n–dimensional vector. Iterative methods aim to construct approximate solutions $x_0, x_1, \ldots, x_k$ converging to the solution x; convergence may not be monotonic. Many modern methods (such as those based on Krylov subspaces) guarantee (in exact arithmetic) $x_k = x$ for some $k \leq n$. This finite termination property has, however, a little practical effect. In real world applications n may be of the order of millions, but we need to achieve convergence to an acceptable accuracy in much fewer steps. Therefore it is important to analyze convergence of iterative methods in dependence on some particular characteristics of the problem. We also need to understand effects of finite precision arithmetic (rounding errors).

We will focus on convergence of the generalized minimal residual method (GMRES). We will briefly recall and discuss some published work and present several recent results.



GMRES applied to a test problem from the Rutherford-Boeing collection. On the left with a zero starting vector and on the right with a random one. In red the residual, in blue the relative error, in green the backward error and in black the loss of orthogonality. Why do you think are black and green almost perfectly symmetric ?

Coffee and desserts are served after the seminar at 15:00 in the lounge, Burnside Hall 1024