McGill Computational Science and Engineering Seminar

Friday January 26th at 14:00 in Burnside Hall room 1205

What is the LU Factorization of Laplace's Equation ?

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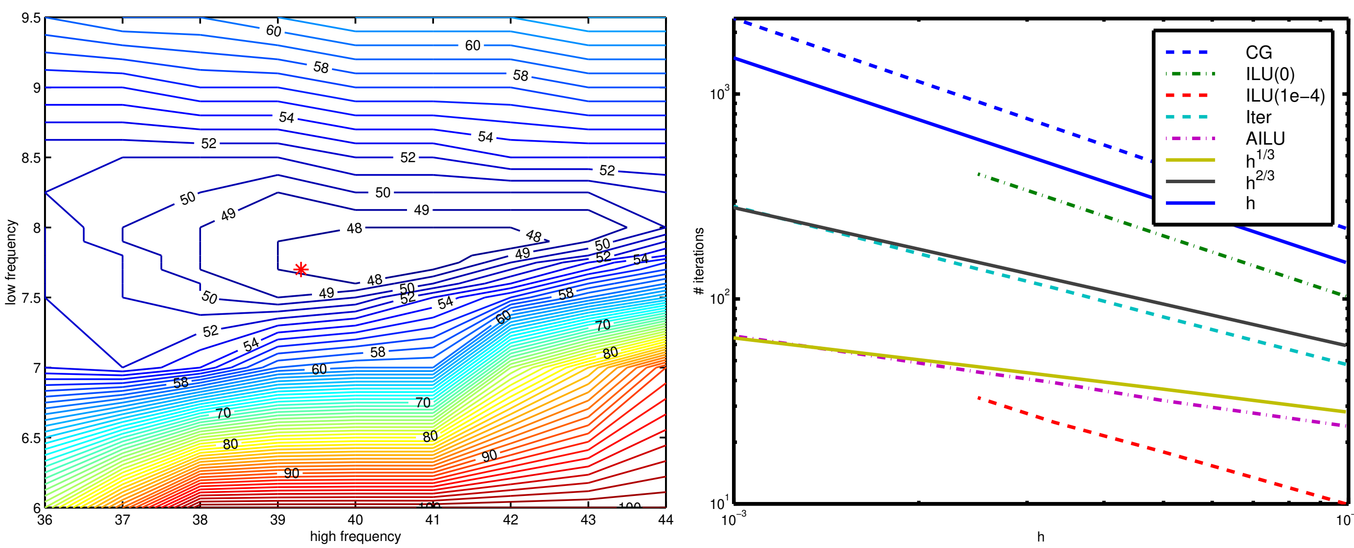
One of the first techniques students are introduced to for solving linear systems of equations Ax=b is Gaussian elimination. This leads to an LU factorization of the system matrix A. In many application this matrix comes from a discretization of a differential operator, so in the limit the matrix becomes again a differential operator. But what happens to the LU factors in the limit ? Do they have a physical interpretation ?

I will show in this talk that the LU factors correspond to parabolic problems if the matrix A is a discretization of an elliptic problem. One of the parabolic problems is an evolution problem that sweeps in one direction over the domain on which the elliptic problem is posed, the other parabolic problem is an evolution problem that sweeps in the opposite direction over the domain. This corresponds to the forward and backward substitution in the triangular solution process with the LU factors. Thus an LU factorization of A corresponds to a parabolic factorization of an elliptic operator.

Using this fundamental relation, I will construct a new ILU preconditioner called AILU (Analytic Incomplete LU) which beats classical ILU preconditioners both initially and asymptotically. This is possible because the AILU factors capture the essential of the continuous operator, something a matrix based ILU can never achieve. I will show numerical results and discuss the advantages and limitations of AILU.



Quality of the analysis in selecting the optimal parameter on the left and better asymptotic performance of AILU on the right

Coffee and desserts are served after the seminar at 15:00 in the lounge, Burnside Hall 1024