McGill Computational Science and Engineering Seminar

Friday February 9th at 14:00 in Burnside Hall room 1205

Unifying Least Squares, Total Least Squares and Data Least Squares Problems

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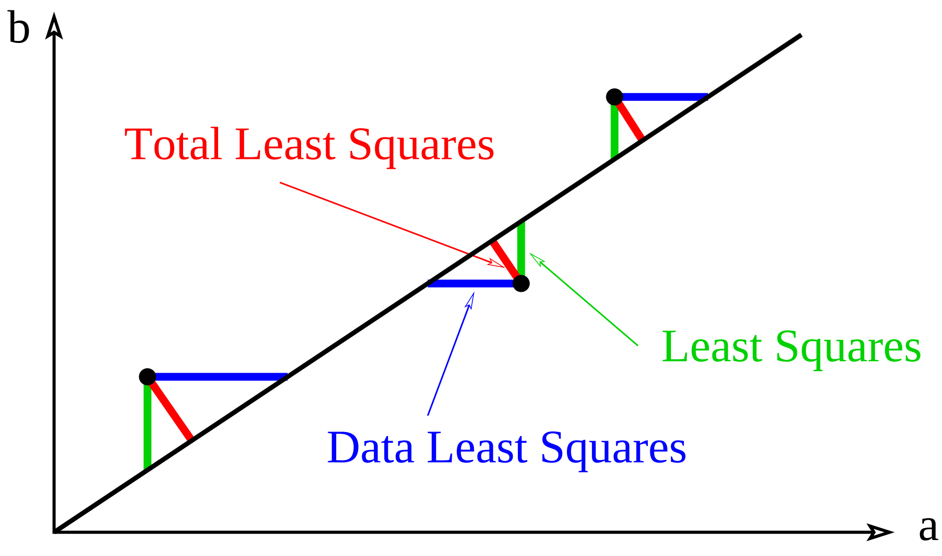
We usually solve overdetermined linear systems $Ax \approx b$ by constructing minimal corrections to b and/or A such that the corrected system is compatible. In least squares (LS) the correction is restricted to b, in data least squares (DLS) it is restricted to A, while in total least squares (TLS) corrections are allowed in b and A. Scaled total least squares (STLS) [B.D. Rao 1997] (who used the terminology "weighted total least squares"), allows corrections to both b and A, where the relative size of the corrections depends on a parameter γ. STLS is a generalization of total least squares (TLS). It becomes TLS when $\gamma = 1$, corresponds to LS when $\gamma \to 0$, and to DLS when $\gamma \to \infty$.

We will give here:

- An elementary introduction to the LS, DLS, TLS and STLS problems.
- A useful formulation of the STLS problem.
- A new criterion that makes the LS, DLS, TLS & STLS theories consistent.
- A theorem that ensures any linear system $Ax \approx b$ can be directly reduced to a minimally dimensioned core system satisfying this criterion.
- The basics of practical algorithms for the LS, DLS, TLS and STLS problems for either dense or large sparse systems.

This last shows that in theory the algorithm LSQR [Paige & Saunders 1982] can be used to solve DLS, TLS and STLS problems, as well as the consistent and LS problems that it is now useful for.



Sums of squares of distances minimized by LS, DLS and TLS

Coffee and desserts are served after the seminar at 15:00 in the lounge, Burnside Hall 1024