The Donut Problem

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Abstract

Given a donut (a torus in $\mathbb{R}^3$ space), how many pieces can you get by cutting the donut using three planes ? The presented solution gives fifteen pieces. Can you verify it ?

The basic idea behind the solution is, that a body with U-shape can be cut into three pieces by a plane. Thus we try with each cut to obtain as many U-shaped pieces as possible. The first cut is shown in Figure 1. We obtain two

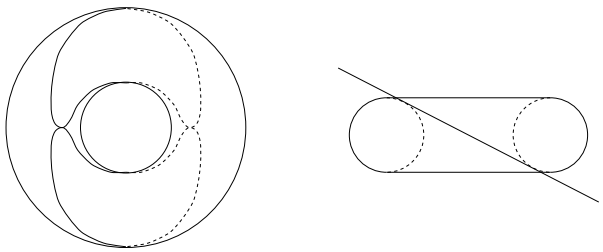

Figure 1: First cut

pieces, both have the desired U-shape. With one cut it is not possible to obtain more than two pieces. The pieces obtained are shown in Figure 2.

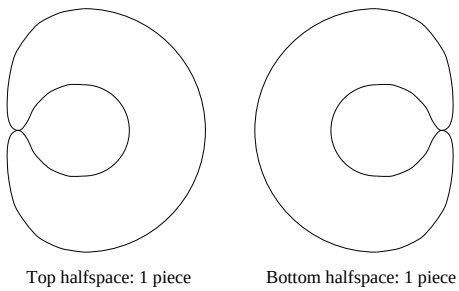

Figure 2: Pieces after the first cut

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The second cut is symmetric to the first one. The goal was to cut both U-shapes in each halfspace into three pieces, the best one can do. Furthermore all the pieces should again have the desired U-shape. The second cut is shown in Figure 3. We obtain six pieces, all having the desired U-shape. With the

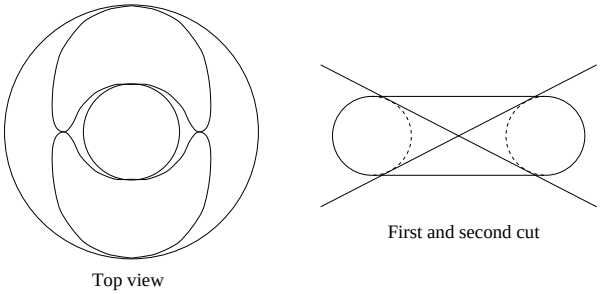

Figure 3: Second cut

second cut it is not possible to obtain more than six pieces, since we can cut both U-shapes into at most three pieces each. The pieces obtained are shown in Figure 4.

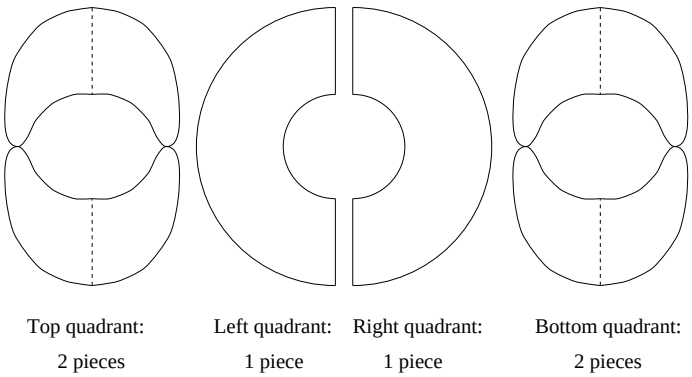

Figure 4: Pieces after the second cut

Now the last cut has to be placed to cut as many of the six U-shaped pieces into as many pieces as possible. The solution leading to fifteen pieces is given by the cut in Figure 5. The resulting pieces are shown in Figure 6.

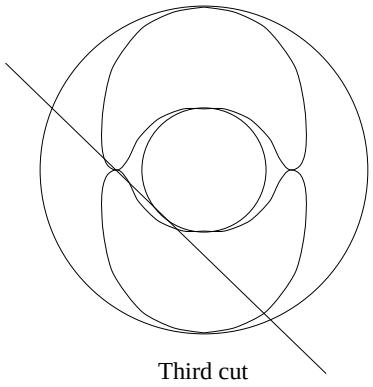

Figure 5: Third cut

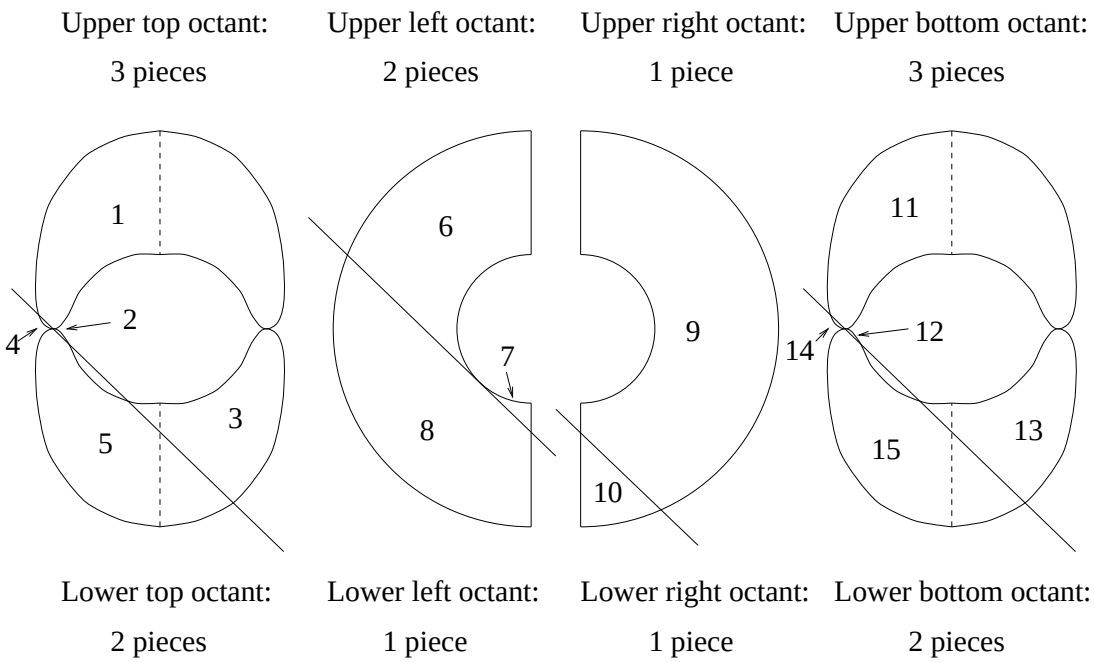

Figure 6: Pieces obtained with the third cut